

# A formal analysis of the outcomes of argumentation-based negotiations

## (Extended Abstract)

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### ABSTRACT

This paper tackles the problem of exchanging arguments in negotiation dialogues, and provides first characterizations of the outcomes of such rich dialogues.

### Categories and Subject Descriptors

I.2.3 [Deduction and Theorem Proving]: Nonmonotonic reasoning and belief revision; I.2.11 [Distributed Artificial Intelligence]: Intelligent agents

### General Terms

Human Factors, Theory

### Keywords

Argumentation, Negotiation

## 1. INTRODUCTION

Negotiation is a process aiming at finding some *compromise* or *consensus* on an issue between two or several agents. Since early nineties, the importance of exchanging arguments during negotiation dialogues has been emphasized and several works have been carried out (see [3] for a survey). The basic idea is to allow agents not only to exchange offers but also reasons that support these offers in order to mutually influence their preferences, and consequently the outcome of the dialogue. These works are unfortunately still preliminary. Before work [1], it was not yet clear how new arguments may have an impact on the agent who receives them. In [1], it has been shown that the theory of an agent may evolve when new arguments are received. However, there is still no characterization of the outputs of an argument-based negotiation. The notion of optimal solution in such dialogues is unclear. This makes it difficult to evaluate the quality of any dialogue protocol.

This paper characterizes the outputs of an argument-based negotiation dialogue. It distinguishes between *local solutions* which are optimal solutions at a given step in a dialogue and *global solutions* which are the ideal solutions.

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## 2. AGENT THEORY

This section presents the argumentation model that is used by each agent for evaluating and comparing offers.

*Definition 1.* An *agent's theory* is a tuple  $\mathcal{T} = (\mathcal{O}, \mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_o, \mathcal{R}, \succeq, \mathcal{F})$  where  $\mathcal{O}$  is a set of *offers*,  $\mathcal{A}_e$  is a set of *epistemic arguments*,  $\mathcal{A}_o$  is a set of *practical arguments*,  $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$  is an *attack* relation,  $\succeq \subseteq \mathcal{A} \times \mathcal{A}$  is a partial *preorder* on  $\mathcal{A}$  and  $\mathcal{F} : \mathcal{O} \mapsto 2^{\mathcal{A}_o}$  s.t.  $\cup \mathcal{F}(o_i) = \mathcal{A}_o$  and for all  $o_i, o_j \in \mathcal{O}$ , if  $o_i \neq o_j$ , then  $\mathcal{F}(o_i) \cap \mathcal{F}(o_j) = \emptyset$ .

Arguments are evaluated using a credulous semantics, like stable semantics proposed in [2].

*Definition 2.* A set  $\mathcal{E} \subseteq \mathcal{A}$  is a *stable extension* of a theory  $\mathcal{T} = (\mathcal{O}, \mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_o, \mathcal{R}, \succeq, \mathcal{F})$  iff: i)  $\nexists a, b \in \mathcal{E}$  s.t.  $a \mathcal{R} b$ , ii)  $\forall a \in \mathcal{A} \setminus \mathcal{E}$ ,  $\exists b \in \mathcal{E}$  such that  $b \mathcal{R} a$  and not  $(a \succ b)$ . Let  $\text{Ext}(\mathcal{T})$  be the set of all stable extensions of  $\mathcal{T}$ .

A status is associated to each offer as follows.

*Definition 3.* Let  $\mathcal{T} = (\mathcal{O}, \mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_o, \mathcal{R}, \succeq, \mathcal{F})$  be an agent theory and  $o \in \mathcal{O}$ . The offer  $o$  is *acceptable* iff  $\exists a \in \mathcal{F}(o)$  s.t.  $a \in \mathcal{E}$ ,  $\forall \mathcal{E} \in \text{Ext}(\mathcal{T})$ . It is *rejected* iff  $\mathcal{F}(o) \neq \emptyset$  and  $\forall a \in \mathcal{F}(o)$ ,  $\nexists \mathcal{E} \in \text{Ext}(\mathcal{T})$  s.t.  $a \in \mathcal{E}$ . It is *non-supported* iff  $\mathcal{F}(o) = \emptyset$ . It is *negotiable* otherwise. Let  $\mathcal{O}_a(\mathcal{T})$  (resp.  $\mathcal{O}_r(\mathcal{T})$ ,  $\mathcal{O}_{ns}(\mathcal{T})$ ,  $\mathcal{O}_n(\mathcal{T})$ ) denote the set of acceptable (resp. rejected, non-supported, negotiable) offers in theory  $\mathcal{T}$ .

