A definition of agent-oriented relevance in modal logic

Stéphanie Roussel, Laurence Cholvy

ONERA - Toulouse DTIM
2 avenue Edouard Belin BP 54025
31055 Toulouse Cedex 4 - FRANCE
stephanie.roussel@onera.fr
laurence.cholvy@onera.fr

Abstract

Cet article étudie la notion de pertinence dans le cadre d'agents coopératifs qui s'échangent des informations. Plus précisément, étant donné un agent qui a un besoin en information, on caractérise en logique multi-modale les informations (possédées par les autres agents) qui sont pertinentes pour lui. On étudie finalement les propriétés et les limites de la pertinence ainsi définie.

Mots-clés *: Pertinence orientée agent, besoin en information, système multi-agents*

Abstract

This article studies the notion of relevance in the context of systems where agents cooperatively exchange pieces of information. More precisely, given an agent who has got some information need, we characterize pieces of information which are relevant to her. This characterization is done in a multi-modal logic. Properties and limits of relevance can therefore be studied.

Key-words*: Agent-oriented relevance, information need, multi-agent systems*

1 INTRODUCTION

The general context of this work is modelling multi-agent systems. We focus on cooperative systems, i.e systems where the agents have to cooperatively act so that the achievement of their individual goals ensures the achievement of the global goal. In particular, the agents have to cooperatively communicate, which means first that exchanged information is easily understandable by the agent who receives it (the receiver), i.e it is expressed in a language she understands and its interpretation does not require too long time nor effort. But more, this implies that the exchanged information is the very one useful for the receiver to fulfil her current individual goal. More precisely, the achievement of the agent's individual goal generates some information needs i.e, requires the acquisition of some pieces of information that we call relevant pieces of information.

In many areas, *relevance* is a key concept. Let us briefly present the main definitions of relevance in the literature and explain what in those definitions could (or could not) be applied to the problem of information exchange in multi-agent systems.

Following Borlund [3], definitions of relevance can be separated into two different groups : system-oriented relevance and agent-oriented relevance.

System-oriented approaches analyze relevance in terms of topicality, aboutness (i.e pieces of information can be represented by a few subjects or topics), matching degrees between a piece of information and a request. Most of the works about this approach can be found in:

- **Information Retrieval area** [5, 6, 14] ; for a given request, an IR system finds in its collection of documents the ones relevant for the request. There are many ways to work with documents and pieces of information they contain (keywords, tree of concepts, ...) but most of the time, relevance is calculated in terms of matching degrees between the document and the request.

  In this approach, although not made explicit, two agents can be considered: one agent, the user, has a request and the other one, the information retrieval system, gives back relevant documents for this request. The representation of the user is generally very poor since it is represented by its request only. Thus, two users (one beginner and one expert in the request area) who send the same request in an information retrieval system will have the same relevant documents returned. Yet, one document can be relevant to the beginner while it contains information which is already known by the expert.

  Moreover, in Information Retrieval area, pieces of information are not really considered. What is manipulated is the document that supports pieces of information.

  Thus, relevance as seen by Information Retrieval area can hardly be applied to multi-agent system.

- **Artificial Intelligence area** [13, 15] ; in the context of propositional logic, notion of relevance has been introduced to characterize some relation between a formula (modelling a sentence) and a set of variables (modelling a subject matter or a topic), or some relation between two formulae together. In this optic, inference knowledge bases can be sped up.

Agents are not really considered in the Artificial Intelligence point of view. The potential agent who is looking for a relation between an information and some subject matter, or trying to infer from a knowledge base, is never mentioned. However, most of the time, there is a set of formulae with respect to which the relation of relevance is defined or in which the inference is made. This set of formulae could match some agent's knowledge base but it is never specified as well.

- **Relevant Logics domain** [2, 21] relevant logics are non-classical logics. These systems have been developed in order to solve the problem of classical logical implication. In fact, many paradoxes derive from this implication.

  Relevant logics do not take into account neither agents nor information need. Most of the systems developed for these logics are quite complex. Thus, these works, as is, could hardly be applied to our context.

On the other hand, agent-oriented approaches try to define a relation between some agent and a piece of information. Thus, relevance is analyzed in terms of agent's utility or informativeness for the agent. In those cases, relevant pieces of information are defined according to agent's information need. This approach is closer to what could characterize relevance in multi-agent systems. Even if most of those works are informal, they highlight different elements in relevance that could be applied to our context.

- In **Information Retrieval area**, Borlund [3] and Mizzaro [18] give a classification of different agent-oriented relevances depending on the chosen user level. For example, the perception by some agent of a piece of information is generally different from the original piece of information. Thus, definitions of relevance for a piece of information or for its interpretation are different.

  Like for system-oriented approaches, information retrieval deals with documents that support pieces of information. It makes it difficult to be applied to multi-agent systems.

- In **Psychology and Linguistic area**, Grice [9] expounds his cooperation principle along with the corresponding maxims. One of the maxim is the relevance maxim and stipulates that one should be relevant in order to be cooperative. Many studies have followed Grice's [11, 20]. In particular, Sperber and Wilson reduce all the Grice's maxims to one and define a cognitive psychological theory, the Relevance Theory, based on the following informal definition: *An input (a sight, a sound, an utterance, a memory) is relevant to an individual when it connects with background information he has available to yield conclusions that matters to him.*

  Sperber's and Wilson's Relevance Theory not only focus on human communication since stimulus for which the relevance is characterized can be a message emitted by an individual towards another one, but also any kind of stimulus. However, it obviously applies to two agents' communication.

They particularly highlight the information need underlying the definition of relevance. For them, this need is multiple as they explicitly mention in [20]: *(Conclusions that matter to an individual are for instance, information that help him) to complete or improve his knowledge base by answering a question he has in mind, improving his knowledge on a certain topic, settling a doubt, confirming a suspicion, correcting a mistaken impression.*

These notions of underlying need and positive effects have to be considered when characterizing relevance in multi-agent systems. In fact, the exchanged piece of information have to be the one needed by the receiver and should have a positive cognitive effect concerning this need.

- In **Philosophy area**, Floridi [8] has developed a subjectivist interpretation of epistemic relevance. In Floridi's theory, *the degree of relevance of a piece of information I towards an agent A is defined as a function of the accuracy of I understood by A as an answer to a query Q, given the probability that Q might be asked by A.*

  Floridi explicitly mentions the receiver, as the agent towards who the information relevance is evaluated but does not mention the sender (the agent who informs the receiver) as the one who could evaluate the degree of relevance.

  Floridi's work explicitly relates the notion of relevance to an hypothetical query the receiver could have asked if he has known that the information answers it. Thus Floridi's theory assumes that the receiver has an information need, that could be represented by this query.

  For Floridi, it is clear that false information (misinformation) cannot be relevant. Worse, false information is deleterious.

In the different agent-oriented approaches, various elements (presence of agents that have some information need, positive cognitive effect, truth of relevant pieces of information) seem to be in accordance to a relevance that could be defined in multi-agent system. The major problem in those approaches is that most of the work presented is informal. Thus, in this present paper, our aim is to contribute to the study of agent-oriented relevance by giving a formal definition. Notice that we limit our contribution to the modelling aspect and do not study any complexity aspect.

To model information need and knowledge of agents we use a widely respected model, the *belief-desire-intention* model (BDI) [22]. This model assumes that an agent is characterized by her mental attitudes, mainly belief, desire and intention. Most formal models based on BDI are modal logics whose modal operators are used to represent the different mental attitudes. The semantic of those operators are generally given by the possible world semantics [4].

This paper is organized as follows. Section 2 presents the multi-modal logic framework we base our work on. Section 3 deals with relevance defined according to an agent's information need. In section 4, we define a hierarchy that characterizes the most relevant pieces of information. In section 5, we compare our proposal with some related work found in AI community. Section 6 addresses some extension of this operator. Finally section 7 concludes this paper.

2 FORMAL FRAMEWORK

The formal framework on which our work is based on is the one defined in [10]. It is a propositional multi-modal logic whose modal operators are belief and intention.

We first present the language. Then, we present the axiomatics of the logic. Concerning semantics, it has been studied in [10]. In this paper, we will only focus on axiomatic aspect.

2.1 Language

The alphabet of our language is based on non logical symbols : a set $\mathcal{A}$ of agents, for every agent a of $\mathcal{A}$, we define two modalities B_a and I_a. We define also the set of logical symbols : a set $\mathcal{V}$ of variables symbols, $\neg$, $\vee$, (and), the constants $\top$ and $\bot$.

Semantic of this logic has been studied in [10]. In this paper, we will only focus on axiomatic aspect.

