Towards Epistemic-Doxastic Planning with Observation and Revision in a Lightweight Logic

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Background and motivation

Lightweight logic of knowledge and belief

Lightweight logic of action

# "Epistemic logic"

narrow sense: logics of knowledge

•  $\mathbf{K}_i \varphi =$  "agent *i* knows that  $\varphi$ "

broad sense: logics of knowledge or of belief

•  $\mathbf{B}_i \varphi =$  "agent *i* beliefs that  $\varphi$ "

"doxastic logics"

all are more complex than propositional logic

- SAT is PSpace-hard
- model checking unfeasible (Kripke models too big)

# "Epistemic doxastic logic"

logics of knowledge and belief

- ▶  $\mathbf{B}_i \varphi \wedge \neg \mathbf{K}_i \varphi =$  "agent *i* beliefs that  $\varphi$  without knowing it"
- "epidox logics"

some conceptual issues: which principles? here:

$$\begin{array}{ll} \mathbf{K}_{i}\varphi \rightarrow \mathbf{B}_{i}\varphi & \mathsf{OK} \\ \mathbf{B}_{i}\varphi \rightarrow \mathbf{K}_{i}\mathbf{B}_{i}\varphi & \mathsf{OK} \\ \mathbf{B}_{i}\mathbf{K}_{i}\varphi \rightarrow \mathbf{K}_{i}\varphi & \mathsf{OK} \\ \mathbf{\sigma}\mathbf{B}_{i}\varphi \rightarrow \mathbf{K}_{i}\neg \mathbf{B}_{i}\varphi & \mathsf{OK} \\ \mathbf{B}_{i}\varphi \rightarrow \mathbf{B}_{i}\mathbf{K}_{i}\varphi & \mathsf{KO!} \text{ (inconsistent with } \neg\mathbf{K}_{i}\varphi \rightarrow \mathbf{K}_{i}\neg\mathbf{K}_{i}\varphi) \end{array}$$

more for the same price: epidox logics are also PSpace complete

# Adding dynamcis

- needed: reasoning about evolution of knowledge and belief!
  - reasoning about actions (cf. epistemic SitCalc)
  - planning (cf. multiagent STRIPS)
- logics of knowledge + action
  - dynamic epistemic logics DEL
  - conceptually nice
    - rich account of who observes what ('event models')
  - but computational problems
    - DEL-based planning undecidable
- logics of belief + action
  - computational problems (v.s.)
  - conceptual problems:
    - action may reveal that some belief is false
    - requires revision of beliefs
    - no good solution in DEL

## Let's restrict the language

logics of knowledge + belief + action inherit difficult problems

conceptually

computationally

first idea: restrict static epidox language

- basically: no knowledge/belief about disjunctions
  - $K_i(p \lor q)$  cannot be expressed
- lightweight epidox logic
- much better computational properties: SAT in NP!
- second idea: restrict language of actions
  - DEL: not very fruitful
    - except special case of fully public actions (PAL)
  - but works better when combined with lightweight epidox logic
  - here: STRIPS-like 'flip-lists' (instead of add- and delete lists)
- will work nicely for planning tasks involving false belief, revision, deception,...

Background and motivation

Lightweight logic of knowledge and belief

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Lightweight logics of knowledge: 'knowing-that' literals [Demolombe&Pozos Parra; Lakemeyer&Lespérance 2012; Muise et al. 2015; 2021]

$$\lambda ::= p \mid \neg \lambda \mid \mathbf{K}_i \lambda$$

formula = boolan combination of epistemic literals
no conjunction or disjunction in scope of epistemic operators
complexity: same as propositional logic
view epistemic atoms as propositional variables
plus theory: ¬(K<sub>i</sub>λ ∧ K<sub>i</sub>¬λ), K<sub>i</sub>K<sub>i</sub>λ ↔ K<sub>i</sub>λ, etc.
cannot express "I know you know more than me"
¬K<sub>i</sub>p ∧ ¬K<sub>i</sub>¬p ∧ K<sub>i</sub>(K<sub>i</sub>p ∨ K<sub>i</sub>¬p)

 but is fundamental in interaction (precondition of questions)
 sequel: 'knowing-whether' primitive instead [Lomuscio; van der Hoek et al.; Gattinger et al.]

