An Empirical and Axiomatic Comparison of Ranking-based Semantics for Abstract Argumentation

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Abstract

Argumentation is the process of evaluating and comparing a set of arguments. A way to compare them consists in using a ranking-based semantics which rank-order arguments from the most to the least acceptable ones. Recently, a number of such semantics have been proposed independently, often associated with some desirable properties. In this work, we provide a thorough analysis of ranking-based semantics in two different ways. The first is an empirical comparison on randomly generated argumentation frameworks which reveals insights into similarities and differences between ranking-based semantics. The second is an axiomatic comparison of all these semantics with respect to the proposed properties aiming to better understand the behavior of each semantics.

Keywords: Argumentation, Ranking-based semantics, Comparison

1 Introduction

Argumentation consists in reasoning with conflicting information based on the exchange and evaluation of interacting arguments. It can be used for modeling dialogue (persuasion,

⋆ This is an extended version of the paper "A Comparative Study of Ranking-based Semantics for Abstract Argumentation" [11] written by the same authors and published in the proceedings of the 30th AAAI Conference on Artificial Intelligence, AAAI 2016.

negotiation), decision, etc. A proof of its appeal is the recent development of online platforms where people participate in debates using argumentation graphs (e.g. Debategraph [1] or Kialo [2]) such representation tools are becoming increasingly popular.

Argumentation has been a very active topic in Artificial Intelligence for more than two decades now. The most popular way to represent argumentation process was proposed by Dung [21] with abstract argumentation frameworks, modelized by directed graphs, where the nodes represent (abstract) arguments, and the edges represent the attacks between them. Given an argumentation framework, one can then examine the question of which set(s) of arguments can be accepted together: answering this question corresponds to defining an argumentation semantics. Various semantics (see [6] for an overview) have been formulated to compute these sets of arguments, called extensions (or labellings [15]), from an argumentation framework. Finally, the acceptability of an argument depends on its membership to a set of extensions in Dung's theory. For example, under the skeptical (resp. credulous) acceptance, an argument is either considered as accepted if it belongs to all (resp. at least one) extension(s) or rejected otherwise.

The extension-based semantics can be used in applications like paraconsistent reasoning. However, there exist some other applications where they are not appropriate. Indeed, some aspects of these semantics, like the existence of multiple extensions, the non-existence of extensions or having only two levels of acceptability (accepted or not accepted), can sometimes be problematic. It is the case, for example, for decision-making problems (see the discussion in [2]) or for online debate platforms, where additional information like votes on arguments and/or on attacks are available (see the discussion in [29]).

An alternative way to evaluate arguments consists in directly reasoning on the arguments themselves by exploiting the topology of the argumentation framework. Following this idea, gradual semantics (assigning numerical acceptability degree to each argument) [9,30,29,19] and ranking-based semantics (returning a ranking on the arguments) [16,2,32,25,4,12] have been proposed. Such semantics address a different question than classical Dung's semantics because they do not provide any indication as to what sets of arguments can be *jointly accepted.* The first semantics belonging to this family of semantics was the h-categorizer semantics introduced by Besnard and Hunter [9] which aimed at capturing the relative strength of arguments in an argument tree by taking into account the attack relations. Since the introduction of this semantics, the number of semantics proposed in the literature has increased steadily, which makes the choice of a semantics difficult for a potential user. This is why we propose in this work two different methods of comparison of the ranking-based semantics. The first one is an experimental comparison where we examine the rankings returned by these semantics on a benchmark of argumentation frameworks, in order to evaluate the degree of similarity between each pair of semantics. The second comparison allows to understand where the similarity and the differences between the rankings come from. To this end, we extend existing axiomatic

[1] `http://debategraph.org`
[2] `https://www.kialo.com`

studies based on the definition of properties and the proof of their satisfaction or not by the existing ranking-based semantics. Regarding the definition of these properties, we generalize some existing properties only defined in the context of a particular semantics and propose new ones which allow us to capture other aspects affecting the diversity between the rankings.

This paper is a substantial development of the initial results presented in [11] and extends this previous work in several ways. Indeed, the main difference is the addition of an empirical comparison of ranking-based semantics on randomly generated argumentation frameworks. The purpose of this comparison is to provide us with insights into similarities and differences between the rankings of arguments returned by the ranking-based semantics. The axiomatic study will then make it possible to explain some of the similarities and differences observed. In addition to the semantics considered in [11], we include seven additional ranking-based semantics which have, meanwhile, been introduced. Finally, two additional properties (Argument Equivalence and Ordinal Equivalence) are taken into consideration in the axiomatic part.
After a refresher on abstract argumentation and on the ranking-based semantics in Section 2, we formally introduce the existing ranking-based semantics in Section 3. In Section 4, we provide the first empirical comparison of ranking-based semantics. Section 5 is devoted to the axiomatic study, where we recall the properties for ranking-based semantics, provide their relationships and check which are satisfied by these semantics. Finally, a more extensive discussion has been included in Section 6. All proofs together with the associated counter-examples have been included in Appendix A.

2 Preliminaries

An abstract argumentation framework [21] is a set of arguments and a binary relation representing attacks between the arguments. Arguments are abstract entities whose internal structure is not specified.

Definition 1 *An* **argumentation framework (AF)** *is a pair $F = \langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a finite and non-empty set of arguments and $\mathcal{R}$ is a binary relation on $\mathcal{A}$, i.e. $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, called the* **attack relation***. So $(x, y) \in \mathcal{R}$, with $x, y \in \mathcal{A}$, means that x attacks y. Let $Arg(F) = \mathcal{A}$.*

Let $\mathbb{AF}$ be the set of all argumentation frameworks. For two AFs $F = \langle \mathcal{A}, \mathcal{R} \rangle$ and $G = \langle \mathcal{A}', \mathcal{R}' \rangle$, we define the union $F \cup G = \langle \mathcal{A} \cup \mathcal{A}', \mathcal{R} \cup \mathcal{R}' \rangle$.

Abstract argumentation frameworks can be represented by directed graphs, where the nodes represent the arguments and the edges represent the attack relation between two arguments. Let us now introduce some useful notions in order to formalize properties of argumentation frameworks.

Notation 1 (Path) *Let $F = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$. A directed* **path** *P from y to x, noted $P(y, x)$, is a sequence $\langle x_0, \ldots, x_n \rangle$ of arguments such that $x_0 = y$, $x_n = x$ and $\forall i$ s.t. $0 \leq i < n, (x_i, x_{i+1}) \in \mathcal{R}$. The length of the path P is n (the number of attacks it is composed of) and is denoted by $l_P = n$.*

According to the length of a path between two arguments, the argument at the beginning of this path can be an attacker and/or a defender (*i.e.*, an argument which attacks an attacker) of the argument at the end of the path.

Notation 2 (Defender/Attacker) *A* **defender** *(resp.* **attacker***) of x is an argument situated at the beginning of an even-length (resp. odd-length) path.*
Let $\mathcal{R}_n(x) = \{y \mid \exists P(y, x) \text{ with } l_P = n\}$ be the multiset of arguments that are bound by a path of length n to the argument x. Thus, an argument $y \in \mathcal{R}_n(x)$ is a ***direct attacker*** *if $n = 1$, a* ***direct defender*** *if $n = 2$, an attacker if n is odd or a defender if n is even. Let us note $\mathcal{R}_+(x) = \bigcup_{n \in 2\mathbb{N}+2} \mathcal{R}_n(x)$ and $\mathcal{R}_-(x) = \bigcup_{n \in 2\mathbb{N}+1} \mathcal{R}_n(x)$ the multiset of all the defenders and all the attackers of x respectively.*

Finally, let us define a particular path, called branch, such that the argument at the beginning of the path is not attacked.

Notation 3 (Root/Branch) *A* **defense root** *(resp.* **attack root***) is a defender (resp. attacker) which is not attacked. Let $\mathcal{B}_n(x) = \{y \in \mathcal{R}_n(x) \mid \mathcal{R}_1(y) = \emptyset\}$ be the multiset of roots that are bounded by a path of length n to the argument x. A path from y to x is a* **defense branch** *(resp.* **attack branch***) if y is a defense (resp. attack) root of x. Let us note $\mathcal{B}_+(x) = \bigcup_{n \in 2\mathbb{N}+2} \mathcal{B}_n(x)$ and $\mathcal{B}_-(x) = \bigcup_{n \in 2\mathbb{N}+1} \mathcal{R}_n(x)$ the multiset of all the defense roots and all the attack roots of x respectively.*

Example 1 *Let $AF_c = \langle \mathcal{A}, \mathcal{R} \rangle$ with $\mathcal{A} = \{a, b, c, d, e, f, g, h, i, j\}$ and $\mathcal{R} = \{(a, b), (b, c), (b, f), (d, f), (d, g), (e, d), (e, h), (e, i), (h, g), (j, i)\}$.*

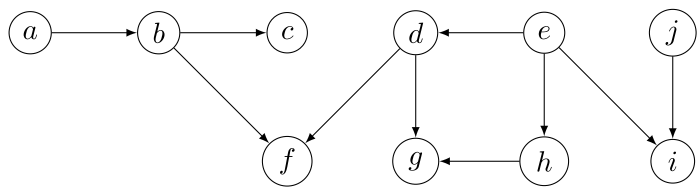


Fig. 1. An argumentation framework AF_c for Example 1.

On the argumentation framework depicted in Fig. 1, we can find some elements as:

- *$\langle e, d, f \rangle$ is a path of length 2 whereas $\langle b, f, d \rangle$ is not a path,*
- *b and d are the direct attackers of f ($\mathcal{R}_1(f) = \{b, d\}$) but they are not attack roots of f because they are attacked by a and e respectively,*
- *e is the only defender, and more precisely the only defense root, of g against two arguments ($\mathcal{R}_2(g) = \{e, e\}$),*
- *$\langle j, i \rangle$ and $\langle a, b \rangle$ are attack branches, whereas $\langle b, f \rangle$ is not, because b is not an attack root of f,*

- $\langle a, b, c\rangle$, $\langle e, d, f\rangle$ *and* $\langle e, h, g\rangle$ *are three defense branches.*

In Dung's framework [21], extension-based semantics have been defined to select sets of arguments, called extensions (or labellings [15]), which can be conjointly accepted (*w.r.t* some criteria depending on the semantics used) for a given argumentation framework. The *acceptability* of an argument (accepted or rejected) depends on its membership to these extensions.

An alternative way to evaluate arguments consists in directly reasoning on the arguments themselves rather than on set of arguments. Following this idea, two kinds of semantics have been introduced in the literature: the ranking-based semantics and the gradual semantics.
Ranking-based semantics aim at (comparatively) evaluating each argument in an argumentation framework. Formally, ranking-based semantics are functions that map each argumentation framework to a ranking on its arguments from the most to the least acceptable ones.

Definition 2 *A* **ranking-based semantics** σ *associates to any* $AF = \langle \mathcal{A}, \mathcal{R}\rangle$ *a ranking* $\succeq^{\sigma}_{AF}$ *on* $\mathcal{A}$*, where* $\succeq^{\sigma}_{AF}$ *is a preorder (a reflexive and transitive relation) on* $\mathcal{A}$*.*

- $a \succeq^{\sigma}_{AF} b$ *means that* a *is at least as acceptable as* b*;*
- $a \simeq^{\sigma}_{AF} b$ *(for* $a \succeq^{\sigma}_{AF} b$ *and* $b \succeq^{\sigma}_{AF} a$*) means that* a *and* b *are equally acceptable;*
- $a \succ^{\sigma}_{AF} b$ *(for* $a \succeq^{\sigma}_{AF} b$ *and* $b \not\succeq^{\sigma}_{AF} a$*) means that* a *is strictly more acceptable than* b*;*
- $a \not\succeq^{\sigma}_{AF} b$ *and* $b \not\succeq^{\sigma}_{AF} a$ *means that* a *and* b *are incomparable.*

We denote by $\sigma(AF)$ *the ranking on* $\mathcal{A}$ *returned by* σ*.*

If there is no ambiguity about the semantics and the argumentation framework in question, we will use $\succeq$ instead of $\succeq^{\sigma}_{\mathrm{AF}}$.

Gradual semantics assign a numerical acceptability degree to each argument, taking into account various criteria from the argumentation framework. This value must be selected among an ordered scale, such as the interval [0, 1], the interval [-1, 1], the set of natural numbers $\mathbb{N}$, the set of positive real numbers $\mathbb{R}^+$, etc. The evaluation is thus numerical instead of ordinal, but the aim is still to evaluate each argument individually.

Clearly, ranking-based semantics and gradual semantics are not independent. Indeed, most of the time the ranking between arguments is based on the comparison of the scores computed with a gradual semantics. In other words, a gradual semantics is used to assign a score to each argument and, as this score belongs to an ordered scale, it is possible to compare them in order to obtain a ranking between arguments. However, most of the scores assigned to each argument only make sense when they are compared with each other. In addition, while it is always possible to build a ranking-based semantics using a gradual semantics, there exist other methods which do not use gradual semantics to build a ranking between arguments as we shall see in the next section. It is why we choose to focus on ranking-based semantics in this paper.

We refer the reader to [1] for a complete overview of the existing families of semantics in abstract argumentation and the differences between these approaches (e.g., definition, outcome, application).

Finally, we need to introduce the notion of lexicographical order between two vectors of real numbers in order to formally define some existing ranking-based semantics.

Definition 3 (Lexicographical order) *Let $V = \langle V_1, V_2, \ldots \rangle$ and $U = \langle U_1, U_2, \ldots \rangle$ be two (finite or infinite) vectors of real numbers such that $len(V) = n$ and $len(U) = m$ with $n, m \in \mathbb{N} \cup \{\infty\}$. We say that:*

- $V \simeq_{lex} U$ *iff* $n = m$ *and* $\forall i, 1 \leq i \leq n$, $V_i = U_i$
- $V \succ_{lex} U$ *iff* $\exists i \geq 1$ *such that* $\forall j, 1 \leq j < i$, $V_j = U_j$ *and one of the following two conditions is satisfied:*
  - $n > m = i - 1$ *(i.e. V_i exists whereas U_i does not exist)*
  - $V_i > U_i$ *(i.e. V_i and U_i both exist and the value at index i of the vector V is strictly greater than the value at index i of the vector U)*
- $V \succeq_{lex} U$ *iff* $V \simeq_{lex} U$ *or* $V \succ_{lex} U$

3 Existing Ranking-based Semantics

In this section, we introduce ranking-based semantics from the literature. Some of these semantics have not been originally defined as ranking-based semantics but rather as gradual semantics. But, as explained in the previous section, we can always induce a ranking-based semantics from a gradual one. However, as a ranking-based semantics associates a unique ranking to any argumentation framework (see Definition 2), the rankings induced by a gradual semantics should be unique too. This is why we leave the social argumentation frameworks [29] and the equational approach [23], which may both return multiple rankings, out of this study. [3]
Please note that, in this section, we only recall the general intuition and formal definitions of existing ranking-based semantics. Thus, we refer the reader to the scientific articles in which these semantics have been defined in order to find more justifications and examples.

3.1 Categoriser-based Ranking Semantics

Originally, Besnard and Hunter [9] proposed a *categoriser* function used for "deductive" arguments, where an argument is structured as a pair $\langle \Phi, \alpha \rangle$, where Φ is a consistent set

[3] It was discovered [5] that the conjecture about the uniqueness of social models only holds up to 3 arguments in the argumentation framework. The study done on SAF semantics in [11] only related to the result obtained when the algorithm introduced in [18] is used. But this result only corresponds to one possible social model.

of formulae, called support or premise, α is a formula, called the claim (or consequent) of the argument such that $\Phi \vdash \alpha$. The categoriser function assigns a value to a tree of such arguments where each value captures the relative strength of an argument taking into account the strength of its attackers which takes into account the strength of its attackers, and so on.

Definition 4 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. The* **categoriser function** $Cat : \mathcal{A} \rightarrow]0, 1]$ *is defined such that $\forall x \in \mathcal{A}$, $Cat(x) = \frac{1}{1+\sum_{y \in \mathcal{R}_1(x)} Cat(y)}$. The values returned by the categoriser function are called the categoriser values. Thus, $Cat(x)$ is the categoriser value of x.*

The categoriser function was initially introduced for argument trees, but Pu et al. [32] proved the existence and uniqueness of such solution for any argumentation framework. In this case, the categoriser values correspond to the solution of the non-linear system of equations with one equation per argument (see Definition 4) and can be computed via a fixed point technique for any argumentation framework. The categoriser-based ranking semantics builds a ranking from the categoriser values obtained. The higher the categoriser value of an argument, the more acceptable the argument.

Definition 5 *The* **Categoriser-based ranking semantics (Cat)** *associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{Cat}_{AF}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq^{Cat}_{AF} y \text{ iff } Cat(x) \geq Cat(y)$$

Example 1 (cont.) *Let us compute the categoriser values of each argument in AF_c (Fig. 1). We have $Cat(a) = Cat(e) = Cat(j) = 1$, $Cat(c) \approx 0.667$, $Cat(b) = Cat(d) = Cat(f) = Cat(g) = Cat(h) = 0.5$, $Cat(i) \approx 0.333$. We obtain the following ranking:*

$$a \simeq^{Cat} e \simeq^{Cat} j \succ^{Cat} c \succ^{Cat} b \simeq^{Cat} d \simeq^{Cat} f \simeq^{Cat} g \simeq^{Cat} h \succ^{Cat} i$$

This semantics assigns high values to arguments with low-valued attackers, with a maximal value of 1 to the non-attacked arguments (like a, e and j). In this way, we can see that even if an argument is always defended (like f and g) it is still attacked anyway. It is why f and g have exactly the same level of acceptability that arguments directly attacked only once but by one stronger argument (like b, d and h).

3.2 Discussion-based Semantics

Amgoud and Ben-Naim [2] have introduced the discussion-based semantics which proposes the comparison of two arguments on the basis of the number of paths leading to them. A distinction is made concerning the polarity of the number of paths computed according to the attack relation meaning (positive for the attackers and negative for the defenders).

Definition 6 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $x \in \mathcal{A}$, and $i \in \mathbb{N} \backslash \{0\}$.*

$$Dis_i(x) = \begin{cases} -|\mathcal{R}_i(x)| & \textit{if } i \textit{ is even} \\ |\mathcal{R}_i(x)| & \textit{if } i \textit{ is odd} \end{cases}$$

The **discussion count of** x *is denoted* $Dis(x) = \langle Dis_1(x), Dis_2(x), \ldots \rangle$.

This semantics was proposed to take into account only the number of attackers/defenders of a given argument, whatever their quality: the fewer attackers and the more defenders an argument, the more acceptable the argument. The method lexicographically ranks the arguments on the basis of the number of attackers and defenders. Concretely, we start by comparing the number of direct attackers of each argument. If some arguments are still equivalent (they have the same number of direct attackers), the size of paths is recursively increased until a difference is found or the threshold [4] is reached.

Definition 7 *The* **Discussion-based semantics (Dbs)** *associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{Dbs}_{AF}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq^{Dbs}_{AF} y \text{ iff } Dis(y) \succeq_{lex} Dis(x)$$

Example 1 (cont.) *Let us compute the discussion count of each argument in AF_c (Fig. 1). We obtain $Dis(a) = Dis(e) = Dis(j) = \langle 0, 0, 0 \rangle$, $Dis(c) = \langle 1, -1, 0 \rangle$, $Dis(b) = Dis(d) = Dis(h) = \langle 1, 0, 0 \rangle$, $Dis(f) = Dis(g) = \langle 2, -2, 0 \rangle$, and $Dis(i) = \langle 2, 0, 0 \rangle$.*

$$a \simeq^{Dbs} e \simeq^{Dbs} j \succ^{Dbs} c \succ^{Dbs} b \simeq^{Dbs} d \simeq^{Dbs} h \succ^{Dbs} f \simeq^{Dbs} g \succ^{Dbs} i$$

During the first step where only the direct attackers are considered, we have three groups of arguments: one contains the non-attacked arguments (a, e and j), one contains the arguments directly attacked once (b, c, d and h) and the last one contains arguments directly attacked twice (f, g and i). Then, during the second step, in some group of arguments with the same level of acceptability, one can distinguish arguments in taking into account the direct defenders. Indeed, c which is defended once by a is now strictly more acceptable than b, d and h which are not defended and, with the same idea, f and g are strictly more acceptable than i. There exists no path of length 3 so the process can be completed.

3.3 Burden-based Semantics

Amgoud and Ben-Naim [2] have also introduced the burden-based semantics which follows a multiple step process. Indeed, instead of computing all the possible paths that lead to an

[4] If there is no cycle, the threshold is equal to the longest branch in the argumentation framework. But if cycles are permitted, the discussion count of some arguments can be infinite because $Dis_i(x)$ evolve cyclically. However, the authors strongly conjecture that there exists a threshold t after which it is no longer possible to distinguish the arguments: if $\forall i \leq t$, $Dis_i(x) = Dis_i(y)$, then $\forall i > t$, $Dis_i(x) = Dis_i(y)$.

argument like the discussion-based semantics does, each argument receives, at each step, a *burden number* which is simultaneously computed on the basis of the burden numbers of their direct attackers at the previous step. Formally, for every argument x, its burden number is initialised to 1, then, for every argument y that attacks x, the burden number of x is increased by a quantity inversely proportional to the burden number of y in the previous step.

Definition 8 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $x \in \mathcal{A}$ and $i \in \mathbb{N}$. The burden number of x at step i is computed as follows:*

$$Bur_i(x) = \begin{cases} 1 & \textit{if } i = 0 \\ 1 + \sum\limits_{y \in \mathcal{R}_1(x)} \frac{1}{Bur_{i-1}(y)} & \textit{otherwise} \end{cases}$$

The **burden vector of** *a is denoted $Bur(x) = \langle Bur_0(x), Bur_1(x), \dots \rangle$.*

Two arguments are lexicographically compared on the basis of their burden numbers. Thus, the idea of this semantics is, like the discussion-based semantics, to consider the number of attackers and defenders of an argument as the main criterion for comparing two arguments.

Definition 9 *The* **Burden-based semantics (Bbs)** *associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{Bbs}_{AF}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq^{Bbs}_{AF} y \text{ iff } Bur(y) \succeq_{lex} Bur(x)$$

Example 1 (cont.) *Let us compute the burden vector of each argument in AF_c (Fig. 1). We obtain $Bur(a) = Bur(e) = Bur(j) = \langle 1, 1, 1, 1 \rangle$, $Bur(c) = \langle 1, 2, 1.5, 1.5 \rangle$, $Bur(b) = Bur(d) = Bur(h) = \langle 1, 2, 2, 2 \rangle$, $Bur(f) = Bur(g) = \langle 1, 3, 2, 2 \rangle$, and $Bur(i) = \langle 1, 3, 3, 3 \rangle$.*

$$a \simeq^{Bbs} e \simeq^{Bbs} j \succ^{Bbs} c \succ^{Bbs} b \simeq^{Bbs} d \simeq^{Bbs} h \succ^{Bbs} f \simeq^{Bbs} g \succ^{Bbs} i$$

The reasons for obtaining this ranking are the same as those described in the example of the Discussion-based semantics. Indeed, as shown in this example, Dbs and Bbs often return the same ranking because they only consider the number of attackers and defenders of arguments.

3.4 α-Burden-based Semantics

Amgoud, Ben-Naim, Doder and Vesic [4] have introduced the α-Burden-based semantics which is a broad class of ranking semantics that allows one to choose to which extent to privilege quality of attacks (i.e. the scores of the direct attackers) over their quantity (i.e. the number of direct attackers) (or vice versa). This principle, called compensation, can be checked when several weak attacks (*i.e.* direct attackers of an argument are attacked) could have the same impact as one strong attack (*i.e.* direct attackers are not attacked). Formally, the formula is quite similar to the one used by the burden-based semantics (see Definition 8) but, in order to satisfy the compensation principle, Amgoud et al. [4] introduce a parameter α, where different values of α give different behaviors (the greater the value of α, the bigger the influence of the quality of attackers).

Definition 10 *Let $\alpha \in]0, +\infty[$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. The function $s_\alpha : \mathcal{A} \to [1, +\infty[$ is defined such that $\forall x \in \mathcal{A}$,*

$$s_\alpha(x) = 1 + \left(\sum_{y \in \mathcal{R}_1(x)} \frac{1}{(s_\alpha(y))^\alpha} \right)^{1/\alpha}$$

The parameter α is both used for the compensation and to ensure the uniqueness of the solution of equations (with one equation per argument) from Definition 10. Indeed, contrary to Bbs where the lexicographical order is used, α-Bbs uses a fixed-point iteration to find the burden number of each argument. Thus, the higher the score s_α of an argument, the less acceptable the argument.

