How to complete regulations in multi-agent systems

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Abstract—In this paper, we deal with regulations that may exist in multiagent systems in order to regulate agent behaviour. More precisely, we discuss two properties of regulations, consistency and completeness. After defining what consistency and completeness mean, we propose a way to consistently complete incomplete regulations. This contribution considers that regulations are expressed in a first order deontic logic. We will focus on particular regulations: information exchange policies.

Index Terms-modal logic, regulations, completeness, consistency

I. INTRODUCTION

In a society of agents, a regulation is a set of statements, or norms, which rule the behaviour of agents by expressing what is obligatory, permitted, forbidden and under which conditions.

Such a regulation is for instance the one which applies in most countries in EU: *smoking is forbidden in any public area except specific places and in such specific places, smoking is permitted*. Another example of regulation is an information exchange policy: it gives the permissions, prohibitions and obligations regarding information communication.

Regulations are means to regulate agent behaviour so that the agents can live together. But in order to be useful, regulations must be *consistent* and, in most cases, they must also be *complete*.

Consistency is a property of regulations that has already been given some attention in the literature. For instance, as for confidentiality policies, consistency allows to avoid cases when the user has both the permission and the prohibition to know something [1]. More generally, according to [2] which studies consistency of general kind of regulations, a regulation is consistent if there is no possible situation which leads an agent to *normative contradictions* or *dilemmas* (a given behaviour is prescribed and not prescribed, or prohibited and not prohibited) and *contrary conflicts* (a given behaviour is prescribed and prohibited).

Completeness of regulations has received much less attention. [1] proposes a definition of completeness between two confidentiality policies, definition which has been adapted in [3] for multilevel security policies. More recently, we have studied the notion of completeness for regulations in [4]. The formal language used in those papers is classical first-order logic (FOL) following the ideas developed in [2], although modal logic is most of the time the formal language chosen for modeling deontic notions. This is the reason why, in this present paper, we aim at using first order modal logic [5] to express regulations in a more elegant manner. Our objective is thus to reformulate the work described in [4] in a first-order modal framework.

This paper is organised as follows. Section II presents the logical formalism used to express regulations, the definitions of consistency and completeness of regulations. Section III focuses on the problem of reasoning with an incomplete regulation. Following the approach that has led to the default logic [6] for default reasoning, we present defaults that can be used in order to complete an incomplete regulation. For each of these sections, results obtained for general regulations are applied to information exchange policies. Finally, section IV is devoted to a discussion and extensions of this work will be mentioned.

II. REGULATIONS

A. Language, semantics and axiomatics

The alphabet of FOSDL (First-Order Standard Deontic Logic) is based on the following sets of non logical symbols: a set \mathcal{P} of predicate symbols, a set \mathcal{F} of function symbols and a modality symbol O representing obligation. The set of functions with arity 0 is called the *constants set* denoted by \mathcal{C} . We define also the following logical symbols: a set \mathcal{V} of variable symbols, \neg , \lor , \forall , (and). We call a *term* a variable or the application of a function symbol to one or several terms. We will use roman uppercase letters as predicate symbols, roman lowercase letters as function symbols and $\{x_1, \ldots, x_i, \ldots\}$ as variable symbols.

Definition 1: The formulae of FOSDL are defined recursively as follows:

- if t_1, \ldots, t_n are terms and P a predicate symbol with arity n, then $P(t_1, \ldots, t_n)$ is a formula of FOSDL.
- if φ is a formula of FOSDL, then $O\varphi$ is a formula of FOSDL.
- if ψ₁ and ψ₂ are formulae of FOSDL and x₁ a variable symbol, then ¬ψ₁, ψ₁ ∨ ψ₂, ∀x₁ ψ₁ are formulae of FOSDL.

 \Box .

We define classically the \land , \rightarrow and \leftrightarrow operators from \neg , \lor and \forall . If ψ_1 , ψ_2 and ψ_3 are FOSDL formulae, $\psi_1 \otimes \psi_2 \otimes \psi_3 \equiv$ $(\psi_1 \land \neg \psi_2 \land \neg \psi_3) \lor (\neg \psi_1 \land \psi_2 \land \neg \psi_3) \lor (\neg \psi_1 \land \neg \psi_2 \land \psi_3).$ The modalities for permission, noted P, and prohibition, noted F, are defined from O in the following way: $F\varphi \equiv O\neg\varphi$ and $P\varphi \equiv \neg O\varphi \land \neg O\neg\varphi$.

