Contrepartie sémantique de la révision locale de croyances par le modèle C-structure

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Résumé : Dans ce papier, nous définissons un nouveau modèle pour la représentation et la révision locale de croyances que nous appelons le modèle C-structure. D'autre part, en utilisant les systèmes de sphères de Grove, nous considérons des contraintes additionnelles sur le calcul de la distance entre les interprétations et nous prouvons que ces contraintes caractérisent précisément la contrepartie sémantique de la révision par le modèle C-structure.

Mots-clés : modèle C-structure, systèmes de sphères de Grove, révision de croyances.

1 Introduction

Agents faced with incomplete, uncertain, and inaccurate information must employ a rational belief revision operation in order to manage belief changes. The agent's epistemic state represents its reasoning process ; belief revision consists of modifying its initial epistemic state in order to maintain consistency, while keeping new information and modifying previous information *as less as possible* (principle of minimal change).

To introduce relevance-sensitivity into belief revision, Parikh (Parikh 99) defined the language splitting model which says that any set of beliefs may be represented as a family of letter-disjoint sets and that revision may be restricted to local portions of the belief corpus (those intersecting with the language of the new epistemic input). In practice, since beliefs do have some overlap, the partition of the main set of beliefs cannot be actually strict. In view of this gap, Parikh's original model for belief revision (and others based on it) (Chopra 01a; Kourousias 07; Parikh 99; Peppas 04) has been extended, by allowing for such overlap, in the B-structure model of (Chopra 00). However, this model is incapable to guarantee a global revision by only a local one, i.e., after revising our beliefs we are not sure if they are globally consistent or not. This fact interferes with the soundness of belief revision.

A new model for belief representation and local belief revision called the C-structure model and also based on such overlap was defined (Doukari 07b). This model based on the containment property defined in (Doukari 07a), allows some overlap between the different belief subsets and preserves all the desirable properties of the language splitting model, in particular soundness of revision operation. Furthermore, this model allows to respect the principle of minimal change when the language splitting (henceforth LS) model fails to do that.

Using Grove's system of spheres construction (Grove 88), we provide a semantics for local revision using the C-structure model, by providing additional constraints based on a distance measurement between interpretations. We prove these constraints characterize the containment property.

The structure of the paper is as follows. In section 2, we provide some preliminaries on the AGM paradigm. In section 3, we define the C-structure model. In section 4, we provide system of spheres semantics for belief revision by the C-structure model.

2 Preliminaries

Throughout this paper, $\mathcal{L}$ is a propositional language defined on some finite set of propositional variables (atoms) $\mathcal{V}$ and the usual connectors ($\neg$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$). $|X|$ denotes the cardinality of the set X. If $\alpha \in \mathcal{L}$ is a sentence, then $\mathcal{V}(\alpha)$ represents the set of variables appearing in α, and similarly for a set of sentences. If V is a subset of $\mathcal{V}$ then $\mathcal{L}(V)$ represents the propositional sublanguage defined over V. $\vdash$ represents the classical inference relation. A literal is a propositional variable or its negation. A clause is a disjunction of literals. A clause c is an implicate of a sentence α iff $\alpha \vdash c$. A clause c is a prime implicate of α iff for all implicates c' of α such that $c' \vdash c$, it is the case that $c \vdash c'$. We denote by $Cove_\alpha$ an arbitrary covering of α, which is a set of prime implicates of α such that for every clause c where $\alpha \vdash c$, there exists $c' \in Cove_\alpha$ such that $c' \vdash c$. $\mathcal{V}(Cove_\alpha)$ is the minimal set of atoms needed to express (a sentence logically equivalent to) α (Herzig 99). This set is unique (Parikh 99).