Definition 11 *The* **α-Burden-based semantics (α-Bbs)** *associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{AF}^{\alpha\text{-}Bbs}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq_{AF}^{\alpha\text{-}Bbs} y \text{ iff } s_\alpha(x) \leq s_\alpha(y)$$

Thus, the greater the value of α, the bigger the influence of the quality of attackers. It is why, for different values of α, the computed ranking can vary.

Example 1 (cont.) *With $\alpha = 0.5$, we have $s_\alpha(a) = s_\alpha(e) = s_\alpha(j) = 1$, $s_\alpha(c) = 1.5$, $s_\alpha(b) = s_\alpha(d) = s_\alpha(h) = 2$, $s_\alpha(f) = s_\alpha(g) = 3$ and $s_\alpha(i) = 5$. Thus, we obtain the following pre-order where, clearly, like Dbs and Bbs, the number of direct attackers of an argument is the main criterion for distinguishing arguments:*

$\alpha = 0.5 \quad a \simeq^{\alpha\text{-}Bbs} e \simeq^{\alpha\text{-}Bbs} j \succ^{\alpha\text{-}Bbs} c \succ^{\alpha\text{-}Bbs} b \simeq^{\alpha\text{-}Bbs} d \simeq^{\alpha\text{-}Bbs} h \succ^{\alpha\text{-}Bbs} f \simeq^{\alpha\text{-}Bbs} g \succ^{\alpha\text{-}Bbs} i$

For example, we can observe that b, which is only attacked once, is better ranked than f which is attacked twice. However, if we increase the value of α, the quality becomes more important than the quantity and we obtain different rankings:

$\alpha = 1 \quad a \simeq^{\alpha\text{-}Bbs} e \simeq^{\alpha\text{-}Bbs} j \succ^{\alpha\text{-}Bbs} c \succ^{\alpha\text{-}Bbs} b \simeq^{\alpha\text{-}Bbs} d \simeq^{\alpha\text{-}Bbs} f \simeq^{\alpha\text{-}Bbs} g \simeq^{\alpha\text{-}Bbs} h \succ^{\alpha\text{-}Bbs} i$

$\alpha = 5 \quad a \simeq^{\alpha\text{-}Bbs} e \simeq^{\alpha\text{-}Bbs} j \succ^{\alpha\text{-}Bbs} c \succ^{\alpha\text{-}Bbs} f \simeq^{\alpha\text{-}Bbs} g \succ^{\alpha\text{-}Bbs} b \simeq^{\alpha\text{-}Bbs} d \simeq^{\alpha\text{-}Bbs} h \succ^{\alpha\text{-}Bbs} i$

Thus, when $\alpha = 1$, we can see that b and f are now equally acceptable (like Cat) while, when $\alpha = 5$, f becomes better ranked than b.

Cayrol and Lagasquie-Schiex [16] have introduced the tuples-based semantics defined as a "global" approach where only the defense and attack branches of an argument are taken into consideration to compare arguments. A structure, called tupled value, is first defined in order to store, for each argument, the set of the lengths of the branches (attack and defense branches are considered separately) leading to this argument.

Definition 12 *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $x \in \mathcal{A}$*.*

- *Let* $v_p(x)$ *be the (ordered) tuple of even integers representing the lengths of all the defense branches of* x*, i.e.* $v_p(x)$ *is the smallest ordered tuple such that* $\forall n \in 2\mathbb{N}, |\mathcal{B}_n(x)| = k \Rightarrow n \in_k v_p(x)$*, where* $\in_k$ *means "appear at least* k *times".*
- *Let* $v_i(x)$ *be the (ordered) tuple of odd integers representing the lengths of all the attack branches of* x*, i.e.* $v_i(x)$ *is the smallest ordered tuple such that* $\forall n \in 2\mathbb{N} + 1, |\mathcal{B}_n(x)| = k \Rightarrow n \in_k v_i(x)$*.*

If x *is non-attacked then* $v_p(x) = (0, 0, \dots) = 0^\infty$ *and* $v_i(x) = ()$*.*
A **tupled value** *for* x *is the pair* $v(x) = [v_p(x), v_i(x)]$*.*

When cycles exist in the AF, there may be no non-attacked argument and thus no branch. The solution proposed in [16] is to consider that a cycle is like an infinity of branches which gives an infinite acyclic graph. However, some tuples can be now infinite and the method to calculate them requires a highly involved process.

Once the tupled values have been computed for each argument, the next step consists in comparing them. To do so, the number of attack and defense branches of two arguments (*i.e.* the length of v_p and v_i) are first compared and, in case of a tie (*i.e.* both arguments have the same number of attack and defense branches), the values inside each tuple (so the length of each branches) are lexicographically compared (see Algorithm 1). Thus, the priority is given to the quantity, and the quality is taken into consideration only if the quantity cannot allow to decide between two arguments.

Let us remark that two arguments can be incomparable. It is the case, for example, if an argument has strictly more attack branches and strictly more defense branches than another one (see line 14 in Algorithm 1). Consequently, this semantics returns a partial ranking between arguments.

Example 1 (cont.) *Let us compute the tupled value of each argument in* AF_c *(Fig. 1). Thus, we have* $v(a) = v(e) = v(j) = [0^\infty, ()]$*,* $v(f) = v(g) = [(2, 2), ()]$*,* $v(c) = [(2), ()]$*,* $v(b) = v(d) = v(h) = [(), (1)]$*, and* $v(i) = [(), (1, 1)]$*. Following Algorithm 1, we obtain the following ranking:*

Algorithm 1 Tuples comparison [16]

Input: $v(a), v(b)$ two tupled values of arguments a and b
Output: A ranking $\succeq^{\mathrm{T}}$ between a and b

if $v_i(a) = v_i(b)$ **and** $v_p(a) = v_p(b)$ **then** $a \simeq^{\mathrm{T}} b$
else
  if $|v_i(a)| = |v_i(b)|$ **and** $|v_p(a)| = |v_p(b)|$ **then**
    if $v_p(a) \preceq_{lex} v_p(b)$ **and** $v_i(a) \succeq_{lex} v_i(b)$ **then** $a \succ^{\mathrm{T}} b$
    else
      if $v_p(a) \succeq_{lex} v_p(b)$ **and** $v_i(a) \preceq_{lex} v_i(b)$ **then** $a \prec^{\mathrm{T}} b$
      else $a \not\succeq^{\mathrm{T}} b$ and $a \not\preceq^{\mathrm{T}} b$
      end if
    end if
  else
    if $|v_i(a)| \geq |v_i(b)|$ **and** $|v_p(a)| \leq |v_p(b)|$ **then** $a \prec^{\mathrm{T}} b$
    else
      if $|v_i(a)| \leq |v_i(b)|$ **and** $|v_p(a)| \geq |v_p(b)|$ **then** $a \succ^{\mathrm{T}} b$
      else $a \not\succeq^{\mathrm{T}} b$ and $a \not\preceq^{\mathrm{T}} b$
      end if
    end if
  end if
end if

$$a \simeq^T e \simeq^T j \succ^T f \simeq^T g \succ^T c \succ^T b \simeq^T d \simeq^T h \succ^T i$$

All the arguments here are comparable. We observe that the non-attacked arguments are ranked highest (a, e and j), followed by the arguments with two defense branches (f and g) which are ranked higher than c which has only one defense branch. Among the lowest ranked arguments, we find arguments with one attack branch (b, d et h) and finally i which has two attack branches.

3.6 Two-person Zero-sum Game Semantics

Matt and Toni [30] compute the strength of an argument using a two-person zero-sum strategic game. This game confronts two players, a proponent and an opponent for a given argument, where the strategies of the players are sets of arguments. For an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and an argument $x \in \mathcal{A}$, the set of strategies for the proponent is all the subsets of arguments that contain x: $S_P(x) = \{P \mid P \subseteq \mathcal{A}, x \in P\}$ and for the opponent it is all the subsets of arguments: $S_O = \{O \mid O \subseteq \mathcal{A}\}$. The goal of the game is to evaluate the interactions between the strategies chosen by the two players. In a classical argumentation framework, the only interaction is the attack relation between arguments, so let us define how a strategy (i.e. a set of arguments) can attack another one.

Definition 13 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $X, Y \subseteq \mathcal{A}$. The set of attacks from X to Y is defined by $Y_{AF}^{\leftarrow X} = \{(x, y) \in X \times Y \mid (x, y) \in \mathcal{R}\}$.*

Thus, the set of attacks from a set of arguments to another one is composed of all the attacks in AF such that an argument from the first set directly attacks an argument from the targeted set. Matt and Toni ensure that, in a dispute, it is better for the proponent of an argument to have more attacks against opponents to this argument and fewer attacks from them. To capture this idea, they introduced the notion of degree of acceptability of a set of argument with respect to another one.

Definition 14 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $X, Y \subseteq \mathcal{A}$. The degree of acceptability of X with respect to Y is given by the following formula:*

$$\phi(X, Y) = \frac{1}{2}\left[1 + f(|Y_{AF}^{\leftarrow X}|) - f(|X_{AF}^{\leftarrow Y}|)\right] \text{ with } f(n) = \frac{n}{n+1}$$

To defend her argument properly, the proponent should avoid contradicting herself, i.e. her opinions should always correspond to sets of arguments that are at least conflict-free. Also, since the opponent's role in the game is to criticize the proponent, the opponent should get a maximal penalty whenever her opinion fails to attack the proponent's. Finally, the game should provide an incentive for the proponent to attack the opponent's opinion with as many attacks as possible and at the same time force her to avoid the opponent's attacks. To implement these principles, Matt and Toni chose to use a reward function to represent the relative degree of acceptability of the players' opinions.

Definition 15 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Given an argument $x \in \mathcal{A}$, $P \in S_P(x)$ (respectively $O \in S_O$) represents a strategy chosen by the proponent (respectively opponent). The rewards of P over O, denoted by $r_{AF}(P, O)$, are defined by:*

$$r_{AF}(P, O) = \begin{cases} 0 & \text{if and only if } \exists x, y \in P, (x, y) \in \mathcal{R} \\ 1 & \text{if and only if } |P_{AF}^{\leftarrow O}| = 0 \\ \phi(P, O) & \text{otherwise} \end{cases}$$

Recall that each player has to change her strategy (if needed) in order to prevent her adversary from adapting her own strategy, and thus getting a better reward. Thus, proponent and opponent have the possibility to use a strategy according to some probability distributions, respectively $p = (p_1, p_2, \ldots, p_m)$ and $q = (q_1, q_2, \ldots, q_n)$, with $m = |S_P|$ and $n = |S_O|$. Thus, for the proponent (respectively opponent), the probability of choosing her i^{th} strategy is equal to p_i (respectively q_i). For each argument $x \in \mathcal{A}$, the proponent's expected payoff $E(x, p, q)$ is then given by $E(x, p, q) = \sum_{j=1}^{n} \sum_{i=1}^{m} p_i q_j r_{i,j}$ with $r_{i,j} = r_{AF}(P_i, O_j)$ where P_i (respectively O_j) represents the i^{th} (respectively j^{th}) strategy of $S_P(x)$ (respectively S_O). The proponent can therefore expect to get at least $\min_q E(x, p, q)$, where the minimum is taken over all the probability distributions q available to the opponent. Hence the proponent can choose a strategy which will guarantee her a reward of $\max_p \min_q E(x, p, q)$. The opposite is also true with $\min_q \max_p E(x, p, q)$. The value of the two-person zero-sum game (2ZG) for an argument x is $s(a) = \max_p \min_q E(x, p, q) = \min_q \max_p E(x, p, q)$.

Definition 16 *The* **ranking-based semantics 2ZG** *associates to any* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq^{2ZG}_{AF}$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$,

$$x \succeq^{2ZG}_{AF} y \text{ iff } s(x) \geq s(y)$$

Example 1 (cont.) *The strength of each argument in* AF_c *(Fig. 1) is* $s(a) = s(e) = s(j) = 1$, $s(c) = s(f) = s(g) = 0.5$, $s(b) = s(d) = s(h) = 0.25$ *and* $s(i) \simeq 0.16$. *Thus, we obtain the following ranking:*

$$a \simeq^{2ZG} e \simeq^{2ZG} j \succ^{2ZG} c \simeq^{2ZG} f \simeq^{2ZG} g \succ^{2ZG} b \simeq^{2ZG} d \simeq^{2ZG} h \succ^{2ZG} i$$

The main observation here is that c *which is attacked once but defended has the same ranking as* f *and* g *which are attacked twice but also defended. However, they are still more acceptable than arguments that are attacked but not defended.*

3.7 Fuzzy Labelling

Da Costa Pereira, Tettamanzi and Villata [19] study how an agent changes her mind in response to new information/arguments. For this, they combine belief revision and argumentation in a single framework close to Dung's framework, called fuzzy argumentation framework, where a degree of trust is first assigned to each argument. Indeed, an argument could come from different sources with a more or less important trustworthiness. Thus, when a new argument is proposed, it has more or less influence on the evaluation of existing arguments according to its degree of trust. Then, to compute the influence of an argument on the others, it is necessary to solve a system of non-linear equations (with an equation for each argument). The obtained values express how much the agent tends to accept an argument coming from not fully trusted agents.
Even if this work does not directly propose a ranking-based semantics, the score obtained by each argument after computation could be used to rank the arguments. In order to compare this semantics with the existing ranking-based semantics in the classical framework, we consider that all arguments are totally trusted.

Definition 17 *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $i \in \mathbb{N}$. *The function* $f : \mathcal{A} \to [0, 1]$ *is defined such that* $\forall x \in \mathcal{A}$,

$$f_i(x) = \begin{cases} 1 & \textit{if } i = 0 \\ \frac{1}{2}(f_{i-1}(x) + (1 - \max\limits_{y \in \mathcal{R}_1(x)} f_{i-1}(y))) & \textit{otherwise} \end{cases}$$

A fuzzy reinstatement labelling for AF is, $\forall x \in \mathcal{A}$, $f(x) = \lim\limits_{i \to \infty} f_i(x)$.

According to the formula, the score of an argument during the step i depends both on its score at the previous step ($\alpha_{i-1}(a)$) and on the score of its direct attacker with the highest score at the previous step ($1 - max(f_{i-1}(b))$). Indeed, its score (and so its acceptability) should not be greater than the degree to which its direct attackers are unacceptable: $f(a) \leq 1 - \max_{b \in \mathcal{R}_1(a)} f(b)$. For instance, an argument with a score of 0 is necessarily attacked by at least one argument with a score of 1.

Definition 18 *The* **Fuzzy labelling (FL)** *associates to any argumentation framework* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq^{FL}{}_{AF}$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$*,*

$$x \succeq^{FL}_{AF} y \text{ iff } f(x) \geq f(y)$$

Example 1 (cont.) *The fuzzy reinstatement labeling applied on* AF_c *(Fig. 1) returns the following values:* $f(a) = f(e) = f(j) = f(c) = f(f) = f(g) = 1$ *and* $f(b) = f(d) = f(h) = f(i) = 0$*. Thus, we obtain the following pre-order:*

$$a \simeq^{FL} e \simeq^{FL} j \simeq^{FL} c \simeq^{FL} f \simeq^{FL} g \succ^{FL} b \simeq^{FL} d \simeq^{FL} h \simeq^{FL} i$$

We observe that all non-attacked and defended arguments are equally acceptable. However, they still rank higher than the non-defended arguments.

3.8 Iterated Graded Defense

The next semantics, introduced by Grossi and Modgil [25,26], proposes a generalisation of Dung's notion of acceptability. The theory is based on two assumptions: (A1) having fewer direct attackers is better than having more; and (A2) having more direct defenders is better than having fewer. To capture this two principles, Grossi and Modgil define a generalisation of the notion of defense initially defined by Dung.
Let x be an argument among a set of arguments $\mathcal{X} \subseteq \mathcal{A}$. Let m be the number of direct attackers of x ($\mathcal{R}_1(x) = \{y_1, \ldots, y_m\}$) and, for each y_i, let n_i be the number of direct attackers of y_i in $\mathcal{X}$ and $n = min(\{n_i\}_{0 \leq i \leq m})$ (all direct attackers have at least n counter-attackers: $\forall y_i \in \mathcal{R}_1(x)$, $|\mathcal{R}_1(y_i)| \geq n$).

Definition 19 *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $m, n \in \mathbb{N}$*. The* **graded defense** *of a subset of arguments* $\mathcal{X} \subseteq \mathcal{A}$ *is:*

$$d_{\substack{m \\ n}}(\mathcal{X}) = \{x \in \mathcal{A} \mid \nexists_{\geq m} y \in \mathcal{A} : [(y, x) \in \mathcal{R} \text{ and } \nexists_{\geq n} z \in \mathcal{A}, (z, y) \in \mathcal{R} \text{ and } z \in \mathcal{X}]\}$$

where $\nexists_{\geq n}$ *means "it does not exist at least n".*

Thus, $d_{\substack{m \\ n}}(\mathcal{X})$ contains the arguments which do not have at least m direct attackers (*i.e.*, which have at most $m - 1$ direct attackers) that are not counter-attacked by at least n arguments in $\mathcal{X}$. For example, $d_{\substack{1 \\ 2}}(\mathcal{X})$ selects the arguments such that none of their direct attackers are directly attacked at most once. Thus, in agreement with the assumptions

(A1) and (A2), the arguments belonging to $d_{\substack{m\\n}}(\mathcal{X})$ are at least as strong as the arguments belonging to $d_{\substack{s\\t}}(\mathcal{X})$ when $m \leq s$ (less direct attackers) and $n \geq t$ (more direct defenders). However, this method can be insufficient on its own to compare arguments: the set of arguments computed that way may be not strong enough to defend itself. It is why this method must be recursively applied until obtaining a "stabilized" set of arguments. Thus, for an ordinal α, the α-fold iteration of $d_{\substack{m\\n}}$ is denoted by $d^{\alpha}_{\substack{m\\n}}$ (with $d^{0}_{\substack{m\\n}}(\mathcal{X}) = \mathcal{X}$, $d^{1}_{\substack{m\\n}}(\mathcal{X}) = d_{\substack{m\\n}}(\mathcal{X})$, $d^{2}_{\substack{m\\n}}(\mathcal{X}) = d_{\substack{m\\n}}(d_{\substack{m\\n}}(\mathcal{X}))$, $\ldots$). A set of arguments is stabilized if and only if there exists an ordinal α such that $d^{\alpha}_{\substack{m\\n}}(\mathcal{X}) = d^{\alpha+1}_{\substack{m\\n}}(\mathcal{X})$. Since $d_{\substack{m\\n}}$ is monotonic the existence of such α is always guaranteed according to the Knaster-Tarski theorem.[5] Thus, the indefinite iteration of $d_{\substack{m\\n}}(\mathcal{X})$ is defined as $d^{*}_{\substack{m\\n}}(\mathcal{X}) = \bigcup_{0 \leq i \leq \alpha} d^{i}_{\substack{m\\n}}(\mathcal{X})$.

Take two arguments x and y, and some fixed set $\mathcal{X}$. Is it the case that every time y is defended through the iteration of some graded defense function of $\mathcal{X}$, x also is? If it is the case then every kind of defense met by y (with respect to $\mathcal{X}$) is also met by x and consequently x is at least more acceptable than y (because x may belong to a more graded defense).

Definition 20 *The* **Iterated Graded Defense semantics (IGD)** *associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{IGD}_{AF}$ on $\mathcal{A}$ with respect to $\mathcal{X} \subseteq \mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq^{IGD}_{AF} y \text{ iff } \forall m, n \geq 0 \textit{ if } y \in d^{*}_{\substack{m\\n}}(\mathcal{X}) \textit{ then } x \in d^{*}_{\substack{m\\n}}(\mathcal{X})$$

Please note that two arguments can be incomparable. Indeed, this occurs when, for two arguments a and b and a subset of arguments $\mathcal{X}$, $a \in d^{*}_{\substack{m\\n}}(\mathcal{X})$ and $a \notin d^{*}_{\substack{s\\t}}(\mathcal{X})$ but $b \notin d^{*}_{\substack{m\\n}}(\mathcal{X})$ and $b \in d^{*}_{\substack{s\\t}}(\mathcal{X})$.

Example 1 (cont.) *Given the argumentation framework AF_c, let us compute the indefinite iteration of the graded defense of the empty set ($\mathcal{X} = \emptyset$) for all the values of $1 \geq m, n \geq 3$. The results are given in the table in Fig. 2. One can remark that f (respectively g) and b (respectively d and h) are incomparable because $f \in d^{*}_{\substack{1\\1}}(\emptyset)$ but $b \notin d^{*}_{\substack{1\\1}}(\emptyset)$ and $b \in d^{*}_{\substack{2\\2}}(\emptyset)$ but $f \notin d^{*}_{\substack{2\\2}}(\emptyset)$. Thus, we obtain the partial preorder represented in Fig. 2 by a Hasse diagram[6] ranking arguments where each argument in $\{b, d, h\}$ is incomparable with each argument in $\{f, g\}$.*

[5] Also called Tarski's fixed point theorem [35].

[6] Concretely, for a partially ordered set $(\mathcal{A}, \succ)$, one represents each argument of $\mathcal{A}$ as a vertex in the diagram and draws a line segment that goes upward from x to y whenever $y \succ x$.

$d_{m}^{*}(\emptyset)$		m		
n		1	2	3
n	1	$\{a, c, e, f, g, j\}$	$\{a, b, c, d, e, f, g, h, j\}$	$\mathcal{A}$
	2	$\{a, e, j\}$	$\{a, b, c, d, e, h, j\}$	$\mathcal{A}$
	3	$\{a, e, j\}$	$\{a, b, c, d, e, h, j\}$	$\mathcal{A}$

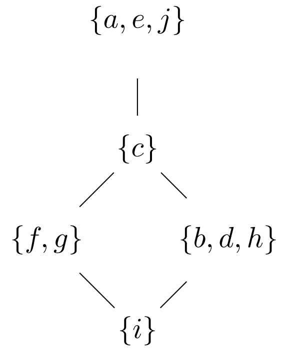


Fig. 2. The indefinite iteration of the graded defense of the empty set for all the values of $m, n \in \{1, 2, 3\}$ and the partial preorder, between arguments of AF_c, returned by IGD semantics and represented by a Hasse diagram.

3.9 Counting Semantics

Pu et al. [33] introduced the counting semantics which allows to rank arguments by counting the number of their respective attackers and defenders. However, contrary to the tuples-based semantics which only focuses on the branches, the counting semantics takes into account a large part of paths that leads to a given argument (and which continues the process even if a difference is found contrary to the Discussion-based semantics). In order to assign a value to each argument, they consider an argumentation framework as a dialogue game between proponents of a given argument x (*i.e.* the defenders of x) and opponents of x (*i.e.* the attackers of x). The idea is that an argument is more acceptable if it has many arguments from proponents and few arguments from opponents.
Formally, a given AF is first converted into a matrix $M_{n\times n}$ (where n is the number of arguments in the AF) which corresponds to the adjacency matrix of the AF (as an AF is a directed graph). The particularity of this matrix is that the matrix product of k copies of M, denoted by M^k, represents, for all the arguments in the AF, their number of defenders (if k is even) or attackers (if k is odd) situated at the beginning of a path of length k. Thanks to this method, they positively count all defenders and negatively count all attackers. Finally, a normalization factor N (*e.g.* the matrix infinite norm [27]) is applied to M in order to guarantee the convergence, and a damping factor α is used to have a more refined treatment on different length of attackers and defenders (*i.e.* shorter attackers and defenders are preferred).

Definition 21 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $\mathcal{A} = \{x_1, \ldots, x_n\}$, $\alpha \in]0, 1[$ be a damping factor and $k \in \mathbb{N}$. The n-dimensional column vector w over $\mathcal{A}$ at step k is defined by,*

$$w_{\alpha}^{k} = \sum_{i=0}^{k} (-1)^i \alpha^i \tilde{M}^i I$$

where $\tilde{M}$ is the normalized matrix such that $\tilde{M} = M/N$ with N as normalization factor and I the n-dimensional column vector containing only 1s.