It must be noticed that our definition of permission does not correspond to the usual definition of permission defined in SDL, but is bilateral.

A formula of FOSDL without modality is said to be *objective*. Terms or formulae of FOSDL without variable symbols are said to be *ground*. A formula of FOSDL without the \lor , \land , \otimes , \rightarrow nor \leftrightarrow connectives is said to be a *literal*.

Finally, we will call a ground substitution any function χ : $\mathcal{V} \to HU$ where HU is the set of all possible instantiations of objective literals. If $\varphi(x)$ is a FOSDL formula with free variable x, $\varphi(\chi(x))$ is the formula φ in which occurrences of x have been replaced by $\chi(x)$.

Semantics is classically based on Kripke models $\langle \mathcal{W}, \mathcal{R}_O, \mathcal{D}, \mathcal{I} \rangle$ where $\langle \mathcal{W}, \mathcal{R}_O, \mathcal{D} \rangle$ is a frame and \mathcal{I} an first-order interpretation on $\langle \mathcal{W}, \mathcal{R}_O, \mathcal{D} \rangle$. Details can be found in [7]. A formal system is also detailed in [7] and it has been proved to be sound and complete in [5].

We will define a *proof* of φ from the set of formulae Σ , noted $\Sigma \vdash \varphi$, as a sequence of formulae such that each one of them is an axiom, a formula of Σ , or produced by the application of an inference rule on previous formula.

In the following, \perp will denote every formula that is a contradiction and \top will denote every formula that is a tautology.

B. Regulation and integrity constraints modelling

In this section we define the notion of regulation. First, we define the notion of rule, which is the basic component of a regulation. In this definition, rules have a general form, in particular they can be conditional.

Definition 2: A rule is a formula of FOSDL of the form $\forall \vec{x} \ l_1 \lor \ldots \lor l_n$ with $n \ge 1$ such that:

- 1) l_n is of the form $O\varphi$ or $\neg O\varphi$ where φ is an objective literal
- 2) $\forall i \in \{1, ..., n-1\}, l_i$ is an objective literal or the negation of an objective literal
- 3) if x is a variable in l_n , then $\exists i \in \{1, ..., n-1\}$ such that l_i is a negative literal and contains the variable x
- 4) $\forall \vec{x}$ denotes $\forall x_1 \dots \forall x_m$ where $\{x_1, \dots, x_m\}$ is the set of free variables appearing in $l_1 \wedge \dots \wedge l_{n-1}$.

 \Box .

In this definition, constraints (1) and (2) allow rules to be conditionals of the form "if such a condition is true then something is obligatory (resp. permitted or forbidden)". Constraint (3) restricts rules to range-restricted formulae. Finally, rules are *sentences*, i.e. closed formulae, as expressed by constraint (4). Notice also that we restrict in the definition of rules the formulae that can be defined as obligatory in the regulation: only objective literals can be obligatory or not obligatory.

We will write $\forall \vec{x} \ l_1 \lor \ldots \lor l_{n-1} \lor P\varphi$ as a shortcut for the two rules $\{\forall \vec{x} \ l_1 \lor \ldots \lor l_{n-1} \lor \neg O\varphi, \forall \vec{x} \ l_1 \lor \ldots \lor l_{n-1} \lor \neg O\varphi\}$.

A regulation is defined as a set of rules.

Let us consider an example which will help us to illustrate our purpose all along the paper.

Example 1: We consider information exchange policy as a specific regulation. This policy rules the behaviour of agents that receive pieces of information. In this particular example, the policy rules the information exchanges between employees in a company.

The language needed is defined in the following. A, B, I, EqtCheck (representing Equipment Check), ExpRisk (representing Explosion Risk), EqtOOO (representing Equipment Out Of Order), Meeting, Quiet and Crisis are 0-arity functions, i.e. constants.

