States and Time in Modal Action Logic

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Contents

Motivation

- States and Time
  Actions frequently describe state transitions. But those take place in time.
- Actions and Agents
  The same action performed by several agents.
- Other aspects
  Knowledge, Communication

Language: Idea

- action terms $a(t)$ or $a(x)$: x variable of the langauge,
- modalities $[a(t)]$ or $[a(x)]$,
- formulas $[a(t)]A(t)$, $\forall x \exists y [a(x, y)]\phi(x, y)$,
- application: describe states and time,
- modality $\Box$ (characterizes any succeeding state)

Language I: FO predicate logic $\mathcal{L}_0$

- a set of variables $x, y, x_1, y_1, \ldots,$
- a set $\boldsymbol{F}$ of function symbols F,
- a set $\boldsymbol{P}$ of predicate symbols P, including $\top$ and $\bot$,
- the logical symbols $\neg$, $\wedge$,$\forall$,
- other symbols $(\vee, \ldots)$ are defined as usual

Language II: Action Terms

Action symbols are special sysmbols

- set A of *action symbols* $a_1, a_2, \ldots$ where $\boldsymbol{A} \cap \boldsymbol{P} = \emptyset$

Action terms are built from action symbols and terms of $\mathcal{L}_0$.

- if a is an action symbol of arity $n \geq 0$ and $t_1, \ldots t_n$ are terms of $\mathcal{L}_0$, then $a(t_1, \ldots t_n)$ is an action term. a is a constant for $n = 0$.

An action term is called *grounded* if no variable occurs free in it.
Example: a, $a_1(c_1, c_2, c_3)$ are grounded, $a_1(x, c_2, y)$ is not grounded.

Language III: Formulas

- If *a* is an action term then [*a*] is an action operator.
- $[\varepsilon]$ is an action operator (empty action operator).
- If ϕ is a formula and [*A*] is an action operator, then $[A]\phi$ is a formula.
- If ϕ is a formula, then $\Box\phi$ is a formula.
- If ϕ is a formula and *x* is a variable, then $\forall x \phi$ is a formula.
- If ϕ and ψ are formulas then $\neg\phi$ and $\phi \wedge \psi$ are formulas

Semantics of Action Logic

$\mathcal{M} = (\mathcal{W}, \{\mathcal{S}_w : w \in \mathcal{W}\}, \mathcal{A}, \mathbf{R}, \tau, \tau')$, where

- $\mathcal{W}$ is a set of *worlds*
- $\forall w \in \mathcal{W}, \mathcal{S}_w = (\mathcal{O}, \mathcal{F}, \mathcal{P})$ classical structure,
  - $\mathcal{O}$ set of individual objects,
  - $\mathcal{F}$ set functions over $\mathcal{O}$
  - $\mathcal{P}$ set predicates over $\mathcal{O}$.
- $\mathcal{A}$ set of *functions* $f : \mathcal{W} \times \underbrace{\mathcal{O} \times \ldots \times \mathcal{O}}_{n} \longrightarrow 2^{\mathcal{W}}, n \in \omega$

Semantics of Action Logic II

- $\mathbf{R} \subseteq \mathcal{W} \times \mathcal{W}$ is a binary accessability relation (characterizing the modal operator). We set $\mathbf{R}(x) = \{y : (x, y) \in \mathbf{R}\}$
- τ is an interpretation function assigning, for every world $w \in \mathcal{W}$, objects from $\mathcal{O}$ to terms of $\mathcal{L}_0$, functions (from $\mathcal{F}$) to function symbols (from $\boldsymbol{F}$) and predicates to predicate symbols. In order to speak about objects from $\mathcal{O}$, we introduce into the language, for every $o \in \mathcal{O}$, a $0-$place function symbol (which we call o, for simplicity)
- τ' is a function assigning action functions to action symbols, such that $arity(\tau'(a)) = arity(a) + 1$

Truth Values for Action Logic

A valuation is defined as follows: Let $t_1, t_2, \ldots, t_n$ be grounded terms and ϕ, ψ grounded formulas.

- if P is an n-ary predicate symbol then $\tau(w, Pt_1, t_2, \ldots, t_n) = \top$ iff $(\tau(w, t_1), \ldots \tau(w, t_n)) \in \tau(w, P)$
- $\tau(w, \neg\phi) = \top$ iff $\tau(w, \phi) = \bot$
- $\tau(w, \phi \wedge \psi) = \top$ iff $\tau(w, \phi) = \tau(w, \psi) = \top$
- $\tau(w, \forall x \phi) = \top$ iff $\forall o \in \mathcal{O}$, $\tau(w, \phi^x_o) = \top$
- $\tau(w, \Box\phi) = \top$ iff $\forall w' \in \mathbf{R}(w)$, $\tau(w', \phi) = \top$
- $\tau(w, [a(t_1, t_2, \ldots, t_n)]\phi) = \top$ iff $\forall w' \in \tau'(a)(w, \tau(w, t_1), \ldots, \tau(w, t_n))$, $\tau(w', \phi) = \top$