Definition 1
The formulae of our language are defined recursively as follows:

- *if p belongs to $\mathcal{V}$ then p is a formula of our language. $\bot$ and $\top$ are formulae of our language.*
- *if a is an agent of $\mathcal{A}$ and φ a formula of our language then $B_a\varphi$ and $I_a\varphi$ are formulae of our language. $B_a\varphi$ is read "agent a believes that φ is true". $I_a\varphi$ is read "agent a intends φ to be true".*
- *if φ_1 and φ_2 are formulae of our language, so are $\neg\varphi$ and $\varphi \vee \varphi_2$.*

If φ_1 and φ_2 are formulae of our language and a some agent of $\mathcal{A}$, we also define the following abbreviations: $\varphi_1 \wedge \varphi_2 \equiv \neg(\neg\varphi_1 \vee \neg\varphi_2)$, $\varphi_1 \rightarrow \varphi_2 \equiv \neg\varphi_1 \vee \varphi_2$, $\varphi_1 \leftrightarrow \varphi_2 \equiv (\varphi_1 \rightarrow \varphi_2) \wedge (\varphi_2 \rightarrow \varphi_1)$, $Bif_a\varphi \equiv B_a\varphi \vee B_a\neg\varphi$.

$\otimes$ is the exclusive disjunction generalized to n formulae i.e. if φ_1, ... φ_n are n ($n \geq 1$) formulae then $\varphi_1 \otimes ... \otimes \varphi_n$ is true if and only if $\varphi_1 \vee ... \vee \varphi_n$

is true and $\forall i, j$ such that $i \neq j$, $\neg(\varphi_i \wedge \varphi_j)$ is true. We define the operator $\bigotimes$ such that

$$\bigotimes_{i=1}^{n} \varphi_i \equiv \varphi_1 \otimes \varphi_2 \otimes ... \otimes \varphi_n$$

For $n = 1$, we have $\bigotimes_{i=1}^{1} \varphi_i \equiv \varphi_1$

Let φ_1, ... φ_n be n ($n \geq 1$) formulae. We define the operator $\bigwedge$ such that

$$\bigwedge_{i=1}^{n} \varphi_i \equiv \varphi_1 \wedge \varphi_2 \wedge ... \wedge \varphi_n$$

For $n = 1$, we have $\bigwedge_{i=1}^{1} \varphi_i \equiv \varphi_1$.

Let φ_1, ... φ_n be n ($n \geq 1$) formulae. We define the operator $\bigvee$ such that

$$\bigvee_{i=1}^{n} \varphi_i \equiv \varphi_1 \vee \varphi_2 \vee ... \vee \varphi_n$$

For $n = 1$, we have $\bigvee_{i=1}^{1} \varphi_i \equiv \varphi_1$

Finally, a formula of our language without any modality is said to be *objective*.

2.2 Axiom system

We now give an axiom system for belief and intention. This axiom system consists of following reasoning rules and axiom schemes. Let a be an agent of $\mathcal{A}$.

- Propositional tautologies and inference rules.
- KD45 for B,

  (BK) $B_a(\varphi \rightarrow \psi) \wedge B_a\varphi \rightarrow B\psi$

  (BD) $B_a\varphi \rightarrow \neg B_a\neg\varphi$

  (B4) $B_a\varphi \rightarrow B_aB_a\varphi$

  (B5) $\neg B_a\varphi \rightarrow B_a\neg B_a\varphi$

- (Nec) Necessitation for B_a, $\frac{\varphi}{B_a\varphi}$
- BI Introspection as follows,

  (I4) $I_a\varphi \rightarrow B_aI_a\varphi$

  (I5) $\neg I_a\varphi \rightarrow B_a\neg I_a\varphi$

  (BI) $I_a\varphi \rightarrow B_a\neg\varphi$

Belief operator is a normal operator. Moreover, we suppose that agent do not have inconsistent beliefs (BD) and that are conscious of what they believe (B4) and what they do not believe (B5).

Intention operator is a non-normal operator.

Moreover, we suppose some relation between belief and intention that we call belief intention introspection. Like in [10], we first suppose **strong realism** (BI), i.e we consider that if some agent intends a proposition to be true then she believes this proposition is false. A consequence of this axiom is that agents cannot intend what they already believe to be true.Finally, we suppose that agents are conscious of what they intend (I4) and what they do not intend (I5).

3 RELEVANCE

3.1 Definition

We define relevance the following way:

Definition 2
Let a be some agent of $\mathcal{A}$, φ a formula and Q a request. φ is said to be relevant *for agent a concerning her request Q iff*

$$I_a Bif_a Q \wedge (B_a(\varphi \to Q) \otimes B_a(\varphi \to \neg Q)) \wedge \varphi$$

This formula is denoted $R_a^Q \varphi$.

This definition can be broken down into three elements:

- **Agent's information need** $I_a Bif_a Q$: As already mentioned, we suppose that the agents that exchange pieces of information have some information needs. These information needs take place in the characterization of relevant pieces of information for those agents i.e we consider that a piece of information cannot be relevant if it is not "linked" to some agent's information needs. This is in accordance with Sperber and Wilson's Relevance Theory presented in introduction [20].

  At first, we suppose that an information need is quite simple and can be modelled the following way: "agent a wants to know if Q or if $\neg Q$, Q being a request".[1] Formally, information need is written $I_a Bif_a Q$, that means agent a wants to know if Q.

- **Agent's beliefs** $B_a(\varphi \to Q) \otimes B_a(\varphi \to \neg Q)$: From her beliefs and the piece of information φ, the agent must be able to answer her request Q, that means she can deduce either Q or $\neg Q$. In order to represent this deduction, we choose logical implication.

[1] In this paper, we do not pay attention to the process that transforms an information need (as it is perceived by the agent) to a formalized request.

Using agent's beliefs in the definition of relevance exempts pieces of information from which the agent cannot deduce anything (concerning her information need) from being relevant to him. For example, some technical data can be relevant to an expert that has the knowledge to deduce something from it whereas it can be inadequate for anyone that is not an expert [2].

If some agent, from a piece of information φ can deduce both Q and $\neg Q$, then φ does not really answer the information need. Using $\otimes$ prevents this case to happen [3].

- **The piece of information truth value** φ: We consider that a false piece of information cannot be relevant. A false piece of information, even if it has a meaning, is false. If we analyze the epistemical relevance in terms of cognitive efforts, misinformation is deleterious. This is in accordance with Floridi's work in the philosophy area ([8]). For example, let us consider some agent who wants to take the train to Paris. This train leaves at 1.05 pm. In this context, telling the agent that the train leaves at 1.15 pm is damaging (as she can miss her train). Then, we cannot consider that the piece of information "The train leaves at 1.15 pm" is relevant to the agent. [4]

The following example illustrates the definition of relevance.

Example 1
Let us consider two agents a and b that have to take a train. Unfortunately, some incidents in train stations can block train and make them be late (modelled by $late$). Let us consider that the piece of information "There are some incidents", modelled by inc, is true.
Agent a needs to know if her train is late or not. Thus, she has the information need $I_a Bif_a Q$. Agent a believes that if there are some incidents, then her train is late. This is modelled by $B_a(inc \to late)$. Thus, in this context, we have :

- $I_a Bif_a(late)$
- $B_a(inc \to late)$
- inc
- *Then, we can deduce* $R_a^{late}(inc)$

[2] considering that both of them have the same information need and that the data are true.

[3] Using $\otimes$ prevents the case where the agent already believes $\neg\varphi$ to happen. Indeed, in this particular case, from $\neg\varphi$, the agent would be able to deduce anything.

[4] In some particular cases, misinformation can be relevant. For example, it is relevant for a teacher to learn that one of his pupils is wrong about some lessons. However, in this case, this is not the wrong lesson itself that is relevant to the teacher but the fact that the pupil is wrong. These considerations on truth value of relevant pieces of information are developed in [8].

That means that information inc is relevant to agent a concerning her request $late$.
Agent b also needs to know if her train is late or not. Her beliefs are different from a's ones. Indeed, agent b only believes that if there are no incidents, then her train is not late. This can be modelled by $B_b(\neg inc \to \neg late)$. [5]
Thus, in this context, we have :

- $I_b Bif_b late$
- $B_b(\neg inc \to \neg late)$
- inc

This time, the piece of information inc is not relevant for agent b as she cannot deduce neither $late$ or $\neg late$.
The information $\neg inc$, which is false in the context, cannot be relevant for agent b as it would allow her to make wrong conclusions about her information need.

3.2 Properties

In this part, we study some properties of the relevance. For that, let us take a an agent of $\mathcal{A}$, Q, Q_1 and Q_2 some objective formulae, φ, φ_1, φ_2 some formulae. The following propositions are theorems of our logic.

Proposition 1

$$\vdash I_a Bif_a Q \to \neg B_a Q \wedge \neg B_a \neg Q$$

If an agent has an information need of the kind "knowing whether Q or $\neg Q$" then this agent does not believe neither Q nor $\neg Q$. That means that agents cannot have information need if they already know the answer to this information need.

Proposition 2

$$\vdash R_a^Q \varphi \to \neg B_a \varphi \wedge \neg B_a \neg \varphi$$

If some piece of information φ is relevant for some agent a, then agent a does not believe neither φ (otherwise she would be already able to answer her information need), or $\neg\varphi$ (because of the operator $\otimes$). Thus, relevant pieces of information cannot be tautologies nor contradiction.