Predicate symbols are: Empl(.) which indicates that a term is an employee, Info(.) which indicates that a term is a piece of information, Topic(.,.) which takes for parameter a piece of information and a topic and indicates what is the topic of the piece of information, Context(.) which indicates the current context in which pieces of information are exchanged, Receive(.,.) which takes for parameter a manager and a piece of information and indicates that a manager receives a piece of information, Send(.,.,.) which takes for parameter a manager, a piece of information and an employee and indicates that the manager tells the employee the piece of information.

Let us now take the three rules (r_0) : "In a quiet context, pieces of information dealing with Explosion Risk are required to be sent to other employees", (r_1) : "In a quiet context, pieces of information dealing with Equipment Check are allowed to be sent to other employees" and (r_2) : "In a quiet context, pieces of information dealing with Equipment Out Of Order are required not to be sent to other employees".

- $\begin{array}{ll} (r_0) & \forall x \forall y \forall i \ Context(Quiet) \land Empl(x) \land Empl(y) \land \\ \neg(x=y) \land Info(i) \land Receive(x,i) \\ \land \ Topic(i, ExpRisk) \to OSend(x,i,y) \end{array}$
- $\begin{array}{ll} (r_1) & \forall x \forall y \forall i \ Context(Quiet) \land Empl(x) \land Empl(y) \land \\ \neg(x=y) \land Info(i) \land Receive(x,i) \\ \land \ Topic(i, EqtCheck) \rightarrow PSend(x,i,y) \end{array}$
- $\begin{array}{ll} (r_2) & \forall x \forall y \forall i \ Context(Quiet) \land Empl(x) \land Empl(y) \land \\ \neg(x=y) \land Info(i) \land Receive(x,i) \\ \land \ Topic(i, EqtOOO) \to FSend(x,i,y)^1 \end{array}$

□.

C. Consistency of regulations

We now define a first notion for regulations, *consistency*. Like in [2], we will say that a regulation is consistent iff we cannot derive dilemmas (like $OStop(x,t) \land P \neg Stop(x,t)$) nor conflicts (like $OStop(x,t) \land FStop(x,t)$). Consistency of a regulation is evaluated under *integrity constraints*, i.e. a set of closed objective formulae which represent for instance physical constraints or domain constraints. In the following, we will note such an integrity constraints set IC. First, we define consistency of a regulation in a particular *state of the world*, which is a complete and consistent set of objective ground literals. Intuitively, states of the world are syntactic representations of classical first-order interpretations. Will define then global consistency for a regulation.

Definition 3 (consistency): Let ρ be a regulation, IC a set of integrity constraints and s a state of the world consistent with IC. ρ is consistent according to IC in s iff ρ , IC, $s \not\vdash \bot$. ρ is consistent according to IC iff for all states of the world s such that s, $IC \not\vdash \bot$ then ρ , IC, $s \not\vdash \bot$.

 \Box .

Example 2: Let us resume example 1. Let us consider that IC contains two constraints: (1) a piece of information has one and only one topic, (2) the context is either Quiet or Crisis. Thus, $IC = \{\forall i \forall x \forall y \ Info(i) \land Topic(i, x) \land Topic(i, y) \rightarrow (x = y), Context(Quiet) \otimes Context(Crisis)\}.$

Let s_0 be the state of the world {Context(Quiet), Empl(A), Empl(B), Info(I), Topic(I, ExpRisk), Receive(A, I)}.

First, s_0 is such that $s_0, IC \not\vdash \bot$. Let us consider a regulation ρ that contains the three rules (r_0) , (r_1) and (r_2) . In this case, $\rho, IC, s_0 \not\vdash \bot$ (because the only deontic literal that can be deduced from ρ, IC and s_0 is OSend(A, I, B)). Thus, ρ is consistent according to IC in s_0 .

 \Box .