Axioms and inference rules of first - order Action Logic

[A0] all of classical logic
[A1] For any action operator $[X]$ all of modal logic K
[A2] For the modal operator $\Box$ all of modal logic $S4$
[A3] $\Box\phi \rightarrow [a]\phi$
[A4] $[\varepsilon]\phi \rightarrow \phi$
[A5] $\forall x\phi \rightarrow \phi^x_c$ for any term c
[A6] $\forall x[X]\phi \leftrightarrow [X]\forall x\phi$
for any modal operator X, with no occurrence of x

Soundness and Completeness of $\mathcal{D}al$

The $\mathcal{D}al$ -logic is sound and complete:

Theorem

$\vdash_{\mathcal{D}al} \phi$ *if and only if* $\models_{\mathcal{D}al} \phi$

Application to Action Systems

Decidable subset of $\mathcal{D}al$: formulas $\forall x \ldots$
Hybrid Representation

- Modellizing state transition aspects of actions
- Modellizing temporal aspects of actions
- Modellizing spatial aspects of actions
- Modellizing agent aspects of actions
- ...

Ordering States

Time axis $\mathcal{T}$, linearly ordered (dense or continuous or discrete). $\mathcal{D}al$ -structure $\mathcal{M}$ determines an "ordering"-relation on the set of its states $\mathcal{W}$, which will be related to the order on $\mathcal{T}$.

Definition

Let $\mathcal{M}$, be a $\mathcal{D}al$ -model. Then $w \prec_0 w'$ iff $\exists a \in \mathcal{A}$ of arity n and there are terms $t_1, \ldots, t_n$, such that $w' \in f(w, t_1, \ldots, t_n)$. Let be $\preceq$ the reflexive and transitive closure of $\prec_0$.

Intuitively $w \prec w'$ if we can "reach" w' from w by performing actions $a_1, a_2, \ldots, a_n$. $\preceq$ is transitive and reflexive.

Linking States to Time

- $time : \mathcal{W} \longrightarrow \mathcal{T}$, where $w \preceq w'$ implies $time(w) \leq time(w')$
- actions operators with complex temporal structures, beginning and ending and duration of actions,
- define preconditions and results of actions to occur at freely determinable time instances before or after the instance when the action occurs.
- When an action a occurs in the state w, $time(w)$ gives us the time point at which a occurs. If the duration of the action is Δ, the time point of the resulting state w' is $time(w') = time(w) + \Delta$.

Action Laws

- *Action terms* as action predicates $a(t, d, \overrightarrow{x})$, where t denotes the instance on which a occurs, d denotes the duration of a, $\overrightarrow{x}$ sequence of the other variables involved
- $move(t, 3, TGV, Marseille, Paris)$ is the action “train TGV goes from Marseille to Paris, the duration being 3 hours”.
- *Action axioms*
  $at(t, x, y) \rightarrow [move(t, d, x, y, z)]at(t + d, x, z)$
- can be instantiated to $at(6, TGV, Marseille) \rightarrow [move(6, 3, TGV, Marseille, Paris)]at(9, TGV, Paris))$,

Action Laws suite

General form of an action law

$$\pi(t_1, \overrightarrow{x_1}) \to [a(t, d, \overrightarrow{x_2})]l(t_2, \overrightarrow{x_3}), where\ \overrightarrow{x_1} \cup \overrightarrow{x_2} \subseteq \overrightarrow{x_3}$$

$\pi(t_1, \overrightarrow{x_1})$ is any FO formula and $l(t_2, \overrightarrow{x_3})$ is a literal

Frame Laws

Idea: fluent f is true either as the result of an action or by persisting over the execution of an action.
Two possibilities:

1. Abductive construction. Extension E_s at state s Add laws $\alpha \rightarrow [a]\alpha$ to E_s as longs as $[a]\neg\alpha \notin E_s$
2. Completion construction (as in Reiter's situation calculus).

Example

Billy and Suzanne throw rocks at a bottle. Suzanne throws first and her rock arrives first. The bottle shatters. When Billy's rock gets to where the bottle used to be, there is nothing there but flying shards of glass. Without Suzanne's throw, the impact of Billy's rock on the intact bottle would have been one of the final steps in the causal chain from Billy's throw to the shattering of the bottle. But, thanks to Suzanne's preempting throw, that impact never happens.

Problems addressed by this example

- A precondition must not only hold before the action is being executed, but it must also continue to hold till the action result can be effective
- Who hits the bottle?
- Who causes the bottle to be broken?
  according to a minimal time difference of throwing or of intensity of throwing ..