It is easy to check that  $\mathcal{O} = \mathcal{O}_a(\mathcal{T}) \cup \mathcal{O}_r(\mathcal{T}) \cup \mathcal{O}_n(\mathcal{T}) \cup \mathcal{O}_{ns}(\mathcal{T})$ . From this partition, a basic ordering  $\succeq$  on the set  $\mathcal{O}$  (i.e.  $\succeq \subseteq \mathcal{O} \times \mathcal{O}$ ) is defined. The idea is that any acceptable offer is preferred to any negotiable offer, any negotiable offer is preferred to any non-supported offer which in turn is preferred to any rejected offer. We abuse notation and write for instance  $\mathcal{O}_a(\mathcal{T}) \succeq \mathcal{O}_n(\mathcal{T})$ .

*Definition 4.* Let  $\mathcal{T} = (\mathcal{O}, \mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_o, \mathcal{R}, \succeq, \mathcal{F})$  be an agent theory.  $\mathcal{O}_a(\mathcal{T}) \succeq \mathcal{O}_n(\mathcal{T}) \succeq \mathcal{O}_{ns}(\mathcal{T}) \succeq \mathcal{O}_r(\mathcal{T})$  hold.

## 3. NEGOTIATION OUTCOMES

We assume that negotiation takes place between two agents, denoted by  $Ag_1$  and  $Ag_2$ . Each agent  $Ag_i$  is equipped with a theory  $\mathcal{T}_i = (\mathcal{O}, \mathcal{A}_i, \mathcal{R}_i, \succeq_i, \mathcal{F}_i)$  which is used for computing the preference relation  $\succeq_i$  on the set  $\mathcal{O}$ . The set  $\mathcal{A}_i$  is a subset of a universal set  $\mathcal{A}_{\mathcal{L}}$  of arguments built from a logical language  $\mathcal{L}$ . Relation  $\mathcal{R}_i$  is a restriction of  $\mathcal{R}_{\mathcal{L}}$  on  $\mathcal{A}_i$  where  $\mathcal{R}_{\mathcal{L}} \subseteq \mathcal{A}_{\mathcal{L}} \times \mathcal{A}_{\mathcal{L}}$ . However, we assume that  $\succeq_i$  is defined over the whole set  $\mathcal{A}_{\mathcal{L}}$ . The two agents are supposed to

share the same set of offers. In order to define the outcomes of a negotiation, we need to define the notion of *dialogue*.

*Definition 5.* A *negotiation dialogue* is a finite sequence of moves  $d = (m_1, \dots, m_l)$  s.t.  $m_i = (x_i, y_i, z_i)$ , where  $x_i$  is either  $Ag_1$  or  $Ag_2$ ,  $y_i \in \mathcal{A}_L \cup \{\theta\}$ ,  $z_i \in \mathcal{O} \cup \{\theta\}$ <sup>1</sup>, and  $y_i \neq \theta$  or  $z_i \neq \theta$ . If  $\forall i = 1, \dots, l$ ,  $y_i = \theta$ , then  $d$  is said *non-argumentative*. It is *argumentative* otherwise.

Note that at each step  $t$  of a dialogue, the theory of each agent may evolve. The original set of arguments is augmented by the new arguments received from the other party, and the attack relation is modified accordingly. We denote by  $\mathcal{T}_i^t = (\mathcal{O}, \mathcal{A}_i^t, \mathcal{R}_i^t, \succeq_i^t, \mathcal{F}_i^t)$  the theory of agent  $i$  at a step  $t$  of a dialogue and  $\mathcal{T}^0$  her theory before the dialogue.

