A Translation-based Approach for Revision of Argumentation Frameworks

Sylvie Coste-Marquis, Sébastien Konieczny, Jean-Guy Mailly, and Pierre Marquis

CRIL
Université d'Artois – CNRS
Lens, France
{coste,konieczny,mailly,marquis}@cril.fr

Abstract. In this paper, we investigate the revision issue for Dung argumentation frameworks. The main idea is that such frameworks can be translated into propositional formulae, allowing the use of propositional revision operators to perform a rational minimal change. Our translation-based approach to revising argumentation frameworks can take advantage of any propositional revision operator $\circ$. Via a translation, each propositional operator $\circ$ can be associated with some revision operators $\star$ suited to argumentation frameworks. Some rationality postulates for the $\star$ operators are presented. If the revision formulae are restricted to formulae about acceptance statuses, some $\star$ operators satisfy these postulates provided that the corresponding $\circ$ operator is AGM.

1 Introduction

In this paper, we investigate the revision issue for abstract argumentation frameworks à la Dung [17]. Such argumentation frameworks are directed graphs, where nodes correspond to arguments and arcs to attacks between arguments. In such frameworks, the status (acceptance) of each argument depends on the chosen acceptability semantics (grounded, preferred, stable – among others).

Change in argumentation frameworks is a very active topic in the argumentation community [9,8,11,3,6,2,10,7,13,15] In [16], a classification of the change operators is given. A change operator can be characterized by the nature of the constraint to enforce and the nature of the change to perform to reach the goal. In this work, we focus on two types of constraint and change: those concerning the *structure* of the argumentation graph, and those concerning the *acceptance statuses* of arguments.

We present a translation-based approach for revising argumentation systems. The aim is to characterize a set $F \star \varphi$ of argumentation systems which corresponds to the revision of the argumentation system F by the revision formula φ. Basically, given a semantics σ, we associate with F a propositional formula $f_\sigma(F)$ which represents it; given the revision formula φ, we take advantage of

AGM revision operators $\circ$ in order to characterize the revision $F \star \varphi$ of F by φ. In a nutshell, the approach consists in revising using $\circ$ the representation of $f_\sigma(F)$ by a propositional formula induced by φ plus some additional constraints on the expected revision. The output is a propositional formula which characterizes the argumentation frameworks which can be interpreted as the revision of F by φ. This paper only presents propositional encodings for Dung's complete and stable semantics, but our revision method can be used with any other acceptability semantics σ, as soon as there is a propositional encoding for arguments acceptance given σ.

Some rationality postulates for the $\star$ operators are presented. We show that if the revision formulae are restricted to formulae about acceptance statuses, some $\star$ operators satisfy these postulates provided that the corresponding $\circ$ operator is AGM.

2 Background

Let us first define formally argumentation frameworks. We only consider the case of finite frameworks. The following notions come from [17].

Definition 1 *An* argumentation framework *(AF) is a pair* $F = \langle A, R \rangle$ *with* A *a finite set of abstract entities called* arguments *and* R *a binary relation on* A *called the* attack relation*.*

The intuitive meaning of an attack $(a, b) \in R$ is that a *defeats* b, so if a is accepted then b has to be rejected. An argument can be defended by another one against a third one: if $(a, b) \in R$ and $(b, c) \in R$, then a defends c against b. These two notions can be extended to sets: $S \subseteq A$ attacks (resp. defends) $a \in A$ if $\exists b \in S$ such that b attacks (resp. defends) a.
To compute the acceptance status of each argument, Dung defines several *acceptability semantics* which leads to sets of arguments (called *extensions*) which can be accepted together. A common point to these semantics is *conflict-freeness*: a set $S \subseteq A$ is *conflict-free* if and only if there is no $a, b \in S$ such that $(a, b) \in R$.

For instance, complete and stable semantics are defined as:

- A conflict-free set $S \subseteq A$ is a *complete extension* of F if and only if S contains every argument that S defends;
- A conflict-free set $S \subseteq A$ is a *stable extension* of F if and only if S attacks every argument that does not belong to S.

Given a semantics σ and a framework F, $Ext_\sigma(F)$ denotes the set of extensions of F. An argument a is skeptically accepted by F with respect to the semantics σ if and only if $\forall \varepsilon \in Ext_\sigma(F)$, $a \in \varepsilon$.

Let us also give a few preliminaries about belief revision. Intuitively, belief revision can be defined as the minimal change to enforce a new information in a logical belief base. It has been characterized in many settings, including the setting of deductively closed theories [1], and the setting of finite propositional belief bases [21]. These works give families of rationality postulates, which are logical properties that a rational revision operator is supposed to satisfy. Katsuno and Mendelzon [21] have proved that propositional revision operators can be characterized through the notion of faithful assignment:

Definition 2 *A faithful assignment is a mapping which associates a propositional formula φ with a total pre-order $\leq_\varphi$ on interpretations such that:*

- *if $\omega \models \varphi$ and $\omega' \models \varphi$, then $\omega \approx_\varphi \omega'$;*
- *if $\omega \models \varphi$ and $\omega' \not\models \varphi$, then $\omega <_\varphi \omega'$;*
- *if $\varphi \equiv \psi$, then $\leq_\varphi = \leq_\psi$.*

Faithful assignments characterize well-behaved revision operators:[1]

Theorem 1. *A KM revision operator $\circ$ satisfies the rationality postulates from [21] if and only if there exists a faithful assignment which associates with every formula φ a total pre-order $\leq_\varphi$, and such that for every formula α:*[2]

$$Mod(\varphi \circ \alpha) = \min(Mod(\alpha), \leq_\varphi)$$

3 A Translation-based Approach

In this section, we explain how to encode an argumentation framework into logical constraints, and which constraints must be added to take into account the main semantics of acceptability. Then we show that classical AGM revision operators can be used to revise an argumentation framework. This idea is reminiscent to the ones considered in [18,12] for other purposes (revising modal or non-classical formulae, and case-based reasoning).