If X is a set of sentences then $Cn(X)$ is the logical closure of X. In particular, X is a theory iff $X = Cn(X)$. For a theory T, B_T denotes a belief base of T, which is a finite set of sentences such that $T = Cn(B_T)$. B_T is a minimal belief base of T iff (i) B_T is a belief base of T, (ii) T is axiomatized by B_T (i.e., $\forall \alpha \in B_T, B_T \setminus \{\alpha\} \not\vdash \alpha$), and (iii) $B_T \subseteq Cove_{\bigwedge_{\alpha \in B_T}}$.

In particular, if B_T is an inconsistent belief base, $M \subseteq B_T$ is a minimal inconsistent subset (MIS) of B_T iff M is inconsistent, and $\forall M' \subset M, M'$ is consistent. We denote the set of all consistent theories of $\mathcal{L}$ by $\mathcal{K}_\mathcal{L}$. A theory T of $\mathcal{K}_\mathcal{L}$ is complete iff $\forall \alpha \in \mathcal{L}, \alpha \in T$ or $\neg\alpha \in T$. We denote by $\mathcal{M}_\mathcal{L}$ the set of all consistent complete theories of $\mathcal{L}$. In the context of system of spheres, consistent complete theories play the role of interpretations (possible worlds). For a set of sentences X of $\mathcal{L}$, $[X]$ represents the set of all interpretations of $\mathcal{L}$ that contain X. For a theory T and a set of sentences X of $\mathcal{L}$, $T + X$ represents the set $Cn(T \cup X)$.

For a sublanguage $\mathcal{L}'$ of $\mathcal{L}$ defined over a subset V' of $\mathcal{V}$, $\overline{\mathcal{L}'} = \mathcal{L}(\mathcal{V} \setminus V')$. $Cn_{\mathcal{L}'}(X)$ for $X \subset \mathcal{L}'$, represents the logical closure of X in $\mathcal{L}'$. When no subscript is present, it is understood that the operation is relevant to the original language $\mathcal{L}$.

In belief revision, much work takes as its starting point the AGM postulates, which appear to capture much of what characterizes rational belief revision (Alchourrón 85). In this framework belief states are represented as theories of $\mathcal{L}$, and the process of belief revision is modeled on a revision function $*$ which is any function from $\mathcal{K}_\mathcal{L} \times \mathcal{L}$ to $\mathcal{K}_\mathcal{L}$, mapping $\langle T, \alpha \rangle$ to $T*\alpha$ that satisfies the AGM postulates (see them in (Alchourrón 85)).

Apart from this axiomatic approach to belief revision, Grove(Grove 88) introduced another construction of revision functions based on a special structure on consistent complete theories, called a system of spheres. Let T be a theory of $\mathcal{L}$, and S_T a collection of sets of interpretations i.e., $S_T \subseteq 2^{\mathcal{M}_{\mathcal{L}}}$. S_T is a system of spheres centered on $[T]$ iff the following conditions are satisfied :

($\mathbf{S1}$). S_T is totally ordered ; if $U, U' \in S_T$ then $U' \subseteq U$ or $U \subseteq U'$.

($\mathbf{S2}$). The smallest sphere in S_T is $[T]$; $[T] \in S_T$ and if $U \in S_T$ then $[T] \subseteq U$.

($\mathbf{S3}$). $\mathcal{M}_{\mathcal{L}} \in S_T$.

($\mathbf{S4}$). $\forall \alpha \in \mathcal{L}$, if there is any sphere in S_T intersecting $[\alpha]$ then there is also a smallest sphere in S_T intersecting $[\alpha]$.

For a system of spheres S_T and a sentence $\alpha \in \mathcal{L}$, the smallest sphere in S_T intersecting α is denoted $C_T(\alpha)$. With any system of spheres S_T, Grove associates a function $f_T : \mathcal{L} \mapsto 2^{\mathcal{M}_{\mathcal{L}}}$ defined as follows : $f_T(\alpha) = [\alpha] \cap C_T(\alpha)$. Consider now a theory T of $\mathcal{L}$ and let S_T be a system of spheres centered on $[T]$. Grove uses S_T to define constructively the process of revising T, by means of the following condition : $(S*) : T * \alpha = \bigcap f_T(\alpha)$.