The following property shows that the theory of an agent does not change in case of non-argumentative dialogues.

**PROPERTY 1.** *If a dialogue  $d = (m_1, \dots, m_l)$  is non-argumentative, then  $\forall j \in \{1, \dots, l\}$  it holds that  $\succeq_1^0 = \succeq_1^j$  and  $\succeq_2^0 = \succeq_2^j$ .*

Let us now analyze the different solutions of a dialogue. The best solution for an agent at a given step of a dialogue is that which suits best her preferences.

*Definition 6.* An offer  $o \in \mathcal{O}$  is an *accepted solution* for agent  $Ag_i$  at step  $t$  of a dialogue  $d$  iff  $o \in \mathcal{O}_a(\mathcal{T}_i^t)$ .

Note that an offer may be accepted for one agent but not for the other. Such offer is certainly not a solution of the dialogue. A local solution at a given step is an offer which is accepted for both agents at that step. We use the term “local” because such an offer is accepted locally in time - it may have been rejected before, or may become rejected after several steps. Such a solution does not always exist.

*Definition 7.* An offer  $o \in \mathcal{O}$  is a *local solution* at a step  $t$  of dialogue  $d$  iff  $o \in \mathcal{O}_a(\mathcal{T}_1^t) \cap \mathcal{O}_a(\mathcal{T}_2^t)$ .

Note that a local solution is not necessarily reached in a dialogue i.e. it is not necessarily the dialogue outcome. In order to be so, an efficient dialogue protocol should be used. The following result characterizes the situation where there exists a local solution.

**PROPERTY 2.** *There exists a local solution iff there exist sets of arguments  $\mathcal{A}'_1 \subseteq \mathcal{A}_1^0$  and  $\mathcal{A}'_2 \subseteq \mathcal{A}_2^0$  s.t.*

$$\mathcal{O}_a(\mathcal{O}, \mathcal{A}'_1 \cup \mathcal{A}'_2, \mathcal{R}_1, \succeq_1, \mathcal{F}_1) \cap \mathcal{O}_a(\mathcal{O}, \mathcal{A}'_1 \cup \mathcal{A}'_2, \mathcal{R}_2, \succeq_2, \mathcal{F}_2) \neq \emptyset.$$

The next result studies the situation when agents do not have to agree on everything but they agree on the arguments related to a given part of the negotiation, which is separated from other problems. If the first agent owns more information than the second, then there exists a dialogue in which the second will agree with the first one.

**PROPERTY 3.** *Let  $\mathcal{A}' \subseteq \mathcal{A}_1^0 \cup \mathcal{A}_2^0$  be s.t.  $\succeq_1|_{\mathcal{A}'} = \succeq_2|_{\mathcal{A}'}$  and let  $\mathcal{A}'$  be not attacked by arguments of  $(\mathcal{A}_1^0 \cup \mathcal{A}_2^0) \setminus \mathcal{A}'$ . If  $\mathcal{A}_1^0 \cap \mathcal{A}' \supseteq \mathcal{A}_2^0 \cap \mathcal{A}'$  and  $\exists a \in \mathcal{F}(o) \cap \mathcal{A}_1^0 \cap \mathcal{A}'$  s.t.  $a$  is accepted in  $\mathcal{T}_1^0$  then there exists a negotiation dialogue  $d = (m_1, \dots, m_l)$  s.t.  $o$  is a local solution at step  $t$ .*

<sup>1</sup>Let  $m = (x, y, z)$  be a move. If  $y = \theta$  (resp.  $z = \theta$ ), this means that no argument (resp. no offer) is uttered.

The next result studies the case when  $\succeq$  is complete and antisymmetric. In this case, we provide a condition under which there exists a local solution.