3.1 A Propositional Encoding

Let us consider a finite set of arguments $A = \{a_1, \dots, a_n\}$ and an argumentation framework $F = \langle A, R \rangle$.

Definition 3 (Propositional language based on A)
- *for $x \in A$, $acc(x)$ is a propositional variable meaning "the argument x is skeptically accepted by the framework F".*
- *for $x, y \in A$, $att(x, y)$ is a propositional variable meaning "the argument x attacks the argument y in the framework F".*
- *for $x \in A$, x is a propositional variable meaning "the argument x belongs to the extension of the framework F which is taken in consideration".*
- *$Prop_A = \{acc(x) | x \in A\} \cup \{att(x, y) | x, y \in A\} \cup \{x | x \in A\}$*
- *$\mathcal{L}_A$ is the propositional language built up from the set of variables $Prop_A$ and the connectives $\neg, \vee, \wedge$.*

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[1] This result is a particular case of Grove's system of spheres [19].

[2] $Mod(\varphi)$ denotes the set of models of the propositional formula φ.
Given a set S and a pre-order $\leq$ on S, $\min(S, \leq) = \{x \in S : \nexists y \in S, y \leq x$ and $x \not\leq y\}$.

An *att*-formula (resp. an *acc*-formula) is a formula from $\mathcal{L}_A$ which contains only variables from $\{att(x,y)|x,y \in A\}$ (resp. $\{acc(x)|x \in A\}$).

Clearly enough, the set of models over $\{att(x,y)|x,y \in A\}$ of an *att*-formula φ_{att} (called *att*-models) corresponds in a bijective way to a set of argumentation frameworks over A: (x,y) belongs to the attack relation R precisely when $att(x,y)$ is true in the model under consideration. It can be formalized through the definition of a mapping from a set of *att* litterals to an argumentation framework:

Definition 4 (Argumentation framework associated with a att-model) *Given a set A of arguments, any $m \subseteq \{att(x,y)|x,y \in A\}$ can be associated with an argumentation framework $arg(m) = \langle A, \{(x,y) \in A \times A | att(x,y) \in m\}\rangle$. This notion can be extended to the set of argumentation frameworks corresponding to a set of att-models: $arg(M) = \{arg(m)|m \in M\}$.*

We also need the following notion of projection:

Definition 5 (att-projection of models and formulae) *Given a set A of arguments, any interpretation m over $\mathcal{L}_A$ can be projected on its att-part: $Proj_{att}(m) = m \cap \{att(x,y)|x,y \in A\}$. This notion can be extended to the projection of a formula $\varphi \in \mathcal{L}_A$: $Proj_{att}(\varphi) = \{Proj_{att}(m)|m \in Mod(\varphi)\}$.*

Then, a formula φ representing argumentation frameworks can be associated with these frameworks by combining these two mappings: $arg(Proj_{att}(\varphi))$.

The other way around, at a shallow level, any $F = \langle A, R\rangle$ can be represented by the formula over $\{att(x,y)|x,y \in A\}$

$$\bigwedge_{(x,y)\in R} att(x,y) \wedge \bigwedge_{(x,y)\notin R} \neg att(x,y)$$

but this translation does not take into account the semantics σ under which F must be interpreted. One clearly needs to consider σ in the encoding. We propose to do it as follows:

Definition 6 (σ-formula of F) *Given an argumentation framework $F = \langle A, R\rangle$ and a semantics σ, the σ-formula of F is*

$$f_\sigma(F) = \bigwedge_{(x,y)\in R} att(x,y) \wedge \bigwedge_{(x,y)\notin R} \neg att(x,y) \wedge th_\sigma(A)$$

where $th_\sigma(A)$ is a logical formula (the σ-theory of A) that encodes the semantics σ.

Now, the question is how to define $th_\sigma(A)$ for some usual semantics. To do so, we take advantage of the logical representation of σ-extensions as proposed in [4]. Let us begin with the stable semantics. It has been proved in [4] that the stable extensions of an argumentation framework $F = \langle\{a_1, \ldots, a_n\}, R\rangle$ are exactly the models of the propositional formula:

$$\bigwedge_{a_k \in A} (a_k \Leftrightarrow \bigwedge_{a_j : (a_j, a_k) \in R} \neg a_j)$$

It is interesting to note that an argument a_i is skeptically accepted by $F = \langle A, R\rangle$ if and only if every model of the previous formula contains a_i:

$$\models [\bigwedge_{a_k \in A} (a_k \Leftrightarrow \bigwedge_{a_j : (a_j, a_k) \in R} \neg a_j) \Rightarrow a_i]$$

or in a simpler way,

$$\forall a_1, \ldots, a_n, [\bigwedge_{a_k \in A} (a_k \Leftrightarrow \bigwedge_{a_j : (a_j, a_k) \in R} \neg a_j) \Rightarrow a_i] \text{ is valid.}$$

In this encoding, it is assumed that the argumentation framework is known. However, one can relax this assumption by taking advantage of the $att(x, y)$ variables:

$$acc(a_i) \Leftrightarrow \forall a_1, \ldots, a_n, [\bigwedge_{a_k \in A} (a_k \Leftrightarrow \bigwedge_{a_j \in A} (att(a_j, a_k) \Rightarrow \neg a_j)) \Rightarrow a_i]$$

This formula encodes a way to compute the skeptically accepted arguments of any argumentation framework built on A given the stable semantics (it proves enough to condition the formula by the literals $att(a_j, a_k)$ corresponding to the attack relation of the given argumentation framework to recover the encoding from [4]).