[5] Agent b only believes $B_b(\neg inc \to \neg late)$ means that this is the only belief of b about inc and $late$, i.e $\neg(B_b(inc \to late)) \wedge \neg(B_b(inc \to \neg late))$

Proposition 3
Let $*$ be some belief revision operator satisfying AGM postulates (postulates 1 to 4) [1]. Bel_a represents the set of beliefs of agent a and $Bel_a * \varphi$ the set of beliefs of agent a after being revised by φ using revision operator $*$. Then, if we have $R_a^Q \varphi$ then either $Q \in Bel_a * \varphi$ or $\neg Q \in Bel_a * \varphi$.

This proposition shows that the deduction operator that we have chosen, logical implication, corresponds to some "basic" belief revision operator. Indeed, if she revises her beliefs with the relevant piece of information, the agent has in her new beliefs set the answer to her information need.

Proposition 4

1. $\vdash I_a Bif_a Q \wedge \neg B_a Q \wedge \neg B_a \neg Q \rightarrow R_a^Q Q \otimes R_a^Q \neg Q$: *one of the pieces of information Q or $\neg Q$ is relevant to agent a concerning her request Q.*

2. $\vdash (Q_1 \leftrightarrow Q_2) \wedge B_a(Q_1 \leftrightarrow Q_2) \rightarrow (R_a^{Q_1}\varphi \leftrightarrow R_a^{Q_2}\varphi)$: *some piece of information that is relevant concerning a request is also relevant concerning a request equivalent to the first one.*

3. $\vdash R_a^Q \varphi \leftrightarrow R_a^{\neg Q} \varphi$: *some piece of information that is relevant concerning a request Q is also relevant concerning the request $\neg Q$.*

4. $\vdash \neg(\varphi_1 \wedge \varphi_2) \rightarrow \neg(R_a^{Q_1}\varphi_1 \wedge R_a^{Q_2}\varphi_2)$: *two conflicting pieces of information cannot both be relevant.*

Proposition 5
$\vdash R_a^Q \varphi \rightarrow \neg B_a R_a^Q \varphi$

If some information φ is relevant to some agent a, then a does not know it. This is due to the truth value of the piece of information contained in the relevance definition. If the agent believes that the piece of information is relevant to her, then she believes this piece of information. If she believes this piece of information, then she can deduce from her set of beliefs the answer to her information need. This is in contradiction with the fact the agent has the information need (relation of strong realism between belief and intention).
At this point, the reader could wonder whether or not this relevance is useful if the agent is not conscious of what is relevant to her. Actually, an agent is conscious of what is potentially relevant to her. A definition for the potential relevance would be the same as the one defined here but without the truth value of the piece of information. An information that is just potentially relevant is not useful because it might be false. In fact, if an agent has some information need, she is conscious of what information could help her solve this need but without knowing the truth value of these pieces of information, she cannot use them.
Moreover, the context we are modelling is communicating agents. The issue

is not to know if the agent is able to determine what pieces of information are relevant to her, but to know if another agent (that possesses some pieces of information) is able to determine them (see Section 7).

Proposition 6
If two pieces of information are equivalent then one being relevant is equivalent to the other one being relevant.

$$\vdash (\varphi_1 \leftrightarrow \varphi_2) \wedge B_a(\varphi_1 \leftrightarrow \varphi_2) \rightarrow (R_a^Q \varphi_1 \leftrightarrow R_a^Q \varphi_2)$$

This proposition shows that our proposal of relevance is syntactic independent. It focuses on the content of pieces of information and not their form. If inc is a relevant piece of information, then so is $(inc \wedge late) \vee (inc \wedge \neg late)$.

Notation. In what follows, we will write $B_a(\varphi_1, \varphi_2/Q)$ instead of $\neg(B_a(\varphi_1 \wedge \varphi_2 \rightarrow Q) \wedge B_a(\varphi_1 \wedge \varphi_2 \rightarrow \neg Q))$. This formula means that agent a believes that φ_1 and φ_2 do not allow to deduce Q and $\neg Q$.

Proposition 7
$\vdash B_a(\varphi_1, \varphi_2/Q) \rightarrow (\varphi_2 \wedge R_a^Q \varphi_1 \rightarrow R_a^Q(\varphi_1 \wedge \varphi_2))$

Proposition 8
$\vdash B_a(\varphi_1, \varphi_2/Q) \rightarrow (R_a^Q \varphi_1 \wedge R_a^Q \varphi_2 \rightarrow R_a^Q(\varphi_1 \vee \varphi_2))$

Those two propositions show that the relevance operator characterizes too many relevant pieces of information. This is illustrated in the following example.

Example 2
Let us take the example of the train that can be late because of incidents. Agent a needs to know if her train is late or not and we suppose that inc is relevant to her.
Let us suppose that the piece of information "it rains", modelled by $rain$ is true in this context. Then, the piece of information $inc \wedge rain$ is relevant to a. Indeed, it contains all necessary elements so that agent a is able to answer her information need. Nevertheless, intuitively, the piece of information inc is more relevant to a than $inc \wedge rain$ because this last one contains the element $rain$ that is not necessary to answer a's information need.

All the pieces of information characterized relevant are "sufficiently" relevant. Indeed, each of them gives an answer to the information need. On the other side, one could consider pieces of information that are "necessarily" relevant, that means the ones without which the agent cannot answer her information need. If we combine the two concepts, we can find, among the "sufficiently relevant" pieces of information, the ones that are the most "necessary". Thus, these most necessary pieces of information are the very ones that are the most relevant.

4 A HIERARCHY FOR RELEVANCE

In this section, we characterize the notion of "necessary relevance". Let $\mathcal{R}_a^Q$ be the set of formulae that are relevant to some agent a concerning her request Q with our operator R_a^Q. We suppose that this set contains only objective formulae [6]. Let us note that formulae of $\mathcal{R}_a^Q$ cannot be tautologies or contradictions [7].

4.1 Case of clauses and cubes

In this subsection, we handle the cases where $\mathcal{R}_a^Q$ are only clauses, then only cubes.

4.1.1 Clauses

Let $\mathcal{R}_a^Q$ be a set of clauses. In terms of "necessary", we can consider that the more precise a clause is, the more "necessary" it is. For example, let us consider the two pieces of information $rain \vee inc$ and inc. Let us suppose that they are both relevant to answer the information need $late$. The piece of information $rain \vee inc$ is less necessary than inc because it contains $rain$ which is not useful to answer the information need and it is more precise that $rain \vee inc$.

More generally, we can define a preorder for clauses.

Definition 3
Let φ_1 and φ_2 be two clauses. We define $\varphi_1 \leq_{Cl} \varphi_2$ iff $\vdash \varphi_2 \rightarrow \varphi_1$

Then, it is possible to define most relevant clauses.

Definition 4
The set of most relevant clauses $\mathcal{R}m^Q_{a\leq_{Cl}}$ is the set $max_{\leq_{Cl}}\mathcal{R}_a^Q$ containing the maxima for $\leq_{Cl}$.

The preorder corresponds to subsumption. That means that most relevant clauses are maxima for subsumption, i.e the most precise clauses.

4.1.2 Cubes

For cubes (conjunction of literals), we can use the dual relation. Let $\mathcal{R}_a^Q$ be a set of cubes. In terms of "necessary", the less precise a piece of information is, the more relevant it is. For example, if both pieces of information $rain \wedge inc$ and inc are relevant, then inc can be considered more relevant because

[6] Of course, not objective formulae could be relevant too but the hierarchy we define in this section only works for objective formulae

[7] This is due to proposition 2

$rain \wedge inc$ contains $rain$ without which it is still possible to answer the information need.

More generally, we can define a preorder for cubes.

Definition 5
Let φ_1 and φ_2 be two cubes. We define $\varphi_1 \leq_{Cu} \varphi_2$ iff $\vdash \varphi_1 \rightarrow \varphi_2$

Then, it is possible to define the most relevant cubes.

Definition 6
The set of most relevant cubes $\mathcal{R}m^Q_{a\leq_{Cu}}$ is the set $max_{\leq_{Cu}}\mathcal{R}^Q_a$ containing the maxima for $\leq_{Cu}$.

Thus, the most relevant cubes are the prime implicants of Q and the prime implicants of $\neg Q$ [8]. It corresponds to cubes that contain only necessary elements to answer the information need.

4.2 General formulae

We can notice that the two preorders defined for clauses and cubes work in an opposite way. For general formulae, a compromise between the two preorders has to be found. The most relevant pieces of information are the ones that are precise enough and that do not contain too many unnecessary elements...

4.2.1 Minimal explanation

For that, we first introduce the notion of minimal explanation. This notion has been used by Lakemeyer [13] in AI field to define relevance. However, the definition of minimal explanation he uses is syntactical [9]. In order to have a more semantic definition, we update the definition by using the notion called "semantical independence" defined in [15].