D. Completeness of regulations

Informally, a regulation is totally complete as soon as it prescribes the behaviour any agent should have in any situation. We can wonder if this definition really makes sense: can or must a regulation take into account all possible situations? Thus, we suggest to define a partial completeness restricted to two ground formulae φ and ψ : φ represents a particular situation in which we want to evaluate the regulation and ψ a predicate ruled by the regulation. Thus, we want a regulation be complete for φ and ψ iff in any situation where φ is true, it is obligatory (resp. permitted, forbidden) that ψ .

This leads to the following definition:

Definition 4: Let IC be a set of integrity constraints, ρ be a regulation consistent according to IC and s a state of the world consistent with IC. Let $\varphi(\vec{x})$ and $\psi(\vec{x})$ two objective formulae, \vec{x} representing free variables in φ and $\psi(\vec{x})$ meaning that the free variables in ψ are a subset of $\vec{x} \cdot \rho$ is $(\varphi(\vec{x}), \psi(\vec{x}))$ -complete according to IC in s for \vdash iff for all ground substitutions χ such that $s \vdash \varphi(\chi(\vec{x}))$, $\rho, s \vdash O\psi(\chi(\vec{x}))$ or $\rho, s \vdash F\psi(\chi(\vec{x}))$ or $\rho, s \vdash P\psi(\chi(\vec{x}))$.

Example 3: Consider ρ , IC and s_0 defined in example 2. s_0 is consistent with IC and $\rho, s \vdash O(Send(A, I, B))$. Let's take $\varphi_0(x, i, y) \equiv Empl(x) \land Empl(y) \land \neg(x = y) \land Info(i) \land Receive(x, i)$ and $\psi_0(x, i, y) \equiv Send(x, i, y)$. $s_0, IC \vdash \varphi_0(A, I, B)$ and $\rho, IC, s_0 \vdash O\psi_0(A, I, B)$. Thus, ρ is $(\varphi_0(x, i, y), \psi_0(x, i, y))$ -complete according to IC in s_0 for \vdash .

Let us now consider the state of the world $s_1 = \{ Empl(A), Empl(B), Info(I), Context(Crisis), Topic(I, ExpRisk), \}$

Receive(A, I))}. s_1 is consistent with IC. $s_1, IC \vdash \varphi_0(A, I, B)$ but $\rho, IC, s_1 \not\vdash O\psi_0(A, I, B)$, $\rho, IC, s_1 \not\vdash P\psi_0(A, I, B)$ and $\rho, IC, s_1 \not\vdash F\psi_0(A, I, B)$. Thus, ρ is $(\varphi_0(x, i, y), \psi_0(x, i, y))$ -incomplete according to IC in s_1 for \vdash . In fact, no rule can be applied as the crisis context is not taken into consideration in the policy ρ .

The previous definition is easily generalized by using all the state of the worlds consistent with IC and by verifying that ρ is complete in each of those states.

 \Box .

Completeness is an important issue for a regulation. For a given situation, without any behaviour stipulated, any behaviour could be observed and thus consequences could be quite important. In order to deal with an incomplete regulation, we could (1) detect the "holes" of the regulation and send them back to the regulation designers so that they can correct them or (2) detect the "holes" of the regulation and apply on those holes some completion rules to correct them. The first solution could be quite irksome to be applied (the number of holes could be quite important and thus correct them one by one quite long). Therefore, we put in place the second solution.

III. REASONING WITH INCOMPLETE REGULATIONS

A. Defaults for completing regulation

Reasoning with incomplete information is a classical problem in logic and artificial intelligence: can we infer something about an information that is not present in a belief base? Several approaches have been defined, but we are here interested in one: default reasoning.

Default logic is a non-monotonous extension of first-order logic introduced by Reiter [6] in order to formalize default reasoning. A default d is a configuration $\frac{P: J_1, \ldots, J_n}{C}$ where P, J_1, \ldots, J_n, C are first-order closed sentences. P is called the *prerequisite* of d, J_1, \ldots, J_n the *justification* of d and C the *consequence* of d. A default theory $\Delta = (D, F)$ is composed of a set of objective closed formulae F (facts) and a set of defaults. Using defaults we obtain *extensions*, i.e. closed sets of formulae that are deduced monotonically and non-monotonically from F.