Altogether, we get:

$$th_{st}(A) = \bigwedge_{a_i \in A} (acc(a_i) \Leftrightarrow \forall a_1, \ldots, a_n, (\bigwedge_{a_k \in A} (a_k \Leftrightarrow \bigwedge_{a_j \in A} (att(a_j, a_k) \Rightarrow \neg a_j)) \Rightarrow a_i))$$

It is well-known that a quantified Boolean formula (QBF) can be transformed into a classical propositional formula through the elimination of quantifications. We keep the notation of our encoding in QBF to keep reasonable the formula size, but it does not prevent from using KM revision operators (Section 3.2).

Example 1 *Let us illustrate these notions on F_1, given on Fig.1.*

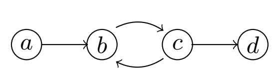


Fig. 1. The argumentation framework F_1

The stable theory of the set of arguments $A = \{a, b, c, d\}$ *is* $th_{st}(A) =$

$$
\begin{aligned}
&acc(a) \Leftrightarrow \forall a, b, c, d, [[(a \Leftrightarrow (att(a,a) \Rightarrow \neg a) \wedge (att(b,a) \Rightarrow \neg b))\\
&\qquad\qquad \wedge (att(c,a) \Rightarrow \neg c)) \wedge (att(d,a) \Rightarrow \neg d))\\
&\quad \wedge (b \Leftrightarrow (att(a,b) \Rightarrow \neg a) \wedge (att(b,b) \Rightarrow \neg b)\\
&\qquad\qquad \wedge (att(c,b) \Rightarrow \neg c) \wedge (att(d,b) \Rightarrow \neg d))\\
&\quad \wedge (c \Leftrightarrow (att(a,c) \Rightarrow \neg a) \wedge (att(b,c) \Rightarrow \neg b)\\
&\qquad\qquad \wedge (att(c,c) \Rightarrow \neg c) \wedge (att(d,c) \Rightarrow \neg d))\\
&\quad \wedge (d \Leftrightarrow (att(a,d) \Rightarrow \neg a) \wedge (att(b,d) \Rightarrow \neg b)\\
&\qquad\qquad \wedge (att(c,d) \Rightarrow \neg c) \wedge (att(d,d) \Rightarrow \neg d))] \Rightarrow a]\\
\wedge \quad &acc(b) \Leftrightarrow \forall a, b, c, d, [\dots]\\
\wedge \quad &acc(c) \Leftrightarrow \forall a, b, c, d, [\dots]\\
\wedge \quad &acc(d) \Leftrightarrow \forall a, b, c, d, [\dots]
\end{aligned}
$$

So the stable formula of F_1 *is given by*

$$th_{st}(A) \wedge \bigwedge_{(a,b) \in R} att(a,b) \wedge \bigwedge_{(a,b) \notin R} \neg att(a,b)$$

Propagating the values of att*-variables allows to deduce the values of* acc*-variables* ($acc(a) = acc(c) = true$*, and* $acc(b) = acc(d) = false$*), and so leads to the set of skeptically accepted arguments* $\{a, c\}$*.*

The complete-theory $th_{co}(A)$ of A can be defined in a similar way. First, let us recall the encoding of the complete extensions given in [4]:

$$\bigwedge_{a_k \in A} [(a_k \Rightarrow \bigwedge_{a_j : (a_j, a_k) \in R} \neg a_j) \wedge (a_k \Leftrightarrow \bigwedge_{a_j : (a_j, a_k) \in R} (\bigvee_{a_l : (a_l, a_j) \in R} a_l))]$$

Using a similar reasoning scheme, we get that:

$$
\begin{aligned}
th_{co}(A) = &\textstyle\bigwedge_{a_i \in A} [acc(a_i) \Leftrightarrow [\forall a_1, \dots, a_n,\\
&\textstyle\bigwedge_{a_k \in A} [(a_k \Rightarrow \bigwedge_{a_j \in A} (att(a_j, a_k) \Rightarrow \neg a_j))\\
&\textstyle\wedge (a_k \Leftrightarrow \bigwedge_{a_j \in A} (att(a_j, a_k) \Rightarrow \bigvee_{a_l \in A} (att(a_l, a_j) \Rightarrow a_l)))]] \Rightarrow a_i]
\end{aligned}
$$

3.2 Encoding Revision Operators with Logical Constraints

One can take advantage of the encodings presented in the previous section to define revision operators for argumentation frameworks, via the use of classical AGM operators. In particular, the KM revision operators $\circ$ defined for propositional logic [21] are suited to the language $\mathcal{L}_A$.

At a first glance, one can consider to revise $f_\sigma(F)$ by the revision formula φ. However, this is not sufficient. Indeed, if the revision formula φ does not correspond to any argumentation framework interpreted under the semantics σ (for instance, when $\varphi = acc(a) \wedge acc(b) \wedge att(a,b)$), then the revised formula will not correspond to any argumentation framework interpreted under σ. Indeed the success postulate $f_\sigma(F) \circ \varphi \models \varphi$ would force φ to be the case.

Such pathological scenarios must be avoided. A way to ensure it consists in revising $f_\sigma(F)$ by $\varphi \wedge th_\sigma(A)$ since the latter formula is logically consistent precisely when there exists at least one argumentation framework interpreted under σ which is compatible with φ.

Finally, the models of the revised formula $f_\sigma(F) \circ (\varphi \wedge th_\sigma(A))$, projected onto the $att(x, y)$ variables, characterize the revised argumentation frameworks.