The **counting model** *of AF is* $w_\alpha = \lim_{k \to +\infty} w_\alpha^k$*. The strength value of* $x_i \in \mathcal{A}$ *is the* i^{th} *component of* w_α*, denoted by* $w(x_i)$*.*

In [31], they deepen their work by presenting some complements about how the damping factor α allows to control the convergence speed of the computation for the counting semantics.
Following the previous definition, for any argumentation framework, the counting model can range the strength value of each argument into the interval $[0, 1]$. Thus, an argument is at least as acceptable as another argument if and only if its strength value is equal or higher than the strength value of the other argument.

Definition 22 *The* **counting semantics (CS)** *associates to any argumentation framework* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq_{AF}^{CS}$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$,

$$x \succeq_{AF}^{CS} y \text{ iff } w(x) \geq w(y)$$

Example 1 (cont.) *Let* $N = ||M||_\infty$ *be the normalization factor (i.e. the maximum absolute row sum of the matrix) and* $\alpha = 0.9$*. The strength value of each argument in* AF_c *(Fig. 1) is* $w(a) = w(e) = w(j) = 1$, $w(c) = 0.7525$, $w(b) = w(d) = w(h) = 0.55$, $w(f) = w(g) = 0.505$, *and* $w(i) = 0.1$*. Thus, we obtain the following ranking:*

$$a \simeq^{CS} e \simeq^{CS} j \succ^{CS} c \succ^{CS} b \simeq^{CS} d \simeq^{CS} h \succ^{CS} f \simeq^{CS} g \succ^{CS} i$$

The main observation here is that an argument attacked once but not defended (like b, d or h) is ranked higher than an argument attacked twice but defended (like f or g).

3.10 Propagation Semantics

Bonzon et al. [12] propose a family of semantics which rely on attackers and defenders (taking into account both quantity and quality) but, unlike the other semantics, also put a stronger emphasis on the non-attacked arguments. As the non-attacked arguments play a key role in the Dung's classical acceptability of an argument, it could be interesting to highlight non-attacked arguments in the process of ranking arguments. Thus, six new semantics based on the idea of propagation are introduced. Each argument has an initial value that depends on its status (non-attacked arguments have a greater value than attacked ones), and then these values are progressively propagated into the argumentation framework. At each propagation step, the polarity of the value changes according to the considered path (negatively if it is an attack path, positively if it is a defense one). The difference between the semantics lies in the method that is chosen to give more importance to the non-attacked arguments at the expense of attacked arguments, and on the choice of considering one or all paths between arguments. Indeed, the use of sets to select the attackers and defenders allows to focus on the arguments at the end of the path whereas the multiset focuses on the several possible paths which could exist between two arguments. Thus, in order to choose one among the two cases, a parameter $\oplus \in \{M, S\}$

is added when the path is selected (see Notation 2 and 3), where M (resp. S) stands for multiset (resp. set). For each argument, the weights from its attackers and defenders are accumulated and stored.

Definition 23 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an AF and $\oplus \in \{M, S\}$. The valuation P of $x \in \mathcal{A}$, at step i, is given by:*

$$P_i^{\epsilon,\oplus}(x) = \begin{cases} v_\epsilon(x) & if\ i = 0 \\ P_{i-1}^{\epsilon,\oplus}(x) + (-1)^i \sum\limits_{y \in \mathcal{R}_i^{\oplus}(x)} v_\epsilon(y) & otherwise \end{cases}$$

where $v_\epsilon : \mathcal{A} \to \mathbb{R}^+$ is a valuation function giving an initial weight to each argument, with $\epsilon \in [0, 1]$ such that $\forall y \in \mathcal{A}$, $v_\epsilon(y) = 1$ if $\mathcal{R}_1^{\oplus}(y) = \emptyset$; $v_\epsilon(y) = \epsilon$ otherwise.
The **Propagation vector of** *x is denoted $P^{\epsilon,\oplus}(x) = \langle P_0^{\epsilon,\oplus}(x), P_1^{\epsilon,\oplus}(x), \dots \rangle$.*

Bonzon et al. propose three possible ways of using the lexicographical order to compare the different propagation vectors. The first one just compares the propagation vectors for a given ϵ which allows to control the impact of the attacked arguments (the closer the value of ϵ is to 1, the higher the impact of attacked arguments).

Definition 24 *Let $\oplus \in \{M, S\}$ and $\epsilon \in]0, 1]$. The ranking-based semantics $Propa_\epsilon^{\epsilon,\oplus}$ associates to any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{AF}^P$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,*

$$x \succeq_{AF}^{P^{\epsilon,\oplus}} y \text{ iff } P^{\epsilon,\oplus}(x) \succeq_{lex} P^{\epsilon,\oplus}(y)$$

Clearly, the non-attacked arguments can have more influence when they propagate their value than the attacked arguments (or similarly the attacked arguments have less influence when they propagate their value). But this influence is variable and depends on the value of ϵ. Indeed, if the value of ϵ is close to 1, then the value propagated by the non-attacked arguments and the value propagated by the attacked arguments are almost the same. This implies, in this case, that the difference for an argument between being attacked (or defended) by a non-attacked argument and being attacked (or defended) by an attacked argument, is weak. And conversely, if the value of ϵ is close to 0, then the influence of the non-attacked arguments will be high. Consequently, for this semantics, two values of ϵ can lead to different rankings.

In order to avoid this, the second semantics splits (with the operator $\cup_s$) and lexicographically compares the influence of the two kinds of arguments.

Definition 25 *The* **shuffle** *$\cup_s$ between two vectors of real numbers $V = \langle V_1, \dots, V_n \rangle$ and $V' = \langle V'_1, \dots, V'_n \rangle$ is defined as $V \cup_s V' = \langle V_1, V'_1, V_2, V'_2, \dots, V_n, V'_n \rangle$.*

The goal of this semantics is to simultaneously look at the result of the two propagation vectors $P^{0,\oplus}$ and $P^{\epsilon,\oplus}$ step by step, using the shuffle operation, starting with the first value of the propagation vector $P^{0,\oplus}$ (*i.e.* the one that only takes into account non-attacked arguments). In the case where two arguments are still equally acceptable, we compare the first value of the propagation vector $P^{\epsilon,\oplus}$. Then, in case of equality, we move to the second step and so on.

Definition 26 *Let* $\oplus \in \{M, S\}$ *and* $\epsilon \in]0, 1]$. *The ranking-based semantics* $Propa_{1+\epsilon}^{\epsilon,\oplus}$ *associates to any* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq_{AF}^{\widehat{P}}$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$,

$$x \succeq_{AF}^{\widehat{P}^{\oplus}} y \text{ iff } P^{0,\oplus}(x) \cup_s P^{\epsilon,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \cup_s P^{\epsilon,\oplus}(y)$$

It is also important to notice that $Propa_{1+\epsilon}$, conversely to $Propa_{\epsilon}$, returns the same ranking whatever the value of ϵ, that removes the problem of choosing "a good" ϵ.

Finally, the last semantics only propagates the weights of non-attacked arguments into the graph (it is possible when $\epsilon = 0$) which gives them the highest priority among the three proposed semantics. And if two arguments are equivalent for this comparison, they are compared using the $Propa_{\epsilon}$ method.

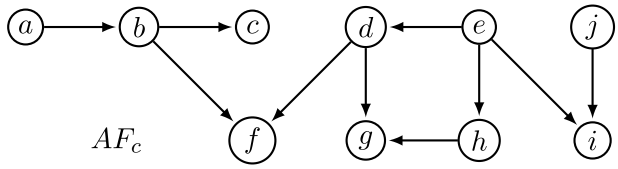
Definition 27 *Let* $\oplus \in \{M, S\}$ *and* $\epsilon \in]0, 1]$. *The ranking-based semantics* $Propa_{1\to\epsilon}^{\epsilon,\oplus}$ *associates to any* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *a ranking* $\succeq_{AF}^{\overline{P}}$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$,

$$x \succeq_{AF}^{\overline{P}^{\oplus}} y \text{ iff } P^{0,\oplus}(x) \succ_{lex} P^{0,\oplus}(y) \text{ or } (P^{0,\oplus}(x) \simeq_{lex} P^{0,\oplus}(y) \textit{ and } P^{\epsilon,\oplus}(x) \succeq_{lex} P^{\epsilon,\oplus}(y))$$

Example 1 (cont.) *All the details for computing and explaining the rankings returned by the different propagation semantics on* AF_c *(Fig. 1) can be found in [12].*

4 An Experimental Comparison of Ranking-based Semantics

As can be easily checked with the rankings obtained with the different ranking-based semantics on AF_c (the AF and the results are given in Fig. 3), the ranking-based semantics we presented here mostly return distinct rankings between arguments. However, this difference concerns a subset of arguments (here b, c, d, f, g, h) and not all the arguments. Conversely, some common behaviors seem to appear between the semantics like the fact that a, e and j are always equally acceptable and more acceptable than all the other arguments (except for the semantics FL) or that i is always ranked last (even if it can be a tie). Thus, it could be interesting to know if such information can be generalized to all the argumentation frameworks. The goal of this section consists in evaluating experimentally how different or similar these ranking-based semantics are. To this purpose, we choose to compute and to compare the ranking of each ranking-based semantics on several randomly generated argumentation frameworks. But before explaining how to compute and compare the different rankings, we choose to exclude some ranking-based semantics from this study. Indeed, it is difficult to compare total and partial preorders (because some arguments could be incomparable), it is why the semantics that return a partial preorder, like IGD [25] and Tuples [16], are excluded. The semantics 2ZG [30] is also excluded from



Semantics	*Ranking between arguments*
FL	$a \simeq c \simeq e \simeq f \simeq g \simeq j \succ b \simeq d \simeq h \simeq i$
2ZG	$a \simeq e \simeq j \succ c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$
Cat	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq f \simeq g \simeq h \succ i$
1-Bbs	
Dbs	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \simeq g \succ i$
Bbs	
0.5-Bbs	
CS	
$\text{Propa}_{\epsilon}^{0.75,M}$	
$\text{Propa}_{\epsilon}^{0.75,S}$	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \succ g \succ i$
5-Bbs	$a \simeq e \simeq j \succ c \succ f \simeq g \succ b \simeq d \simeq h \succ i$
$\text{Propa}_{\epsilon}^{0.3,M}$	
$\text{Propa}_{1+\epsilon}^{\epsilon,M}$	
$\text{Propa}_{\epsilon}^{0.3,S}$	$a \simeq e \simeq j \succ c \succ f \succ g \succ b \simeq d \simeq h \succ i$
$\text{Propa}_{1+\epsilon}^{\epsilon,S}$	
$\text{Propa}_{1\to\epsilon}^{\epsilon,S}$	$a \simeq e \simeq j \succ f \succ c \succ g \succ b \simeq d \simeq h \succ i$
Tuples	$a \simeq e \simeq j \succ f \simeq g \succ c \succ b \simeq d \simeq h \succ i$
$\text{Propa}_{1\to\epsilon}^{\epsilon,M}$	
IGD	$a \simeq e \simeq j \succ c \succ f \simeq g \succ i$ $a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ i$

Fig. 3. Rankings obtained on AF_c with some existing ranking-based semantics.

this study because, according to the authors, this semantics can only be used for argumentation frameworks with less than a dozen of arguments (see [30]). Indeed, the size of the players strategy spaces grows exponentially fast with the total number of arguments in the argumentation framework considered so when it contains more than twelve arguments, the computation becomes almost impossible.

4.1 Benchmarks

To experimentally compare the rankings returned by the ranking-based semantics, we considered a significantly large experimental setting with a great variety of benchmarks. This set is separated into two parts: the first one contains AFs similar to those used in online debates and the second one contains AFs generated from standard random AF generators (Erdös-Rényi, Barabasi-Albert and Watts-Strogatz).

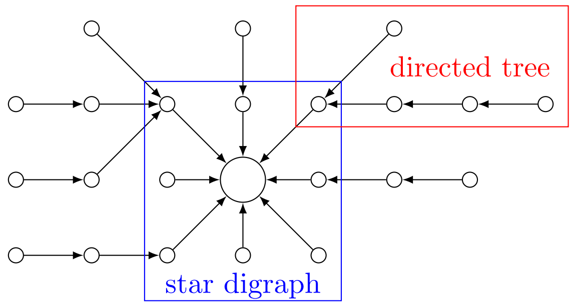


Fig. 4. A debate graph.

4.1.1 Debate Graphs

As mentioned in the introduction and in the literature (e.g. [22,1]), ranking-based semantics (and gradual semantics) seem to be a promising tool to evaluate online debates. We therefore chose to randomly generate argumentation frameworks [7] with the same characteristics as the most popular online debates. This type of graph is characterised by a specific argument (the issue) which plays the role of the main question of the debate and several branches converging towards this argument with the condition that all arguments must be connected to this issue. In order to build such a graph (an instance is illustrated in Fig. 4), we create a new generator using several tools from the networkx package [8]. First, a directed star graph with $n + 1$ nodes is created where the central node is the target argument connected to n outer nodes (i.e. n arguments which directly attack the target argument). For each node representing a branch of the star, a directed tree containing m additional nodes is generated where edges are oriented into this node. For our experiment, we generated 10000 debate graphs whose value of the integer n varies randomly between 6 and 15 and whose value of the integer m varies randomly between 1 and 6. In the following, we collectively refer to this group of AFs as `debateGraph10000`. Fig. 5 shows the distribution of the number of arguments in the benchmark `debateGraph10000`.

4.1.2 Random Graphs

We also consider an experimental setting representing three different models used during the ICCMA competition (`http://argumentationcompetition.org/`) as a way to generate random argumentation graphs:

(1) the Erdös-Rényi model (ER) which generates graphs by randomly selecting attacks between arguments;

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[7] Ideally, we would have liked to create a set of AFs from real online debates but actual online debates have additional features (e.g. a support relation, or votes on arguments) that are not taken into account by the semantics studied in this paper.

[8] `https://networkx.org`

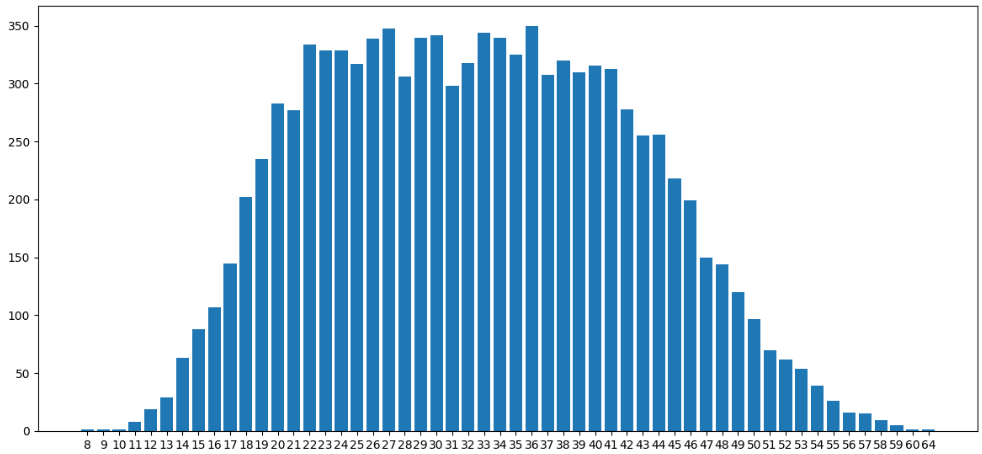

Fig. 5. Histogram showing the distribution of the number of arguments in the benchmark `debateGraph10000`. The average number of arguments is 32.5176.

(2) the Barabasi-Albert model (BA) which provides networks, called scale-free networks, with a structure in which some nodes have a huge number of links, but in which nearly all nodes are connected to only a few other nodes; and
(3) the Watts-Strogatz model (WS) which produces graphs which have small-world network properties, such as high clustering and short average path lengths.

The generation of these three types of AFs was done by the AFBenchGen2 generator [17]. We generated a total of 9460 AFs almost evenly distributed between the three models (3000 AFs for the WS model and 3230 AFs for the ER and BA model). For each model, the number of arguments varies among $\texttt{Arg} = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. The parameters used to generate graphs are as follows: for ER, 19 random instances for each (numArg, probAttacks) in $\texttt{Arg} \times \{0.15, 0.2, \ldots, 0.95\}$; for BA, 17 random instances for each (numArg, probCycles) in $\texttt{Arg} \times \{0, 0.05, 0.1, \ldots, 0.9\}$; for WS, (numArg, probCycles, β, $\mathcal{K}$) in $\texttt{Arg} \times \{0.25, 0.5, 0.75\} \times \{0, 0.25, 0.5, 0.75, 1\} \times \{k \in 2\mathbb{N}\ s.t.\ 2 \leq k \leq |\texttt{Arg}| - 1\}$. We refer the reader to [17] for the meaning of the parameters.
In the following, we collectively refer to the group of AFs generated using the Erdös-Rényi model (resp. Barabasi-Albert model and Watts-Strogatz model) as `rER` (resp. `rBA` and `rWS`). Finally, the notation `randomAF` refers to the union of these three groups.

4.2 Comparison of Two Rankings

Let us now detail how we compare these rankings in order to represent the concordance of the ranking-based semantics. A way to compare these semantics on the basis of the rankings previously computed consists in using the Kendall's tau coefficient [28]. This value corresponds to the total number of rank disagreements over all unordered pairs of arguments between two rankings from distinct semantics. It therefore allows us to obtain a dissimilarity degree between two rankings.

Definition 28 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and σ_1,σ_2 be two ranking-based semantics. The* **Kendall's tau coefficient** *between $\sigma_1(AF)$ and $\sigma_2(AF)$ is calculated as follow:*

$$K(\sigma_1(AF), \sigma_2(AF)) = \frac{\sum_{\{i,j\} \in \mathcal{A}} \overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF))}{0.5 \times |\mathcal{A}| \times (|\mathcal{A}| - 1)}$$

with:

- $\overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 0$ *if* $i \succ_{AF}^{\sigma_1} j$ *and* $i \succ_{AF}^{\sigma_2} j$*, or* $i \prec_{AF}^{\sigma_1} j$ *and* $i \prec_{AF}^{\sigma_2} j$*, or* $i \simeq_{AF}^{\sigma_1} j$ *and* $i \simeq_{AF}^{\sigma_2} j$*,*
- $\overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 1$ *if* $i \succ_{AF}^{\sigma_1} j$ *and* $i \prec_{AF}^{\sigma_2} j$ *or vice versa,*
- $\overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 0.5$ *if* $i \succ_{AF}^{\sigma_1} j$ *or* $i \prec_{AF}^{\sigma_1} j$ *and* $i \simeq_{AF}^{\sigma_2} j$ *or vice versa.*

For example, the Kendall's tau coefficient between the rank orders $\tau_{\sigma_1} = a \succ_{\sigma_1} b \simeq_{\sigma_1} c \succ_{\sigma_1} d \succ_{\sigma_1} e$ and $\tau_{\sigma_2} = a \succ_{\sigma_2} c \succ_{\sigma_2} d \succ_{\sigma_2} b \succ_{\sigma_2} e$ is 0.15 because $\sum_{\{i,j\} \in \mathcal{A}} \overline{K_{i,j}}(\tau_{\sigma_1}, \tau_{\sigma_2}) = 1.5$ (since the orders disagree on pair $\{b, d\}$ and become strict on pair $\{b, c\}$) and $|\mathcal{A}| = 5$.
A Kendall's tau coefficient of 1 between two rankings means that both rankings are opposite (*i.e.* $\forall x, y \in \mathcal{A}$, if $x \succ^{\sigma_1} y$ then $y \succ^{\sigma_2} x$) while a Kendall's tau coefficient of 0 means that both rankings are identical. So, the smaller the Kendall's tau coefficient between two rankings, the higher their similarity.

4.3 Results

From the rankings computed for each the argumentation framework in input, we compute the Kendall's tau coefficient between all pairs of rankings. [9] Finally, for each pair of ranking-based semantics, we average the Kendall's tau coefficients computed from rankings for each argumentation framework and multiply the result by 100 to obtain a percentage of dissimilarity. All the results are given in a symmetric matrix (Table 1). Thus, the biggest dissimilarity degree between two ranking-based semantics is observed between the discussion-based semantics (Dbs) and the fuzzy labellings (FL) with a value of 25.43%. FL clearly stands out from the other semantics with a degree of dissimilarity always greater than 24%. But, globally, the others ranking-based semantics seems to share a solid common basis with a dissimilarity degree often smaller than 10%.
In order to better represent the "closeness" between these ranking-based semantics, from the previous matrix, we compute a dendrogram, which is a graphical representation of the results of hierarchical cluster analysis. In our case, the method used is a stepwise algorithm for n semantics which merges two semantics or clusters with the least dissimilarities at each step until obtaining a unique cluster. Several operators exist [34] to compute the

[9] The code and benchmarks are available online at
`https://github.com/jeris90/comparison_rankingsemantics.git`

	Cat	Bbs	Dbs	0.3-Bbs	1-Bbs	10-Bbs	FL	CS	$\textbf{Propa}_{\epsilon}^{0.5,S}$	$\textbf{Propa}_{\epsilon}^{0.5,M}$	$\textbf{Propa}_{1+\epsilon}^{0.5,S}$	$\textbf{Propa}_{1+\epsilon}^{0.5,M}$	$\textbf{Propa}_{1\rightarrow\epsilon}^{0.5,S}$	$\textbf{Propa}_{1\rightarrow\epsilon}^{0.5,M}$
Cat	0	1.88	2.00	1.56	0	2.21	25.09	1.55	2.84	1.94	3.38	2.48	7.63	7.27
Bbs	1.88	0	0.77	2.48	1.88	2.13	25.37	1.44	2.43	1.22	3.01	1.79	7.44	6.79
Dbs	2.00	0.77	0	2.49	2.00	2.40	25.43	0.97	2.45	0.67	3.04	1.25	7.48	6.25
0.3-**Bbs**	1.56	2.48	2.49	0	1.56	3.28	25.33	2.08	3.68	2.93	4.24	3.49	8.44	8.22
1-**Bbs**	0	1.88	2.00	1.56	0	2.21	25.09	1.55	2.84	1.94	3.38	2.48	7.63	7.27
10-**Bbs**	2.21	2.13	2.40	3.28	2.21	0	24.55	2.46	2.97	1.76	2.40	1.19	6.17	5.46
FL	25.09	25.37	25.43	25.33	25.09	24.55	0	25.35	24.40	24.94	24.16	24.71	24.32	25.06
CS	1.55	1.44	0.97	2.08	1.55	2.46	25.35	0	2.71	1.47	3.30	2.05	7.62	6.94
$\textbf{Propa}_{\epsilon}^{0.5,S}$	2.84	2.43	2.45	3.68	2.84	2.97	24.40	2.71	0	1.86	0.60	2.43	5.06	7.40
$\textbf{Propa}_{\epsilon}^{0.5,M}$	1.94	1.22	0.67	2.93	1.94	1.76	24.94	1.47	1.86	0	2.41	0.60	6.84	5.60
$\textbf{Propa}_{1+\epsilon}^{0.5,S}$	3.38	3.01	3.04	4.24	3.38	2.40	24.16	3.30	0.60	2.41	0	1.84	4.47	6.82
$\textbf{Propa}_{1+\epsilon}^{0.5,M}$	2.48	1.80	1.25	3.49	2.48	1.19	24.71	2.05	2.43	0.60	1.84	0	6.28	5.03
$\textbf{Propa}_{1\rightarrow\epsilon}^{0.5,S}$	7.63	7.44	7.48	8.44	7.63	6.17	24.32	7.62	5.06	6.84	4.47	6.28	0	2.45
$\textbf{Propa}_{1\rightarrow\epsilon}^{0.5,M}$	7.27	6.79	6.25	8.22	7.27	5.46	25.06	6.94	7.40	5.60	6.82	5.03	2.45	0

Table 1
Average of Kendall's tau coefficients on `debateGraph10000` and `randomAF`. The darker the color of the cell, the greater the dissimilarity between the two ranking-based semantics.

distance between the new cluster and the other clusters like the single link (minimum), complete link (maximum), group average, median, etc. However, a few number of inputs make the differences negligible between these methods, so we choose the average method to compute the dendrogram illustrated in Fig. 6.
On this dendrogram, the height of the branch between two clusters indicates how different they are from each other: the greater the height, the greater the difference. Four groups emerge from this study: one containing the semantics Dbs, Bbs and CS (which have a dissimilarity degree always smaller than 1.5%), another one containing the semantics Cat, 0.3-Bbs and 1-Bbs (which have a dissimilarity degree always smaller than 1.56%), another one containing $Propa_{\epsilon}^{0.5,S}$ and $Propa_{1+\epsilon}^{S}$ (which have a dissimilarity degree equals to 0.6%), and the last one containing 10-Bbs and and $Propa_{\epsilon}^{0.5,M}$ and $Propa_{1+\epsilon}^{M}$ (which have a dissimilarity degree always smaller than 1.76%). The propagation semantics $Propa_{1\rightarrow\epsilon}$ seem closer to the third group of semantics with a dissimilarity degree between 4% and 6% with all these ranking-based semantics. Among these groups, one can observe that some semantics are very close like Bbs and Dbs with a dissimilarity value of 0.77%. An important observation is that the categoriser-based semantics and the α-Burden-based semantics always returns the same ranking (their dissimilarity degree is 0%) when $\alpha = 1$, as noticed in [4].