Here, we are not interested in the fact that a given objective formula ψ is believed but in the fact that a given regulation deduces that it is obligatory, forbidden or tolerated (those cases are the only ones due to the D axiom of O). In the following, let IC be a set of integrity constraints, ρ be a consistent regulation according to IC and s be a state of the world consistent with IC. Let $\varphi(\vec{x})$ and $\psi(\vec{x})$ be two objective formulae verifying definition 4.

Definition 5: Let $E_F(\vec{x})$, $E_P(\vec{x})$ and $E_O(\vec{x})$ be three objective formulae such that their respective set of free variables is in \vec{x} . We define a set of configuration as follows: $(\rho(\vec{x}) \wedge E_P(\vec{x}) + E\psi(\vec{x}))$

$$(DF_{\varphi,\psi}) \quad \frac{\varphi(x) \wedge E_F(x) : F\psi(x)}{F\psi(\vec{x})}$$
$$(DP_{\varphi,\psi}) \quad \frac{\varphi(\vec{x}) \wedge E_F(\vec{x}) : F\psi(\vec{x})}{P\psi(\vec{x})}$$

$$(DO_{\varphi,\psi}) \quad \frac{\varphi(\overrightarrow{x}) \wedge E_O(\overrightarrow{x}) : O\psi(\overrightarrow{x})}{O\psi(\overrightarrow{x})}$$

A $(\varphi(\vec{x}), \psi(\vec{x}))$ -completeness default theory for ρ and s is a default theory $\Delta_{\rho,s}(\varphi(\vec{x}), \psi(\vec{x}))$ whose surface form is given by $(\{DF_{\varphi,\psi}, DP_{\varphi,\psi}, DO_{\varphi,\psi}\}, \rho \cup s).$

□.

We can complete an incomplete regulation so that $\psi(\vec{x})$ is forbidden $(DF_{\varphi,\psi})$, permitted $(DP_{\varphi,\psi})$ or obligatory $(DO_{\varphi,\psi})$ depending on $E_F(\vec{x})$, $E_P(\vec{x})$ and $E_O(\vec{x})$. We define a new inference relation \vdash_* defined as follows:

Definition 6: Let γ be a formula of FOSDL. $\rho, s \vdash_* \gamma$ iff $\gamma \in \bigcup E_{\Delta_{\rho,s}(\varphi(\vec{x}),\psi(\vec{x}))}$ where $\bigcup E_{\Delta_{\rho,s}(\varphi(\vec{x}),\psi(\vec{x}))}$ is the union of all extensions of $\Delta_{\rho,s}(\varphi(\vec{x}),\psi(\vec{x}))$.

□.

 \Box .

 \Box .

B. Consistency and completeness of the completed regulation

In order to define consistency and completeness with \vdash_* , we extend the previous definitionsby using \vdash_* instead of \vdash in those definitions. To distinguish the new notions of consistency and completeness from the old ones, we will use * as a prefix (for instance we will write "*-consistency") or write explicitly "for \vdash_* " (for instance, we will write "consistent for \vdash_* ").

Proposition 1: Let us consider a set of integrity constraints IC, a regulation ρ consistent according to IC and a state of the world s consistent with IC and such that $\rho \cup s$ is consistent. Let $\varphi(\vec{x})$ and $\psi(\vec{x})$ be two objective formulae verifying definition 4 and $\Delta_{\rho,s}(\varphi(\vec{x}), \psi(\vec{x}))$ the corresponding default theory.

The following propositions are equivalent:

for every vector a of ground terms, if s ⊢ φ(a), ρ, s ⊭
Oψ(a), ρ, s ⊭ Pψ(a) and ρ, s ⊭ Fψ(a) (i.e. ρ is not (φ(a), ψ(a))-complete in s), then s ⊢ E_F(a) ⊗
E_P(a) ⊗ E_F(a).

2)
$$\rho$$
 is consistent and $(\varphi(\vec{x}), \psi(\vec{x}))$ -complete for \vdash_* in s.

The proof is given in[7].

This proposition characterizes necessary and sufficient conditions for the defaults to consistently complete an incomplete regulation. More precisely, this proposition says that if every time the regulation does not prescribe a behaviour one and only one E_i is true, then the defaults consistently complete the regulation (because one and only one default is applied for a particular $\psi(\vec{a})$).