Grove showed the class of functions generated from systems of spheres by means of $(S*)$, is precisely the family of the functions satisfying the AGM postulates.

We now recall the main definitions and results of the C-structure model.

3 The C-structure model (Doukari 07b)

The C-structure model uses disjoint sublanguages to define a set of *cores* of a given language, each surrounded by a *covering* of atoms. The concept of covering allows some degree of overlap between the sublanguages defined over the coverings.

Definition 1
$\{V_1, ..., V_n\}$ *is a set of cores of* $\mathcal{L}$ *iff it is a partition of* $\mathcal{V}$.

Example 1
Let the language $\mathcal{L}$ *be built from the propositional variables* a, b, c, d. *Let* T *be an arbitrary theory of* $\mathcal{L}$, *axiomatized by the minimal belief base* $B_T = \{\neg a \vee b, \neg b \vee c, \neg c \vee b, \neg c \vee d\}$. *The set* $\{\{a\}, \{b\}, \{c\}, \{d\}\}$ *is a set of cores of* $\mathcal{L}$.

To order the atoms of $\mathcal{L}$, we use the following relevance relation from (Chopra 01b).

Definition 2
Let T *be a theory of* $\mathcal{L}$. *We say that two atoms,* p *and* q, *are directly relevant wrt* B_T, *denoted by* $R(p, q, B_T)$ *(or by* $R_0(p, q, B_T)$*), iff* $\exists \alpha \in B_T$ *s.t.,* $p, q \in \mathcal{V}(\alpha)$. *Two atoms* p, q *are* k*-relevant wrt* B_T, *denoted by* $R_k(p, q, B_T)$, *if* $\exists p_0, p_1, ..., p_{k+1} \in \mathcal{V}$ *s.t. :* $p_0 = p$ *;* $p_{k+1} = q$ *; and* $\forall i \in \{0, ..., k\}, R(p_i, p_{i+1}, B_T)$.

In Example 1, we find : $R(a, b, B_T)$, $R_1(a, c, B_T)$, $R_2(a, d, B_T)$, etc.

To define clearly the extent of overlapping between the various sublanguages, we define a *distance* between variables.

Definition 3
Suppose two atoms $p, q \in \mathcal{V}$, T is a theory of $\mathcal{L}$. The distance between p, q wrt B_T, denoted by $dist(p, q, B_T)$, is defined as follows :

$$dist(p, q, B_T) = \begin{cases} 0 & \textit{if } p = q \\ min\{k : R_k(p, q, B_T)\} + 1 & \textit{if } k \textit{ exists} \\ \infty & \textit{otherwise.} \end{cases}$$

In Example 1 : $dist(a, b, B_T) = 1$, $dist(a, c, B_T) = 2$, $dist(a, d, B_T) = 3$, etc.

We now define the notion of a covering, parametrized by its *thickness* :

Definition 4
Let $\{V_1, ..., V_n\}$ be a set of cores of $\mathcal{L}$ and T be a theory of $\mathcal{L}$. $Cov_k(V_i, B_T)$ is a covering whose thickness is equal to k of V_i wrt B_T iff : $Cov_k(V_i, B_T) \subseteq \mathcal{V}$; and $\forall p \in \mathcal{V}, if \exists q \in V_i$ s.t., $dist(p, q, B_T) \leq k$ then $p \in Cov_k(V_i, B_T)$.

For example, the set of coverings with thickness 1 corresponding to the set of cores $\{\{a\}, \{b\}, \{c\}, \{d\}\}$ wrt B_T (Example 1) is : $\{\{a, b\}, \{a, b, c\}, \{b, c, d\}, \{c, d\}\}$.

In order to parametrize a C-structure by a particular thickness, we require a definition of the size of a MIS :

Definition 5
Let B_T and $B'_{T'}$ be two belief bases such that $\mathcal{V}(B'_{T'}) \subseteq \mathcal{V}(B_T)$ and $B'_{T'}$ is inconsistent. The size of the MIS M of $B'_{T'}$ wrt B_T, $Size(M, B_T) = max\{dist(a, b, B_T) : a, b \in \mathcal{V}(M)\}$.