Definition 7
Let φ be an objective formula. φ is said to be in Negation Normal Form (NNF) if and only if only propositional symbols are in the scope of an occurrence of $\neg$ in φ [10]. $Lit(\varphi)$ denotes the the set of literals occurring in the NNF of φ.

For example, the NNF form of $\varphi = \neg((\neg a \wedge b) \vee c)$ is $(a \vee \neg b) \wedge \neg c$. Then, $Lit(\varphi) = \{a, \neg b, \neg c\}$.

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[8] We remind that an implicant is prime if it stops to be an implicant when it has one literal less

[9] Indeed, he uses CNF form of a formula. But for a given formula, the CNF form is not unique

[10] Formulae $\varphi \rightarrow \psi$ should be changed into $\neg\varphi \vee \psi$ and $\varphi \leftrightarrow \psi$ into $(\neg\varphi \vee \psi) \wedge (\neg\psi \vee \varphi)$

Definition 8
Let φ be an objective formula, l a literal.
φ is said to be syntactically Lit-dependent on l (resp. syntactically Lit-independent from l) if and only if $l \in Lit(\varphi)$ (resp. $l \notin Lit(\varphi)$).

Definition 9
Let φ be an objective formula, l a literal and L a subset of literals.
φ is said to be Lit-independent from l, denoted $l \not\mapsto \varphi$, if and only if there exists a formula Σ such that $\Sigma \equiv \varphi$ and Σ is syntactically Lit-independent from l. Otherwise, φ is said to be Lit-dependent on l, denoted $l \mapsto \varphi$. Given a language, the set of all literals of this language such that $l \mapsto \varphi$ is denoted by $DepLit(\varphi)$.

Example 3
Let $\varphi = (a \wedge \neg b \wedge (a \vee b))$. We have $DepLit(\varphi) = \{a, \neg b\}$. Note that φ is Lit-independent from b because it is equivalent to $\Sigma = (a \wedge \neg b)$, in which b does not appear positively.

Now, let us give the definition of minimal explanation.

Definition 10
Let Δ be a finite set of objective formulae, and α and β be two objective formulae.
β is an explanation of α if and only if $\vdash B\Delta \rightarrow B(\beta \rightarrow \alpha)$ and $\not\vdash B\Delta \rightarrow B(\neg\beta)$.
β is a minimal explanation of α if and only if β is an explanation of α and there is no explanation β' of α such that $DepLit(\beta') \subset DepLit(\beta)$.

4.2.2 Application

From this minimal explanation, we can define what are the most relevant formulae.

For that, let us suppose that Δ is the subset of Bel_a (agent's a belief base) containing only objective formulae [11].

Let $\mathcal{R}_a^Q$ be the set of relevant formulae. For all φ in $\mathcal{R}_a^Q$, we have $B_a(\varphi \rightarrow Q)$ or $B_a(\varphi \rightarrow \neg Q)$ and $\neg B_a(\neg\varphi)$, that means that φ is an explanation of Q or $\neg Q$.

Definition 11
The set of most relevant formulae $\mathcal{R}m_a^Q$ is the subset of $\mathcal{R}_a^Q$ containing the minimal explanations of Q and the minimal explanations of $\neg Q$.

Example 4
Let us consider the following set of relevant pieces of information to agent a concerning her request Q:$\mathcal{R}_a^Q = \{inc \wedge rain, inc \vee strike, strike\}$. Then $\mathcal{R}m_a^Q = \{strike, inc \wedge rain\}$.

[11] If $B_a\varphi$ then $\varphi \in Bel_a$ and if $B_a\varphi$ and φ is objective then $\varphi \in \Delta$

With this preorder and our notion of relevance, necessary [12] and sufficient relevant pieces of information can be characterized.

Proposition 9

- *if $\mathcal{R}_a^Q$ contains only clauses, then $\mathcal{R}m^Q_{a\leq_{Cl}} \subseteq \mathcal{R}m^Q_a$.*
- *if $\mathcal{R}_a^Q$ contains only cubes, then $\mathcal{R}m^Q_{a\leq_{CU}} \subseteq \mathcal{R}m^Q_a$.*

This proposition shows that the concept of necessary with respect to minimal explanation is consistent with the preorders previously defined for clauses and cubes.

Of course, according to a different definition of "necessary" for a piece of information, we could have a different set of most relevant pieces of information.

Finally, even if a formal link remains to be done for relating the generation of most relevant information and abduction, we think that we will be able to take benefit from abductive reasoning methods. For instance,we think that the SOL-Resolution ([12]) could be applied in our case if we select only pieces of information which are considered as true (this production fied is stable).

5 COMPARISON TO SOME RELATED WORK

By using the notion of minimal explanation, we can compare the relevance defined in this paper with the one defined by Lakemeyer [13].

5.1 Definition

In [13], Lakemeyer defines many relevances and compared his notion of relevance with many concepts of literature ([17], [16]). Nevertheless, in this paper, we will focus on the relevance that is the closest to ours [13].

First of all, let us remind a few definitions and notations used in [13].

- a formula or a set of formulae Δ *mentions* an atom p if p or $\neg p$ appears in Δ.
- a *subject matter* π is a set of atoms.
- $\pi_\Delta = \{p \text{ — } p \textit{ is an atom mentioned in } \Delta\}$
- A formula is trivial if it is locally equivalent to a tautology.

Then, Lakemeyer defines L-relevance the following way:

[12]in respect to minimal explanation

[13]To avoid confusion between relevance defined in this paper and Lakemeyer's one, we will write Lakemeyer's relevance "L-relevance".

Definition 12
A subject matter π is L-relevant *for α with respect to Δ (denoted $RX_{\Delta}(\pi, \alpha)$) if and only if there is an minimal explanation of α that is not trivial and that mentions some atom p such that $p \in \pi$.*

Intuitively, some subject matter is relevant for a formula if it contains atoms that are necessary to the explanation of this formula.

Example 5
Let us consider the following set: $\Delta = \{inc \wedge strike \rightarrow late, inc \wedge strike \wedge rain \rightarrow late\}$. In this set, the minimal explanation of $late$ is $inc \wedge strike$. Thus, any subject matter containing either inc or $strike$ is L-relevant to $late$ with respect to Δ.

In fact, L-relevance characterize the "necessarily" relevant formulae. The "sufficient" aspect is not considered whereas it is the base of our relevance.

5.2 Comparison

In order to study the link between L-relevance and relevance, we have to consider some agent and her set of beliefs. This set reduced to objective formulae corresponds to Lakemeyer's formulae set Δ [14]. Formulae for which we look for an explanation are the agent's request and its negation. The subject matter potentially relevant concerning those formulae is the set of atoms of the potentially relevant formula φ.

Proposition 10
Let φ and Q be two objective formulae. Let suppose that this agent a needs to find out if Q or $\neg Q$.
We note $\mathcal{R}_a^Q$ the set of relevant formulae.
If $\varphi \in \mathcal{R}_a^Q$, then there is one minimal explanation of Q or of $\neg Q$ that mentions at least one atom of φ, that means that we have L-relevance ($RX_{\Delta_a}(\pi_\varphi, Q) \vee RX_{\Delta_a}(\pi_\varphi, \neg Q)$).

If a piece of information is relevant concerning a request, then it contains necessary elements to explain the request or the request's negation. This means that relevance (as defined in this paper) implies L-relevance.

The comparison between both relevances is even stronger when considering the set of most relevant pieces of information $\mathcal{R}m_a^Q$. In fact, where L-relevance simply expresses that there exists a minimal explanation, relevance, when reduced to the most necessary formulae, characterizes directly what are those minimal explanations.

[14] If $B_a\varphi$ then $\varphi \in Bel_a$ and if $B_a\varphi$ and φ is objective then $\varphi \in \Delta$

Proposition 11
Let φ be a formula of $\mathcal{R}m_a^Q$. Not only there is an minimal explanation of Q or $\neg Q$ that mentions at least one atom of φ $(RX_\Delta(\pi_\varphi, Q) \vee RX_\Delta(\pi_\varphi, \neg Q))$ but φ is one minimal explanation of Q or $\neg Q$.

This shows that if a piece of information is "sufficiently" and "necessarily" relevant, then not only it is L-relevant but we can characterize this L-relevance more precisely than Lakemeyer because the minimal explanation he refers to is the relevant piece of information.

6 Information need generalized

6.1 Definition

Information need considered in this paper is "agent a wants to know if Q or if $\neg Q$". It can be extended to the following information need: "agent a wants to know if Q_1 or if Q_2 ... or if Q_n", $Q_1, ...Q_n$ being mutually exclusive objective formulae.

Thus, we can extend relevance the following way :

Definition 13
Let $\mathbf{Q}$ be a set of n $(n \geq 1)$ objective formulae Q_i exclusive to each other. φ is relevant concerning the set $\mathbf{Q}$ for a iff

$$I_a \bigotimes_{i=1}^{n} B_a Q_i \wedge \bigotimes_{i=1}^{n} B_a(\varphi \rightarrow Q_i) \wedge \varphi$$

This formula is denoted $R_a^{\mathbf{Q}}\varphi$.