This empirical comparison allowed us to validate the hypotheses that despite some differences between the semantics (only two semantics return exactly the same ranking whatever the input AF), they seem to have similar behaviours when building the ranking of these arguments. Our aim is now to axiomatically compare these semantics in order to explain the common features and the differences observed in this section.

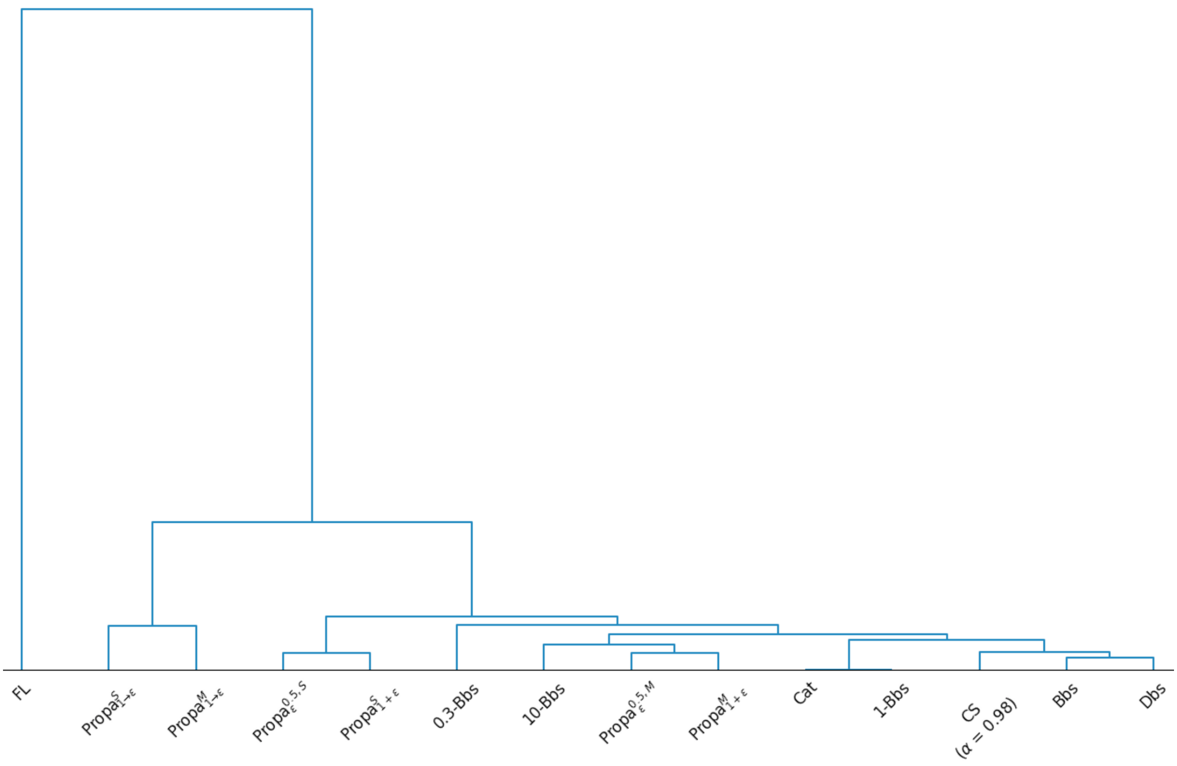

Fig. 6. Dendrogram representing the relationships between the ranking-based semantics studied in this work.

5 Properties for Ranking-based Semantics

In the last few years, a lot of properties have been proposed in different papers [16,30,2], allowing to better understand the behavior of the ranking-based semantics. Please note that we do not claim that all of these properties are mandatory (we will see later that some of them are incompatible), but we just list them and check which ones are satisfied by the existing ranking-based semantics. So in this section, we first recall the existing properties in the literature. Then we introduce new ones allowing to capture more information on the ranking semantics, and finally, check the incompatibilities/dependencies between the properties.

5.1 Existing Properties

Let us begin to introduce the basic idea and the formal definition of existing properties. For each property, we give the name, the intuitive explanation, the abbreviation and the formal definition. [10] Unless stated explicitly, all the properties are defined for a ranking-based semantics σ, $\forall AF \in \mathbb{AF}$ and $\forall a, b \in Arg(AF)$.

[10] For more information about a particular property, we point the reader to the paper where the property has been introduced.

Let us first introduce the notion of isomorphism:

Definition 29 *An* **isomorphism** γ *between two argumentation frameworks* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and AF' =* $\langle \mathcal{A}', \mathcal{R}' \rangle$ *is a bijective function* $\gamma : \mathcal{A} \rightarrow \mathcal{A}'$ *such that* $\forall x, y \in \mathcal{A}$, $(x, y) \in \mathcal{R}$ *iff* $(\gamma(x), \gamma(y)) \in \mathcal{R}'$. *With a slight abuse of notation, we will note* $AF' = \gamma(AF)$.

Abstraction [2] The ranking on arguments does not depend on the arguments' identities.
(Abs) Let $AF, AF' \in \mathbb{AF}$. For any isomorphism γ s.t. $AF' = \gamma(AF)$, we have $a \succeq^{\sigma}_{\mathrm{AF}} b$ iff $\gamma(a) \succeq^{\sigma}_{\mathrm{AF'}} \gamma(b)$

Let us now define the notion of connected component that we will use in the next property:

Definition 30 *The* **connected components** *of an AF are the set of largest subgraphs of AF, denoted by* $cc(AF)$, *where two arguments are in the same component of AF if and only if there is some path (ignoring the direction of the edges) between them.*

Independence [30,2] The ranking between two arguments a and b should be independent of any argument that is neither connected to a nor to b.
(In) $\forall AF' \in cc(AF), \forall a, b \in Arg(AF'),\ a \succeq^{\sigma}_{\mathrm{AF'}} b \Leftrightarrow a \succeq^{\sigma}_{\mathrm{AF}} b$

We may have expectations regarding the best and worst arguments that we may find in an AF:

Void Precedence [16,30,2] A non-attacked argument should be strictly more acceptable than an attacked argument.
(VP) $\mathcal{R}_1(a) = \emptyset$ and $\mathcal{R}_1(b) \neq \emptyset \Rightarrow\ a \succ^{\sigma}_{\mathrm{AF}} b$

Self-Contradiction [30] An argument that attacks itself should be strictly less acceptable than an argument that does not.
(SC) $(a, a) \notin \mathcal{R}$ and $(b, b) \in \mathcal{R}\ \Rightarrow\ a \succ^{\sigma}_{\mathrm{AF}} b$

The following local properties only focus on the direct attackers, or defenders, of arguments:

Cardinality Precedence [2] The greater the number of direct attackers for an argument, the weaker the level of acceptability of this argument.
(CP) $|\mathcal{R}_1(a)| < |\mathcal{R}_1(b)| \Rightarrow\ a \succ^{\sigma}_{\mathrm{AF}} b$

Quality Precedence [2] The greater the acceptability of one direct attacker for an argument, the weaker the level of acceptability of this argument.
(QP) $\exists c \in \mathcal{R}_1(b)$ s.t. $\forall d \in \mathcal{R}_1(a),\ c \succ^{\sigma} d \Rightarrow\ a \succ^{\sigma}_{\mathrm{AF}} b$

Before defining the next properties, we need to introduce a relation that compares sets of arguments on the basis of their rankings:

Definition 31 [2] *Let $\geq_S$ be a ranking on a set of arguments $\mathcal{A}$. For any $S_1, S_2 \subseteq \mathcal{A}$, $S_1 \geq_S S_2$ iff there exists an injective mapping f from S_2 to S_1 such that $\forall a \in S_2, f(a) \succeq a$. And $S_1 >_S S_2$ iff $S_1 \geq_S S_2$ and ($|S_2| < |S_1|$ or $\exists a \in S_2, f(a) \succ a$).*

Counter-Transitivity [2] If the direct attackers of b are at least as numerous and acceptable as those of a, then a is at least as acceptable as b.
(CT) $\mathcal{R}_1(b) \geq_S \mathcal{R}_1(a) \Rightarrow a \succeq^{\sigma}_{\text{AF}} b$

Strict Counter-Transitivity [2] If the direct attackers of b are at least as numerous and acceptable as those of a and either the direct attackers of b are strictly more numerous or acceptable than those of a, then a is strictly more acceptable than b.
(SCT) $\mathcal{R}_1(b) >_S \mathcal{R}_1(a) \Rightarrow a \succ^{\sigma}_{\text{AF}} b$

Defense Precedence [2] For two arguments with the same number of direct attackers, a defended argument should be strictly more acceptable than a non-defended argument.
(DP) $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)|, \mathcal{R}_2(a) \neq \emptyset$ and $\mathcal{R}_2(b) = \emptyset \Rightarrow \; a \succ^{\sigma}_{\text{AF}} b$

Definition 32 [2] *The* **defense** *of a is* **simple** *iff every defender of a attacks exactly one direct attacker of a. The defense of a is* **distributed** *iff every direct attacker of a is attacked by at most one argument.*

Distributed-Defense Precedence [2] A defense where each defender attacks a distinct attacker is better than any other.
(DDP) $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)|$ and $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)|$, if the defense of a is simple and distributed and the defense of b is simple but not distributed, then $a \succ^{\sigma}_{\text{AF}} b$

5.2 Generalized Properties

Cayrol and Lagasquie-Schiex [16] introduced properties checking if some change related to the branches in an argumentation framework can improve or degrade the ranking of one argument. Indeed, what is the effect on the acceptability of a given argument with an additional attack branch? Is the effect the same if it is a defense branch? Does the length of the branch matter? Such questions seem interesting to answer in order to better understand the behavior of semantics. However, these properties have been proposed informally, in the context of the tuples-based semantics. This is why we propose a formal definition of these properties, that generalizes them for any argumentation framework. First of all, let us introduce how we formally define the addition of an attack branch and the addition of a defense branch to an argument.

Definition 33 *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$ be an argument. The defense branch added to x is $P_+(x) = \langle \mathcal{A}', \mathcal{R}' \rangle$, with $\mathcal{A}' = \{x_0, \ldots, x_n\}$ such that $n \in 2\mathbb{N}$, $x_0 = x$, $\mathcal{A} \cap \mathcal{A}' = \{x\}$, and $\mathcal{R}' = \{(x_i, x_{i-1}) \mid i \leq n\}$. The attack branch added to x, denoted $P_-(x)$ is defined similarly except that the sequence is of odd length (i.e. $n \in 2\mathbb{N} + 1$).*

In order to evaluate the impact of an additional attack (or defense) branch on a given argument x of an argumentation framework AF, we "clone" this AF with an isomorphism γ. Then, we can modify the argumentation framework $\gamma(AF)$ and analyse the impact on $\gamma(x)$ compared to x.
Thus, the following properties are defined $\forall AF, AF^\gamma \in \mathbb{AF}$ such that there exists an isomorphism γ with $AF^\gamma = \gamma(AF)$, and $\forall a \in Arg(AF)$. We use AF^γ as a clone of AF.

Let us first check the consequences of the addition of an attack or a defense branch on an argument with the three following properties.

Addition of an Attack Branch. Adding an attack branch to any argument decreases its level of acceptability.
(+AB) If $AF^\star = AF \cup AF^\gamma \cup P_-(\gamma(a))$, then $a \succ^\sigma_{\text{AF}^\star} \gamma(a)$

Strict addition of a Defense Branch. Adding a defense branch to any argument increases its level of acceptability.
(⊕DB) If $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(a))$, then $\gamma(a) \succ^\sigma_{\text{AF}^\star} a$

Addition of a Defense Branch. It could make sense to treat differently non-attacked arguments. It is why, in [16], this property is defined in a more specific way: adding a defense branch to any attacked argument increases its level of acceptability.
(+DB) If $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(a))$ and $\mathcal{R}_1(a) \neq \emptyset$, then $\gamma(a) \succ^\sigma_{\text{AF}^\star} a$

Let us now define the properties based on the increase of the length of a branch. Formally, increasing the length of a branch consists in adding a defense branch [11] to the argument at the beginning of the branch.

Increase of an Attack branch. Increasing the length of an attack branch of an argument increases its level of acceptability.
(↑AB) If $b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(b))$, then $\gamma(a) \succ^\sigma_{\text{AF}^\star} a$

[11] We add here a defense branch in order to leave the "role" of the branch unchanged: a defense (respectively attack) branch stays a defense (respectively attack) branch.

Increase of a Defense branch. Increasing the length of a defense branch of an argument decreases its level of acceptability.
(↑DB) If $b \in \mathcal{B}_+(a)$, $b \notin \mathcal{B}_-(a)$ and $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(b))$, then $a \succ^\sigma_{\mathrm{AF}^\star} \gamma(a)$

5.3 Additional Properties

To this set of properties from the literature we want to add some other interesting properties.

The first one, called *Total*, allows to make a distinction between the semantics which return a total preorder or a partial preorder between arguments. Indeed, too many incompatibilities can be problematic, especially if we want to use argumentation for decision-making or for the online debate platforms (see the discussion in [29]), the users could be requested to give arguments for or against two opposite topics in order to compare them and know which one is the most popular. Thus, it could be frustrating for the users to obtain an incomparability between both arguments after spending time deliberating. In this case, one will prefer to select a semantics that satisfies Total.

Total. All pairs of arguments can be compared.
(Tot) $a \succeq^\sigma_{\mathrm{AF}} b$ *or* $b \succeq^\sigma_{\mathrm{AF}} a$

In order to introduce the next property, let us define the ancestor's graph of an argument which contains all its attackers and defenders and their relation:

Definition 34 *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $a \in \mathcal{A}$*. The* ***ancestor's graph*** *of* a *is denoted by* $Anc(a) = \langle \mathcal{A}', \mathcal{R}' \rangle$ *with* $\mathcal{A}' = \{a\} \cup \mathcal{R}_+(a) \cup \mathcal{R}_-(a)$ *and* $\mathcal{R}' = \{(a_1, a_2) \in \mathcal{R} \mid a_1 \in \mathcal{A}'$ *and* $a_2 \in \mathcal{A}'\}$*.*

Argument Equivalence states that the acceptability of an argument depends only on (the structure of) its attackers and defenders. This property is related to a well-known property satisfied by the classical semantics, called *Directionality* [6], which states that an argument can only be affected by arguments following the direction of the attacks (i.e. an argument a cannot be affected by another argument b if there exists no path from b to a).

Argument Equivalence If there exists an isomorphism between the ancestors' graph of two arguments, then they are equally acceptable.
(AE) For any isomorphism γ s.t. $Anc(a) = \gamma(Anc(b))$ then $a \simeq^\sigma_{\mathrm{AF}} b$
Please note that the reverse is not true because two arguments can be equally acceptable but with different ancestors' graphs.

The property *Non-attacked Equivalence* is a particular case of Argument Equivalence because it focuses on the comparison between the non-attacked arguments. Indeed, if the arguments are affected only by the arguments in their ancestors' graph, then the non-attacked arguments should be unaffected by the remaining part of the argumentation framework (because they have no attackers or defenders). Thus, they should have the same ranking. If one agrees with this idea then Non-attacked Equivalence must be satisfied.

Non-attacked Equivalence All the non-attacked arguments should have the same rank.
(NaE) $\mathcal{R}_1(a) = \emptyset$ *and* $\mathcal{R}_1(b) = \emptyset \Rightarrow a \simeq^{\sigma}_{\text{AF}} b$

Another intuitive possibility to detect when two arguments are equally acceptable consists in just taking into account their direct attackers.

Ordinal Equivalence If two arguments have the same number of direct attackers and for each direct attacker of one argument, there exists a direct attacker of the other argument such that both are equally acceptable then the two arguments are equally acceptable too.
(OE) If there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall c \in \mathcal{R}_1(a), c \simeq^{\sigma}_{\text{AF}} f(c)$ then $a \simeq^{\sigma}_{\text{AF}} b$

The last property describes the behavior adopted by a semantics concerning the notion of defense, which plays a key role in the obtained ranking. Indeed, in Fig. 1, while c, which is defended once, is always more acceptable than b which is directly attacked by a non-attacked argument, if we compare b and f, which has two distinct defense branches, we can remark, that for some semantics, b is either more acceptable (e.g. Dbs), equally acceptable (e.g. Cat) or less acceptable (e.g. $Propa_{1\to\epsilon}$) than f. However, existing properties which concern the defense (e.g. DP, DDP, +DB) are not able to say if a defense cancels the effect of an attack or just weakens this attack. It is why we introduce the property *Attack vs Full Defense.*

Attack vs Full Defense. An argument without any attack branch is ranked higher than an argument only attacked by one non-attacked argument.
(AvsFD) AF is acyclic, $|\mathcal{B}_-(a)| = 0, |\mathcal{R}_1(b)| = 1$ and $|\mathcal{R}_2(b)| = 0 \Rightarrow a \succ^{\sigma}_{\text{AF}} b$

For example, as illustrated in Fig. 7, the property states that an argument which is (only) attacked once by a non-attacked argument (it is the case of b only attacked by b_1) is worse than an argument that has any number of attacks that all belong to defense branches (it is the case of a which has four defense branches and no attack branch).

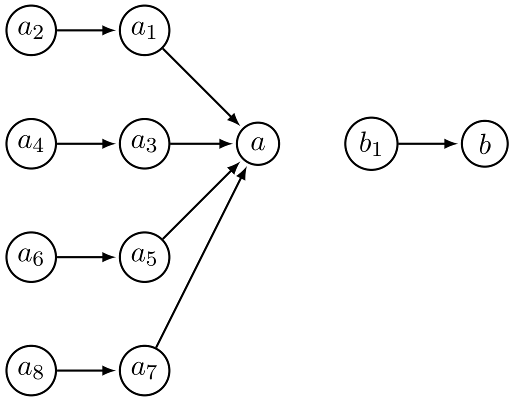

Fig. 7. AF that illustrates the property AvsFD.

5.4 Relationships between Properties

Each property studied in this paper aims to capture a particular principle. However, some of them focus on the same aspect of the argumentation framework (*e.g.* direct attackers, the number of defenders). Thus, one can wonder whether some overlaps exist between them. Conversely, one can also wonder whether some additional incompatibilities exist. To this purpose, we continue the work initiated in [2,8] about the incompatibilities and the dependencies between properties. All the results obtained in this section are summed up in Figure 8.

Let us first recall when two properties are incompatible (*i.e.* they cannot be simultaneously satisfied).

Definition 35 *Two properties are* ***incompatible*** *if there exists an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $x, y \in \mathcal{A}$ such that when one property states that $x \succ_{AF} y$, the other property states that $y \succeq_{AF} x$.*

The next proposition recalls some results, and proves new ones, about the incompatibility of some properties.

Proposition 1 *For every ranking-based semantics, the following pairs of properties are incompatible:*

(1) Cardinality Precedence (CP) and Quality Precedence (QP) [2]
(2) Self-Contradiction (SC) and Cardinality Precedence (CP) [8]
(3) Self-Contradiction (SC) and Counter-Transitivity (CT) [8]
(4) Self-Contradiction (SC) and Strict Counter-Transitivity (SCT) [8]
(5) Cardinality Precedence (CP) and Attack vs Full Defense (AvsFD)
(6) Cardinality Precedence (CP) and Addition of a Defense Branch (+DB)
(7) Cardinality Precedence (CP) and Strict Addition of a Defense Branch (⊕DB)
(8) Void Precedence (VP) and Strict Addition of a Defense Branch (⊕DB)
(9) Argument Equivalence (AE) and Self-Contradiction (SC)

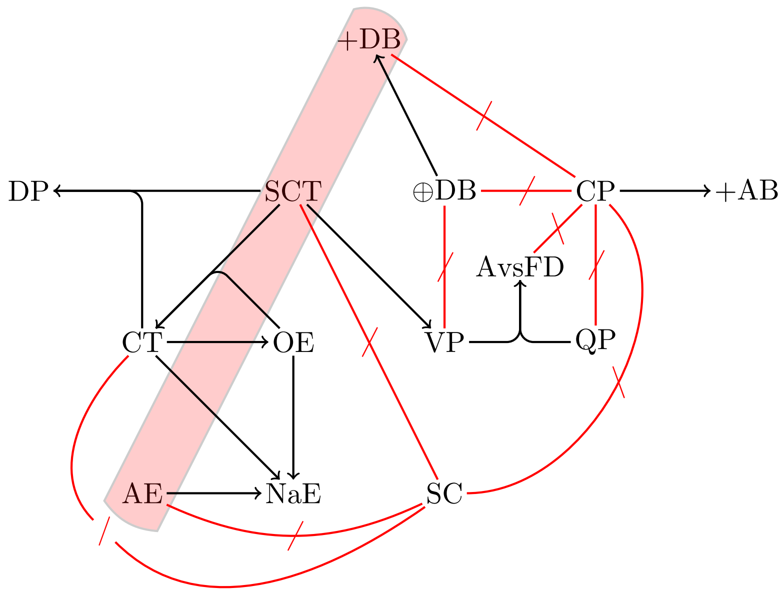

Fig. 8. Graph which represents the relation between properties ($X \rightarrow Y$ means that X implies Y, $X \not\!\!-\!\!- Y$ means that X and Y are not compatible and the properties into the red rectangle cannot be simultaneously satisfied).

Proposition 2 *No ranking-based semantics can simultaneously satisfy Addition of a Defense Branch (+DB), Strict Counter-Transitivity (SCT) and Argumentation Equivalence (AE).*

Some of these results are not surprising. Indeed, some properties have different views on the notion of defense (see the discussion in the previous section when we introduced the property AvsFD). It is the case, for example, with the properties CP and SCT which consider that any additional (defense or attack) branch should have a negative effect on a given argument while +DB or ⊕DB state that an additional defense branch should have a positive impact on this argument.

Then, let us first define when a property implies another property.

Definition 36 *A property P **implies** another property Q if and only if for any ranking-based semantics σ, if σ satisfies P then σ satisfies Q.*

The next proposition recalls some results, and proves new ones, about the implication between properties.

Proposition 3

(1) Strict Counter-Transitivity (SCT) implies Void Precedence (VP) [2]
(2) Counter-Transitivity (CT) and Strict Counter-Transitivity (SCT) imply Defense Precedence (DP) [2]
(3) Counter-Transitivity (CT) implies Non-attacked Equivalence (NaE)

(4) Counter-Transitivity (CT) implies Ordinal Equivalence (OE)
(5) Strict Counter-Transitivity (SCT) and Ordinal Equivalence (OE) imply Counter-Transitivity (CT)
(6) Strict Addition of Defense Branch (⊕DB) implies Addition of a Defense Branch (+DB)
(7) Argument Equivalence (AE) implies Non-attacked Equivalence (NaE)
(8) Ordinal Equivalence (OE) implies Non-attacked Equivalence (NaE)
(9) Void Precedence (VP) and Quality Precedence (QP) imply Attack vs Full Defense (AvsFD)
(10) Cardinality Precedence (CP) implies Addition of an Attack Branch (+AB)

Interestingly, even if each property aims to catch a particular behavior, some of them remain connected. For example, if the properties SCT and OE are both satisfied, then one can directly consider VP, DP, CT and NaE satisfied too.

5.5 Properties × Ranking-based Semantics

We are now able to check which properties are satisfied by the ranking-based semantics studied in this work. Recall that, among these ranking-based semantics, some of them (*e.g.* α-Bbs, the propagation semantics) are configurable with one or several parameters. Thus, two values of a parameter could give different rankings. It is why we consider that a property is satisfied by these ranking-based semantics only if the property is satisfied for all the values of a parameter.