**PROPERTY 4.** *Let  $\succeq_1$  and  $\succeq_2$  be complete and antisymmetric preorders. If there exist sets  $\mathcal{A}'_1 \subseteq \mathcal{A}_1^0$  and  $\mathcal{A}'_2 \subseteq \mathcal{A}_2^0$ ,  $\exists o \in \mathcal{O}$ ,  $\exists a_1 \in (\mathcal{A}_1^0 \cup \mathcal{A}'_2) \cap \mathcal{F}(o)$ ,  $\exists a_2 \in (\mathcal{A}_2^0 \cup \mathcal{A}'_1) \cap \mathcal{F}(o)$ , s.t.  $\nexists$  odd chain of attacks  $x_1 \mathcal{R}_L x_2, x_2 \mathcal{R}_L x_3, \dots, x_{2k+1} \mathcal{R}_L a_1$  with  $x_1, x_2, \dots, x_{2k} \in \mathcal{A}'_1 \cup \mathcal{A}'_2$  and  $x_1 >_1 x_2 >_1 \dots >_1 a_1$  and  $\nexists$  odd chain of attacks  $y_1 \mathcal{R}_L y_2, y_2 \mathcal{R}_L y_3, \dots, y_{2k+1} \mathcal{R}_L a_2$  with  $y_1, y_2, \dots, y_k \in \mathcal{A}'_2 \cup \mathcal{A}'_1$  and  $y_1 >_2 y_2 >_2 \dots >_2 a_{2k}$ , then there exists a local solution.*

The two previous solutions are time-dependent. An offer may, for instance, be a local solution at step  $t$  but not at step  $t + 1$ . In what follows, we propose two other solutions (one for a single agent and one for a dialogue) which are not time-dependent. They represent respectively the *optimal solution* for an agent and the *ideal solution* of a dialogue. An offer is an optimal solution for an agent iff she would choose that offer if she had access to all arguments owned by all agents.

*Definition 8.* An offer  $o \in \mathcal{O}$  is an *optimal solution* for agent  $Ag_i$  iff  $o \in \mathcal{O}_a(\mathcal{T})$  where  $\mathcal{T} = (\mathcal{O}, \mathcal{A}_1^0 \cup \mathcal{A}_2^0, \mathcal{R}_i, \succeq_i, \mathcal{F}_i)$  with  $\mathcal{R}_i \subseteq (\mathcal{A}_1^0 \cup \mathcal{A}_2^0) \times (\mathcal{A}_1^0 \cup \mathcal{A}_2^0)$ .

The following property shows that if an offer is optimal for an agent, then there exists a dialogue in which that solution is accepted for that agent at a given step.

**PROPERTY 5.** *If  $o$  is an optimal solution for an agent, then there exists a dialogue  $d = (m_1, \dots, m_l)$  s.t.  $o$  is accepted for that agent at step  $l$ .*

If both agents agree when all information has been exchanged, they can obtain an ideal solution.

*Definition 9.* An offer  $o \in \mathcal{O}$  is an *ideal solution* iff  $o \in \mathcal{O}_a(\mathcal{O}, \mathcal{A}_1^0 \cup \mathcal{A}_2^0, \mathcal{R}_1^0 \cup \mathcal{R}_2^0, \succeq_1, \mathcal{F}_1) \cap \mathcal{O}_a(\mathcal{O}, \mathcal{A}_1^0 \cup \mathcal{A}_2^0, \mathcal{R}_1^0 \cup \mathcal{R}_2^0, \succeq_2, \mathcal{F}_2)$ .

The next property shows that if an ideal solution exists, then it is a local solution for a dialogue.

**PROPERTY 6.** *If  $o$  is an ideal solution then there exists a dialogue  $d = (m_1, \dots, m_l)$  s.t.  $o$  is a local solution at step  $l$ .*

It is natural to expect that for two agents with same beliefs and goals an exchange of arguments can ameliorate the chance of finding a solution. Moreover, if the first agent has more information, he can influence the second one.

**PROPERTY 7.** *Let  $\succeq_1 = \succeq_2$ ,  $\mathcal{A}_1^0 \supseteq \mathcal{A}_2^0$ . If  $o$  is an accepted solution for  $Ag_1$  at step  $t = 0$ , then  $o$  is an ideal solution.*

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