# Knowledge/belief about a proposition

- 'know whether' has no belief-counterpart in natural language (just as the other 'know wh' modalities) [Egré, 2008]
- therefore:

 $\mathbf{KA}_i \varphi$  = "agent *i* has knowledge about  $\varphi$ "  $\mathbf{BA}_i \varphi$  = "agent *i* has belief about  $\varphi$ " 'About' modalities: expressivity

1. 'belief about': weaker [Fan et al., 2015]

$$\mathbf{BA}_{i}\varphi \leftrightarrow \mathbf{B}_{i}\varphi \lor \mathbf{B}_{i}\neg\varphi \\
 \mathbf{B}_{i}\varphi \leftrightarrow ?$$

2. 'knowledge about': equi-expressive

$$\begin{split} \mathbf{KA}_{i}\varphi \leftrightarrow \mathbf{K}_{i}\varphi \vee \mathbf{K}_{i}\neg\varphi \\ \mathbf{K}_{i}\varphi \leftrightarrow \varphi \wedge \mathbf{KA}_{i}\varphi \end{split}$$

but:

- 'knowledge about' can express things more succinctly [van Ditmarsch et al., 2014]
- equivalent presentations may lead to new insights

'Knowledge about' atoms [Herzig et al., 2015, Cooper et al., 2021]

grammar:

$$\alpha ::= p \mid \mathbf{KA}_i \alpha$$

where  $p \in Prop$ 

- formula = boolan combination of epistemic atoms
- can express some disjunctions in scope of epistemic operator:

 $\mathbf{K}_{i}(\mathbf{K}_{j}p \vee \mathbf{K}_{j} \neg p)$ 

expressed as

 $\mathbf{K}_{i} \mathbf{K} \mathbf{A}_{j} p$  $= \mathbf{K} \mathbf{A}_{j} p \wedge \mathbf{K} \mathbf{A}_{i} \mathbf{K} \mathbf{A}_{j} p$ 

'Knowledge about' atoms: computation

- basically: epistemic atoms can be viewed as propositional logic variables
  - take care of introspection:  $\mathbf{KA}_i\mathbf{KA}_i\alpha$  valid
  - simple solution: forbid repetitions
- complexity of reasoning: same as propositional logic
  - satisfiability NP-complete
- can be extended by an operator 'common knowledge about' [Herzig&Perrotin, AiML 2020; forthcoming]

Lightweight logics of knowledge: dynamics

'dual use' of knowledge about atoms [Cooper et al., AIJ 2020]:

- $\mathbf{KA}_i \alpha = \text{agent } i \text{ sees truth value of } \alpha$
- KA<sub>i</sub>α = agent *i* sees truth value changes of α (except if action makes KA<sub>i</sub>α false)

STRIPS-like actions: preconditions + pos./neg. effects

complexity of planning: same as propositional logic

plan existence PSPACE-complete

# Lightweight logics of belief?

knowledge-about atoms 'work' because there are 4 independent combinations of p and KA<sub>i</sub>p:

$$\begin{array}{c|c} p \land \mathbf{KA}_i p & \neg p \land \mathbf{KA}_i p \\ p \land \neg \mathbf{KA}_i p & \neg p \land \neg \mathbf{KA}_i p \end{array}$$

in terms of knowledge-that:

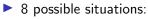
$$\begin{array}{c|c} p \land \mathbf{K}_i p & \neg p \land \mathbf{K}_i \neg p \\ p \land \neg \mathbf{K}_i p \land \neg \mathbf{K}_i \neg p & \neg p \land \neg \mathbf{K}_i p \land \neg \mathbf{K}_i \neg p \end{array}$$

for belief: 6 possible doxastic situations

$$\begin{array}{c|c}
p \land \mathbf{B}_i p & \neg p \land \mathbf{B}_i \neg p \\
p \land \neg \mathbf{B}_i p \land \neg \mathbf{B}_i \neg p & \neg p \land \neg \mathbf{B}_i p \land \neg \mathbf{B}_i \neg p \\
p \land \mathbf{B}_i \neg p & \neg p \land \mathbf{B}_i p
\end{array}$$

requires 3 dimensions ⇒ cannot be independent

Three dimensions of epidox situations



$$\begin{array}{ll} p \land \mathbf{K}_i p & \neg p \land \mathbf{K}_i \neg p \\ p \land \mathbf{B}_i p \land \neg \mathbf{K}_i p & \neg p \land \mathbf{B}_i \neg p \land \neg \mathbf{K}_i \neg p \\ p \land \neg \mathbf{B}_i p \land \neg \mathbf{B}_i \neg p & \neg p \land \neg \mathbf{B}_i p \land \neg \mathbf{B}_i \neg p \\ p \land \mathbf{B}_i \neg p & \neg p \land \mathbf{B}_i p \end{array}$$

▶  $8 = 2^3 \implies$  which are the 3 dimensions?