Definition 7 (Translation-based revision) *Let $\circ$ be a KM revision operator. For any semantics σ, any argumentation framework $F = \langle A, R\rangle$ and any formula $\varphi \in \mathcal{L}_A$, the associated* translation-based revision operator $\star$ *is given by:*

$$F \star \varphi = arg(Proj_{att}(f_\sigma(F) \circ (\varphi \wedge th_\sigma(A))))$$

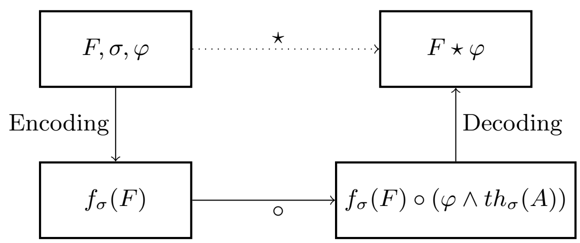


Fig. 2. Schematic explanation of the revision process

The decoding process is performed by the functions arg and $proj_{att}$ defined previously (Definition 4, Definition 5).

Let us instanciate this general definition of translation-based revision operators, using distances [3] between the interpretations over $\mathcal{L}_A$.

Definition 8 (Distance-based revision) *Let d be a distance between interpretations over $\mathcal{L}_A$. Given a formula $\psi \in \mathcal{L}_A$, the pre-order $\leq_\psi$ is defined by*

$$\omega \leq_\psi \omega' \text{ if and only if } d(\omega, Mod(\psi)) \leq d(\omega', Mod(\psi))$$

For any formulae $\psi, \alpha \in \mathcal{L}_A$, the distance-based KM revision operator $\circ_d$ is defined by

$$Mod(\psi \circ_d \alpha) = \min(Mod(\alpha), \leq_\psi)$$

Then, the distance-based AF revision operator $\star_d$ *is defined by*

$$F \star_d \varphi = arg(Proj_{att}(f_\sigma(F) \circ_d (\varphi \wedge th_\sigma(A))))$$

[3] We call a *distance* a function d such that (1) $d(x, y) = 0$ iff $x = y$; (2) $d(x, y) = d(y, x)$; (3) $d(x, z) \leq d(x, y) + d(y, z)$. In fact, we only need pseudo-distances: (3) is not required. Such a pseudo-distance d can be extended to a "distance" between an interpretation and a set of interpretations: $d(\omega, \Omega) = \min_{\omega' \in \Omega} d(\omega, \omega')$.

Depending on the revision operator $\circ$ used, the concept of minimal change in the argumentation framework can vary. A first option is to consider minimal change on the arguments statuses more important than minimal change on the attack relation.
To perform this kind of change, we can consider a weighted Dalal-like operator (see [14,21] for details about Dalal's revision operator) which ensures minimal change on the *acc* variables. This kind of revision operator is a particular distance-based revision operator:

Definition 9 (Arguments statuses minimal revision) *Let A be a set of arguments, let $N = |A|^2 + 1$. The acceptance-weighted distance d^{acc} between interpretations is defined by*[4]

$$\begin{aligned} d^{acc}(I_1, I_2) = N \times & \textstyle\sum_{a \in A}(I_1(acc(a)) \oplus I_2(acc(a))) \\ & + \textstyle\sum_{a,b \in A}(I_1(att(a,b)) \oplus I_2(att(a,b))) \end{aligned}$$

The arguments statuses minimal revision operator $\star_d^{acc}$ *is the distance-based revision operator based on the distance d^{acc}.*

The weight on $acc(x)$ variables is chosen in such a way that changing the value of every $att(x,y)$ variable is still cheaper than changing the value of a single $acc(x)$ variable.

Conversely, we can define a Dalal-like revision operator which requires minimal change on the attack relation. Here the weights are chosen to ensure that changing the value of every $acc(x)$ variable is cheaper than changing the value of a single $att(x,y)$ variable:

Definition 10 (Attacks minimal revision) *Let A be a set of arguments, let $N = |A|+1$. The attacks-weighted distance d^{att} between interpretations is defined by*

$$\begin{aligned} d^{att}(I_1, I_2) = & \textstyle\sum_{a \in A}(I_1(acc(a)) \oplus I_2(acc(a))) \\ & + N \times \textstyle\sum_{a,b \in A}(I_1(att(a,b)) \oplus I_2(att(a,b))) \end{aligned}$$

The attacks minimal revision operator $\star_d^{att}$ *is the distance-based revision operator based on the distance d^{att}.*

Interestingly, the addition of new arguments is allowed.

Definition 11 (Open world revision) *Given $F = \langle A, R \rangle$ an AF, B a non-empty set of arguments such that $A \cap B = \emptyset$, $\varphi \in \mathcal{L}_{A \cup B}$ a formula and $\circ$ a KM revision operator. The associated* open world revision operator $\star_B$ *is defined as:*

$$F \star_B \varphi = arg(Proj_{att}(f_\sigma(F) \circ (\varphi \wedge th_\sigma(A \cup B))))$$

[4] The *exclusive or* $\oplus$ is the binary operation on Boolean variables defined by $x \oplus y \equiv (x \vee y) \wedge (\neg x \vee \neg y)$.

Here, new arguments and new attacks between them or between new and old arguments can be added.

More generally, one can constrain the revision process: some integrity constraints can be required for a particular application (because a given attack is known to hold for sure or because a given argument has to be skeptically accepted, and so cannot change during the revision):

Definition 12 (Constrained revision) *Given $F = \langle A, R \rangle$ an AF, $\varphi, \mu \in \mathcal{L}_A$ formulae and $\circ$ a KM revision operator. The associated μ-*constrained revision operator *is*

$$F \star_\mu \varphi = arg(Proj_{att}(f_\sigma(F) \circ (\varphi \wedge th_\sigma(A) \wedge \mu)))$$

Here are some examples of integrity constraints μ which can prove useful:

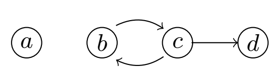
- $\bigwedge_{a \in A} \neg att(a, a)$ is useful when self-attacking arguments are not allowed [13];
- $\bigwedge_{(a,b) \in R} att(a, b) \wedge \bigwedge_{(a,b) \notin R} \neg att(a, b)$ is useful when attacks between former arguments must be preserved but attacks involving new arguments can be added [11].