Thus, a relevant piece of information answers the information need by allowing the agent to deduce one of the Q_i.

This definition can be broken down into the same three elements:

- The information need "agent a wants to know if Q_1 or Q_2 or ... or Q_n" is formalized the following way : $I_a \bigotimes_{i=1}^{n} B_a Q_i$.
- Agent a, from φ and her set of beliefs can deduce either Q_1, or Q_2 or ... or Q_n. This is represented by $\bigotimes_{i=1}^{n} B_a(\varphi \rightarrow Q_i)$.
- The piece of information φ still needs to be true.

Relevance for a general information need is illustrated in the following example.

Example 6
Let us consider some agent a that needs to know on which platform her train leaves. We suppose that there are 3 platforms in the train station. The train

leaves on platform 1 (resp 2 and 3) is modelled by p_1 (resp. p_2 and p_3). The set $\{p_1, p_2, p_3\}$ is denoted $\mathbf{Q}$.

Let us suppose that agent a believes that her train is a high speed train modelled by HS. Let us consider that piece of information $HS \rightarrow p_1$ (meaning that if the train is a high speed train, then it leaves on platform 1) is true in our context.

- $I_a(B_a p_1 \otimes B_a p_2 \otimes B_a p_3)$
- $HS \rightarrow p_1$
- $B_a(HS)$

Then, we have $R_a^{\mathbf{Q}}(HS \rightarrow p_1)$. Indeed, the piece of information $HS \rightarrow p_1$ allows agent a to deduce on which platform of the set $\mathbf{Q}$ her train leaves.

For $n = 2$, we get back to relevance as it has been studied all along the paper.

For $n = 1$, a piece of information is relevant for a concerning a set containing one objective formula Q if :

- agent a intends to believe that Q
- in agent a's set of beliefs, $\varphi \rightarrow Q$
- φ is true.

In that case, agent a has a particular information need : he wants to believe a formula. Thus, any true piece of information confirming Q is relevant.

As well, any true piece of information that was believed as false by the agent is relevant concerning any request he has i.e $I_a B_a Q \wedge B_a(\neg\varphi) \wedge \varphi \rightarrow R_a^{\{Q\}}\varphi$.

We do not develop this case in this paper. In what follows, we consider that $n \geq 2$.

6.2 Properties

Most of the properties stand for this extended relevance. We consider some agent a, a set of objectives and exclusive formulas $\mathbf{Q} = \{Q_i\}_{i=1\ldots n}$ such that $n \geq 2$ and some formulae φ, φ_1 and φ_2.

Proposition 12

$$\vdash I_a \bigotimes_{i=1}^{n} B_a Q_i \rightarrow \bigwedge_{i=1}^{n} \neg B_a Q_i$$

Proposition 13

$$\vdash R_a^{\mathbf{Q}}\varphi \rightarrow \neg B_a\neg\varphi \wedge \neg B_a\varphi$$

Proposition 14
Let $*$ be some belief revision operator satisfying AGM postulates (postulates 1 to 4) [1]. If we have $R_a^Q\varphi$ then there exists one and only one $k \in \{1...n\}$ such that $Q_k \in Bel_a * \varphi$.

Proposition 15

$$\vdash \bigotimes_{i=1}^{n} Q_i \wedge I_a \bigotimes_{i=1}^{n} B_a Q_i \wedge \bigwedge_{i=1}^{n} \neg B_a(\neg Q_i) \rightarrow \bigotimes_{i=1}^{n} R_a^{\mathbf{Q}} Q_i$$

The hypothesis $\bigotimes_{i=1}^{n} Q_i$ is necessary to make sure than at least one request Q_i is true and this is necessary for one Q_i to be relevant.

Proposition 16

$$\vdash R_a^{\mathbf{Q}}\varphi \rightarrow \neg B_a R_a^{\mathbf{Q}}\varphi$$

The explanation is the same as for the previous definition of relevance.

As well as the previous definition, the extended relevance characterises too many relevant pieces of information. In the same way, some hierarchy can be defined.

6.3 Partial relevance

With this extended definition, a relevant piece of information answers the information need by allowing the agent to deduce one of the Q_i. But there are pieces of information that can allow the agent to filter out some of the possibilities Q_i. In that case, even if the piece of information is not sufficiently relevant (as it does not answer the information need), it is partially relevant. Thus, we express this partial relevance the following way.

Definition 14
Let $\mathbf{Q}$ be a set of n ($n \geq 1$) objective formulae Q_i exclusive to each other. Let $\mathbf{P} = \{Q_1, ...Q_p\}_{1 \leq p \leq n}$ be a subset of $\mathbf{Q}$. φ is $\mathbf{P}$-partially relevant for agent a concerning the set $\mathbf{Q}$" iff

$$I_a \bigotimes_{i=1}^{n} B_a Q_i \wedge \varphi \wedge \bigwedge_{i=1}^{p} B_a(\varphi \rightarrow \neg Q_i)$$

This formula is denoted $PR_a^{\mathbf{P},\mathbf{Q}}\varphi$.

Example 7
Consider the agent of example 6 that wants to know on which platform her train leaves. a *believes her train is a high speed train modelled by* HS *and there are three platforms. Her train leaves on platform 1 (resp 2 and 3) is modelled by* p_1 *(resp.* p_2 *and* p_3*).* $\mathbf{Q}$ *is the set* $\{p_1, p_2, p_3\}$*.*

Let us consider the piece of information $HS \to \neg p_2$ *modelling the fact that if the train is a high-speed train, then it does not leave on platform 2. We suppose that this piece of information is true in our context. Let* $\mathbf{P}$ *be the set* $\{p_2\}$*.*

In this context, we have :

- $I_a(B_a p_1 \otimes B_a p_2 \otimes B_a p_3)$
- $B_a(HS)$
- $HS \to \neg p_2$

Then, we can deduce that $PR_a^{\mathbf{P},\mathbf{Q}}(HS \to \neg p_2)$*, i.e the piece of information* $HS \to \neg p_2$ *is* $\{p_2\}$*-partially relevant for* a *concerning the set* $\{p_1, p_2, p_3\}$*.*

Even if $HS \to \neg p_2$ *is not sufficient to deduce one of the platform, it allows the agent to filter out one of the possibilities.*

We consider some agent a, a set of objectives and exclusive formulas $\mathbf{Q} = \{Q_i\}_{i=1\ldots n}$ such that $n \geq 2$ and some formula φ.

The case when there is a formula φ such that $PR_a^{\mathbf{Q},\mathbf{Q}}\varphi$ can happen. Let us illustrate that case on two small examples :

- Let a be some agent who wants to know if her train leaves on platform 1, 2 or 3. The piece of information p_4 representing the fact that the train leaves on platform 4, if it is true, is such that $PR_a^{\mathbf{Q},\mathbf{Q}}p_4$.
- Let b be another agent who wants to know if her train leaves on platform 1, 2 or 3. Agent b does not know anything about her train but she knows more about the station. In particular, she believes that:
  - high speed trains do not leave neither on platform 1 or platform 2, $B_b(HS \to \neg p_1 \wedge \neg p_3)$
  - trains going to Amsterdam do not leave on platform 3, $B_b(Ams \to \neg p_3)$.

  Let us suppose that agent b is wrong about her second belief, i.e today (exceptionally) trains going to Amsterdam leave on platform 3.

  In that case, the true piece of information $HS \wedge Ams$ is such that $PR_a^{\mathbf{Q},\mathbf{Q}}(HS \wedge Ams)$.

Of course, there would be many ways to define partial relevance. Another possible partial relevance would be the one that characterizes the pieces of information that are necessary but not sufficient to answer the information need. This is more or less what is characterized by L-relevance.

7 CONCLUSION

The main contribution of this work is the definition and the formalization of a notion of agent-oriented relevance. Given an agent that has some information need, we have defined, in the BDI framework, a new operator that characterizes pieces of information that are relevant to her. Thus, a piece of information is relevant to some agent that has an information need if the agent has this information need, if the piece of information answers the information need and finally if the piece of information is true.

However, this relevance operator characterizes too many relevant pieces of information. Nevertheless, in that case, it is possible to define a hierarchy to determine the most relevant pieces of information. In this hierarchy, some compromise has to be found between "being precise" and "being concise". The minimal explanation is such a compromise.

With this compromise, we can compare our relevance with Lakemeyer's one that can be seen as a characterization of necessarily relevant formulae.

This work can be extended in many ways.

First, let us come back to cooperative communication.

In [7], Demolombe proposes that an agent b communicates cooperatively with respect to another agent a for a piece of information φ if and only if if b believes that φ is true then b informs a about it. The question is : for which φ should agent b be cooperative for with respect to a ? We can propose an answer to this question by using the notions of relevance introduced here and define "b is relevantly cooperative with respect to a if and only if there is a request Q such that if b believes that φ is maximally relevant for a concerning Q then she informs a about φ".