Proposition 4 *The properties that are satisfied by each ranking-based semantics (the other properties are not satisfied by the corresponding ranking-based semantics):*

- *The categoriser-based ranking semantics (Cat) satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.*
- *The discussion-based semantics (Dbs) satisfies Abs, In, VP, DP, CT, SCT, CP, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.*
- *The burden-based semantics (Bbs) satisfies Abs, In, VP, DP, CT, SCT, CP, DDP, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.*
- *Let $\alpha \in]0, +\infty[$. The α-burden-based semantics (α-Bbs) satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.*
- *The fuzzy labeling (FL) satisfies Abs, In, CT, QP, Tot, NaE, AE, OE and AvsFD.*
- *Let $\alpha \in]0, 1[$. The counting semantics (CS) satisfies Abs, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.*
- *The tuples-based semantics (Tuples) satisfies Abs, In, VP, +DB, ↑AB, ↑DB, +AB, NaE, AE, OE and AvsFD.*
- *The ranking-based semantics 2ZG satisfies Abs, In, VP, SC, Tot, NaE and AvsFD.*
- *The iterated graded defense semantics (IGD) satisfies Abs, In, VP, +AB, NaE and AE.*
- *The ranking-based semantics $Propa_{\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, ↑AB, ↑DB, +AB,*

Tot, NaE and AE. When $\oplus = M$, $Propa_{\epsilon}^{\epsilon,M}$ also satisfies CT, SCT and OE.
- *The ranking-based semantics $Propa_{1+\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, ↑AB, ↑DB, +AB, Tot, NaE, AE and AvsFD. When $\oplus = M$, $Propa_{1+\epsilon}^{\epsilon,M}$ satisfies CT, SCT and OE.*
- *The ranking-based semantics $Propa_{1\to\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, +DB, ↑AB, ↑DB, +AB, Tot, NaE, AE and AvsFD. When $\oplus = M$, $Propa_{1\to\epsilon}^{\epsilon,M}$ satisfies OE.*

We also checked what are the properties satisfied by the usual Dung's grounded semantics which is the only semantics to return an unique extension. The idea is to give some hints on the compatibility of these properties with classical semantics.

Proposition 5 *The grounded semantics (Gr) satisfies Abs, In, Tot, NaE, AE and AvsFD. The other properties are not satisfied.*

Properties	Cat	Dbs	Bbs	α-Bbs	FL	CS	$Propa_{\epsilon}$	$Propa_{1+\epsilon}$	$Propa_{1\to\epsilon}$	Tuples	2ZG	IGD	Gr
Abs	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
In	✓	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	✓
VP	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	✓	×
DP	✓	✓	✓	✓	×	✓	✓	✓	✓	×	×	×	×
CT	✓	✓	✓	✓	✓	✓	✓$_M$	✓$_M$	×	×	×	×	×
SCT	✓	✓	✓	✓	×	✓	✓$_M$	✓$_M$	×	×	×	×	×
CP	×	✓	✓	×	×	×	×	×	×	×	×	×	×
QP	×	×	×	×	✓	×	×	×	×	×	×	×	×
DDP	×	×	✓	×	×	×	×	✓	✓	×	×	×	×
SC	×	×	×	×	×	×	×	×	×	×	✓	×	×
⊕DB	×	×	×	×	×	×	×	×	×	×	×	×	×
+DB	×	×	×	×	×	×	×	×	✓	✓	×	×	×
↑AB	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	×	×	×
↑DB	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	×	×	×
+AB	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	×	×	×
Tot	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	×	✓
NaE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
AE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	✓
OE	✓	✓	✓	✓	✓	✓	✓$_M$	✓$_M$	✓$_M$	✓	×	×	×
AvsFD	×	×	×	×	✓	×	×	✓	✓	✓	✓	×	✓

Table 2
Properties satisfy by the studied ranking semantics. A cross × means that the property is not satisfied, symbol ✓ means that the property is satisfied and the shaded cells highlight the results already proven in the literature. Symbol ✓$_M$ only concerns the propagation semantics and means that the property is satisfied only when $\oplus = M$.

6 Discussion

6.1 Observations

Several observations can be made regarding these axioms and the results reported in Table 2:

Some axioms seem to be widely accepted and shared by almost all semantics. It is the case with the properties Abs, In, VP, ↑AB, ↑DB, +AB, Tot, NaE and AE.

We recall that the input is a Dung's abstract argumentation framework without information about the nature of arguments, so only the attacks have to be taken into account, hence the importance of Abs. Concerning the property Independence (In), it seems difficult to explain the fact that an argument can influence others arguments without an existing link between them. The only semantics which does not satisfy this property is the counting semantics (CS) which needs the maximal indegree to guarantee the convergence. As seen on the running example, all the semantics consider the non-attacked arguments as the best arguments in an argumentation framework (VP). Nevertheless, there are situations where VP does not seem appropriate (e.g. see the study of protocatalepsis in the context of persuasion [14]). With the exception of 2ZG for the AE property, NaE and AE are also satisfied by all semantics. This is a kind of compatibility principle with usual Dung's semantics (the grounded semantics satisfies them too) where only your attackers should impact your ranking, not the arguments you attack. It is also interesting to note that almost all the semantics satisfy the property Total allowing a direct utilization in real applications wanting distinguish all the arguments. A last property satisfied by almost all semantics is +AB, which states that adding an attack branch towards an argument degrades its ranking. This also seems to be a perfectly natural requirement for ranking semantics: the more you are attacked, the worse you are. Furthermore, this property is one of the main reasons (in addition to the non-satisfaction of the properties ↑AB and ↑DB as well) to explain the difference observed in section 4 between the semantics FL which does not satisfy it and all the other semantics. Indeed, FL extends the complete semantics by considering varying degrees of acceptability (rather than the three classical ones: in, out and undec) and thus does not take into account the number of attackers, while all the other semantics do.

Some axioms are very discriminatory and provide a rough classification of semantics. If some incompatibilities between properties exist, some other properties (like (S)CT, AvsFD or +DB) allow us to separate the semantics into sub-classes groups. Indeed, different approaches (without being incompatible) concerning the defense are considered by these properties. The semantics that satisfy AvsFD take care of the whole branches of attack/defense. Whereas for the semantics that satisfy (S)CT, a defense branch (that still ends by an attack towards the argument) always penalizes it. Such properties reveal the elements in an argumentation framework causing differences between the rankings.

More specific properties. As mentioned already, the axioms operate at different levels. There are 'local' axioms (*e.g.* CP, QP, DP, DDP, (S)CT) focusing on the direct attackers (or defenders) which can be justified in some situations but seem hardly general (and sometimes impossible to reconcile with some more global properties, as Proposition 1 shows). And properties related to 'change' (*e.g.* ⊕DB, +DB, ↑AB, ↑DB, +AB) seem very appealing because they specify how the ranking should be affected on the basis of the comparison of attack and defense branches. They allows, for example, to categorize the semantics according to the behavior towards some basic requirements like the defense with +DB (see the previous observation). Another example of their interest is that, in focusing on the two semantics Tuples* and 2ZG, it seems clear that the properties related to "change" satisfied by these two semantics allow to distinguish these two semantics while it is not the case with the existing properties.

Defining axiomatically the worst arguments is not obvious. Interestingly, while all semantics agree axiomatically on which arguments should be the best in an argumentation framework (VP), there is no consensus regarding the *worst* arguments. SC is very interesting in that respect. It makes the observation that a self-contradicting argument is intrinsically flawed, without even requiring other arguments to defeat it. But as can be observed none of the semantics comply with it, except that of Matt and Toni (2ZG) who introduced the property. The explanation is that all semantics consider that an argument that attacks itself is a path like the other ones. So an argument which attacks itself (and by no other argument) is better than an argument which is attacked several times. On the other hand, another possibility is when the properties +AB and $\uparrow$AB are satisfied together. Indeed, one can consider the worst argument as the one which is directly attacks by a maximum (+AB) of non-attacked arguments ($\uparrow$AB).

The interplay of axioms is often instructive. In section 5.4, we have identified some implications and incompatibilities between axioms. Let us focus, for example, on the relation of incompatibility between VP and $\oplus$DB. One can easily remark that $\oplus$DB is more general than +DB, and in a sense more natural: the property is stated for *any* cases, it does not treat some arguments (the non-attacked arguments here) differently. But it contradicts VP in this case. +DB is a less "systematic" property (it was the original one proposed in [16]) but is compatible with VP: if one accepts that non-attacked arguments should be the best (VP), then adding a defense branch cannot *always* improve the situation of a given argument.

This set of axioms is yet to be augmented. This can be observed with the semantics Categorizer, α-Burden-based semantics and $Propa_\epsilon$ which satisfy the same set of properties, whereas they have quite different definitions and behaviors as it is revealed in Section 4. This means that at least one logical property (if it exists) is lacking in order to discriminate these operators.

Towards an application-oriented axiomatic analysis. Let us recall that we do not claim that all of the properties presented in Section 5 are required. However, at this level of abstraction, they allow us to compare and better understand the ranking-based semantics. Indeed, while our objective has been to offer the broadest possible picture of ranking-based semantics by presenting a large catalogue of properties, we certainly believe that the relevance of each axiom should be ultimately evaluated with respect to the application at hand. Depending on the context, a designer may only focus a subset of axioms, or even challenge a specific property often assumed in existing semantics (as for instance in the case of persuasion where VP may not be desirable [14]). In line with the work initiated in [37] for gradual semantics, it would be interesting to target the mandatory properties for some practical aspects of argumentation (persuasion, negotiation, online debate, etc.).

6.2 Link between Extension-based Semantics and Ranking-based Semantics

One can note that Abs, In, AE, NaE, Tot and AvsFD are satisfied by the grounded semantics. However, among the properties widely accepted by the ranking-based semantics, VP and +AB are not satisfied by the grounded semantics. An explanation is related to the fact that the extension-based semantics (and in particular the grounded semantics) consider that the impact of an attack from an argument to another one is drastic. In other words, the grounded semantics falsifies VP and +AB because an attack can "kill" another argument (see the discussion in [2]) while the ranking-based semantics suppose that an attack does not "kill" but can just weaken the attacked argument. However, these two principles are not totally incompatible with grounded semantics to some extent. Indeed, as suggested in [36], a weak version of Void Precedence, which states that non-attacked arguments should be at least as acceptable as (and not strictly more acceptable) attacked arguments, can also be defined.
weak Void Precedence (wVP) [36] A ranking-based semantics σ satisfies weak Void Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) \neq \emptyset$ then $x \succeq^{\sigma}_{\text{AF}} y$.
Clearly, Void Precedence implies weak Void Precedence so all the semantics which satisfy VP also satisfy wVP. But it is interesting to note that the grounded semantics satisfies wVP because the non-attacked arguments are always accepted but can be equal to some other attacked arguments. Following the same reasoning, the weak version of some other existing properties [12] can be defined and satisfied by the grounded semantics. Let us check which of them are satisfied by the grounded semantics.

Proposition 6 *The grounded semantics satisfies the weak version of VP, DP, QP, DDP, SC, ⊕DB, +DB, ↑DB, ↑AB and +AB.*

It is clear that in this case the grounded semantics satisfied many more properties. This is due to the fact that there exist only two levels of acceptability (accepted/rejected) to evaluate the arguments. Thus, as the proofs show, if an argument is considered as accepted (resp. rejected) then it will always be considered more (resp. less) acceptable regardless of the acceptability of the other argument. But in general, these weak properties are less interesting for the ranking-based semantics precisely because of the many levels of acceptability. They can nevertheless make sense when they are combined. Indeed, as explained in [13], one may opt for two kinds of evaluations of arguments: at the level of sets of arguments (with extension-based or labelling-based semantics) or at the level of single arguments (with ranking-based or gradual semantics). These two ways to evaluate the information encoded in an argumentation framework are interesting, and target different

[12] Defining the weak version of a property means replacing the strict comparison operator between two arguments with a strict or equal comparison operator without changing the conditions (as is done for VP and wVP where $x \succ^{\sigma}_{\text{AF}} y$ for VP becomes $x \succeq^{\sigma}_{\text{AF}} y$ for wVP).

kinds of application. They share however the same goal because they evaluate the acceptability of the arguments. In other words, both can be used to extract some information about the status/strength/situation of (sets of) arguments. Thus, instead of seeing these approaches as mutually exclusive, some works propose to combine them. Indeed, existing works [36,24,20,13,10] propose constraining the rankings to be compatible with the acceptance status of the arguments. The basic idea is to first compute the extensions (or labellings) in an argumentation framework for a given semantics before distinguishing the arguments with the same level of acceptability by using a ranking-based semantics. Bonzon et al. [13] also propose different ways to decrease the number of extensions returned by extension-based semantics, in order to allow more inferences, thanks to ranking-based semantics. Indeed, a ranking-based semantics often uses criteria that differ from those used by the extension-based semantics (e.g. the number and the quality of attackers and defenders of each argument) in order to evaluate the arguments. These criteria are used to select the "best" extensions among the set of extensions returned by an extension-based semantics.

7 Conclusion

In this work we proposed a comparative study of existing ranking-based semantics which received more and more attention these last years. It turns out that these ranking-based semantics exhibit quite different behaviors, even on simple argumentation frameworks with few arguments, but also share some "natural" principles. These observations have been generalized by our experimental study aiming to compare the rank orders returned by some existing semantics on randomly generated argumentation frameworks. To understand the origin of these differences and similarities, we group together the properties proposed in the literature and the new ones that we propose in order to check which ones are satisfied by the existing semantics. Thus, some properties (Abs, In, VP, ↑AB, ↑DB, +AB, Tot, NaE and AE) satisfied by almost all the semantics confirm the common bases shared by the semantics. Conversely, in addition to the incompatibilities that we revealed in this work, some properties (SCT and AvsFD) discriminates two subclasses of semantics which partially explain the observed differences. Our analysis is applied to existing semantics which provide a good base for comparison, and thus, with the rising number of ranking semantics, any new semantics could be inspected through the same lens.
However, there is still work needed on the topic because some semantics share the same set of satisfied properties whereas our experimental comparison clearly highlights differences in the rankings that they return. Future work could be inspired by some works [3,7] done on the properties for the gradual semantics. Indeed, some of these properties can be related to properties defined in this work for ranking-based semantics. For example, a property related to the void precedence (VP) property is the maximality property defined in [3] which states that if an argument is not attacked, its overall strength should be maximal. However, it is important to recall that when a gradual semantics can be used to rank arguments, the reverse is not true.

The main goal of these axiomatic approaches is to characterize classes of semantics with respect to a set of properties. Thus it will be possible, among other things, to identify families of semantics that have not been explored yet. Another perspective is to provide tools to built semantics from principles accepted by the user, as suggested in [8].

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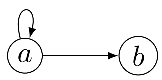
A Proofs

Proof (Proposition 1 (page 32)) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $a, b \in \mathcal{A}$ and σ be a ranking semantics.*

(1) See [2]
(2) See [8]
(3) See [8]
(4) See [8]
(5) Let us suppose that $|\mathcal{R}_1(b)| = 1$, $|\mathcal{R}_2(b)| = 0$ and $|\mathcal{B}_-(a)| = 0$. If σ satisfied AvsFD then $a \succ^\sigma_{AF} b$. However, there is no restriction about the number of defense branches of a, so there exists cases where $|\mathcal{R}_1(a)| > |\mathcal{R}_1(b)|$. In these cases, the property CP says that $a \prec^\sigma_{AF} b$ which contradicts AvsFD.
(6) Let $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(a))$ be an argumentation framework such that $AF^\gamma = \gamma(AF)$ and a is attacked ($\mathcal{R}_1(a) \neq \emptyset$). In $AF^\star$, $\gamma(a)$ has one more defense branch (and so one more direct attacker) than a ($|\mathcal{R}_1(a)| < |\mathcal{R}_1(\gamma(a))|$). If σ satisfies CP then $a \succ^\sigma_{AF^\star} \gamma(a)$ whereas if +DB is satisfied then $\gamma(a) \succ^\sigma_{AF^\star} a$.
(7) Same proof that +DB except that a can be non-attacked too.
(8) Let $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(a))$ be an argumentation framework such that $AF^\gamma = \gamma(AF)$ and a is a non-attacked argument ($\mathcal{R}_1(a) = \emptyset$). If σ satisfies $\oplus$DB then $\gamma(a) \succ^\sigma_{AF^\star} a$ whereas if σ satisfies VP then $a \succ^\sigma_{AF^\star} \gamma(a)$ because $\gamma(a)$ becomes attacked ($\mathcal{R}_1(a) = \emptyset$ and $\mathcal{R}_1(\gamma(a)) \neq \emptyset$).
(9) In the AF illustrated in Fig. A.1, it is clear that it exists an isomorphism between



AE : $a \simeq b$
SC : $b \succ a$

Fig. A.1. Incompatibility between AE and SC.

the ancestor's graph of a and b (which is an infinite line of arguments) so, according to the property AE, a and b should be equally acceptable ($a \simeq b$). In addition, the argument a attacks itself contrary to b so, according to the property SC, b should be strictly more acceptable than a ($b \succ a$). Thus, according to the definition of the incompatibility between two properties, AE and SC are incompatible.

■

Proof (Proposition 2 (page 33)) *Let AF, AF^γ be two argumentation framework such that there exists an isomorphism γ between AF and AF^γ ($AF^\gamma = \gamma(AF)$) and σ be a ranking semantics.*
According to AE, each argument and its image are equally acceptable ($\forall x \in Arg(AF)$, $x \simeq^\sigma_{AF \cup AF^\gamma} \gamma(x)$).
Let $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(a))$ and $a \in Arg(AF)$ an attacked argument ($\mathcal{R}_1(a) \neq \emptyset$). If σ satisfies +DB then $\gamma(a) \succ^\sigma_{AF^\star} a$ whereas if σ satisfies SCT then $a \succ^\sigma_{AF^\star} \gamma(a)$ because an injective function f exists from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(\gamma(a))$ such that $\forall c \in \mathcal{R}_1(a), f(c) \succeq^\sigma_{AF^\star} c$

(thanks to the property AE) and $|\mathcal{R}_1(\gamma(a))| > |\mathcal{R}_1(a)|$. ■

Proof (Proposition 3 (page 33)) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework,* $a, b \in \mathcal{A}$ *and* σ *be a ranking semantics.*

(1) See [2]

(2) See [2]

(3) Let us suppose that a *and* b *are non-attacked* ($\mathcal{R}_1(a) = \mathcal{R}_1(b) = \emptyset$)*. As the set of direct attackers is empty for* a *and* b*, it is clear that there exists an injective function from* $\mathcal{R}_1(a)$ *to* $\mathcal{R}_1(b)$ *(resp. from* $\mathcal{R}_1(b)$ *to* $\mathcal{R}_1(a)$*) such that* $\forall c \in \mathcal{R}_1(a), f(c) \succeq^{\sigma}_{AF} c$ *(resp.* $\forall c \in \mathcal{R}_1(b), f(c) \succeq^{\sigma}_{AF} c$*). As* σ *satisfies CT, so* $a \simeq^{\sigma}_{AF} b$ *(because* $a \succeq^{\sigma}_{AF} b$ *and* $b \succeq^{\sigma}_{AF} a$*) in agreement with the property NaE.*

(4) Let us suppose that there exists a bijective function f *from* $\mathcal{R}_1(a)$ *to* $\mathcal{R}_1(b)$ *such that* $\forall c \in \mathcal{R}_1(a), c \simeq^{\sigma}_{AF} f(c)$ (i.e. $c \succeq^{\sigma}_{AF} f(c)$ *and* $f(c) \succeq^{\sigma}_{AF} c$*). By definition, a bijective function is also an injective function, so* f *is injective and* $\mathcal{R}_1(b) \geq_S \mathcal{R}_1(a)$ *(because* $\forall c \in \mathcal{R}_1(a), f(c) \succeq^{\sigma}_{AF} c$*). As* σ *satisfies CT, one can conclude that* $a \succeq^{\sigma}_{AF} b$*. But the existence of the bijective function* f *implies that there also exists an injective function* g *(*$g = f^{-1}$*) from* $\mathcal{R}_1(b)$ *to* $\mathcal{R}_1(a)$*. So with the same reasoning, one can conclude that* $b \succeq^{\sigma}_{AF} a$*. By definition,* $a \succeq^{\sigma}_{AF} b$ *and* $b \succeq^{\sigma}_{AF} a$ *implies that* $a \simeq^{\sigma}_{AF} b$ *in agreement with OE.*

(5) Let us suppose that there exists an injective function f *from* $\mathcal{R}_1(a)$ *to* $\mathcal{R}_1(b)$ *such that* $\forall c \in \mathcal{R}_1(a), f(c) \succeq^{\sigma}_{AF} c$ *and that* σ *satisfies SCT and OE*[13]*. Let us show that for all* a, b *which satisfy this condition then* $a \succeq^{\sigma}_{AF} b$*.*

   (a) If $|\mathcal{R}_1(b)| > |\mathcal{R}_1(a)|$ *or* $\exists c \in \mathcal{R}_1(a), f(c) \succ^{\sigma}_{AF} c$ *then according to SCT we have* $a \succ^{\sigma}_{AF} b$*. By definition,* $a \succ^{\sigma}_{AF} b$ *is equivalent to* $a \succeq^{\sigma}_{AF} b$ *and* $b \not\succeq^{\sigma}_{AF} a$*, so CT is satisfied.*

   (b) If $|\mathcal{R}_1(b)| = |\mathcal{R}_1(a)|$ *and* $\nexists c \in \mathcal{R}_1(a), f(a) \succ^{\sigma}_{AF} a$ *then* $\forall c \in \mathcal{R}_1(a), f(a) \simeq^{\sigma}_{AF} a$*. But, as* $|\mathcal{R}_1(b)| = |\mathcal{R}_1(a)|$ *then* f *is also surjective so* f *is bijective and* $\forall c \in \mathcal{R}_1(a), f(c) \simeq^{\sigma}_{AF} c$*, so according to OE, we have* $a \simeq^{\sigma}_{AF} b$*. By definition,* $a \simeq^{\sigma}_{AF} b$ *is equivalent to* $a \succeq^{\sigma}_{AF} b$ *and* $b \succeq^{\sigma}_{AF} a$*, so CT is satisfied.*

(6) Obvious because +DB is a particular case of $\oplus$*DB (if it is true for all the arguments then it is also true for the attacked arguments).*

(7) Obvious because if a *and* b *are non-attacked, then they have the same ancestors' graph which is empty. Thus according to AE, they are equally acceptable (*$a \simeq^{\sigma} b$*) in agreement with NaE.*

(8) Let us suppose that a *and* b *are non-attacked* ($\mathcal{R}_1(a) = \mathcal{R}_1(b) = \emptyset$)*. As the set of direct attackers is empty for* a *and* b*, it is clear that there exists an bijective function* f *from* $\mathcal{R}_1(a)$ *to* $\mathcal{R}_1(b)$ *such that* $\forall c \in \mathcal{R}_1(a), c \simeq^{\sigma}_{AF} f(c)$*. As* σ *satisfies OE, so* $a \simeq^{\sigma}_{AF} b$ *in agreement with NaE.*

(9) Let us suppose that $|\mathcal{B}_-(a)| = 0$ *which means that* a *is either not attacked or attacked but defended. Let us also assume that* $|\mathcal{R}_1(b)| = 1$ *and* $|\mathcal{R}_2(b)| = 0$*. According to the*

[13] We point out that the result of the paper [11] saying that SCT implies CT is incomplete. Indeed, it lacked the very special case where attacking arguments can be equally acceptable, hence the need to include OE.

property AvsFD, in this case, a is strictly more acceptable than b ($a \succ^{\sigma}_{AF} b$). We will show that when the properties VP and QP are satisfied we obtain the same result.
Let $\mathcal{R}_1(a) = \{a_1, \ldots, a_n\}$ and $\mathcal{R}_1(b) = \{b_1\}$.
<u>$\mathcal{R}_1(a) = \emptyset$</u>*: From VP, we have $a \succ^{\sigma}_{AF} b$ because $\mathcal{R}_1(a) = \emptyset$ and $\mathcal{R}_1(b) \neq \emptyset$.*
<u>$\mathcal{R}_1(a) \neq \emptyset$</u>*: By VP, $\forall a_i \in \mathcal{R}_1(a)$, $b_1 \succ^{\sigma}_{AF} a_i$ because $\mathcal{R}_1(b_1) = \emptyset$. So, by QP, we have $a \succ^{\sigma}_{AF} b$.*

(10) Let $AF^\star = AF \cup AF^\gamma \cup P_-(\gamma(a))$ be an argumentation framework such that $AF^\gamma = \gamma(AF)$. In $AF^\star$, $\gamma(a)$ has one more attack branch (and so one more direct attacker) than a ($|\mathcal{R}_1(a)| < |\mathcal{R}_1(\gamma(a))|$). As σ satisfies CP, so $a \succ^{\sigma}_{AF^\star} \gamma(a)$ in agreement with +AB.