Of course, the KM revision operator used to define $\star_B$ or $\star_\mu$ can take advantage of a weighted distance to ensure minimal change of arguments statuses or minimal change of the attack relation.

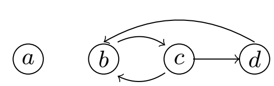
Depending on the situation, it can also be useful to consider a single argumentation framework as result of the revision process. This amounts to selecting one model of the projected formula. Several criteria can be used to do so; for space reasons, we will not detail them is this paper. Let us now illustrate two of the previously defined revision operators.

Example 2 *Let us revise the argumentation framework F_1, given on Fig.1, by the revision formula $\varphi = acc(a) \wedge \neg att(a, b)$, meaning that we want to change F_1 to have a skeptically accepted without a attacking b.*

F_1's single stable extension is $\{a, c\}$, so a is already skeptically accepted, but φ is not satisfied because a attacks b. All possible results of attack minimal revision and argument minimal revision are given respectively on Fig.3(a) and Fig.3(b). F_2's stable extensions are $\{\{a, c\}\{a, b, d\}\}$, so a is the only skeptically accepted



(a) F_2: Attack minimal revision of F_1



(b) F_3: Arguments statuses minimal revision of F_1

Fig. 3. Results of F_1 revisions

argument. With respect to acceptance statuses, the difference between F_1 and F_2 is 1, and there is also 1 attack different between them ((a, b) is removed).

The single stable extension of F_3 is $\{a, c\}$, so there is no difference between F_1 and F_3 with respect to acceptance statuses. The difference only concerns the attack relation ((a, b) is removed and (d, b) is added).

4 Rationality Postulates in the *acc* Case

In this section, we focus on constraints expressing an information about skeptically accepted arguments.
$Sc_\sigma(F)$ correspond to the *skeptical consequences* of the argumentation framework F with respect to the semantics σ. Formally, it is defined as $\{\bigcap_{\varepsilon \in Ext_\sigma(F)} \varepsilon\}$. We generalize this notion to $Sc_\sigma(S) = \bigcup_{F \in S} Sc_\sigma(F)$ where S is a set of argumentation frameworks. We call this set the skeptical consequences of S.

The satisfaction of *acc*-formulae can be defined with respect to a set of arguments. Let $\varepsilon \subseteq A$ and φ an *acc*-formula. The concept of *satisfaction* of φ by ε, noted $\varepsilon \mid\!\sim \varphi$, is defined inductively as follows:

- If $\varphi = acc(a)$ with $a \in A$, then $\varepsilon \mid\!\sim \varphi$ iff $a \in \varepsilon$,
- If $\varphi = (\varphi_1 \wedge \varphi_2)$, $\varepsilon \mid\!\sim \varphi$ iff $\varepsilon \mid\!\sim \varphi_1$ and $\varepsilon \mid\!\sim \varphi_2$,
- If $\varphi = (\varphi_1 \vee \varphi_2)$, $\varepsilon \mid\!\sim \varphi$ iff $\varepsilon \mid\!\sim \varphi_1$ or $\varepsilon \mid\!\sim \varphi_2$,
- If $\varphi = \neg\psi$, $\varepsilon \mid\!\sim \varphi$ iff $\varepsilon \not\mid\!\sim \psi$.

Then for any argumentation framework F, any set S of argumentation frameworks on A, and any semantics σ, we say that:

- φ is *skeptically accepted* w.r.t. F, noted $F \mid\!\sim_\sigma \varphi$, if $\forall \varepsilon \in Sc_\sigma(F)$, $\varepsilon \mid\!\sim \varphi$.
- φ is *rejected* w.r.t. F in the remaining case.
- φ is *skeptically accepted* w.r.t. S, noted $S \mid\!\sim_\sigma \varphi$, if $\forall \varepsilon \in Sc_\sigma(S)$, $\varepsilon \mid\!\sim \varphi$.
- φ is *rejected* w.r.t. S in the remaining case.

Each ε in the set $\mathcal{S}(\varphi) = \{\varepsilon \subseteq A | \varepsilon \mid\!\sim \varphi\}$ is a possible set of skeptically accepted arguments with respect to a framework which accepts the formula φ.
A formula φ is said to be *acc*-consistent if and only if $\mathcal{S}(\varphi) \neq \emptyset$.
Two formulae φ and ψ are said to be *acc*-equivalent, noted $\varphi \equiv_{acc} \psi$, if and only if $\mathcal{S}(\varphi) = \mathcal{S}(\psi)$.

Let us now point out an adaptation of KM's postulates:

(AS1) $Sc_\sigma(F \star \varphi) \subseteq \mathcal{S}(\varphi)$
(AS2) If $Sc_\sigma(F) \cap \mathcal{S}(\varphi) \neq \emptyset$, then $Sc_\sigma(F \star \varphi) = Sc_\sigma(F) \cap \mathcal{S}(\varphi)$
(AS3) If φ is *acc*-consistent, then $Sc_\sigma(F \star \varphi) \neq \emptyset$
(AS4) If $\varphi \equiv_{acc} \psi$, then $Sc_\sigma(F \star \varphi) = Sc_\sigma(F \star \psi)$
(AS5) $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi) \subseteq Sc_\sigma(F \star (\varphi \wedge \psi))$
(AS6) If $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi) \neq \emptyset$, then $Sc_\sigma(F \star (\varphi \wedge \psi)) \subseteq Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi)$

The first postulate is the *success* postulate: the result of the revision must satisfy the formula φ. **(AS2)** requires the skeptical consequences to stay the same ones if the input framework already satisfies φ. The third postulate states that revising a framework by a consistent formula cannot lead to an inconsistent result (such an inconsistent result is identified by an empty set of skeptical consequences). **(AS4)** states that revising by equivalent formulae leads to the same result. The last two postulates constrain the behavior of the revision operator when revising by a conjunction of formulae.