This means that agent b has to know if φ is relevant for agent a concerning a request Q. In our language, this is modelled the following way :

$$B_b R_a^Q \varphi \leftrightarrow B_b I_a Bif_a(Q) \wedge B_b \varphi$$

$$\wedge B_b(B_a(\varphi \rightarrow Q) \otimes B_a(\varphi \rightarrow \neg Q))$$

That means that

- agent b believes that agent a has this information need
- agent b believes that agent a can answer her information need with the relevant piece of information
- agent b believes that the piece of information is true.

We can note at this point that agent b can be wrong in many ways about the relevance of a piece of information: if she is wrong about the agent's a information need, or about agent's a beliefs or about the piece of information agent b believes to be true is wrong.

Thus, it is possible to propose a formal definition for cooperativity [19]: an agent is cooperative if and only if she informs the others only about what she thinks relevant for them.

Secondly, in this paper, we do not consider the transition to individual goals to information need. Nevertheless, if the individual goal can be formalized, it would be possible to define relevance directly for this goal.

Finally, it would also be interesting to consider other needs than information need. For example, some agent could have a verification need. In this case, any piece of information in accordance or in contradiction with the agent's believes (in a given domain) would be relevant. In the same way, we could consider a completion need: some agent needs to know any piece of information that concerns a subject.

PROOFS

Proof 1
We first prove that $I_a Bif_a Q \to \neg B_a Q$ is a theorem.

1. $\vdash I_a Bif_a Q \wedge B_a Q \to I_a Bif_a Q \wedge B_a B_a Q$ ((B4))
2. $\vdash I_a Bif_a Q \wedge B_a Q \to I_a Bif_a Q \wedge B_a(B_a Q \vee B_a \neg Q)$ (1. and (BK))
3. $\vdash I_a Bif_a Q \wedge B_a Q \to \neg B_a(B_a Q \vee B_a \neg Q) \wedge B_a(B_a Q \vee B_a \neg Q)$ (2. and (BI))
4. $\vdash I_a Bif_a Q \wedge B_a Q \to \bot$ (3. and (BD))
5. $\vdash I_a Bif_a Q \to B_a Q$ (4.)

In this proof, Q and $\neg Q$ have a symetrical role. Then, by replacing Q by $\neg Q$, we can prove that $I_a Bif_a Q \to \neg B_a \neg Q$. $\square$

Proof 2
(1) First, let us prove that $R_a^Q \varphi \to \neg B_a \varphi$ is a theorem of our logic.

1. $\vdash R_a^Q \varphi \wedge B_a \varphi \to I_a Bif_a Q \wedge (B_a(\varphi \to Q) \otimes B_a(\varphi \to \neg Q)) \wedge B_a \varphi$ (Def. 2)
2. $\vdash R_a^Q \varphi \wedge B_a \varphi \to \neg B_a Q \wedge \neg B_a \neg Q \wedge ((B_a(\varphi \to Q) \wedge \neg B_a(\varphi \to \neg Q)) \vee (\neg B_a(\varphi \to Q) \wedge B_a(\varphi \to \neg Q)) \wedge B_a \varphi$ (Prop. 1 and 1.)
3. $\vdash R_a^Q \varphi \wedge B_a \varphi \to \neg B_a Q \wedge \neg B_a \neg Q \wedge ((B_a \varphi \wedge B_a(\varphi \to Q)) \vee (B_a \varphi \wedge B_a(\varphi \to \neg Q))$ (2.)
4. $\vdash R_a^Q \varphi \wedge B_a \varphi \to \neg B_a Q \wedge \neg B_a \neg Q \wedge (B_a Q \vee B_a \neg Q)$ (3. and (BK))
5. $\vdash R_a^Q \varphi \wedge B_a \varphi \to \bot$ (4. and (BD))
6. $\vdash \neg(R_a^Q \varphi \wedge B_a \varphi)$ (5.)
7. $\vdash R_a^Q \varphi \to \neg B_a \varphi$ (6.)

(2) Then, let us prove that $R_a^Q \varphi \to \neg B_a \neg \varphi$ is a theorem of our logic

1. $\vdash R_a^Q \varphi \wedge B_a \neg \varphi \to R_a^Q \varphi \wedge B_a(\varphi \vee Q) \wedge B_a(\varphi \vee \neg Q)$ ((BK) and (Nec))
2. $\vdash R_a^Q \varphi \wedge B_a \neg \varphi \to (B_a(\varphi \to Q) \otimes B_a(\varphi \to \neg Q)) \wedge B_a(\varphi \to Q) \wedge B_a(\varphi \to \neg Q)$(1.)

3. $\vdash R_a^Q\varphi \wedge B_a\neg\varphi \to \bot$ (2.)
4. $\vdash R_a^Q\varphi \to \neg B_a\neg\varphi$ (3.) $\square$

Proof 3
Let us first present the first four postulates. Let Bel_a be the belief base of agent a. [15] The revision of Bel_a by φ is written $Bel_a * \varphi$ and $Bel_a + \varphi$ is the deductive closure of $Bel_a \cup \{\varphi\}$. AGM postulates are

(G*1) $Bel_a * \varphi$ is a belief set (a set which is deductively closed)

(G*2) $\varphi \in Bel_a * \varphi$

(G*3) $Bel_a * \varphi \subset Bel_a + \varphi$

(G*4) if $\neg\varphi \notin Bel_a$ then $Bel_a + \varphi \subset Bel_a * \varphi$

Let us suppose that $R_a^Q\varphi$ is true. Then we have $\neg B_a\varphi$. That means that $\varphi \notin Bel_a$. Thus, revising Bel_a by φ is equivalent to doing a expansion of Bel_a by φ. That means that $\varphi \in Bel_a * \varphi$.
From $R_a^Q\varphi$, we can deduce that either $B_a(\varphi \to Q)$ or $B_a(\varphi \to \neg Q)$. This two formulae are in the set $Bel_a * \varphi$ too. Then, we can deduce that either $Q \in Bel_a * \varphi$ or $\neg Q \in Bel_a * \varphi$. $\square$

Proof 4
(1). Let us show that $\vdash I_aBif_aQ \wedge \neg B_aQ \wedge \neg B_a\neg Q \to R_a^QQ \otimes R_a^Q\neg Q$. Let us notice that $R_a^QQ \otimes R_a^Q\neg Q$ is equivalent to $(R_a^QQ \vee R_a^Q\neg Q) \wedge \neg(R_a^QQ \wedge R_a^Q\neg Q)$. We first show that $I_aBif_aQ \to \neg(R_a^QQ \wedge R_a^Q\neg Q)$ is a theorem, then we show that $I_aBif_aQ \wedge \neg B_aQ \wedge \neg B_a\neg Q \to R_a^QQ \vee R_a^Q\neg Q$ is a theorem.

1. $\vdash I_aBif_aQ \wedge (R_a^QQ \wedge R_a^Q\neg Q) \to I_aBif_aQ \wedge Q \wedge \neg Q$ (Def. 2)
2. $\vdash I_aBif_aQ \wedge (R_a^QQ \wedge R_a^Q\neg Q) \to \bot$ (1.)
3. $\vdash I_aBif_aQ \to \neg(R_a^QQ \wedge R_a^Q\neg Q)$ (2.)
4. $\vdash I_aBif_aQ \wedge \neg B_aQ \wedge \neg B_a\neg Q \wedge \neg R_a^QQ \wedge \neg R_a^Q\neg Q \to I_aBif_aQ \wedge (\neg I_aBif_aQ \vee \neg Q \vee (B_a(Q \to Q) \wedge B_a(Q \to \neg Q)) \vee (\neg B_a(Q \to Q) \wedge \neg(B_a(Q \to \neg Q)))) \wedge (\neg I_aBif_a\neg Q \vee Q \vee (B_a(\neg Q \to Q) \wedge B_a(\neg Q \to \neg Q)) \vee (\neg B_a(\neg Q \to Q) \wedge \neg(B_a(\neg Q \to \neg Q))))$ (Def. 2)
5. $\vdash I_aBif_aQ \wedge \neg B_aQ \wedge \neg B_a\neg Q \wedge \neg R_a^QQ \wedge \neg R_a^Q\neg Q \to I_aBif_aQ \wedge \neg Q \wedge Q$ because $B_a(Q \to Q)$ is a theorem (Nec), $B_a(Q \to \neg Q)$ is false because of $\neg B_a\neg Q$
6. $I_aBif_aQ \wedge \neg B_aQ \wedge \neg B_a\neg Q \wedge \neg R_a^QQ \wedge \neg R_a^Q\neg Q \to \bot$ (4.)
7. $I_aBif_aQ \wedge \neg B_aQ \wedge \neg B_a\neg Q \to R_a^QQ \vee R_a^Q\neg Q$ (5.)