In Example 1, let $M = \{a \wedge \neg b, a \rightarrow b\}$ be a MIS of $B'_{T'} = B_T \cup \{a \wedge \neg b\}$, so $Size(M, B_T) = 1$.

When we want to construct a C-structure C on a belief base B_T, we only require that the thickness of coverings of cores (value of k) should be (at least) equal to the maximal size of MISs which may exist in B_T.

Definition 6
Let T be a theory defined in $\mathcal{L}$ and B_T an arbitrary belief base of T. The set $C = \{(V_1, Cov_k(V_1, B_T), T_1), ..., (V_n, Cov_k(V_n, B_T), T_n)\}$ is a C-structure of T iff : (i) *$\{V_1, ..., V_n\}$ is a set of cores of $\mathcal{L}$,* (ii) *$Cov(C) = \{Cov_k(V_1, B_T), ..., Cov_k(V_n, B_T)\}$ is a corresponding set of coverings wrt B_T s.t., $\forall i \in \{1, ..., n\} \forall \alpha \in \mathcal{L}(Cov_k(V_i, B_T))$, if $B_T \cup \{\alpha\}$ is inconsistent, then $\forall M$ a MIS of $B_T \cup \{\alpha\}$, $Size(M, B_T) \leq k$, and* (iii) *$\forall T_i, T_i = Cn_{\mathcal{L}(Cov_k(V_i, B_T))}(\mathcal{L}(Cov_k(V_i, B_T)) \cap T)$. C is called an atomic C-structure of T iff $\forall i \in \{1, ..., n\}$, $|V_i| = 1$*

We obtain the following C-structure corresponding to Example 1 by assuming that the maximal size of eventual exiting MISs in B_T is 1 (condition (ii) of Definition 6) : $\{(\{a\}, \{a, b\}, Cn_{\mathcal{L}(\{a,b\})}(\{\neg a \vee b\})), (\{b\}, \{a, b, c\}, Cn_{\mathcal{L}(\{a,b,c\})}(\{\neg a \vee b, \neg b \vee c, \neg c \vee b\})), (\{c\}, \{b, c, d\}, Cn_{\mathcal{L}(\{b,c,d\})}(\{\neg b \vee c, \neg c \vee b, \neg c \vee d\})), (\{d\}, \{c, d\}, Cn_{\mathcal{L}(\{c,d\})}(\{\neg c \vee d\}))\}$.

Now, we can formalize the local revision property called containment property.

(Containment Property) : Let T be a theory of $\mathcal{L}$, B_T an arbitrary belief base of T, and $C = \{(V_1, Cov_k(V_1, B_T), T_1), ..., (V_n, Cov_k(V_n, B_T), T_n)\}$ a C-structure of T. If $\alpha \in \mathcal{L}(Cov_k(V_i, B_T))$ and $\mathcal{V}(Cove_\alpha) \cap V_i \neq \emptyset$ for some i, then : $T * \alpha = (T_i \circ \alpha) + ((\bigcup_{j=0}^{n} T_j) \setminus T_i)$ where $\circ$ is a revision operator of the sublanguage $\mathcal{L}(Cov_k(V_i, B_T))$.

4 Semantics for the Containment Property

Consider a C-structure $C = \{(V_1, Cov_k(V_1, B_T), T_1), ..., (V_n, Cov_k(V_n, B_T), T_n)\}$ of the theory T. Moreover, let α be any sentence in $\mathcal{L}(Cov_k(V_1, B_T))$ such that $\mathcal{V}(Cove_\alpha) \cap V_1 \neq \emptyset$. According to the containment property, anything outside T_1 in T will not be affected during the revision of the theory T by α. More formally, this leads to the following condition, which is equivalent to the containment property.