Similar postulates have been proposed in [13]. The main difference concerns the semantics of revision formulae. In [13], argumentation frameworks are revised by propositional formulae the satisfaction of which is defined with respect to the extensions. For instance, $a \vee b$ means "a or b must be in every extension" (and so, this formula is satisfied by a framework the extensions of which are $E = \{\{a\}, \{b\}\}$). Whereas here, formulae deal with the skeptical consequences of the framework, i.e. the intersection of the extensions. So the formula $acc(a) \vee acc(b)$ means "a must be in every extension or b must be in every extension", and is not satisfied by the set of extensions E.
More generally, the difference between our postulates and those expressed in [13] is the object of the constraint they give: in [13], the postulates give some constraint on the expected extensions of the output of the revision process, while the current postulates concern the set of skeptically accepted arguments.

The following proposition explains how to define a rational revision operator from any pseudo-distance between sets of arguments.

Proposition 1 *Given a pseudo-distance d between sets of arguments and an argumentation framework F, $\leq_F^d$ denotes the total pre-order between sets of arguments defined by: $\varepsilon_1 \leq_F^d \varepsilon_2$ iff $d(\varepsilon_1, Sc_\sigma(F)) \leq d(\varepsilon_2, Sc_\sigma(F))$.*
The pseudo-distance based revision operator $\star_d$ *which satisfies*

$$Sc_\sigma(F \star_d \varphi) = \min(\mathcal{S}(\varphi), \leq_F^d)$$

satisfies the postulates ***(AS1)*** *-* ***(AS6)****.*

Proof. **(AS1)** is satisfied from the definition of the operator.

If $Sc_\sigma(F) \cap \mathcal{S}(\varphi) \neq \emptyset$, then obviously $\forall \varepsilon \in Sc_\sigma(F) \cap \mathcal{S}(\varphi)$, $\varepsilon \in Sc_\sigma(F)$, and $d(\varepsilon, Sc_\sigma(F)) = 0$. Any ε' which is not in $Sc_\sigma(F) \cap \mathcal{S}(\varphi)$ either does not satisfy φ (and so does not belong to $\mathcal{S}(\varphi)$), or does not belong to $Sc_\sigma(F)$ (and so $d(\varepsilon', Sc_\sigma(F)) > 0$). So $\min(\mathcal{S}(\varphi), \leq_F^d) = Sc_\sigma(F) \cap \mathcal{S}(\varphi)$, which leads to **(AS2)**.

If φ is acc-consistent, $\mathcal{S}(\varphi) \neq \emptyset$, so $\min(\mathcal{S}(\varphi), \leq_F^d) \neq \emptyset$. So **(AS3)** holds.

$\varphi \equiv_{acc} \varphi$ can be rewritten $\mathcal{S}(\varphi) = \mathcal{S}(\psi)$, which leads to $\min(\mathcal{S}(\varphi), \leq_F^d) = \min(\mathcal{S}(\psi), \leq_F^d)$. It is enough to prove **(AS4)**.

If $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi) = \emptyset$, **(AS5)-(AS6)** are satisfied. We suppose now that $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi) \neq \emptyset$.

We first prove the inclusion $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi) \subseteq Sc_\sigma(F \star \varphi \wedge \psi)$. By *reductio ad absurdum*, suppose that $\exists \varepsilon \in Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\varphi \wedge \psi)$ such that $\varepsilon \notin Sc_\sigma(F \star \varphi \wedge \psi)$, also written as $\varepsilon \in \min(\mathcal{S}(\varphi), \leq_F^d) \cap \mathcal{S}(\psi)$ and $\varepsilon \notin \min(\mathcal{S}(\varphi \wedge \psi), \leq_F^d)$. From the first part, we deduce $\varepsilon \in \mathcal{S}(\varphi \wedge \psi)$. However, ε is not a minimal element in this set with respect to $\leq_F^d$. Consequently, $\exists \varepsilon' \in \mathcal{S}(\varphi \wedge \psi)$ such that $\varepsilon' <_F^d \varepsilon$. From the definition of $\mathcal{S}(\varphi \wedge \psi)$, $\varepsilon' \in \mathcal{S}(\varphi)$ holds. This contradicts $\varepsilon \in \min(\mathcal{S}(\varphi), \leq_F^d)$. So $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\varphi \wedge \psi) \subseteq Sc_\sigma(F \star \varphi \wedge \psi)$, **(AS5)** holds.