## Which epistemic-doxastic situations?

two new modalities:

 $TBA_{i} p = (p \land B_{i} p) \lor (\neg p \land B_{i} \neg p)$ = "i has a true belief about p"  $MBA_{i} p = (B_{i} p \land \neg K_{i} p) \lor (B_{i} \neg p \land \neg K_{i} \neg p)$ = "i has a mere belief about p" = "i has a falsifiable belief about p" = "i has a belief about p but does not know whether p"

insensitive to negation:

 $\mathbf{TBA}_i \neg p \leftrightarrow \mathbf{TBA}_i p$  $\mathbf{MBA}_i \neg p \leftrightarrow \mathbf{MBA}_i p$ 

Epistemic-doxastic situations: 3 dimensions



▶ 2<sup>3</sup> epistemic-doxastic situations:

$p \wedge TBA_i p \wedge \neg MBA_i p$	$\neg p \land TBA_i p \land \neg MBA_i p$
$p \wedge TBA_i p \wedge MBA_i p$	$ eg p \land TBA_i p \land MBA_i p$
$p \land \neg TBA_i p \land \neg MBA_i p$	$\neg p \land \neg TBA_i p \land \neg MBA_i p$
$p \land \neg TBA_i p \land MBA_i p$	$ eg p \land \neg TBA_i p \land MBA_i p$

needs getting used to, but is natural...

Example: the Sally-Ann Test

false belief task [Wimmer and Perner, 1983, Baron-Cohen et al., 1985]

1. Sally puts the marble in the basket

 $\textbf{TBA}_{\mathcal{S}}\, b \wedge \neg \textbf{MBA}_{\mathcal{S}}\, b$ 

2. Sally goes out for a walk

 $\textbf{TBA}_{\mathcal{S}}\, b \wedge \textbf{MBA}_{\mathcal{S}}\, b$ 

3. Ann takes the marble out of the basket and puts it into the box

 $\neg \textbf{TBA}_{\mathcal{S}} \, b \land \textbf{MBA}_{\mathcal{S}} \, b$ 

## Full expressivity

## knowledge:

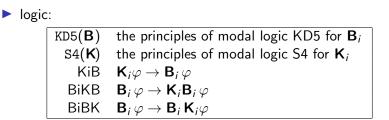
# $\begin{aligned} \mathsf{KA}_{i}\varphi &\leftrightarrow \mathsf{TBA}_{i}\varphi \wedge \neg \mathsf{MBA}_{i}\varphi \\ \mathsf{K}_{i}\varphi &\leftrightarrow \mathsf{TBA}_{i}\varphi \wedge \neg \mathsf{MBA}_{i}\varphi \wedge \varphi \end{aligned}$



 $\begin{aligned} \mathbf{B}\mathbf{A}_{i}\varphi \leftrightarrow \mathbf{T}\mathbf{B}\mathbf{A}_{i}\varphi \lor \mathbf{M}\mathbf{B}\mathbf{A}_{i}\varphi \\ \mathbf{B}_{i}\varphi \leftrightarrow (\varphi \land \mathbf{T}\mathbf{B}\mathbf{A}_{i}\varphi) \lor (\neg \varphi \land \neg \mathbf{T}\mathbf{B}\mathbf{A}_{i}\varphi \land \mathbf{M}\mathbf{B}\mathbf{A}_{i}\varphi) \end{aligned}$ 

... remember:  $\mathbf{B}_i \varphi$  cannot be expressed with  $\mathbf{BA}_i$  alone

## An epistemic-doxastic logic



belief definable from knowledge [Lenzen, 1978, Lenzen, 1995]:

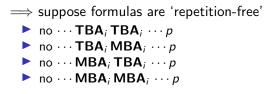
 $\mathbf{B}_i \varphi \leftrightarrow \neg \mathbf{K}_i \neg \mathbf{K}_i \varphi$ 

 ▶ alternative axiomatisation: S4.2(K) plus B<sub>i</sub> φ ↔ ¬K<sub>i</sub>¬K<sub>i</sub>φ
 ▶ complexity of satisfiability: PSPACE-complete [Shapirovsky, 2004, Chalki et al., 2021]

## Reducing modalities

reduction of consecutive modal operators to length 1:

 $TBA_{i} TBA_{i} \varphi \leftrightarrow TBA_{i} \varphi \vee \neg MBA_{i} \varphi$  $MBA_{i} TBA_{i} \varphi \leftrightarrow MBA_{i} \varphi$  $TBA_{i} MBA_{i} \varphi \leftrightarrow \neg MBA_{i} \varphi$  $MBA_{i} MBA_{i} \varphi \leftrightarrow MBA_{i} \varphi$ 



Lightweight epistemic-doxastic fragments: the idea

epidox atoms:

$$\alpha ::= p \mid \mathsf{TBA}_i \, \alpha \mid \mathsf{MBA}_i \, \alpha$$

repetition-free

## Theorem

If  $\varphi$  is a boolean combination of repetition-free epidox atoms then the following are equivalent:

- φ is valid in epistemic-doxastic logic;
- φ is propositionally valid.