(**C**). If $C = \{(V_1, Cov_k(V_1, B_T), T_1), ..., (V_n, Cov_k\ (V_n, B_T), T_n)\}$ is a C-structure of T, $\alpha \in \mathcal{L}(Cov_k\ (V_1, B_T))$ and $\mathcal{V}(Cove_\alpha) \cap V_1 \neq \emptyset$, then : (i) if $\forall i \in \{2, ..., n\}$, $\mathcal{L}(Cov_k\ (V_1, B_T)) \cap \mathcal{L}(Cov_k\ (V_i, B_T)) = \emptyset$, then : $T \cap \mathcal{L}(Cov_k(V_1, B_T)) = (T * \alpha) \cap \mathcal{L}(Cov_k(V_1, B_T))$; (ii) otherwise : $((\bigcup_{i=0}^{n} T_i) \setminus T_1) \subseteq T * \alpha$.

Condition (C) is straightforward : when revising a theory T by a sentence α, the part of T that is not related to α is not affected by the revision ; we do not remove any information from it since the size of MISs is limited by k. However, we can deduce more consequences because the existence of the overlap between the parts related and unrelated to α. The following result shows (C) is equivalent to the containment property.

Theorem 1
Let $*$ be a revision function satisfying the AGM postulates $(T * 1)$–$(T * 8)$. Then $*$ satisfies the containment property iff $*$ satisfies (C).

4.1 The special case of Complete Theories

Let T be a consistent complete theory, and let S_T be a system of spheres centered on $[T]$. The intended meaning of S_T is that it represents comparative similarity between possible worlds i.e., the further away a world is from the center of S_T, the less similar it is to $[T]$. However, none of the conditions (S1)–(S4) indicate how similarity between worlds should be measured. In (Peppas 00), a specific criterion of similarity is considered, originally introduced in the context of reasoning about action with Winslett's Possible Models Approach (PMA) (Winslett 88).

This criterion, called PMA's criterion of similarity, measures "distance" between worlds based on propositional variables. In particular, let w, w' be any two interpretations of $\mathcal{L}$. By $Diff(w, w')$ we denote the set of propositional variables that have different truth values in the two interpretations i.e., $Diff(w, w') = \{v_i \in \mathcal{V} : v_i \in w$ and $v_i \notin w'\} \cup \{v_j \in \mathcal{V} : v_j \notin w$ and $v_j \in w'\}$. A system of spheres S_T is a PMA system of spheres iff it satisfies the following condition (Peppas 00) :

(PS) For any two consistent complete theories w and w', if $Diff\ (T, w) \subset Diff\ (T, w')$ then there is a sphere $U \in S_T$ that contains w but not w'.

4.1.1 Condition (C) and Systems of Spheres

In our case, condition (PS) is the counterpart of (C) in the realm of systems of spheres.

Theorem 2
Let $*$ be a revision function satisfying the AGM postulates $(T{*}1)$–$(T{*}8)$, T a consistent complete theory of $\mathcal{L}$, and S_T the system of spheres centered on $[T]$, corresponding to $*$ by means of $(S*)$. Then $*$ satisfies (C) at T iff S_T satisfies (PS).

What is appealing about Theorem 2 is that it characterizes (C), not in terms of some technical non-intuitive condition, but rather by a natural constraint on similarity between interpretations, that in fact predates (C) and was motivated independently in a different context (Winslett 88). Moreover, as we will show in the next section, the essence of this characterization of (C) in terms of constraints on similarity, carries over into the general case of incomplete theories (albeit with some modifications).

4.2 The General Case

To elevate Theorem 2 to the general case, we need to extend the definition of $Diff$ to cover comparisons between an interpretation w and an arbitrary, possibly incomplete, theory T. Our generalization of $Diff$ is based on the comparison between an interpretation w and a C-structure C which gives us the minimal subsets of atoms, wrt C, such that w does not satisfy the subtheories of T defined over these subsets (w is not a possible world of them), and it satisfies the subtheories of T defined over the complements of these subsets (w is a possible world of them). The usual distance definition between w and T (Winslett 88) wrt set inclusion fails to compute that.