(2). Let us prove that $\vdash (Q_1 \leftrightarrow Q_2) \wedge B_a(Q_1 \leftrightarrow Q_2) \to (R_a^{Q_1}\varphi \leftrightarrow R_a^{Q_2}\varphi)$

1. $\vdash (Q_1 \leftrightarrow Q_2) \wedge B_a(Q_1 \leftrightarrow Q_2) \wedge R_a^{Q_1}\varphi \to (Q_1 \leftrightarrow Q_2) \wedge B_a(Q_1 \leftrightarrow Q_2) \wedge I_a(B_aQ_1 \vee B_a\neg Q_1) \wedge Q_1 \wedge (B_a(\varphi \to Q_1) \otimes B_a(\varphi \to \neg Q_1))$ (Def. 2 and (BK))
2. $\vdash (Q_1 \leftrightarrow Q_2) \wedge B_a(Q_1 \leftrightarrow Q_2) \wedge R_a^{Q_1}\varphi \to I_a(B_aQ_2 \vee B_a\neg Q_2) \wedge Q_2 \wedge (B_a(\varphi \to Q_2) \otimes B_a(\varphi \to \neg Q_2))$ (1.)
3. $\vdash (Q_1 \leftrightarrow Q_2) \wedge B_a(Q_1 \leftrightarrow Q_2) \wedge R_a^{Q_1}\varphi \to R_a^{Q_2}\varphi$ (2.)

[15] We suppose here that this belief base is deductively closed.

(3). Let us prove that $\vdash R_a^Q\varphi \leftrightarrow R_a^{\neg Q}\varphi$

1. $\vdash I_aBif_a\neg Q \leftrightarrow I_aBif_aQ$
2. $\vdash (B_a(\varphi \to Q) \otimes B_a(\varphi \to \neg Q)) \leftrightarrow (B_a(\varphi \to \neg Q) \otimes B_a(\varphi \to Q))$
3. Finally, $\vdash R_a^Q\varphi \leftrightarrow R_a^{\neg Q}\varphi$ (1. and 2.)

(4). Let us prove that $\vdash \neg(\varphi_1 \wedge \varphi_2) \to \neg(R_a^Q\varphi_1 \wedge R_a^Q\varphi_2)$. For that, we prove that $\vdash (R_a^Q\varphi_1 \wedge R_a^Q\varphi_2) \to \varphi_1 \wedge \varphi_2$ which comes directly from Def. 2. $\square$

Proof 5
We prove that $B_aR_a^Q\varphi \wedge R_a^Q\varphi$ is a contradiction.

1. $\vdash R_a^Q\varphi \to \neg B_a\varphi$ (Prop. 2)
2. $\vdash R_a^Q\varphi \to B_a\neg B_a\varphi$ (1. and (B5))
3. $\vdash B_aR_a^Q\varphi \to B_a\varphi$ (Def. 2 and (BK))
4. $\vdash B_aR_a^Q\varphi \to B_aB_a\varphi$ (3. and (B4))
5. $\vdash B_aR_a^Q\varphi \wedge R_a^Q\varphi \to B_a\neg B_a\varphi \wedge B_aB_a\varphi$ (2. and 4.)
6. $\vdash B_aR_a^Q\varphi \wedge R_a^Q\varphi \to \bot$ (5. and (BD)) $\square$

Proof 6
φ_1 and φ_2 clearly have a symmetric role. Thus, we just prove that $\vdash (\varphi_1 \leftrightarrow \varphi_2) \wedge B_a(\varphi_1 \leftrightarrow \varphi_2) \to (R_a^Q\varphi_1 \to R_a^Q\varphi_2)$.

1. $\vdash (B_a(\varphi_1 \to Q) \otimes B_a(\varphi_1 \to \neg Q)) \wedge B_a(\varphi_1 \leftrightarrow \varphi_2) \to B_a(\varphi_2 \to Q) \otimes B_a(\varphi_2 \to \neg Q)$ ((BK))
2. $\vdash R_a^Q\varphi_1 \wedge (\varphi_1 \leftrightarrow \varphi_2) \to \varphi_2 \wedge I_aBif_aQ$
3. $\vdash (\varphi_1 \leftrightarrow \varphi_2) \wedge B_a(\varphi_1 \leftrightarrow \varphi_2) \wedge R_a^Q\varphi_1 \to \varphi_2 \wedge I_aBif_aQ \wedge B_a(\varphi_2 \to Q) \otimes B_a(\varphi_2 \to \neg Q)$ (1. and 2.)
4. $\vdash (\varphi_1 \leftrightarrow \varphi_2) \wedge B_a(\varphi_1 \leftrightarrow \varphi_2) \wedge R_a^Q\varphi_1 \to R_a^Q\varphi_2$ (3. and Def. 2) $\square$

Proof 7

1. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge B_a(\varphi_1 \to Q) \wedge \neg B_a(\varphi_1 \to \neg Q) \to B_a(\varphi_1 \wedge \varphi_2 \to Q) \wedge \neg B_a(\varphi_1 \wedge \varphi_2 \to \neg Q))$
2. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge \neg B_a(\varphi_1 \to Q) \wedge B_a(\varphi_1 \to \neg Q) \to \neg B_a(\varphi_1 \wedge \varphi_2 \to Q) \wedge B_a(\varphi_1 \wedge \varphi_2 \to \neg Q))$ (1.)
3. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge (B_a(\varphi_1 \to Q) \otimes B_a(\varphi_1 \to \neg Q)) \to B_a(\varphi_1 \wedge \varphi_2 \to Q) \otimes B_a(\varphi_1 \wedge \varphi_2 \to \neg Q)$ (2.)
4. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge \varphi_2 \wedge R_a^Q\varphi_1 \to I_aBif_aQ \wedge (\varphi_1 \wedge \varphi_2) \wedge B_a(\varphi_1 \wedge \varphi_2 \to Q) \otimes B_a(\varphi_1 \wedge \varphi_2 \to \neg Q)$ (3.)
5. $\vdash B_a(\varphi_1, \varphi_2/Q) \to (\varphi_2 \wedge R_a^Q\varphi_1 \to R_a^Q(\varphi_1 \wedge \varphi_2))$ (4.)

Proof 8

1. $\vdash B_a(\varphi_1 \vee \varphi_2 \to \neg Q) \to B_a(\varphi_1 \wedge \varphi_2 \to \neg Q)$
2. $\vdash \neg B_a(\varphi_1 \wedge \varphi_2 \to \neg Q) \to \neg B_a(\varphi_1 \vee \varphi_2 \to \neg Q)$ (1.)
3. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge B_a(\varphi_1 \to Q) \wedge B_a(\varphi_2 \to Q) \wedge \neg B_a(\varphi_1 \to \neg Q) \wedge \neg B_a(\varphi_2 \to \neg Q) \to B_a(\varphi_1 \vee \varphi_2 \to Q) \wedge \neg B_a(\varphi_1 \vee \varphi_2 \to \neg Q)$ (2.)
4. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge B_a(\varphi_1 \to Q) \wedge B_a(\varphi_2 \to \neg Q) \to \bot$ (3.)

5. $\vdash B_a(\varphi_1, \varphi_2/Q) \wedge (B_a(\varphi_1 \to Q) \otimes B_a(\varphi_1 \to \neg Q)) \wedge (B_a(\varphi_2 \to Q) \otimes B_a(\varphi_2 \to \neg Q)) \to B_a(\varphi_1 \vee \varphi_2 \to Q) \otimes B_a(\varphi_1 \vee \varphi_2 \to \neg Q)$ (4.)
6. $\vdash B_a(\varphi_1, \varphi_2/Q) \to (R_a^Q \varphi_1 \wedge R_a^Q \varphi_2 \to R_a^Q(\varphi_1 \vee \varphi_2))$ (5.)

Proof 9
(1). Let us consider a clause φ_1 of $\mathcal{R}m^Q_{a \leq Cl}$. Let us suppose that $\varphi_1 \notin \mathcal{R}m^Q_a$. That means there exists a clause φ_2 such that $DepLit(\varphi_2) \subset DepLit(\varphi_1)$. φ_1 and φ_2 are objective clauses so $DepLit(\varphi_2) \subset DepLit(\varphi_1)$ means that there exists a disjunction of literals α such that $\varphi_1 \leftrightarrow \varphi_2 \vee \alpha$. We can deduce that $\varphi_2 <_{Cl} \varphi_1$. This means that φ_1 should not be in $\mathcal{R}m^Q_{a \leq Cl}$ and this is in contradiction with the first hypothesis. We can deduce that $\varphi_1 \in \mathcal{R}m^Q_a$.
(2). Let us consider a cube φ_1 of $\mathcal{R}m^Q_{a \leq Cu}$. Let us suppose that $\varphi_1 \notin \mathcal{R}m^Q_a$. That means there exists a cube φ_2 such that $DepLit(\varphi_2) \subset DepLit(\varphi_1)$. φ_1 and φ_2 are objective cubes so $DepLit(\varphi_2) \subset DepLit(\varphi_1)$ means that there exists a conjunction of literals α such that $\varphi_1 \leftrightarrow \varphi_2 \wedge \alpha$. We can deduce that $\varphi_2 <_{Cu} \varphi_1$. This means that φ_1 should not be in $\mathcal{R}m^Q_{a \leq Cu}$ and this is in contradiction with the first hypothesis. We can deduce that $\varphi_1 \in \mathcal{R}m^Q_a$. □

Proof 10
Let φ be a formula of $\mathcal{R}^Q_a$.