■

Proof (Proposition 4 (page 34))

Categoriser-based ranking semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [32].

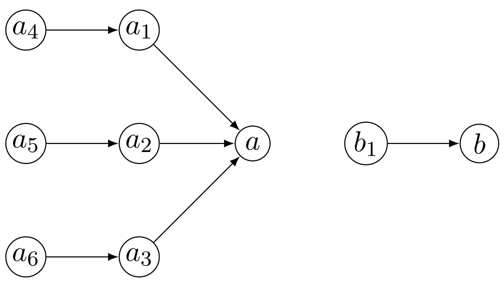
(OE) *OE is implied by CT which is satisfied.*
(NaE) *NaE is implied by OE which is satisfied.*
(AE) *According to the definition of the categoriser function, the categoriser value of an argument is computed from the categoriser values of its direct attackers which depend themselves of the categoriser values of their direct attackers and so on. So the only arguments which directly or indirectly impact a given argument x are the attacker and the defender of x ($x \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x)$),* i.e. *the arguments in its ancestors' graph.*
Pu et al. [32, Theorem 1] show that for every argumentation framework there always exists a unique categoriser valuation, which means that two AFs with the same topology assign the same value to their arguments (and so have the same ranking). So if two arguments x and y have the same ancestors' graph (and it is the case because there exists an isomorphism between $Anc(x)$ and $Anc(y)$) then $Cat(x) = Cat(y)$ and so $x \simeq^{Cat} y$, in agreement with the property.
(Tot) *The categoriser-based ranking semantics guarantees a comparison between all the arguments because all arguments have a score between 0 and 1 which is a totally ordered set of real number and [32, Theorem 1] ensures the existence of a result. So all pairs of arguments can be compared.*
(+AB,↑AB,↑DB) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists: $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments such that $\mathcal{R}_1(a) = \{a_1, \ldots, a_n\}$ and $\mathcal{R}_1(\gamma(a)) = \{\gamma(a_1), \ldots, \gamma(a_n)\}$. As the semantics satisfies AE, each argument and its image are equally acceptable and, consequently, have the same score: $\forall x \in \mathcal{A}, Cat(x) = Cat(\gamma(x))$.*
<u>*+AB*</u> *Let us add an attack branch to $\gamma(a)$ where the argument b is the direct attacker of*

$\gamma(a)$ belonging to the new attack branch. If a is not attacked, then, according to the property VP which is satisfied, we have $a \succ^{Cat} \gamma(a)$ because $\gamma(a)$ is now attacked by b. If a is attacked then the result is the same. Indeed, the score of b is strictly positive ($Cat(b) > 0$) because the function $f(x) = \frac{1}{1+x}$ cannot be equal to 0, so we have:

$$Cat(a_1) + \cdots + Cat(a_n) + 0 < Cat(\gamma(a_1)) + \cdots + Cat(\gamma(a_n)) + Cat(b)$$
$$\frac{1}{1 + Cat(a_1) + \cdots + Cat(a_n)} > \frac{1}{1 + Cat(\gamma(a_1)) + \cdots + Cat(\gamma(a_n)) + Cat(b)}$$
$$Cat(a) > Cat(\gamma(a))$$

Consequently, we have $a \succ^{Cat} \gamma(a)$ in agreement with the property.
$\uparrow$AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N}+1$: $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a\rangle$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. As the property VP is satisfied, the score of $\gamma(b)$, which is now attacked, becomes smaller than the score of b (so $b \succ^{Cat} \gamma(b)$). Combining with the fact that SCT is satisfied, then $\gamma(b_{n-1}) \succ^{Cat} b_{n-1}$. With the same reasoning, we obtain $b_{n-2} \succ^{Cat} \gamma(b_{n-2})$ and so on until that $\gamma(a) \succ^{Cat} a$, in agreement with the property.
$\uparrow$DB The same reasoning as the proof for $\uparrow$AB can be used.

(+DB) *Incompatible with SCT and AE which are satisfied.*
(SC) *Incompatible with AE which is satisfied.*
($\oplus$DB) *Incompatible with VP which is satisfied.*
(AvsFD) *To show that the categoriser-based ranking semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework from Fig. A.2. The property says that a should be strictly more acceptable than b because a has only*



$$Cat(a_4) = Cat(a_5) = 1$$
$$Cat(a_6) = Cat(b_1) = 1$$
$$Cat(a_1) = Cat(a_2) = 0.5$$
$$Cat(a_3) = Cat(b) = 0.5$$
$$Cat(a) = 0.4$$

$$a_4 \simeq^{\text{Cat}} a_5 \simeq^{\text{Cat}} a_6 \simeq^{\text{Cat}} b_1 \succ^{\text{Cat}} a_1 \simeq^{\text{Cat}} a_2 \simeq^{\text{Cat}} a_3 \simeq^{\text{Cat}} b \succ^{\text{Cat}} a$$

Fig. A.2. The categoriser-based ranking semantics falsifies the property AvsFD.

defense branches while b has exactly one direct attacker and no defense branch. But in using the categoriser-based ranking semantics, b is strictly more acceptable than a, contradicting the property.

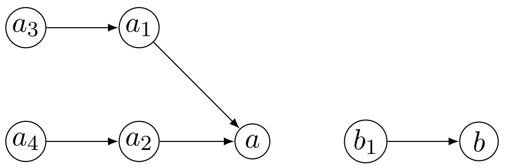
Discussion-based semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality

Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [2].

(OE) *OE is implied by CT which is satisfied.*
(NaE) *NaE is implied by OE which is satisfied.*
(AE) *It is clear that, following the definition, the discussion count of an argument only depends on the attackers and the defenders of this argument, and so only on the arguments in its ancestors' graph. So if two arguments x and y have the same ancestors' graph (and it is the case because there exists an isomorphism between $Anc(x)$ and $Anc(y)$) then it is obvious to say that $\forall i \in \mathbb{N} \backslash \{0\}$, $|\mathcal{R}_i(x)| = |\mathcal{R}_i(y)|$. Consequently, $Dis(x) = Dis(y)$ which implies that $x \simeq^{Dbs} y$, in agreement with the property.*
(Tot) *The discussion-based ranking semantics guarantees a comparison between all the arguments because $\succeq^{Dbs}$ is total [2, Definition 2].*
(+AB) *+AB is implied by CP which is satisfied.*
(↑AB,↑DB) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists: $AF = \gamma(AF')$. As the semantics satisfies AE, each argument and its image are equally acceptable and so have the same score: $\forall x \in \mathcal{A}, Dis(x) = Dis(\gamma(x))$ (*i.e. $\forall i > 0$, $Dis_i(x) = Dis_i(\gamma(x))$*).*
↑AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N}+1$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. So, $\forall i \leq n$, $Dis_i(a) = Dis_i(\gamma(a))$ but during the step $n+1$, $\gamma(a)$ has now one additional defender ($|\mathcal{R}_{n+1}(\gamma(a))| > |\mathcal{R}_{n+1}(a)|$) so $Dis_{n+1}(\gamma(a)) = -|\mathcal{R}_{n+1}(\gamma(a))| < -|\mathcal{R}_{n+1}(a)| = Dis_{n+1}(a)$. Consequently, $Dis(a) \succ_{lex} Dis_{n+1}(\gamma(a))$ implies that $\gamma(a) \succ^{Dbs} a$, in agreement with the property.
↑DB The reasoning is similar to the proof of ↑AB except that the length of the branch from b to a is $n \in 2\mathbb{N}$. So $Dis_{n+1}(\gamma(a)) = |\mathcal{R}_{n+1}(\gamma(a))| > |\mathcal{R}_{n+1}(a)| = Dis_{n+1}(a)$ which implies that $Dis(\gamma(a)) \succ_{lex} Dis_{n+1}(a)$ and $a \succ^{Dbs} \gamma(a)$, in agreement with the property.

(+DB) *Incompatible with SCT and AE which are satisfied.*
(⊕DB) *Incompatible with VP which is satisfied.*
(SC) *Incompatible with AE which is satisfied.*
(AvsFD) *To show that the discussion-based semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework from Fig. A.3. The*



$Dis(a_3) = Dis(a_4) = Dis(b_1) = \langle 0, 0, 0 \rangle$
$Dis(a_1) = Dis(a_2) = Dis(b) = \langle 1, 0, 0 \rangle$
$Dis(a) = \langle 2, -2, 0 \rangle$

$$a_3 \simeq^{\text{Dbs}} a_4 \simeq^{\text{Dbs}} b_1 \succ^{\text{Dbs}} a_1 \simeq^{\text{Dbs}} a_2 \simeq^{\text{Dbs}} b \succ^{\text{Dbs}} a$$

Fig. A.3. The discussion-based semantics falsifies the property AvsFD.

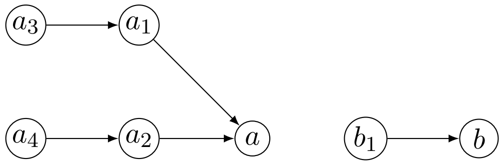
property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But in using the *discussion-based semantics, b is strictly more acceptable than a, contradicting the property.*

Burden-based semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [2].

(OE) *OE is implied by CT which is satisfied.*
(NaE) *NaE is implied by OE which is satisfied.*
(AE) *According to its definition, the burden number of an argument is computed from the burden number of its direct attackers which depend themselves of the burden number of their direct attackers and so on. So the only arguments which directly or indirectly impact a given argument x are the attacker and the defender of x* $(x \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x))$, i.e. *the arguments in its ancestors' graph. So if two arguments x and y have the same ancestors' graph (and it is the case because there exists an isomorphism between* $Anc(x)$ *and* $Anc(y)$*) then* $\forall i \in \mathbb{N}, Bur_i(x) = Bur_i(y)$*. Indeed, it is obviously true when* $i = 0$ *(see the definition), when* $i = 1$ *because they have the same number of direct attackers, when* $i = 2$ *because their direct attackers are attacked by the same number of arguments and so on. Consequently,* $x \simeq^{Bbs} y$*, in agreement with the property.*
(Tot) *The burden-based ranking semantics guarantees a comparison between all the arguments because* $\succeq^{Bbs}$ *is total [2, Definition 2].*
(+AB) *+AB is implied by CP which is satisfied.*
(↑AB,↑DB) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ *be two argumentation frameworks such that an isomorphism* γ *exists:* $AF = \gamma(AF')$*. As the semantics satisfies AE, each argument and its image are equally acceptable and so have the same burden vector:* $\forall x \in \mathcal{A}, Bur(x) = Bur(\gamma(x))$ (i.e. $\forall i \geq 0,\ Bur_i(x) = Bur_i(\gamma(x))$*).*
↑AB *Let us suppose* $\exists b \in \mathcal{B}_-(a),\ b \notin \mathcal{B}_+(a)$ *and consider a branch from b to a with a length of* $n \in 2\mathbb{N}+1$*:* $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a \rangle$*. Let us now add a defense branch to the non-attacked argument* $\gamma(b)$*. As the property VP is satisfied,* $\gamma(b)$*, which is now attacked, becomes less acceptable than b which is non-attacked* ($b \succ^{Bbs} \gamma(b)$)*. Combining with the fact that SCT is satisfied, then* $\gamma(b_{n-1}) \succ^{Bbs} b_{n-1}$*. With the same reasoning,* $b_{n-2} \succ^{Bbs} \gamma(b_{n-2})$ *and so on until that* $\gamma(a) \succ^{Bbs} a$*, in agreement with the property.*
↑DB *The proof is similar to the one of ↑AB.*

(+DB) *Incompatible with SCT and AE which are satisfied.*
(⊕DB) *Incompatible with VP which is satisfied.*
(SC) *Incompatible with AE which is satisfied.*
(AvsFD) *To show that the burden-based semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework from Fig. A.4. The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But in using the burden-*



$Bur(a_3) = Bur(a_4) = Bur(b_1) = \langle 1,1,1,1\rangle$
$Bur(a_1) = Bur(a_2) = Bur(b) = \langle 1,2,2,2\rangle$
$Bur(a) = \langle 1,3,2,2\rangle$

$$a_3 \simeq^{\text{Bbs}} a_4 \simeq^{\text{Bbs}} b_1 \succ^{\text{Bbs}} a_1 \simeq^{\text{Bbs}} a_2 \simeq^{\text{Bbs}} b \succ^{\text{Bbs}} a$$

Fig. A.4. The burden-based semantics falsifies the property AvsFD.

based semantics, b is strictly more acceptable than a, contradicting the property.

α-burden-based semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP) (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [4].

(OE) *OE is implied by CT which is satisfied.*
(NaE) *NaE is implied by OE which is satisfied.*
(AE) *According to its definition, the burden number of an argument is computed from the burden number of its direct attackers which depend themselves of the burden number of their direct attackers and so on. So the only arguments which directly or indirectly impact a given argument x are the attacker and the defender of x* $(x \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x))$, i.e. *the arguments in its ancestors' graph. Let* $AF = \langle \mathcal{A}, \mathcal{R}\rangle$ *be an argumentation framework and* $a, b \in \mathcal{A}$ *such that there exists an isomorphism* γ *between* $Anc_{AF}(a)$ *and* $Anc_{AF}(b)$*. In [4, Theorem 1], the authors ensure that the solution of a system of equations exists and is unique. So it is clear that the systems of equations from* $Anc_{AF}(a)$ *and from* $Anc_{AF}(b)$ *are similar (because there exists an isomorphism) and have the same solution. Consequently,* $\forall a' \in Anc_{AF}(a)$ *then* $s_\alpha(a') = s_\alpha(\gamma(a'))$*. It is particularly true for a and b, so* $s_\alpha(a) = s_\alpha(b)$ *which means that* $a \simeq^{\alpha\text{-}Bbs}_{AF} b$.
(Tot) *The* α*-burden-based semantics guarantees a comparison between all the arguments because* $\succeq^{\alpha\text{-}Bbs}$ *is total [4, Definition 2].*
(+AB,↑AB,↑DB) *Let* $AF = \langle \mathcal{A}, \mathcal{R}\rangle$ *and* $AF' = \langle \mathcal{A}', \mathcal{R}'\rangle$ *be two argumentation frameworks such that an isomorphism* γ *exists:* $AF = \gamma(AF')$*. Let* $a \in \mathcal{A}$ *and its image* $\gamma(a) \in \mathcal{A}'$ *be two arguments such that* $\mathcal{R}_1(a) = \{a_1, \ldots, a_n\}$ *and* $\mathcal{R}_1(\gamma(a)) = \{\gamma(a_1), \ldots, \gamma(a_n)\}$*. As the semantics satisfies AE, each argument and its image are equally acceptable and so have the same score:* $\forall x \in \mathcal{A}, s_\alpha(x) = s_\alpha(\gamma(x))$.
+AB Let us add an attack branch to $\gamma(a)$ *where the argument b is the direct attacker of* $\gamma(a)$ *belonging to the new attack branch. If a is not attacked, then, according to the property VP which is satisfied, we have* $a \succ^{\alpha\text{-}Bbs} \gamma(a)$ *because* $\gamma(a)$ *is now attacked by b. If a is attacked then the result is the same. Indeed, the score of b is strictly positive* $(s_\alpha(b) > 0)$

because the domain of s_α *is* $[1, \infty[$ *, so we have* $\forall \alpha \in]0, \infty[$*:*

$$\frac{1}{(s_\alpha(a_1))^\alpha} + \cdots + \frac{1}{(s_\alpha(a_n))^\alpha} + 0 < \frac{1}{(s_\alpha(\gamma(a_1)))^\alpha} + \cdots + \frac{1}{(s_\alpha(\gamma(a_n)))^\alpha} + \frac{1}{(s_\alpha(b))^\alpha}$$

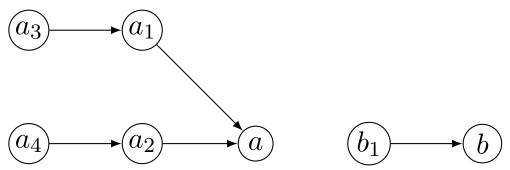
$$\left(\frac{1}{(s_\alpha(a_1))^\alpha} + \cdots + \frac{1}{(s_\alpha(a_n))^\alpha}\right)^{1/\alpha} < \left(\frac{1}{(s_\alpha(\gamma(a_1)))^\alpha} + \cdots + \frac{1}{(s_\alpha(\gamma(a_n)))^\alpha} + \frac{1}{(s_\alpha(b))^\alpha}\right)^{1/\alpha}$$

$$1 + \left(\frac{1}{(s_\alpha(a_1))^\alpha} + \cdots + \frac{1}{(s_\alpha(a_n))^\alpha}\right)^{1/\alpha} < 1 + \left(\frac{1}{(s_\alpha(\gamma(a_1)))^\alpha} + \cdots + \frac{1}{(s_\alpha(\gamma(a_n)))^\alpha} + \frac{1}{(s_\alpha(b))^\alpha}\right)^{1/\alpha}$$

$$s_\alpha(a) < s_\alpha(\gamma(a))$$

Consequently, we have $a \succ^{\alpha\text{-}Bbs} \gamma(a)$ *in agreement with the property.*
$\uparrow$*AB Let us suppose* $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ *and consider a branch from* b *to* a *with a length of* $n \in 2\mathbb{N}+1$*:* $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a\rangle$*. Let us now add a defense branch to the non-attacked argument* $\gamma(b)$*. As the property VP is satisfied, the score of* $\gamma(b)$*, which is now attacked, becomes greater than the score of* b *(so* $b \succ^{\alpha\text{-}Bbs} \gamma(b)$*). Combining with the fact that SCT is satisfied, then* $\gamma(b_{n-1}) \succ^{\alpha\text{-}Bbs} b_{n-1}$*. With the same reasoning, we obtain* $b_{n-2} \succ^{\alpha\text{-}Bbs} \gamma(b_{n-2})$ *and so on until* $\gamma(a) \succ^{\alpha\text{-}Bbs} a$*, in agreement with the property.*
$\uparrow$*DB The reasoning is similar to the proof of* $\uparrow$*AB.*

(+DB) *Incompatible with SCT and AE which are satisfied.*
(⊕DB) *Incompatible with VP which is satisfied.*
(SC) *Incompatible with AE which is satisfied.*
(AvsFD) *To show that the* α*-burden-based semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework from Fig. A.5. The*



If $\alpha = 1$:

$$s_\alpha(a_3) = s_\alpha(a_4) = s_\alpha(b_1) = 1$$
$$s_\alpha(a_1) = s_\alpha(a_2) = s_\alpha(b) = 2$$
$$s_\alpha(a) = 2$$

$$a_3 \simeq^{\alpha\text{-Bbs}} a_4 \simeq^{\alpha\text{-Bbs}} b_1 \succ^{\alpha\text{-Bbs}} a_1 \simeq^{\alpha\text{-Bbs}} a_2 \simeq^{\alpha\text{-Bbs}} b \simeq^{\alpha\text{-Bbs}} a$$

Fig. A.5. The α-burden-based semantics falsifies the property AvsFD.

property says that a *should be strictly more acceptable than* b *because* a *has only defense branches while* b *has exactly one direct attacker and no defense branch. But in using the* α*-burden-based semantics with* $\alpha = 1$*,* a *and* b *are equally acceptable, contradicting the property.*

Fuzzy labeling

The results concerning the property Total (Tot) can be found in [19][Definition 9].

(Abs) *The nature of an argument is not used in the computation of its score. Only the attack relation is needed (see definition 17).*

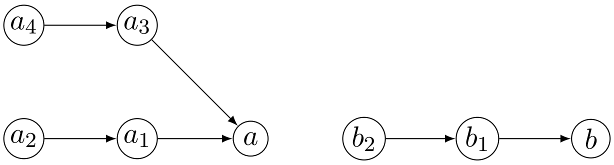
(In) *Obvious because, according to the definition 17, an argument only depends on the score of its direct attacker, which depends on the score of its direct attackers and so on.*
(QP) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\forall x, y \in \mathcal{A}$. *Suppose that* $\exists y' \in \mathcal{R}_1(y)$ *such that* $\forall x' \in \mathcal{R}_1(x)$, $y' \succ^{FL}_{AF} x'$ *which implies that* $f(y') > f(x')$. *So* $\max_{y' \in \mathcal{R}_1(y)} f(y') > \max_{x' \in \mathcal{R}_1(x)} f(x')$. *According to the definition 17, we obtain* $f(y) < f(x)$ *and thus* $x \succ^{FL}_{AF} y$.
(CT) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $\forall x, y \in \mathcal{A}$. *Suppose that it exists an injective function* f *from* $\mathcal{R}_1(y)$ *to* $\mathcal{R}_1(x)$ *such that* $\forall z \in \mathcal{R}_1(y)$, $f(z) \succeq^{FL} z$ *which implies that* $f(f(z)) \geq f(z)$. *So* $\max_{f(z) \in \mathcal{R}_1(x)} f(f(z)) \geq \max_{z \in \mathcal{R}_1(y)} f(z)$. *According to the definition 17, we obtain* $f(y) \geq f(x)$ *and thus* $y \succeq^{FL}_{AF} x$.
(OE) *OE is implied by CT which is satisfied.*
(NaE) *NaE is implied by OE which is satisfied.*
(AvsFD) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework where* $x \in \mathcal{A}$ *is attacked by only one non-attacked argument and* $y \in \mathcal{A}$ *has no attack branch. The non-attacked arguments have a score of 1 which implies that all the arguments directly attacked by them have a score of 0, so* $f(x) = 0$. *If* y *is non-attacked then* $f(y) = 1$. *So,* $f(y) = 1 > 0 = f(x)$ *which implies that* $y \succ^{FL}_{AF} x$. *If* y *is attacked then it is clear that* $f(y) > 0$ *because it cannot have a direct attacker with a score of 1 (otherwise one of its branch will be an attack branch but it is not the case because it has only defense branches). So* $f(y) > f(x)$ *implies that* $y \succ^{FL}_{AF} x$, *in agreement with the property.*
(AE) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $a, b \in \mathcal{A}$ *such that there exists an isomorphism* γ*: Anc(a) =* γ*(Anc(b)). Following the definition 17, a and b begin with the same score:* $f_0(a) = f_0(b) = 1$. *If* a *and* b *are not attacked then* $a \simeq^{FL}_{AF} b$ *because NaE is satisfied. Otherwise, they are both directly attacked so* $f_1(a) = f_1(b) = 0.5$. *In order to prove that* $\forall i \in \mathbb{N}$, $f_i(a) = f_i(b)$ *(which implies that a and b are equally acceptable), we must show that* $\forall i \in \mathbb{N}, \max_{c \in \mathcal{R}_1(a)} f_i(c) = \max_{c' \in \mathcal{R}_1(b)} f_i(c')$.
Let us prove that in using a reductio ad absurdum. Let us suppose that $\exists i > 2$, *s.t.* $\max_{c \in \mathcal{R}_1(a)} f_i(c) > \max_{c' \in \mathcal{R}_1(b)} f_i(c')$ *and* $\forall j < i, \max_{c \in \mathcal{R}_1(a)} f_j(c) = \max_{c' \in \mathcal{R}_1(b)} f_j(c')$. *Let us consider an argument* $c \in argmax_{c \in \mathcal{R}_1(a)} f_i(c)$. *It is clear that* $f_i(c) > f_i(\gamma(c))$ *(otherwise* $\max_{c \in \mathcal{R}_1(a)} f_i(c)$ *should be equal to* $\max_{c' \in \mathcal{R}_1(a)} f_i(c')$*). But these are the non-attacked arguments which influence the score of an argument according to the value of i:*

- *i is even* $\Rightarrow$ $f_i(c) > 0.5$ *and* $f_i(\gamma(c)) = 0.5$, *then* $\exists x \in \mathcal{R}_{i-1}(c)$ *and* $x \notin \mathcal{R}_{i-1}(\gamma(c))$ *s.t.* $\mathcal{R}_1(x) = \emptyset$.
- *i is odd* $\Rightarrow$ $f_i(c) = 0.5$ *and* $f_i(\gamma(c)) < 0.5$, *then* $\exists x \in \mathcal{R}_{i-1}(\gamma(c))$ *and* $x \notin \mathcal{R}_{i-1}(c)$ *s.t.* $\mathcal{R}_1(x) = \emptyset$.

In both cases, there exists a non-attacked argument at the beginning of a path of length $i - 1$ *for one argument but not for the other one. But it is impossible because there exists an isomorphism between Anc(a) and Anc(b), so we have* $\forall i \in \mathbb{N}$, $f_i(a) = f_i(b) \Rightarrow \forall i f_i(a) = f_i(b) \Rightarrow f(a) = f(b) \Rightarrow a \simeq^{FL} b$.