Definition 7
Let C be a consistent C-structure constructed on the belief base B_T, and w an interpretation. $Diff(C, w) = min\{|\bigcup_{r\in R}| : R$ is a subset of $Cov(C)$ s.t., for some $\alpha \in \mathcal{L}\left(\bigcup_{r\in R}\right), Cn(B_T) \vdash \alpha$, and $w \vdash \neg\alpha$; and $\forall\alpha \in Cn(B_T\backslash(B_T\cap\mathcal{L}(\bigcup_{r\in R}))), w \vdash \alpha\}$.

Minimality is required for the elements of $Diff(C, w)$ as the coverings of C overlap and are not disjoint.

To guarantee the satisfaction of the minimal change principle, we use, to compute the difference between a (possibly incomplete) theory T of $\mathcal{L}$ and an interpretation w, only atomic C-structures constructed on minimal belief bases of T :

Definition 8
Let T be a consistent theory of $\mathcal{L}$ (possibly incomplete) and w an interpretation. $Diff$ $(T, w) = \bigcap Diff\ (C, w)$, s.t., C is an atomic C-structure of T constructed on B_T a minimal belief base of T.

It is not hard to verify that in the special case of a consistent complete theory, the above definition of $Diff$ collapses to the one given in Section 4.1.

From Example 1, Table 1 illustrates the computation of $Diff(T, w_i)$ for all $w_i \in \mathcal{M}_{\mathcal{L}} \setminus [T]$. In this table we are representing interpretations as sequences of literals rather

w_i	$Diff(C, w_i)$	$Diff(T, w_i)$
$w_1 = abc\overline{d}$	{{c,d}}	{c,d}
$w_2 = ab\overline{c}d$	{{a,b,c},{b,c,d}}	{b,c}
$w_3 = ab\overline{c}\overline{d}$	{{a,b,c},{b,c,d}}	{b,c}
$w_4 = a\overline{b}cd$	{{a,b,c}}	{a,b,c}
$w_5 = a\overline{b}c\overline{d}$	{{a,b,c,d}}	{a,b,c,d}
$w_6 = a\overline{b}\overline{c}d$	{{a,b}}	{a,b}
$w_7 = a\overline{b}\overline{c}\overline{d}$	{{a,b}}	{a,b}
$w_8 = \overline{a}bc\overline{d}$	{{c,d}}	{c,d}
$w_9 = \overline{a}b\overline{c}d$	{{a,b,c},{b,c,d}}	{b,c}
$w_{10} = \overline{a}b\overline{c}\overline{d}$	{{a,b,c},{b,c,d}}	{b,c}
$w_{11} = \overline{a}\overline{b}cd$	{{a,b,c},{b,c,d}}	{b,c}
$w_{12} = \overline{a}\overline{b}c\overline{d}$	{{b,c,d}}	{b,c,d}

TAB. 1 – $Diff(T, w_i)$ computation of Example1

than theories ; moreover the negation of a propositional variable p is denoted $\overline{p}$. $[T] = \{abcd, \overline{a}bcd, \overline{a}\overline{b}\overline{c}d, \overline{a}\overline{b}\overline{c}\overline{d}\}$.

If T is incomplete, then for any interpretation w such that $w \in [T]$, $Diff(T, w) = \emptyset$. Moreover, for any interpretation w', $Diff(T, w') \subseteq \bigcup_{w \in [T]} Diff(w, w')$.

4.2.1 Condition (C) and Systems of Spheres

(Peppas 04) showed that condition (PS) does not correspond to the revision by the LS model in the general case of arbitrary theories. Indeed, (PS) does not correspond to (C) for a system of spheres S_T related to a theory T which is not necessarily complete, since the LS model is a special case of the C-structure model. To see this, refer to the counter-example given in (Peppas 04).