If $Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi) \neq \emptyset$, let us suppose $\exists \varepsilon \in Sc_\sigma(F \star \varphi \wedge \psi)$ such that $\varepsilon \notin Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi)$. $\varepsilon \in \min(\mathcal{S}(\varphi \wedge \psi), \leq^d_F) \Rightarrow \varepsilon \in \mathcal{S}(\varphi \wedge \psi) \Rightarrow \varepsilon \in \mathcal{S}(\psi)$ holds. From this and $\varepsilon \notin Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi)$, we deduce $\varepsilon \notin Sc_\sigma(F \star \varphi)$. Since we suppose that the intersection is non-empty, $\exists \varepsilon' \in Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi)$. In particular, ε' satisfies φ and ψ, i.e. $\varepsilon' \in \mathcal{S}(\varphi) \cap \mathcal{S}(\psi) = \mathcal{S}(\varphi \wedge \psi)$. From $\varepsilon \in Sc_\sigma(F \star \varphi \wedge \psi) = \min(\mathcal{S}(\varphi \wedge \psi), \leq^d_F)$ and $\leq^d_F$ is a total relation, $\varepsilon \leq^d_F \varepsilon'$. As $\varepsilon' \in Sc_\sigma(F \star \varphi) = \min(\mathcal{S}(\varphi), \leq^d_F)$, $\varepsilon \in \min(\mathcal{S}(\varphi), \leq^d_F)$. It is a contradiction. So $Sc_\sigma(F \star \varphi \wedge \psi) \subseteq Sc_\sigma(F \star \varphi) \cap \mathcal{S}(\psi)$ holds.

The previous proposition gives a sufficient condition to prove that a pseudo-distance based revision operator satisfies the rationality postulates. From this proposition, we prove that the arguments statuses minimal revision operator (restricted to the *acc*-case) satisfies the postulates, through a reduction of this operator to a pseudo-distance based revision operator as described in Prop. 1.

Proposition 2 *The arguments statuses minimal revision operator satisfies the postulates* ***(AS1)-(AS6)****.*

Proof. Let us show that the arguments statuses minimal revision operator is a pseudo-distance based revision operator. We define $Proj_{acc}$ as the counterpart of $Proj_{att}$ to project the formulae on their *acc*-part.
$F \star^{acc}_D \varphi = arg(Proj_{att}(f_\sigma(F) \circ^{acc}_D (\varphi \wedge th_\sigma(A))))$ leads to

$$\begin{aligned} Sc_\sigma(F \star^{acc}_D \varphi) &= Proj_{acc}(f_\sigma(F) \circ^{acc}_D (\varphi \wedge th_\sigma(A))) \\ &= Proj_{acc}(\min(Mod(\varphi \wedge th_\sigma(A)), \leq_F^{d_H^{acc}})) \end{aligned}$$

Let us prove that projecting the minimal models of $\varphi \wedge th_\sigma(A)$ leads to the minimal sets of skeptically accepted arguments. The models of $\varphi \wedge th_\sigma(A)$ are the propositional representations of argumentation frameworks which satisfy φ, so it is obvious that the projection of the models on the *acc* variables allows to obtain a subset of $\mathcal{S}(\varphi)$. Let us show that these sets of arguments are minimal with respect to $\leq^d_F$:
Given $m \in \min(Mod(\varphi \wedge th_\sigma(A)), \leq_F^{d_H^{acc}})$, we have $d_H^{acc}(m, Mod(f_\sigma(F))$ is minimal. $f_\sigma(F)$ has a single model m_F, so $d_H^{acc}(m, m_F)$ is minimal. In other words,

$$(|A|^2+1)\sum_{a \in A}(m(acc(a)) \oplus m_F(acc(a))) + \sum_{a,b \in A}(m(att(a,b)) \oplus m_F(att(a,b)))$$

is minimal. Let us suppose that the *acc* part of the distance is not minimal, i.e. there exists m' such that

$$(|A|^2+1)\sum_{a \in A}(m'(acc(a)) \oplus m_F(acc(a))) < (|A|^2+1)\sum_{a \in A}(m(acc(a)) \oplus m_F(acc(a)))$$

In the extreme case when $\sum_{a,b \in A}(m(att(a,b)) \oplus m_F(att(a,b))) = 0$ and $\sum_{a,b \in A}(m'(att(a,b)) \oplus m_F(att(a,b))) = |A|^2$,

$$\begin{aligned} &(|A|^2+1)\textstyle\sum_{a \in A}(m'(acc(a)) \oplus m_F(acc(a))) \\ &\qquad + \textstyle\sum_{a,b \in A}(m'(att(a,b)) \oplus m_F(att(a,b))) \\ &< (|A|^2+1)\textstyle\sum_{a \in A}(m(acc(a)) \oplus m_F(acc(a))) \\ &\qquad + \textstyle\sum_{a,b \in A}(m(att(a,b)) \oplus m_F(att(a,b))) \end{aligned}$$

is ensured by the weight $|A|^2 + 1$ on the acc part. By *reductio ad absurdum*, we proved that the acc part of $d_H^{acc}(m, m_F)$ is minimal, i.e., $d_H(Proj_{acc}(m), Sc_\sigma(F))$ is minimal, with d_H the Hamming distance [20]. It implies

$$\begin{aligned} Sc_\sigma(F \star_D^{acc} \varphi) &= Proj_{acc}(\min(Mod(\varphi \wedge th_\sigma(A)), \leq_F^{d_F^{acc}})) \\ &= \min(\mathcal{S}(\varphi), \leq_F^{d_H}) \end{aligned}$$

From Prop. 1, $\star_D^{acc}$ satisfies the postulates **(AS1)-(AS6)**.

5 Conclusion

In this paper, we studied a way to benefit from the well-known logical revision operators from Katsuno and Mendelzon's work $\circ$ to define revision operators $\star$ for abstract argumentation frameworks.
This approach is particularly interesting due to the ability of our revision operators to enforce both *structural* and *acceptability* constraints. Depending on the underlying operator $\circ$, the operator $\star$ ensures minimal change either on the acceptance statuses, or on the attack relation. Moreover, these operators can encode some change operators defined in some recent related works [11].
We have also stated some rationality postulates inspired by the classical AGM framework, and proved that under the constraint that revision formulae only deal with acceptability, a revision operator $\star$ based on an AGM operator $\circ$ satisfies our postulates.