## Corollary

Satisfiability of boolean combinations of epidox atom is in NP. Plan existence is in PSpace. Background and motivation

Lightweight logic of knowledge and belief

Lightweight logic of action

## Adding actions

action = precondition + conditional effects

- precondition = boolean combination of epidox atoms
- effects = epidox atoms that are flipped

 $\varphi \triangleright \pm \alpha =$  "if  $\varphi$  is true then  $\alpha$  changes its truth value"

• restriction to atoms  $\alpha$  of depth  $\leq 2$ 

express STRIPS action with add-list P<sup>+</sup> and delete-list P<sup>-</sup>:

$$\{p \triangleright \pm p : p \in P^-\} \cup \{\neg p \triangleright \pm p : p \in P^+\}$$

## Direct and indirect effects

## direct effects:

- ▶ either on the world (prop.var.s) ⇒ ontic actions
- ▶ or on knowledge/belief ⇒ epistemic actions
  - $1. \ observation \ change/sensing$
  - 2. communication (future work)
- indirect effects:
  - are always epistemic (change knowledge/belief)
  - derived from direct effects
  - depending on agents' observation status

## Ontic actions

direct effects = set of conditional effects

$$\{\varphi_1 \triangleright \pm p_1, \ldots, \varphi_n \triangleright \pm p_n\}$$

modify the world = the propositional variables  $p_k$ 

the main principle deriving indirect effects:

(*M*) 
$$\varphi_k \wedge \mathsf{MBA}_i p_k \triangleright \pm \mathsf{TBA}_i p_k$$

other principles deriving second-order indirect effects ....

Epistemic actions: starting individual observation

i starts to observe propositional variable p (without learning about others' belief change):

 $startobs^1(i, p)$ 

direct effects: i has knowledge about p

1. add **TBA**<sub>*i*</sub> *p*:

 $\neg$ **TBA**<sub>*i*</sub>  $p \triangleright \pm$ **TBA**<sub>*i*</sub> p

2. delete MBA<sub>i</sub> p:

 $MBA_i p \triangleright \pm MBA_i p$ 

indirect effects (obtained via Principle (M)):

 $\{\neg \mathsf{TBA}_i \ p \land \mathsf{MBA}_j \ \mathsf{TBA}_i \ p \triangleright \pm \mathsf{TBA}_j \ \mathsf{TBA}_i \ p : \ j \neq i \} \cup \\ \{\mathsf{MBA}_i \ p \land \mathsf{MBA}_j \ \mathsf{MBA}_i \ p \triangleright \pm \mathsf{TBA}_j \ \mathsf{MBA}_i \ p : \ j \neq i \}$ 

## Epistemic actions: starting group observation

group J starts to observe propositional variable p, learning that the other members of J also do so:

 $\mathtt{startobs}^2(J,p)$ 

direct effects:

- every  $i \in J$  has knowledge about p:
  - 1. add **TBA**<sub>*i*</sub> p, for  $i \in J$
  - 2. delete **MBA**<sub>i</sub> p, for  $i \in J$
- every  $i \in J$  has knowledge about **TBA**<sub>j</sub> p, for  $j \in J$ :
  - 1. add TBA<sub>j</sub> TBA<sub>i</sub> p
  - 2. delete MBA<sub>j</sub> TBA<sub>i</sub> p
- every  $i \in J$  has knowledge about **MBA**<sub>i</sub> p, for  $j \in J$ :

. . .

- 1. add TBA<sub>j</sub> MBA<sub>i</sub> p
- 2. delete MBA<sub>j</sub> MBA<sub>i</sub> p
- indirect effects (obtained via Principle (M)):

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Epistemic actions: ceasing to observe a fact

group J ceases to observe propositional variable p, learning that the other members of J also do so:

stopobs(i, p)

direct effect: knowledge about p becomes mere belief

 $\mathsf{TBA}_i p \land \neg \mathsf{MBA}_i p \triangleright \pm \mathsf{MBA}_i p$ 

inertia of beliefs

when Sally leaves the room her knowledge about the marble becomes a mere belief

. . .

more realistic: decaying beliefs

indirect effects (obtained via Principle (M)):

Epistemic actions: ceasing to observe another agent

group J ceases to observe propositional variable p, learning that the other members of J also do so:

stopobs(i, j, p)

. . .

direct effects: ...

indirect effects (obtained via Principle (M)):

# Epidox planning

just as in classical planning:

- initial state = set of epidox atoms
- goal = boolean combination of epidox atoms
- examples:

...

- Sally-Ann test as a planning task (goal: induce Sally's false belief)
- variants of the grapevine domain
- tasks involving correction of false beliefs
- tasks involving deception
- Theorem

An epidox planning task is solvable iff it is propositionally solvable.

# Conclusion: lightweight planning with epidox logic

- lightweight fragment of epistemic-doxastic logic
  - 'true belief about' and 'mere belief about' modalities
  - repetition-free epistemic-doxastic atoms
  - same complexity as propositional logic

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