Despite its failure to generalize, (PS) should not be disregarded altogether. It can still serve as a guide in formulating the appropriate counterpart(s) of (C) for the general case ; as we prove later in this section, the two general conditions (Q1) and (Q2) that correspond to (C) are both in the spirit of (PS) (and surprisingly, they collapse to (PS) in the special case of complete theories).

To formulate the conditions (Q1) and (Q2), we need to define concepts related to the notion of distance between an interpretation and an incomplete theory (Peppas 04).

Definition 9
Let w, w' be interpretations, and let T be a theory of $\mathcal{L}$. The interpretations w and w' are external T-duals iff $Diff(T, w) = Diff(T, w')$ and $w \cap (\mathcal{V} \setminus Diff(T, w)) = w' \cap (\mathcal{V} \setminus Diff(T, w'))$.

Multiple T-duals (external and internal ones as we will see later) add more structure to a system of spheres, and render condition (PS) too strong for the general case. The possibility of placing external T-duals in different spheres, opens up new ways of ordering interpretations that still induce containment property revision functions without fulfilling entirely the demands of (PS). Let us elaborate on this point and define the notion of w'-cover :

Definition 10

Let T be a theory of $\mathcal{L}$, let w, w' be two interpretations s.t., $Diff\ (T, w) \subset Diff\ (T, w')$, and let w'' be an external T-dual of w. The interpretation w'' is the w'-cover for w at T denoted by $\vartheta_T(w, w')$, iff $w'' \cap Diff(T, w) = w' \cap Diff(T, w)$.

In Table 1, w_{11} is the w_4-cover for w_9 at T (T is the theory of Example 1).

The notion of "cover" will be used to weaken (PS). In particular, consider the condition (Q1) below :

(**Q1**). If $Diff(T, w) \subset Diff(T, w')$ then there is a sphere $V \in S_T$ that contains $\vartheta_T(w, w')$ but not w'.

Condition (Q1) formalizes the intuition mentioned earlier about weakening (PS) with the aid of external T-duals. It is not hard to show that (PS) entails (Q1), and that (Q1) collapses to (PS) when the initial theory T is complete. Moreover, (Q1) is strictly weaker than (PS).

Now, condition (Q1) alone does not suffice to guarantee the satisfaction of the containment property ; from something too strong for (C) (condition (PS)), we have now moved to something too weak. Consider the following counter-example given in (Peppas 04) : the language $\mathcal{L}$ is built over three propositional variables a, b, c, the initial theory T is $T = Cn(\{a \leftrightarrow b\})$, and the system of spheres S_T centered on $[T]$ is the following :

$$\begin{array}{ccccccc} abc & & & & & & \\ ab\overline{c} & & & & & & a\overline{b}\overline{c} \\ \overline{a}\overline{b}c & \leq & a\overline{b}c & \leq & \overline{a}bc & \leq & \overline{a}b\overline{c} \\ \overline{a}\overline{b}\overline{c} & & & & & & \end{array}$$

In this example all the interpretations outside $[T]$ (i.e. in $\mathcal{M}_{\mathcal{L}} \setminus [T]$) differ from $[T]$ on precisely the same propositional variables, namely on $\{a, b\}$. Consequently S_T satisfies (Q1) since its antecedent $Diff(T, w) \subset Diff(T, w')$ never holds for $w, w' \notin [T]$. Yet despite the compliance with (Q1), the revision function $*$ induced from S_T violates (C) at T (simply consider the revision of T by $a \wedge \neg b$).

To secure the correspondence with (C), condition (Q1) needs to be complimented with a second condition, (Q2). This second condition uses the notion of internal T-dual.

Definition 11

Let w, w' be interpretations, and let T be a theory of $\mathcal{L}$. The interpretations w and w' are internal T-duals iff $Diff(T, w) = Diff(T, w')$, and $w \cap Diff(T, w) = w' \cap Diff(T, w')$.

In Table 1, w_2, w_3, w_9, and w_{10} are internal T-duals.