- If $\varphi \in \mathcal{R}m^Q_a$, then φ is a minimal explanation of Q or of $\neg Q$ that mentions at least one atom of φ.
- Otherwise, there exists a minimal explanation ψ. ψ is an explanation of Q or $\neg Q$ and $DepLit(\psi) \subset DepLit(\varphi)$. Let l be a literal of $DepLit(\psi)$. l is also a literal of $DepLit(\varphi)$. Thus, there exists a minimal explanation of Q or of $\neg Q$ that mentions at least one atom of φ. □

Proof 11
This is the first part of the proof of last proposition (Prop. 10).

Proof 12
Let k be an integer between 1 and n. Let us prove that $\vdash I_a \bigotimes_{i=1}^n B_a Q_i \to \neg B_a Q_k$. Q_1, ... Q_n are mutually exclusive, so $\bigotimes_{i=1}^n B_a Q_i$ is equivalent to $\bigvee_{i=1}^n B_a Q_i$ by (Nec).

1. $\vdash I_a \bigotimes_{i=1}^n B_a Q_i \wedge B_a Q_k \to B_a \neg(\bigvee_{i=1}^n B_a Q_i) \wedge B_a Q_k$ ((BI))
2. $\vdash I_a \bigotimes_{i=1}^n B_a Q_i \wedge B_a Q_k \to B_a \neg(\bigvee_{i=1}^n B_a Q_i) \wedge B_a B_a Q_k$ (1. and (B4))
3. $\vdash I_a \bigotimes_{i=1}^n B_a Q_i \wedge B_a Q_k \to B_a \neg B_a Q_k \wedge B_a B_a Q_k$ (2.)
4. $\vdash I_a \bigotimes_{i=1}^n B_a Q_i \wedge B_a Q_k \to \bot$ (3. and (BD))
5. $\vdash I_a \bigotimes_{i=1}^n B_a Q_i \to B_a Q_k$ (4.) □

Proof 13
(1). Let us prove that $\vdash R_a^{\mathbf{Q}} \varphi \to \neg B_a \neg \varphi$

1. $\vdash R_a^{\mathbf{Q}} \varphi \wedge B_a \neg \varphi \to R_a^{\mathbf{Q}} \varphi \wedge B_a(\neg \varphi \vee Q_1) \wedge B_a(\neg \varphi \vee Q_2)$ ((BK) and (Nec))
2. $\vdash R_a^{\mathbf{Q}} \varphi \wedge B_a \neg \varphi \to (B_a(\varphi \to Q_1) \otimes B_a(\varphi \to Q_2)) \wedge B_a(\varphi \to Q_1) \wedge B_a(\varphi \to Q_2)$ (1.)
3. $\vdash R_a^{\mathbf{Q}} \varphi \wedge B_a \neg \varphi \to \bot$ (2.)
4. $\vdash \neg(R_a^{\mathbf{Q}} \varphi \wedge B_a \neg \varphi)$ (3.)
5. $\vdash R_a^{\mathbf{Q}} \varphi \to \neg B_a \neg \varphi$ (4.)

(2). Let us prove that $R_a^{\mathbf{Q}}\varphi \rightarrow \neg B_a\varphi$ is a theorem of our logic.

1. $\vdash R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi \rightarrow I_a \bigotimes_{i=1}^{n} B_aQ_i \wedge \bigotimes_{i=1}^{n} B_a(\varphi \rightarrow Q_i) \wedge B_a\varphi$ (Def. 13) 2
2. $\vdash R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi \rightarrow \bigwedge_{i=1}^{n} \neg B_aQ_i \wedge \bigvee_{i=1}^{n}(B_a(\varphi \rightarrow Q_i) \wedge \bigwedge_{k\neq i} \neg B_a(\varphi \rightarrow Q_k))) \wedge B_a\varphi$ (Prop. 12 and 1.)
3. $\vdash R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi \rightarrow \bigwedge_{i=1}^{n} \neg B_aQ_i \wedge \bigvee_{i=1}^{n}(B_a(\varphi \rightarrow Q_i) \wedge B_a\varphi \wedge \bigwedge_{k\neq i} \neg B_a(\varphi \rightarrow Q_k)))$ (2.)
4. $\vdash R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi \rightarrow \bigwedge_{i=1}^{n} \neg B_aQ_i \wedge \bigvee_{i=1}^{n}(B_aQ_i \wedge \bigwedge_{k\neq i} \neg B_a(\varphi \rightarrow Q_k)))$ (3. and (BK))
5. $\vdash R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi \rightarrow \bigvee_{i=1}^{n}(B_aQ_i \wedge \neg B_aQ_i \wedge \bigwedge_{k\neq i} \neg B_a(\varphi \rightarrow Q_k)))$
6. $\vdash R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi \rightarrow \perp$ (4. and (BD))
7. $\vdash \neg(R_a^{\mathbf{Q}}\varphi \wedge B_a\varphi)$ (5.)
8. $\vdash R_a^{\mathbf{Q}}\varphi \rightarrow \neg B_a\varphi$ (6.) $\square$

Proof 14
Let us suppose that $R_a^{Q}\varphi$ is true. Then we have $\neg B_a\varphi$. That means that $\varphi \notin Bel_a$. Thus, revising Bel_a by φ is equivalent to doing a expansion of Bel_a by φ. That means that $\varphi \in Bel_a * \varphi$.
From $R_a^{Q}\varphi$, we can deduce that $\bigotimes_{i=1}^{n} B_a(\varphi \rightarrow Q_i)$. That means that there exists one and only $k \in \{1, ..., n\}$ such that $B_a(\varphi \rightarrow Q_k)$. For any $i \neq k$, we have $\neg B_a(\varphi \rightarrow Q_i)$. These formulae are in the set $Bel_a * \varphi$ too. Then, we can deduce that $Q_k \in Bel_a * \varphi$. $\square$

Proof 15

1. For $i \in \{1, ..., n\}, \vdash B_a(Q_i \rightarrow Q_i)$ ((Nec))
2. For $i, j \in \{1, ..., n\}, \vdash B_a(Q_i \rightarrow Q_i) \wedge B_a(Q_i \rightarrow Q_j) \rightarrow B_a\neg Q_i$ because Q_i and Q_j are mutually exclusive.
3. For $i, j \in \{1, ..., n\}, \vdash \neg B_a(\neg Q_i) \rightarrow \neg B_a(Q_i \rightarrow Q_j)$ (2.)
4. For $i \in \{1, ..., n\}, \vdash \neg B_a(\neg Q_i) \rightarrow \bigwedge_{j=1, j\neq i}^{n} \neg B_a(Q_i \rightarrow Q_j)$ (3.)
5. For $i \in \{1, ..., n\}, \vdash \neg B_a(\neg Q_i) \rightarrow \bigotimes_{j=1}^{n} B_a(Q_i \rightarrow Q_j)$ (4.)
6. $\vdash \bigwedge_{i=1}^{n} \neg B_a(\neg Q_i) \rightarrow \bigwedge_{i=1}^{n} \bigotimes_{j=1}^{n} B_a(Q_i \rightarrow Q_j)$ (5.)
7. $\vdash \bigwedge_{i=1}^{n} \neg R_a^{\mathbf{Q}}Q_i \rightarrow \neg I_a \bigotimes_{i=1}^{n} B_aQ_i \vee \bigwedge_{i=1}^{n} \neg Q_i \wedge \bigwedge_{i=1}^{n} \neg \bigotimes_{i=j}^{n} B_a(Q_i \rightarrow Q_j)$ (6.)
8. $\vdash \bigwedge_{i=1}^{n} \neg R_a^{\mathbf{Q}}Q_i \wedge \bigotimes_{i=1}^{n} Q_i \wedge I_a \bigotimes_{i=1}^{n} B_aQ_i \wedge \bigwedge_{i=1}^{n} \neg B_a(\neg Q_i) \rightarrow \perp$ (7.)
9. Let i and j be in $\{1, ..., n\}$, $\vdash \neg(R_a^{\mathbf{Q}}Q_i \wedge R_a^{\mathbf{Q}}Q_i)$ because Q_i and Q_j are mutually exclusive.
10. Finally, $\vdash \bigotimes_{i=1}^{n} Q_i \wedge I_a \bigotimes_{i=1}^{n} B_aQ_i \wedge \bigwedge_{i=1}^{n} \neg B_a(\neg Q_i) \rightarrow \bigotimes_{i=1}^{n} R_a^{\mathbf{Q}}Q_i$ $\square$ (8. and 9.)

Proof 16
This is exactly the same proof as Proof 5

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