As a future work, several possibilities are opened. First, this paper only presents the logical characterization of skeptical acceptance under the stable and complete semantics. It would be interesting to define a similar characterization of skeptical acceptance under other semantics, this can be done thanks to the encoding method defined in [4,22,23,5]. Another interesting result would be to define the credulous σ-theory for these semantics σ. We are also interested in enforcing the result of the revision to belong to a particular subclass of argumentation frameworks, as the acyclic argumentation frameworks.
Another point for further studies is the axiomatic characterization of revision operators. We proved that arguments statuses minimal revision satisfies some rationality postulates in the case of acceptability revision constraints, but it would be interesting to know if some other kinds of operators satisfy these postulates, and to know if some other kinds of revision constraints can be characterized.
At last, we plan to encode our revision operators into a SAT-based software. The propositional setting of our operators is particularly well-suited to SAT solvers, so this approach is very promising from a computational point of view.

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References

1. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change : Partial meet contraction and revision functions. Journal of Symbolic Logic 50, 510–530 (1985)
2. Baumann, R.: What does it take to enforce an argument? minimal change in abstract argumentation. In: Proceedings of the European Conference on Artificial Intelligence (ECAI'12). pp. 127–132 (2012)
3. Baumann, R., Brewka, G.: Expanding argumentation frameworks: Enforcing and monotonicity results. In: Proceedings of the Third International Conference on Computational Models of Argument (COMMA'10). pp. 75–186 (2010)
4. Besnard, P., Doutre, S.: Checking the acceptability of a set of arguments. In: Proceedings of the 10th International Workshop on Non-Monotonic Reasoning (NMR'04). pp. 59–64 (2004)
5. Besnard, P., Doutre, S., Herzig, A.: Encoding Argument Graphs in Logic. In: International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU), Montpellier, France. Springer (2014), to appear
6. Bisquert, P., Cayrol, C., de Saint-Cyr, F.D., Lagasquie-Schiex, M.C.: Change in argumentation systems: Exploring the interest of removing an argument. In: Proceedings of the International Conference on Scalable Uncertainty Management (SUM'11). Lecture Notes in Computer Science, vol. 6929, pp. 275–288. Springer (2011)
7. Bisquert, P., Cayrol, C., de Saint-Cyr, F.D., Lagasquie-Schiex, M.C.: Enforcement in argumentation is a kind of update. In: Seventh International Conference on Scalable Uncertainty Management (SUM 2013). pp. 30–43 (2013)
8. Boella, G., Kaci, S., van der Torre, L.: Dynamics in argumentation with single extensions: Abstraction principles and the grounded extension. In: Proceedings of the European Conferences on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'09). Lecture Notes in Computer Science, vol. 5590, pp. 107–118. Springer (2009)
9. Boella, G., Kaci, S., van der Torre, L.: Dynamics in argumentation with single extensions: attack refinement and the grounded extension. In: Proceedings of the International Conference on Autonomous Agents and Multiagents Systems (AAMAS'09). pp. 1213–1214 (2009)
10. Booth, R., Kaci, S., Rienstra, T., van der Torre, L.: A logical theory about dynamics in abstract argumentation. In: In Proceedings of the 7th International Conference on Scalable Uncertainty Management (SUM'13). pp. 148–161 (2013)
11. Cayrol, C., de Saint-Cyr, F.D., Lagasquie-Schiex, M.C.: Change in abstract argumentation frameworks: Adding an argument. Journal of Artificial Intelligence Research 38, 49–84 (2010)
12. Cojan, J., Lieber, J.: Belief revision-based case-based reasoning. In: Richard, G. (ed.) ECAI-2012 Workshop Similarity and Analogy-based Methods in AI. pp. 33–39. Montpellier, France (2012)
13. Coste-Marquis, S., Konieczny, S., Mailly, J.G., Marquis, P.: On the revision of argumentation systems: Minimal change of arguments statuses. In: 14th International Conference on Principles of Knowledge Representation and Reasoning (KR'2014). Vienna (july 2014), to appear
14. Dalal, M.: Investigations into a theory of knowledge base revision: Preliminary report. In: Proceedings of the Seventh National Conference on Artificial Intelligence (AAAI'88). pp. 475–479 (1988)

15. Doutre, S., Herzig, A., Perrussel, L.: A dynamic logic framework for abstract argumentation. In: Proceedings of the 14th International Conference on Principles of Knowledge Representation and Reasoning (KR 2014). pp. 62–71 (2014)
16. Doutre, S., Perrussel, L.: On Enforcing a Constraint in Argumentation (11th European Workshop on Multi-Agent Systems (EUMAS 2013), Toulouse) (2013)
17. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming, and n-person games. Artificial Intelligence 77(2), 321–357 (1995)
18. Gabbay, D., Rodrigues, O., Russo, A.: Revision by translation. In: Proceedings of the Seventh International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU'98). vol. Information, Uncertainty and Fusion, pp. 3–32 (1998)
19. Grove, A.: Two modellings for theory change. Journal of Philosophical Logic 17, 157–170 (1988)
20. Hamming, R.W.: Error detecting and error correcting codes. Bell System Technical Journal 29(2), 147–160 (1950)
21. Katsuno, H., Mendelzon, A.O.: Propositional knowledge base revision and minimal change. Artificial Intelligence 52, 263–294 (1991)
22. Nieves, J., Osorio, M., Corts, U.: Inferring preferred extensions by minimal models. In: Workshop on Argumentation and Non-Monotonic Reasoning. pp. 114–124 (2007), workshop at Logic Programming and Non-Monotonic Reasonning 2007 (LPNMR07)
23. Nofal, S., Atkinson, K., Dunne, P.: Algorithms for decision problems in argument systems under preferred semantics. Artificial Intelligence 207, 23–51 (2014)