(SC) *Incompatible with CT which is satisfied.*
(CP) *Incompatible with QP which is satisfied.*
(SCT) *To show that FL does not satisfy the property Strict Counter-Transitivity (SCT),*

consider the argumentation framework from Fig. A.6. The property says that b should be



$$f(a_2) = f(a_4) = f(b_2) = f(a) = f(b) = 1$$
$$f(a_1) = f(a_3) = f(b_1) = 0$$

$$a_2 \simeq^{\mathrm{FL}} a_4 \simeq^{\mathrm{FL}} b_2 \simeq^{\mathrm{FL}} a \simeq^{\mathrm{FL}} b \succ^{\mathrm{FL}} a_1 \simeq^{\mathrm{FL}} a_3 \simeq^{\mathrm{FL}} b_1$$

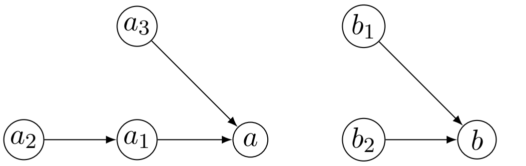
Fig. A.6. The fuzzy labeling falsifies the properties SCT, VP, +DB and ⊕DB.

strictly more acceptable than a because it exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq b'$ ($a_1 \succeq^{FL} b_1$ because $a_1 \simeq^{FL} b_1$) so $\mathcal{R}_1(a) \geq_S^{FL} \mathcal{R}_1(b)$ and $|\mathcal{R}_1(a)| > |\mathcal{R}_1(b)|$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(VP) *To show that FL does not satisfy the property Void Precedence (VP), consider the argumentation framework from Fig. A.6. Void Precedence says that a_2 should be strictly more acceptable than a because a_2 is a not attacked ($\mathcal{R}_1(a_2) = \emptyset$) while a is attacked ($\mathcal{R}_1(a) \neq \emptyset$). But the semantics considers that a_2 and a are equally acceptable, contradicting the property.*

(+DB, ⊕DB) *To show that FL does not satisfy the property Addition of Defense Branch (+DB) and the property Strict addition of Defense Branch (⊕DB), consider the argumentation framework from Fig. A.6. Both properties say that a should be strictly more acceptable than b because a has two defense branches while b has only one defense branch. But the semantics considers that a and b are equally acceptable, contradicting both properties.*

(DP) *To show that the fuzzy labeling does not satisfy the property Defense Precedence (DP), consider the argumentation framework from Fig. A.7. Defense Precedence states*



$$f(a_2) = f(a_3) = f(b_1) = f(b_2) = 1$$
$$f(a_1) = f(a) = f(b) = 0$$

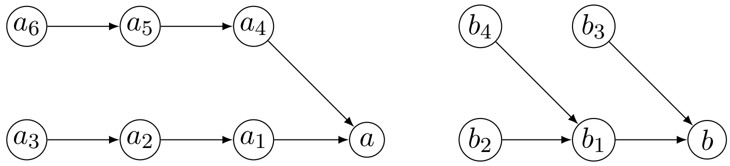
$$a_2 \simeq^{\mathrm{FL}} a_3 \simeq^{\mathrm{FL}} b_1 \simeq^{\mathrm{FL}} b_2 \succ^{\mathrm{FL}} a_1 \simeq^{\mathrm{FL}} a \simeq^{\mathrm{FL}} b$$

Fig. A.7. The fuzzy labeling falsifies the properties DP and +AB.

that a should be strictly more acceptable than b because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ and $|\mathcal{R}_2(a)| = 1 > 0 = |\mathcal{R}_2(b)|$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(+AB) *To show that FL does not satisfy the property Addition of Attack Branch (+AB), consider the argumentation framework from Fig. A.7. The property says that a_1 should be strictly more acceptable than b because b has two attack branches while a_1 has one attack branch. But the semantics considers that a_1 and b are equally acceptable, contradicting the property.*

(DDP) *To show that FL does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework from Fig. A.8. The property says that*

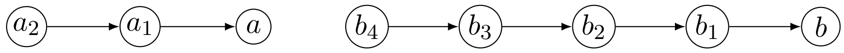


$$f(a_1) = f(a_3) = f(a_4) = f(a_6) = f(b_2) = f(b_3) = f(b_4) = 1$$
$$f(a_2) = f(a_5) = f(b_1) = f(a) = f(b) = 0$$
$$a_3 \simeq^{\text{FL}} a_6 \simeq^{\text{FL}} b_2 \simeq^{\text{FL}} b_3 \simeq^{\text{FL}} b_4 \simeq^{\text{FL}} a_1 \simeq^{\text{FL}} a_4 \succ^{\text{FL}} a_2 \simeq^{\text{FL}} a_5 \simeq^{\text{FL}} b \simeq^{\text{FL}} a \simeq^{\text{FL}} b_1$$

Fig. A.8. The fuzzy labeling falsifies the property DDP.

a should be strictly more acceptable than b because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ and the defense of a is simple and distributed while the defense of b is simple but not distributed. But the semantics considers that a and b are equally acceptable, contradicting the property.

(↑AB) *To show that FL does not satisfy the property Increase of Attack branch (↑AB), consider the argumentation framework from Fig. A.9. The property says that b_1 should be*



$$f(a_2) = f(b_4) = f(b_2) = f(a) = f(b) = 1$$
$$f(a_1) = f(b_3) = f(b_1) = 0$$
$$a_2 \simeq^{\text{FL}} b_4 \simeq^{\text{FL}} b_2 \simeq^{\text{FL}} a \simeq^{\text{FL}} b \succ^{\text{FL}} a_1 \simeq^{\text{FL}} b_3 \simeq^{\text{FL}} b_1$$

Fig. A.9. The fuzzy labeling falsifies the properties ↑DB and ↑AB.

strictly more acceptable than a_1 because the length of the attack branch of b_1 is greater than the length of the attack branch of a_1. But in using the semantics, a_1 and b_1 are equally acceptable, contradicting the property.

(↑DB) *To show that FL does not satisfy the property Increase of Defense branch (↑DB), consider the argumentation framework from Fig. A.9. The property says that a should be strictly more acceptable than b because the length of the defense branch of b is greater than the length of the defense branch of a. But in using the semantics, a and b are equally acceptable, contradicting the property.*

Counting semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [33].

(OE) *OE is implied by CT which is satisfied.*
(NaE) *NaE is implied by OE which is satisfied.*
(AE) *Use the matrix approach ensures that the score of an argument only depends on its attackers and defenders. Thus, the score of an argument is the same in focusing on its ancestors' graph as in the full argumentation framework with the same normalization factor. If there exists an isomorphism between the ancestors' graph of x and y then the topology*

of the argumentation frameworks $Anc(x)$ and $Anc(y)$ are identical, which implies that the adjacency matrix of $Anc(x)$ and $Anc(y)$ are identical too. Pu et al. guarantee that the counting model always exists and is unique so the counting model is the same for $Anc(x)$ and $Anc(y)$. So $w(x) = w(y)$ which implies that $x \simeq^{CS} y$.

(Tot) *According to [33, Theorem 1], the counting model ranges the strength value of each argument into the interval $[0, 1]$ and converges to a unique solution. The interval $[0, 1]$ is a totally ordered set of real number so all its values can be compared using $\geq$. Consequently, all the arguments can be compared too.*

(+AB,↑AB,↑DB) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists: $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments. As the property AE is satisfied, we can say that $\forall x \in \mathcal{A}$, $x \simeq^{CS}_{AF} \gamma(x)$ because if there exists an isomorphism between AF and AF′, it is also true for the subgraphs of AF and more precisely for the ancestors' graph of each argument. Consequently, $a \simeq^{CS}_{AF \cup AF'} \gamma(a)$ and $\mathcal{R}_1(\gamma(a)) \geq^{CS}_{S} \mathcal{R}_1(a)$.*

<u>+AB</u> If we add an attack branch $P^-(\gamma(a))$ to $\gamma(a)$ then we still have $\mathcal{R}_1(\gamma(a)) \geq^{CS}_{S} \mathcal{R}_1(a)$ but $|\mathcal{R}_1(\gamma(a))| > |\mathcal{R}_1(a)|$ which implies that $\mathcal{R}_1(\gamma(a)) >^{CS}_{S} \mathcal{R}_1(a)$. As the property SCT is satisfied then $a \succ^{CS}_{AF^*} \gamma(a)$, in agreement with the property.

<u>↑AB</u> Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N} + 1$: $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a \rangle$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. As the property VP is satisfied, the score of $\gamma(b)$, which is now attacked, becomes lower than the score of b (so $b \succ^{CS}_{AF^*} \gamma(b)$). Combining with the fact that SCT is satisfied, then $\gamma(b_{n-1}) \succ^{CS}_{AF^*} b_{n-1}$. With the same reasoning, we obtain $b_{n-2} \succ^{CS}_{AF^*} \gamma(b_{n-2})$ and so on until $\gamma(a) \succ^{CS}_{AF^*} a$, in agreement with the property.

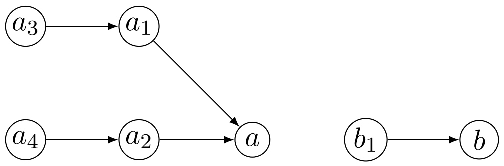
<u>↑DB</u> The reasoning is similar to the proof of ↑AB.

(+DB) *Incompatible with SCT and AE which are satisfied.*

(⊕DB) *Incompatible with VP which is satisfied.*

(SC) *Incompatible with AE which is satisfied.*

(AvsFD) *To show that the counting semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework from Fig. A.10. The property*



If $\alpha = 0.5$:
$w(a_3) = w(a_4) = w(b_1) = 1$
$w(a_1) = w(a_2) = w(b) = 0.75$
$w(a) = 0.5625$

$$a_3 \simeq^{CS} a_4 \simeq^{CS} b_1 \succ^{CS} a_1 \simeq^{CS} a_2 \simeq^{CS} b \succ^{CS} a$$

Fig. A.10. The counting semantics falsifies the property AvsFD.

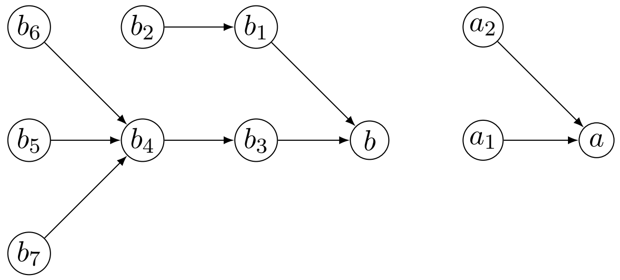
says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But the counting semantics considers that b is strictly more acceptable than a, contradicting the property.

Tuples-based semantics

The results concerning the properties Void Precedence (VP), Addition of Defense Branch (+DB), Addition of Attack Branch (+AB), Increase of Attack branch (↑AB), Increase of Defense branch (↑DB) and Total (Tot) can be found in [16].

(Abs) *If there exists an isomorphism γ between two argumentation frameworks AF and AF' then they have the same structure. So for each argument, its image has exactly the same number of branches with the same length which implies that an argument and it image have the same tupled value: $\forall x \in Arg(AF), v(x) = v(\gamma(x))$. Thus, following Algorithm 1, for all arguments $a, b \in Arg(AF)$, as $v(a) = v(\gamma(a))$ and $v(b) = v(\gamma(b))$, it is clear that if $a \succeq^{T}_{AF} b$ then we also have $\gamma(a) \succeq^{T}_{AF'} \gamma(b)$.*
(In) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $AF' \in cc(AF)$ with $a, b \in Arg(AF')$ and $c \notin Arg(AF')$. The tupled value of an argument is only computed from its attack and defense roots which necessarily belongs to the same component as the arguments. So as there exists no path between a (respectively b) and c, then c cannot be a root of a (respectively b). Consequently, it cannot influence the ranking between a and b.*
(OE) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $a, b \in \mathcal{A}$ such that there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall z \in \mathcal{R}_1(a), z \simeq^{\sigma}_{AF} f(z)$. According to Algorithm 1, two arguments are equally acceptable if they have the same tupled value so $\forall z \in \mathcal{R}_1(a), v(z) = v(f(z))$. The original definition [16, Definition 10] computes the tupled value of each argument on the basis of the tupled value of its direct attackers. Because the tupled values of the direct attackers of a and b are the same, then they obtain the same tupled value ($v(a) = v(b)$) which implies that $a \simeq^{T}_{AF} b$.*
(NaE) *NaE is implied by OE which is satisfied.*
(AE) *Obvious because if a and b have the same ancestors' graph, then they have the same number of branches with a length of 1, the same number of branches with a length of 2 and so on. So $v_p(a) = v_p(b)$ and $v_i(a) = v_i(b) \Rightarrow v(a) = v(b) \Rightarrow a \simeq^{T}_{AF} b$.*
(AvsFD) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $a, b \in \mathcal{A}$ such that $|\mathcal{B}_{-}(b)| = 0$, $|\mathcal{R}_1(a)| = 1$ and $|\mathcal{R}_2(a)| = 0$. We have $|v_i(a)| = 1$ and $|v_p(a)| = 0$ because $v(a) = [(), (1)]$. Concerning b, its tupled-value respects the following criteria: $|v_i(b)| = 0$ and $|v_p(b)| \geq 0$. If b is not attacked, then $b \succ^{T}_{AF} a$ because VP is satisfied. If b is attacked (but defended), then $|v_p(b)| > 0$, then we have $|v_i(a)| > |v_i(b)|$ and $|v_p(a)| < |v_p(b)|$ and, according to Algorithm 1, b is strictly more acceptable than a ($b \succ^{T}_{AF} a$).*

(⊕DB) *Incompatible with VP which is satisfied.*
(SC) *Incompatible with AE which is satisfied.*
(DP) *To show that the tuples-based semantics does not satisfy the property Defense Precedence (DP), consider the argumentation framework from Fig. A.11. The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ and $|\mathcal{R}_2(a)| = 0 < 2 = |\mathcal{R}_2(b)|$. But we obtain two incomparable tuples: $v(a) = [(), (1, 1)]$ and $v(b) = [(2), (3, 3, 3)]$ (see Algorithm 1 Case 7 [16]: $|v_i(a)| < |v_i(b)|$ and $|v_p(a)| < |v_p(b)|$), so a and b are incomparable, contradicting the property.*
(QP) *To show that the tuples-based semantics does not satisfy the property Quality Precedence (QP), consider the argumentation framework from Fig. A.11. The property says that b should be more acceptable that a because $a_2 \succeq^{T} b_1$ and $a_2 \succeq^{T} b_3$ (a_1 can also be used).*



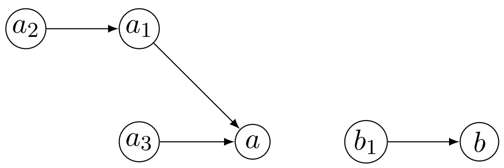
$$v(a_1) = v(a_2) = v(b_2) = [0^\infty, ()]$$
$$v(b_5) = v(b_6) = v(b_7) = [0^\infty, ()]$$
$$v(b_1) = [(), (1)]$$
$$v(a) = [(), (1, 1)]$$
$$v(b_4) = [(), (1, 1, 1)]$$
$$v(b_3) = [(2, 2, 2), ()]$$
$$v(b) = [(2), (3, 3, 3)]$$

$$a_1 \simeq^{\mathrm{T}} a_2 \simeq^{\mathrm{T}} b_2 \simeq^{\mathrm{T}} b_5 \simeq^{\mathrm{T}} b_6 \simeq^{\mathrm{T}} b_7 \succ^{\mathrm{T}} b_3 \succ^{\mathrm{T}} b_1 \succ^{\mathrm{T}} a \succ^{\mathrm{T}} b_4$$
$$a_1 \simeq^{\mathrm{T}} a_2 \simeq^{\mathrm{T}} b_2 \simeq^{\mathrm{T}} b_5 \simeq^{\mathrm{T}} b_6 \simeq^{\mathrm{T}} b_7 \succ^{\mathrm{T}} b_3 \succ^{\mathrm{T}} b \succ^{\mathrm{T}} b_4$$
$$a \not\succeq^{\mathrm{T}} b \text{ and } b \not\succeq^{\mathrm{T}} a$$
$$b_1 \not\succeq^{\mathrm{T}} b \text{ and } b \not\succeq^{\mathrm{T}} b_1$$

Fig. A.11. The tuples-based semantics falsifies the properties DP and QP.

But, using the tuples-based semantics, a and b are incomparable, contradicting the property.

(CT) *To show that the tuples-based semantics does not satisfy the property Counter-Transitivity (CT), consider the argumentation framework from Fig. A.12. The definition*



$$v(a_2) = v(a_3) = v(b_1) = [0^\infty, ()]$$
$$v(a_1) = v(b) = [(), (1)]$$
$$v(a) = [(2), (1, 1)]$$

$$a_2 \simeq^{\mathrm{T}} a_3 \simeq^{\mathrm{T}} b_1 \succ^{\mathrm{T}} a \succ^{\mathrm{T}} a_1 \simeq^{\mathrm{T}} b$$

Fig. A.12. The tuples-based semantics falsifies the properties CT, SCT and CP.

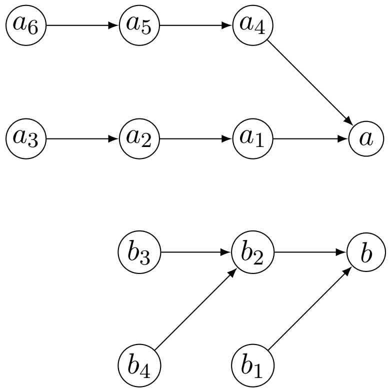
says that b should be at least as acceptable as a because there exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq^T b'$. Indeed, we have $\mathcal{R}_1(b) = \{b_1\}$ and $\mathcal{R}_1(a) = \{a_1, a_3\}$ and $a_3 \succeq^T b_1$. But, using the tuples-based semantics, a is strictly more acceptable than b, contradicting the property.

(SCT) *To show that the tuples-based semantics does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework from Fig. A.12. The property says that b should be strictly more acceptable than a because it exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq^T b'$ and $|\mathcal{R}_1(b)| < |\mathcal{R}_1(a)|$. Indeed, we have $\mathcal{R}_1(b) = \{b_1\}$ and $\mathcal{R}_1(a) = \{a_1, a_3\}$ (so $|\mathcal{R}_1(b)| = 1 < 2 = |\mathcal{R}_1(a)|$) and $a_3 \succeq^T b_1$. But, using the tuples-based semantics, a is strictly more acceptable than b, contradicting the property.*

(CP) *To show that the tuples-based semantics does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework from Fig. A.12. The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = 2 > 1 = |\mathcal{R}_1(b)|$. But, using the tuples-based semantics, a is strictly more acceptable than b, contradicting the property.*

(DDP) *To show that the tuples-based semantics does not satisfy the property Distributed-*

Defense Precedence (DDP), consider the argumentation framework from Fig. A.13. The



$$v(a_3) = v(a_6) = [0^\infty, ()]$$
$$v(b_1) = v(b_3) = v(b_4) = [0^\infty, ()]$$
$$v(a_2) = v(a_5) = [(), (1)]$$
$$v(a_1) = v(a_4) = [(2), ()]$$
$$v(b_2) = [(), (1, 1)]$$
$$v(a) = [(), (3, 3)]$$
$$v(b) = [(2, 2), (1)]$$
$$b \succ^{\mathrm{T}} a$$

Fig. A.13. The tuples-based semantics falsifies the property DDP.

property says that a should be strictly more acceptable than b because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ *and the defense of a is simple and distributed while the defense of b is simple but not distributed. But, using the tuples-based semantics, b is strictly more acceptable than a, contradicting the property.*

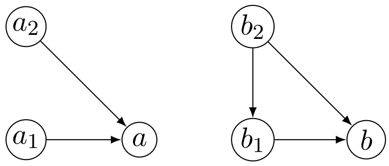
Ranking-based semantics 2ZG

The results concerning the properties Independence (In), Void Precedence (VP) and Self-Contradiction (SC) can be found in [30].

(Abs) *The nature of an argument is not used in the computation of its score. Only the attack relation is needed.*
(Tot) *This semantics guarantees a comparison between all the arguments because the score of an argument* $a \in \mathcal{A}$ *is such that* $s(a) \in [0, 1]$ *which is a totally ordered set. In [30], they ensure the existence of a value v thanks to the minimax theorem (von Neumann 1928). So all the arguments can be compared.*
(NaE) *Obvious because the non-attacked arguments have a score of 1 (see [30, Proposition 4]) which is the maximal value so they always have the same score.*
(AvsFD) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework and* $a, b \in \mathcal{A}$ *be two arguments where b is attacked by a non-attacked argument and a has no attack branch. The value of the zero-sum game for b is* $v(b) = 0.25$. *For a, we can say that this argument belongs to the set of stable (and so admissible) extension because it has only defense branches. Moreover, the Proposition 5 [30] says that if an argument belong to a stable extension (which is unique here then its strength is greater or equal to* $\frac{1}{2}$*) then* $v(a) \geq \frac{1}{2}$. *Consequently, we have* $v(b) = 0.25 < 0.5 \leq v(a)$ *and so* $a \succ b$.

(AE) *Incompatible with SC which is satisfied.*
($\oplus$DB) *Incompatible with VP which is satisfied.*
(DP) *To show that the ranking-based semantics 2ZG does not satisfy the property Defense*

Precedence (DP), consider the argumentation framework from Fig. A.14. The property

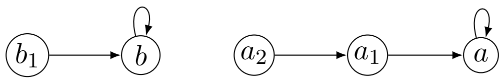


$s(a_1) = s(a_2) = s(b_2) = 1$
$s(b_1) = 0.25$
$s(a) = s(b) \simeq 0.16$

$$a_1 \simeq^{\mathrm{2ZG}} a_2 \simeq^{\mathrm{2ZG}} b_2 \succ^{\mathrm{2ZG}} b_1 \succ^{\mathrm{2ZG}} a \simeq^{\mathrm{2ZG}} b$$

Fig. A.14. The ranking-based semantics 2ZG falsifies the property DP.

says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ but $|\mathcal{R}_2(a)| = 0 < 1 = |\mathcal{R}_2(b)|$. But in using the semantics, a and b are equally acceptable, contradicting the property.
(QP) *To show that the ranking-based semantics 2ZG does not satisfy the property Quality Precedence (QP), consider the argumentation framework from Fig. A.15. The property*

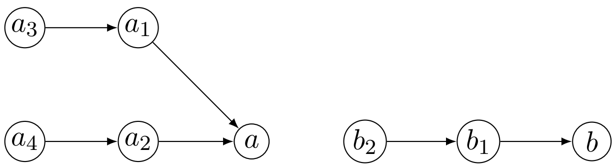


$$a_2 \simeq^{\mathrm{2ZG}} b_1 \succ^{\mathrm{2ZG}} a_1 \succ^{\mathrm{2ZG}} a \simeq^{\mathrm{2ZG}} b$$

$s(a_2) = s(b_1) = 1$
$s(a_1) = 0.25$
$s(a) = s(b) = 0$

Fig. A.15. The ranking-based semantics 2ZG falsifies the property QP.

says that a should be strictly more acceptable than b because $b_1 \succ^{2ZG} a_1$ and $b_1 \succ^{2ZG} a$. But the semantics considers that a and b are equally acceptable, contradicting the property.
(CP) *To show that the ranking-based semantics 2ZG does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework from Fig. A.16. The*

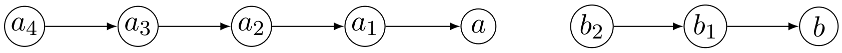


$s(a_3) = s(a_4) = s(b_1) = 1$
$s(a_1) = s(a_2) = 0.25$
$s(a) = s(b) = 0.5$

$$a_3 \simeq^{\mathrm{2ZG}} a_4 \simeq^{\mathrm{2ZG}} b_2 \succ^{\mathrm{2ZG}} a \simeq^{\mathrm{2ZG}} b \succ^{\mathrm{2ZG}} a_1 \simeq^{\mathrm{2ZG}} a_2 \simeq^{\mathrm{2ZG}} b_1$$

Fig. A.16. The ranking-based semantics 2ZG falsifies the properties CP and +DB.

property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = 2 > 1 = |\mathcal{R}_1(b)|$. But in using the semantics, a and b are equally acceptable, contradicting the property.
(+DB) *To show that the ranking-based semantics 2ZG does not satisfy the property Addition of Defense Branch (+DB), consider the argumentation framework from Fig. A.16. The property says that a should be strictly more acceptable than b because a has one more defense branch than b. But in using the semantics, a and b are equally acceptable, contradicting the property.*
(↑DB) *To show that the ranking-based semantics 2ZG does not satisfy the property Increase of Defense branch (↑DB), consider the argumentation framework from Fig. A.17.*



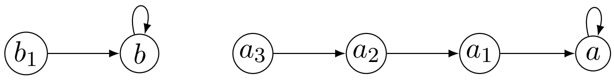
$s(a_4) = s(b_2) = 1$, $s(a_1) \simeq 0.386$, $s(a_3) = s(b_1) = 0.25$ and $s(a) = s(b) = s(a_2) = 0.5$

$$a_3 \simeq^{2ZG} b_1 \succ^{2ZG} a \simeq^{2ZG} b \simeq^{2ZG} a_2 \succ^{2ZG} a_1 \succ^{2ZG} a_3 \simeq^{2ZG} b_1$$

Fig. A.17. The ranking-based semantics 2ZG falsifies the property ↑DB.