Clearly, for any theory T and any two interpretations w, w', if w and w' are both internal and external T-duals, then they are identical.

We now proceed with the presentation of condition (Q2), which together with (Q1), brings about the correspondence with (C1). In the following condition T is an arbitrary consistent theory of $\mathcal{L}$, S_T is a system of spheres centered on $[T]$, and w, w' are interpretations.

($\mathbf{Q2}$). If w and w' are internal T-duals, then they belongs to the same spheres in S_T ; i.e., for any sphere $V \in S_T, w \in V$ iff $w' \in V$.

In the special case that T is complete, no interpretation w has internal or external T-duals (other than itself). Consequently, in that case (Q1) reduces to (PS), while (Q2) degenerates to a vacuous condition.

The promised correspondence between (C) and the two conditions (Q1) and (Q2) is given by the theorem below :

Theorem 3
Let $*$ be a revision function satisfying $(T * 1)$–$(T * 8)$. Let T be a consistent theory of $\mathcal{L}$, and S_T a system of spheres centered on $[T]$, that corresponds to $*$ by means of $(S*)$. Then $*$ satisfies (C) at T iff S_T satisfies (Q1)–(Q2).

It should be noted that in the case of overlapping theories T, the LS model cannot avoid the counter-intuitive effect of throwing away all non-tautological beliefs in T whenever the new information is inconsistent with T, regardless of whether these beliefs can be kept or not. For example, the system of spheres S_T centered on $[T]$ (T is the theory of Example 1), which is based on the LS model (see in (Peppas 04) for all details on S_T construction) is composed only of two spheres : the sphere $[T]$ and the sphere $\mathcal{M}_{\mathcal{L}}$, the set of all consistent complete theories of $\mathcal{L}$. However, systems of spheres based on the C-structure model (satisfying the two conditions (Q1) and (Q2)) allows us to avoid such undesirable systems of spheres. To see this, consider one system of spheres S'_T corresponding to the theory T of Example 1, and satisfying the two conditions (Q1) and (Q2) as given below. Then, consider the revision of T by $a \wedge \neg b$ using S_T and S'_T. By S_T, the result is $Cn(a \wedge \neg b)$.

$$\begin{array}{ccccccc}
 & & w_1 : abc\overline{d} & & & & \\
 & & w_2 : ab\overline{c}d & & & & \\
abcd & & w_3 : ab\overline{c}\overline{d} & & & & \\
\overline{a}bcd & & w_6 : a\overline{b}\overline{c}d & & w_4 : a\overline{b}cd & & \\
\overline{a}\overline{b}\overline{c}d & \leq & w_7 : a\overline{b}\overline{c}\overline{d} & \leq & w_{12} : \overline{a}\overline{b}c\overline{d} & \leq & w_5 : a\overline{b}c\overline{d} \\
\overline{a}\overline{b}\overline{c}\overline{d} & & w_8 : \overline{a}bc\overline{d} & & & & \\
 & & w_9 : \overline{a}b\overline{c}d & & & & \\
 & & w_{10} : \overline{a}b\overline{c}\overline{d} & & & & \\
 & & w_{11} : \overline{a}\overline{b}cd & & & &
\end{array}$$

Theorems 1 and 3 provide immediately the following theorem that provides semantics for the containment property.

Theorem 4
Let $*$ be a revision function satisfying the AGM postulates $(T * 1)$–$(T * 8)$. Let T be a consistent theory of $\mathcal{L}$, and S_T a system of spheres centered on $[T]$, that corresponds to $*$ by means of $(S*)$. Then $*$ satisfies the containment property iff S_T satisfies (Q1)–(Q2).

5 Conclusion

In this paper, based on Grove's system of spheres construction, we provided semantics for local revision using the C-structure model by considering additional constraints on measuring distance between interpretations. We also proved these constraints characterize precisely the containment property.

In future work we intend to carry out a thorough study of this property by generalizing our results to belief merging.

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