Example 4: Consider the state of the world = $\{Empl(A),$ Empl(B),Context(Crisis), s_1 Info(I),Topic(I, ExpRisk),Receive(A, I))from the last example. ρ is incomplete in s_1 for $Empl(A) \land Empl(B) \land Receive(A, I)$ $\varphi_0(A, I, B)$ \equiv $\psi_0(A, I, B)$ Send(A, I, B).Let's take and \equiv $E_F(x, i, y) = Context(Crisis) \land (Topic(I, EqtOOO) \lor$ $Topic(I, EqtChk) \lor Topic(I, Meeting)), E_P(x, i, y) = \bot$ and $E_O(x, i, y) = Context(Crisis) \wedge Topic(I, ExpRisk),$ then $s_1 \vdash E_O(A, I, B)$. Thus, ρ is consistent and $(\varphi_0(x, i, y), \psi_0(x, i, y))$ -complete for \vdash_* in s_1 .

Even if this necessary and sufficient condition is interesting in theory, it is not really useful for practical purposes. In fact, to verify that this condition is satisfied, we would have to detect every "hole" in the regulation. This detection is an operation we want to avoid. Finding more general conditions that are still sufficient but not necessary for the completion rules to consistently complete the regulation is possible, see [7]. Another alternative would be to take fixed E_i . For example, we could take one E_i equal to \top and the two others to \bot which lead to either highly or dimmed secured systems depending on the E_x chosen to be equivalent to \top .

IV. CONCLUSION

In this paper, we have addressed the problem of analysing consistency and completeness of regulations which may exist in a society of agents in order to rule their behaviour. We have defined a modal logical framework and showed how to express regulations, consistency and completeness within this framework. We also dealt with incomplete regulations and proposed a way for completing them by using defaults. We have established several results which show when these defaults consistently complete a regulation. These notions have been illustrated on an example of information exchange policy using multiple contexts.

This framework has now to be tested on a real-world problem. The example used in this paper is really simple, particularly considering the number of agents and actions involved. It may be interesting for instance to use ad hoc firstorder languages like description logics to model a domain, to show that the framework is sufficiently expressive to be applied on large systems. Given this objective, notice that complexity and decidability problems remain to be tackled. Furthermore, in order to deal with intelligent agents, we have to extend it by considering more notions, among them time and action.Finally, we developed a really simple model of the deontic notions by using SDL and lots of classical problem in deontic logic are not handled here: norms with exceptions, contrary-to-duties, collective obligations etc. Another extension of this work will be to define a logic that can deal with these problems.

REFERENCES

- P. Bieber and F. Cuppens, "Expression of confidentiality policies with deontic logic," in *Deontic logic in computer science: normative system* specification. John Wiley and Sons, 1993, pp. 103–121.
- [2] L. Cholvy, "Checking regulation consistency by using SOL-resolution," in *International Conference on Artificial Intelligence and Law*, 1999, pp. 73–79.
- [3] F. Cuppens and R. Demolombe., "A modal logical framework for security policies," in *Lectures Notes in Artificial Intelligence*, vol. 1325. Springer, 1997, p. 1997.
- [4] L. Cholvy and S. Roussel, "Consistency and completeness of regulations," in Proceedings of the third International Workshop on Normative Multiagent Systems (NORMAS'08), 2008, pp. 51–65.
- [5] M. Fitting and R. L. Mendelsohn, *First-order modal logic*. Kluwer Academic, 1999.
- [6] R. Reiter, "A logic for default reasoning," Artificial Intelligence, vol. 13, no. 1,2, 1980.
- [7] C. Garion, S. Roussel, and L. Cholvy, "A modal logic for reasoning on consistency and completeness of regulations," in *Normative Multi-Agent Systems*, ser. Dagstuhl Seminar Proceedings, G. Boella, P. Noriega, G. Pigozzi, and H. Verhagen, Eds., no. 09121. Dagstuhl, Germany: Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, Germany, 2009. [Online]. Available: http://drops.dagstuhl.de/opus/volltexte/2009/1904