Formalization

- continuous (or dense) time axis, $[0, \infty[$
- $T(t, d, p)$, person p throws a stone at instance t and the result occurs at instance $t + d$
- $H(t, p)$ person p hits at instance t
- $BB(t)$ bottle is broken at instance t

$$
\begin{array}{ll}
(1) & \Box(\neg BB(t+d) \rightarrow [T(t,d,p)]H(t+d,p)) \\
(2) & \Box(H(t,p) \rightarrow BB(t+d_1))d_1 \text{ very small constant} \\
(3) & \Box(BB(t) \rightarrow \forall t'(t < t' \rightarrow BB(t'))) \\
(4) & \neg BB(0)
\end{array}
$$

Scenario 1

Suzanne throws at instance 0 and Billy throws some instance later

$$
\begin{array}{ll}
(5) & < T(0, d_s, suzy) > \top \\
(6) & < T(t_1, d_b, billy) > \top
\end{array}
$$

Scenario 1 suite

The moment when the bottle can be hit (and broken) after Suzanne's throw ($d_s + d_1$) occurs before Billy's stone could possibly hit the bottle $t_1 + d_b$.

(7) $d_s + d_1 < t_1 + d_b$
(8) $\Box(BB(d_s + d_1) \rightarrow BB(t_1 + d_b))$ from (3) and (7)
(9) $\neg BB(d_s)$ by persistency from (4)
(10) $[T(0, d_s, suzy)]H(d_s, suzy)$ from (1) and (9)
(11) $[T(0, d_s, suzy)]BB(d_s + d_1)$ from (2), (10)
(12) $[T(0, d_s, suzy)]BB(t_1 + d_b)$ from (11), (8), K and (A2)

Scenario 2

Billy's stone hits the bottle, which breaks, **before** Suzanne's stone could possibly hit the bottle.

(13) $t_1 + d_b + d_1 < d_s$
(14) $\Box(BB(t_1 + d_b + d_1) \rightarrow BB(d_s))$ from (3) and (13)
(15) $\neg BB(t_1 + d_b)$ by persistency from (4), see (9)
(16) $[T(t_1, d_b, billy)]H(t_1 + d_b, billy)$ from (1) and (15)
(17) $[T(t_1, d_b, billy)]BB(t_1 + d_b + d_1)$ from (2), (16), K and (A2)
(18) $[T(t_1, d_b, billy)]BB(d_s)$ from (14), (17), K and (A2)

Scenario 3

Suzanne's and Billy's stone hit the bottle precisely at the same moment.

(19) $t_1 + d_b = d_s$
(20) $\neg BB(t_1 + d_b) \wedge \neg BB(d_s)$ by persistency from (4), see (9)
(21) $[T(0, d_s, suzy)]H(d_s, suzy)$ like (10)
(22) $[T(t_1, d_b, billy)]H(t_1 + d_b, billy)$ as (16)
(23) $[T(0, d_s, suzy)]BB(d_s + d_1)$ from (21)
(24) $[T(t_1, d_b, billy)]BB(t_1 + d_s b + d_1)$ from (22))

In this case, both stones hit the bottle which breaks as a result of Suzanne's throw and Billy's throw.

Outlook

- Application to Planning: derive temporal constraints from action laws
- Consider other decidable languages
- Multi-Agents environment:
  - action term $a(i, \overrightarrow{x})$ meaning “agent *i* performs action *a*”
  - agent interaction (see previous work by (Giordano/Martelli/Schwind using DLTL)
  - common knowledge
  - communication

Application to Multi-agent information exchange

- Any set of agents $i, j, \ldots$
- action term $a(i, \overrightarrow{x})$ meaning “agent i performs action a
- $[K_i]$ meaning what agent i knows
- $[ask_{i,j}]$ meaning agent i asks a question to agent j
- $[tell_{i,j}]$ meaning‘agent i gives some information to agent j
- $[trust_{i,j}]$ meaning agent i trusts agent j (believes the information given by agent j)
- $[update_i]$ meaning agent i adds some new information to his knowledge base

Knowledge in Multi-agent systems

- $\forall i[K_i]\phi$ Everybody knows ϕ
- $\forall i[K_i][K_1]\phi$ Everybody knows that agent 1 knows ϕ

Related work

- Related work on first order modal logic by Lars Thalmann: any term is a modal operator and he quantifies over modal operators. Representation of Thlamann's approach in $\mathcal{D}al$: introduce one action symbol *a* and replace every Thalmann-formula $[x]\phi$ by $[a(x)]\phi$
- Relation to Hybrid Modal Logics (Blackburn, ...): Hybrid logics allow to quantify over worlds, while we quantify over terms occurring within action operators
- Decidability issues: (more interesting, more general) decidable subclasses

Related work

- Grove and Halpern: sorted logic where formula $P(x)$ in the scope of $[x]$ must not have free variables of the agent sort
- Meyer (et al) applications to MAS