The property says that b should be strictly more acceptable than a because the defense branch of a is longer than the defense branch of b. But in using the semantics, a and b are equally acceptable, contradicting the property.

(↑AB) *To show that the ranking-based semantics 2ZG does not satisfy the property Increase of Attack branch (↑AB), consider the argumentation framework from Fig. A.18. The property says that a should be strictly more acceptable than b because the attack branch of a is longer than the attack branch of b. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.*

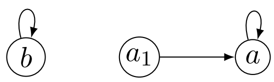


$s(b_1) = s(a_3) = 1$
$s(a_2) = 0.25$
$s(a_1) = 0.5$
$s(a) = s(b) = 0$

$$b_1 \simeq^{2ZG} a_3 \succ^{2ZG} a_1 \succ^{2ZG} a_2 \succ^{2ZG} a \simeq^{2ZG} b$$

Fig. A.18. The ranking-based semantics 2ZG falsifies the property ↑AB.

(+AB) *To show that the ranking-based semantics 2ZG does not satisfy the property Addition of Attack Branch (+AB), consider the argumentation framework from Fig. A.19. The property says that b should be strictly more acceptable than a because a has one more attack branch that b. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.*



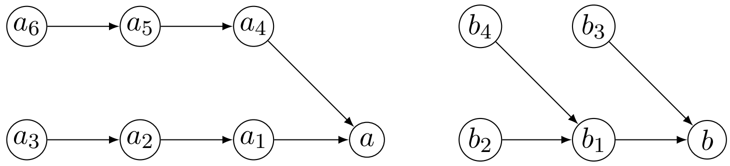
$s(a_1) = 1$
$s(a) = s(b) = 0$

$a_1 \succ^{2ZG} a \simeq^{2ZG} b$

Fig. A.19. The ranking-based semantics 2ZG falsifies the property +AB.

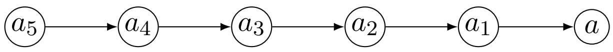
(DDP) *To show that the ranking-based semantics 2ZG does not satisfy the property Distributed Defense Precedence (DDP), consider the argumentation framework from Fig. A.20. The definition says that a should be strictly more acceptable than b because they have the same number of direct attackers ($|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$) and the same number of direct defenders ($|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$) but the defense of a is simple and distributed whereas the defense of b is simple and not distributed. But in using the semantics, b is strictly more acceptable than a, contradicting the property.*

(OE) *To show that the ranking-based semantics 2ZG does not satisfy the property Ordinal Equivalence (OE), consider the argumentation framework from Fig. A.21. The property says that a and a_2 should be equally acceptable because there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(a_2)$ such that $\forall b \in \mathcal{R}_1(a)$, $f(b) \simeq^{2ZG} b$ ($a_3 \simeq^{2ZG} a_1$). But in using the semantics, a is strictly more acceptable than a_2, contradicting the property.*



$$s(a_3) = s(a_6) = s(b_2) = s(b_3) = s(b_4) = 1$$
$$s(a_2) = s(a_5) = s(b) = 0.25$$
$$s(a_1) = s(a_4) = 0.5$$
$$s(b_1) = s(a) = 0.1667$$
$$a_3 \simeq^{\mathrm{2ZG}} a_6 \simeq^{\mathrm{2ZG}} b_2 \simeq^{\mathrm{2ZG}} b_3 \simeq^{\mathrm{2ZG}} b_4 \succ^{\mathrm{2ZG}} a_1 \simeq^{\mathrm{2ZG}} a_4 \succ^{\mathrm{2ZG}} b \simeq^{\mathrm{2ZG}} a_2 \simeq^{\mathrm{2ZG}} a_5 \succ^{\mathrm{2ZG}} a \simeq^{\mathrm{2ZG}} b_1$$

Fig. A.20. The ranking-based semantics 2ZG falsifies the property Distributed-Defense Precedence.



$$a_5 \succ^{\mathrm{2ZG}} a_1 \simeq^{\mathrm{2ZG}} a_3 \succ^{\mathrm{2ZG}} a \succ^{\mathrm{2ZG}} a_2 \succ^{\mathrm{2ZG}} a_4$$

$$s(a_5) = 1$$
$$s(a_4) = 0.25$$
$$s(a_3) = s(a_1) = 0.5$$
$$s(a_2) \simeq 0.3863$$
$$s(a) = 0.425$$

Fig. A.21. The ranking-based semantics 2ZG falsifies the property OE.

(CT) *To show that the ranking-based semantics 2ZG does not satisfy the property Counter-Transitivity (CT), consider the argumentation framework from Fig. A.21. The property says that* a_2 *should be at least as acceptable as* a *because there exists an injective function* f *from* $\mathcal{R}_1(a_2)$ *to* $\mathcal{R}_1(a)$ *such that* $\forall b \in \mathcal{R}_1(a_2)$, $f(b) \succeq b$ ($a_1 \simeq^{2ZG} a_3$ *which implies* $a_1 \succeq^{2ZG} a_3$) *so* $\mathcal{R}_1(a) \geq_S^{2ZG} \mathcal{R}_1(a_2)$. *But the semantics considers that* a *is strictly more acceptable than* a_2, *contradicting the property.*

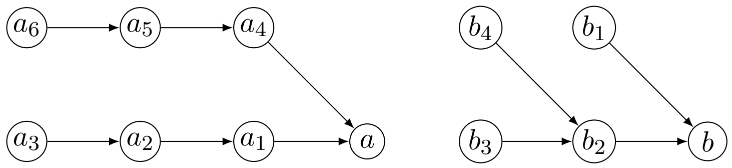
(SCT) *To show that the ranking-based semantics 2ZG does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework from Fig. A.21. The property says that* a_3 *should be strictly more acceptable than* a_1 *because it exists an injective function* f *from* $\mathcal{R}_1(a_3)$ *to* $\mathcal{R}_1(a_1)$ *such that* $\forall b \in \mathcal{R}_1(a_3)$, $f(b) \succeq b$ ($a_2 \succeq^{2ZG} a_4$) *and especially* $a_2 \succ^{2ZG} a_4$, *so* $\mathcal{R}_1(a_1) >_S^{2ZG} \mathcal{R}_1(a_3)$. *But the semantics considers that* a *and* b *are equally acceptable, contradicting the property.*

Iterated Graded Defense semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP), Addition of an Attack Branch (+AB), Strict Addition of a Defense Branch (⊕DB), Addition of a Defense Branch (+DB), Increase of an Attack branch (↑AB), Increase of a Defense branch (↑DB), Total (Tot) and Non-attacked Equivalence (NaE) can be found in [25,26].

(AE) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $a, b \in \mathcal{A}$ *such that there exists an isomorphism* γ: $Anc(a) = \gamma(Anc(b))$. *This isomorphism implies that* $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)|$ *and* $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)|$. *So* *according to the definition 19, the graded defense is computed from the number of direct attackers and the number of direct defenders of arguments. So* $\forall n, m \in \mathbb{N}^*$, $a \in \underset{n}{d_m}(\mathcal{X}) \Leftrightarrow b \in \underset{n}{d_m}(\mathcal{X})$*. Following the same reasoning, as* $\forall i \in \mathbb{N}$, $|\mathcal{R}_i(a)| = |\mathcal{R}_i(b)|$ *then* $\forall n, m \in \mathbb{N}^*$, $\forall j \in \mathbb{N}$, $a \in \underset{n}{d^j_m}(\mathcal{X}) \Leftrightarrow b \in \underset{n}{d^j_m}(\mathcal{X})$*. Let us recall that the indefinite iteration of* $\underset{n}{d_m}(\mathcal{X})$ *is defined as* $\underset{n}{d^*_m}(\mathcal{X}) = \bigcup_{0 \leq i \leq \alpha} \underset{n}{d^i_m}(\mathcal{X})$*. So, we can say that* $\forall n, m \in \mathbb{N}^*$, $a \in \underset{n}{d^*_m}(\mathcal{X}) \Leftrightarrow b \in \underset{n}{d^*_m}(\mathcal{X})$*, and by definition,* $a \simeq^{IGD}_{AF} b$.

(DDP) *To show that the iterated graded defense semantics does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework from Fig. A.22. The property says that a should be strictly more acceptable than b because* $|\mathcal{R}_1(a)| =$



$$b_1 \simeq^{\text{IGD}} b_3 \simeq^{\text{IGD}} b_4 \simeq^{\text{IGD}} a_3 \simeq^{\text{IGD}} a_6 \succ^{\text{IGD}} a_1 \simeq^{\text{IGD}} a_4 \succ^{\text{IGD}} a_2 \simeq^{\text{IGD}} a_5 \succ^{\text{IGD}} b \succ^{\text{IGD}} a \succ^{\text{IGD}} b_2$$

Fig. A.22. The iterated graded defense semantics falsifies the property DDP.

$|\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ *and the defense of a is simple and distributed while the defense of b is simple but not distributed. But the semantics considers that b is strictly more acceptable than a, contradicting the property.*
(AvsFD) *To show that the iterated graded defense semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework from Fig. A.23. The property says that a should be strictly more acceptable than b because a has only*

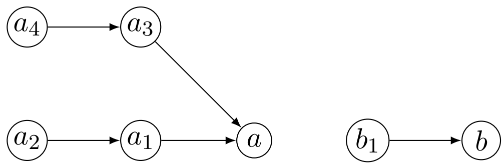
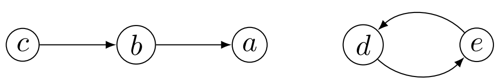


Fig. A.23. The iterated graded defense semantics falsifies the property AvsFD.

defense branches while b has exactly one direct attacker and no defense branch. But the semantics considers that a and b are incomparable because $a \in \underset{1}{d^*_1}(\emptyset)$ *but* $a \notin \underset{2}{d^*_2}(\emptyset)$ *while* $b \notin \underset{1}{d^*_1}(\emptyset)$ *but* $b \in \underset{2}{d^*_2}(\emptyset)$*, which contradicts the property.*
(OE) *To show that the iterated graded defense semantics does not satisfy the property Ordinal Equivalence (OE), consider the argumentation framework from Fig. A.24. The*



$$c \succ^{\text{IGD}} a \succ^{\text{IGD}} b \simeq^{\text{IGD}} d \simeq^{\text{IGD}} e$$

Fig. A.24. The grounded semantics falsifies the property OE.

property states that a and e should be equally acceptable ($a \simeq^{IGD} e$) *because there exists a* *bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(e)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \simeq^{IGD} a'$ (b and d are equally acceptable). But in using the semantics, a is strictly more acceptable than e, which contradicts the property.*

Propagation semantics

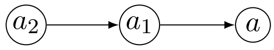
The results can be found in [11].

■

Proof (Proposition 5 (page 35))

(Abs) *The nature of an argument is not used in the computation of its score. Only the attack relation is needed.*
(In) *This is directly connected with the property Directionality [6] which says that if an argument a attacks an argument b then a affects b and not vice versa. So the only arguments which have a direct or indirect impact on an argument are the arguments belonging to its ancestors' graph. So an argument such that there exists no path to (or from) another argument, cannot influence the score of this argument.*
(Tot) *All the arguments are either accepted or not accepted so a comparison between two arguments is always guaranteed.*
(NaE) *All the non-attacked argument are accepted, so they are all equally acceptable.*
(AE) *This is directly connected with the property Directionality [6] which says that if an argument a attacks an argument b then a affects b and not vice versa. So the only arguments which have a direct or indirect impact on an argument are the arguments belonging to its ancestors' graph. So two arguments with the "same" ancestors' graph are equally acceptable.*
(AvsFD) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an acyclic argumentation framework with $a, b \in \mathcal{A}$ be two arguments where b is attacked by a non-attacked argument and a has no attack branch. It is clear that b is not accepted because it is directly attacked by a non-attacked argument which is accepted. If a is not attacked then it is directly accepted. And if it is attacked then it is accepted too by the grounded semantics because AF is acyclic (so there exists at least one non-attacked argument) and as a has no attack branch so all its direct attackers are not accepted. So, in every cases, a is accepted whereas b is rejected. Consequently, a is more acceptable than b, in agreement with the property.*

(SC) *Incompatible with AE which is satisfied.*
(VP) *To show that the grounded semantics does not satisfy the property Void Precedence (VP), consider the argumentation framework from Fig. A.25. Void Precedence says that a_2*

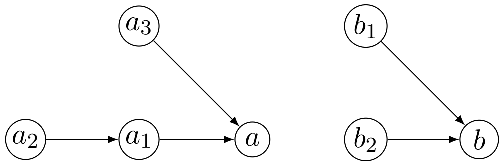


$$\mathcal{E}_{gr}(AF) = \{a_2, a\}$$
$$a_2 \simeq^{gr} a \succ^{gr} a_1$$

Fig. A.25. The grounded semantics falsifies the property VP.

should be strictly more acceptable than a ($a_2 \succ^{gr} a$) because a_2 is a not attacked while a is

attacked by a_1. But the grounded semantics considers that a_2 and a are equally acceptable (a_2 and a are both accepted), contradicting the property.
(DP) To show that the grounded semantics does not satisfy the property Defense Precedence (DP), consider the argumentation framework from Fig. A.26. Defense Precedence

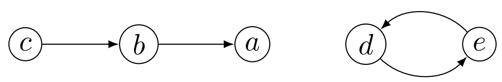


$$\mathcal{E}_{gr}(AF) = \{a_2, a_3, b_1, b_2\}$$

$$a_2 \simeq^{gr} a_3 \simeq^{gr} b_1 \simeq^{gr} b_2 \succ^{gr} a_1 \simeq^{gr} a \simeq^{gr} b$$

Fig. A.26. The grounded semantics falsifies the properties DP, SCT and CP.

says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ and $|\mathcal{R}_2(a)| = 1 > 0 = |\mathcal{R}_2(b)|$. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.
(SCT) To show that the grounded semantics does not satisfy the property Strict-Counter Transitivity (SCT), consider the argumentation framework from Fig. A.26. The property says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because there exists an injective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \succeq a'$ ($b_1 \succeq^{gr} a_3$ and $b_2 \succeq^{gr} a_1$) and especially $b_2 \succ^{gr} a_1$, so $\mathcal{R}_1(b) >^{gr}_S \mathcal{R}_1(a)$. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.
(CP) To show that the grounded semantics does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework from Fig. A.26. The property says that a_1 should be strictly more acceptable than b because $|\mathcal{R}_1(b)| = 2 > 1 = |\mathcal{R}_1(a_1)|$. But the grounded semantics considers that a_1 and b are equally acceptable, contradicting the property.
(CT) To show that the grounded semantics does not satisfy the property Counter Transitivity (CT), consider the argumentation framework from Fig. A.27. The property says



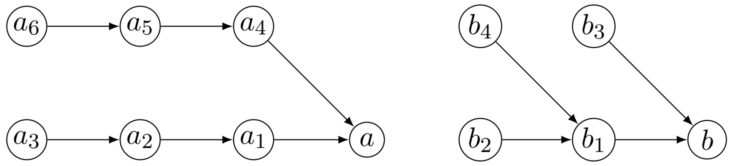
$$\mathcal{E}_{gr}(AF) = \{a, c\}$$
$$a \simeq^{gr} c \succ^{gr} b \simeq^{gr} d \simeq^{gr} e$$

Fig. A.27. The grounded semantics falsifies the properties CT, QP and OE.

that e should be at least as acceptable than a ($e \succeq^{gr} a$) because there exists an injective function f from $\mathcal{R}_1(e)$ to $\mathcal{R}_1(a)$ such that $\forall e' \in \mathcal{R}_1(e)$, $f(e') \succeq e'$ ($b \simeq^{gr} d$ which implies $b \succeq^{gr} d$) so $\mathcal{R}_1(a) \geq^{gr}_S \mathcal{R}_1(e)$. But the grounded semantics considers that a is strictly more acceptable than e, contradicting the property.
(QP) To show that the grounded semantics does not satisfy the property Quality Precedence (QP), consider the argumentation framework from Fig. A.27. The property says that e should be strictly more acceptable than b ($e \succ^{gr} b$) because $c \succ^{gr} d$. But the grounded semantics considers that e and b are equally acceptable, contradicting the property.
(OE) To show that the grounded semantics does not satisfy the property Ordinal Equivalence (OE), consider the argumentation framework from Fig. A.27. The property says that a and e should be equally acceptable ($a \simeq^{gr} e$) because there exists a bijective function f

from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(e)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \simeq^{gr} a'$ (b and d are both rejected). But in using the grounded semantics, a is accepted whereas e is rejected ($a \succ^{gr} e$), contradicting the property.

(DDP) *To show that the grounded semantics does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework from Fig. A.28. The*



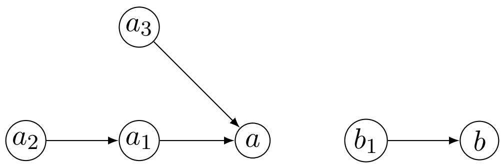
$$\mathcal{E}_{gr}(AF) = \{a_3, a_6, b_2, b_3, b_4, a_1, a_4\}$$

$$a_3 \simeq^{gr} a_6 \simeq^{gr} b_2 \simeq^{gr} b_3 \simeq^{gr} b_4 \simeq^{gr} a_1 \simeq^{gr} a_4 \succ^{gr} a_2 \simeq^{gr} a_5 \simeq^{gr} b \simeq^{gr} a \simeq^{gr} b_1$$

Fig. A.28. The grounded semantics falsifies the property DDP.

property says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ and the defense of a is simple and distributed while the defense of b is simple but not distributed. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.

(+DB,$\oplus$DB) *To show that the grounded semantics does not satisfy the property Addition of Defense Branch (+DB) and the property Strict addition of Defense Branch ($\oplus$DB), consider the argumentation framework from Fig. A.29. Both properties say that a should*



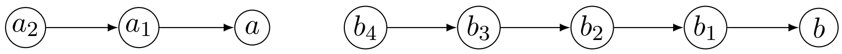
$$\mathcal{E}_{gr}(AF) = \{a_2, a_3, b_1\}$$

$$a_2 \simeq^{gr} a_3 \simeq^{gr} b_1 \succ^{gr} a_1 \simeq^{gr} a \simeq^{gr} b$$

Fig. A.29. The grounded semantics falsifies the properties +DB and $\oplus$DB.

be strictly more acceptable than b ($a \succ^{gr} b$) because a has one defense branch and b has no defense branch. But the grounded semantics considers that a and b are equally acceptable, contradicting both properties.

($\uparrow$AB) *To show that the grounded semantics does not satisfy the property Increase of Attack branch ($\uparrow$AB), consider the argumentation framework from Fig. A.30. The property*



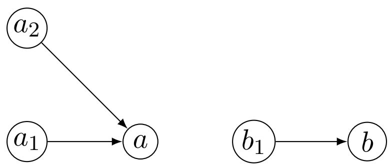
$$\mathcal{E}_{gr}(AF) = \{a_2, a, b_4, b_2, b\} \qquad a_2 \simeq^{gr} a \simeq^{gr} b_4 \simeq^{gr} b_2 \simeq^{gr} b \succ^{gr} a_1 \simeq^{gr} b_3 \simeq^{gr} b_1$$

Fig. A.30. The grounded semantics falsifies the properties $\uparrow$AB and $\uparrow$DB.

says that b_1 should be strictly more acceptable than a_1 because the length of the attack branch of b_1 is greater than the length of the attack branch of a_1. But the grounded semantics considers that a_1 and b_1 are equally acceptable, contradicting the property.

($\uparrow$DB) *To show that the grounded semantics does not satisfy the property Increase of Defense branch ($\uparrow$DB), consider the argumentation framework from Fig. A.30. The property says that a should be strictly more acceptable than b because the length of the defense*

branch of b is greater than the length of the defense branch of a. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.
(+AB) *To show that the grounded semantics does not satisfy the property Addition of Attack Branch (+AB), consider the argumentation framework from Fig. A.31. The prop-*



$$\mathcal{E}_{gr}(AF) = \{a_1, a_2, b_1\}$$

$$a_1 \simeq^{gr} a_2 \simeq^{gr} b_1 \succ^{gr} a \simeq^{gr} b$$

Fig. A.31. The grounded semantics falsifies the property +AB.

erty says that b should be strictly more acceptable than a ($b \succ^{gr} a$) because a has one attack branch while b has two attack branches. But the grounded semantics considers that a and b are equally acceptable, contradicting the property. ■

Proof (Proposition 6 (page 35))
(wVP) *A non-attacked argument is always accepted while an attacked argument can be accepted or rejected. Consequently, a non-attacked argument is always at least as acceptable as an attacked argument, in agreement with the property.*

(wDP) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $x, y \in \mathcal{A}$ such that $|\mathcal{R}_1(x)| = |\mathcal{R}_1(y)|, \mathcal{R}_2(x) \neq \emptyset$ and $\mathcal{R}_2(y) = \emptyset$. Argument y is clearly always rejected because it is only attacked by at least one non-attacked argument. So if x is accepted then $x \succ^{gr} y$ and if x is rejected then $x \simeq^{gr} y$. In both cases, $x \succeq^{gr} y$, in agreement with the property.*

(wQP) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $x, y \in \mathcal{A}$. Suppose that $\exists y' \in \mathcal{R}_1(y)$ such that $\forall x' \in \mathcal{R}_1(x)$, $y' \succ^{gr} x'$. It means that y' is accepted which implies that y is rejected. So if x is accepted then $x \succ^{gr} y$ and if x is rejected then $x \simeq^{gr} y$. In both cases, $x \succeq^{gr} y$, in agreement with the property.*

(wDDP) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $x, y \in \mathcal{A}$ such that the defense of x is simple and distributed and the defense of y is simple but not distributed. Clearly, the fact that the defense of y is not distributed implies that y is always attacked by at least one non-attacked argument. Thus, y is always rejected. So if x is accepted then $x \succ^{gr} y$ and if x is rejected then $x \simeq^{gr} y$. In both cases, $x \succeq^{gr} y$, in agreement with the property.*

(wSC) *Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $x, y \in \mathcal{A}$ such that x does not attack itself and y attacks itself. Clearly, y is rejected under the grounded semantics. So if x is accepted then $x \succ^{gr} y$ and if x is rejected then $x \simeq^{gr} y$. In both cases, $x \succeq^{gr} y$, in agreement with the property.*

(w⊕DB, w+DB, w↑DB, w↑AB) *These actions will have no influence on the acceptability of the targeted argument. Indeed, if an argument x is accepted, adding (or increasing) a defense branch to x does not change the acceptability of x, i.e. x is still accepted. And, if x is rejected, then adding (or increasing) a defense branch will not change its acceptability, i.e. will not impact the origin of the non-acceptation of x. Idem for ↑AB.*

(w+AB) *Let* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ *be two argumentation frameworks such that an isomorphism* γ *exists:* $AF = \gamma(AF')$*. Let* $x \in \mathcal{A}$ *and its image* $\gamma(x) \in \mathcal{A}'$ *be two arguments. Let us add an attack branch to* $\gamma(x)$*. Clearly,* y *is now always rejected because it is (directly or indirectly) attacked by an accepted argument. So if* x *is accepted then* $x \succ^{gr} \gamma(x)$ *and if* x *is rejected then* $x \simeq^{gr} \gamma(x)$*. In both cases,* $x \succeq^{gr} \gamma(x)$*, in agreement with the property.*

■