Model Counting at CRIL

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Introduction: Model counting

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Model Counting (#*SAT*)

Input: CNF-formula F

Output: Number of satisfying assignments of F

- generic #P-hard counting problem
- applications, e.g.
  - probabilistic graphical models
  - stochastic planning

On the Hardness of $\#SAT$

- state-of-the-art practical solvers for $\#SAT$ far slower than SAT-solvers

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$$\mathsf{PH} \subseteq \mathsf{P}^{\#SAT[1]}.$$

- in words: $\#SAT$ at least as hard as all problem in polynomial hierarchy

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$\Rightarrow$ extremely hard problem (theory and practice)

Interaction Between Variables: Primal Graphs

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- variables are vertices
- edge between variables that share clauses

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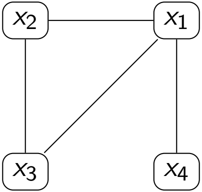
Example

- $(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_4) \wedge (\neg x_2 \vee x_3) \wedge (x_1)$



Example: #SAT on trees

5

Observation

Tree-shaped #SAT is tractable.

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- compute $S(v, i)$ for each vertex v and $i \in \{0, 1\}$:
  $S(v, i)$: number of solutions of formula rooted in v in which v takes value i

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- for every leaf v:

$$S(v, 0) = S(v, 1) = 1$$

$\#SAT$ on trees, example

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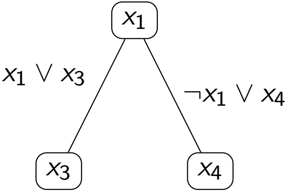

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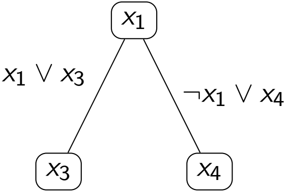
$\#SAT$ on trees, example



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$$S(x_1, 0) = S(x_3, 1) \cdot \big(S(x_4, 0) + S(x_4, 1)\big)$$
$$S(x_1, 1) = \big(S(x_3, 0) + S(x_3, 1)\big) \cdot S(x_4, 1)$$

#SAT on trees, example

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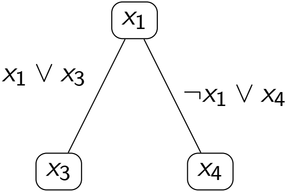

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also works in general

Structural tractability of $\#SAT$

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Which other graphs give tractable instances of $\#SAT$?

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Structural tractability of #*SAT*

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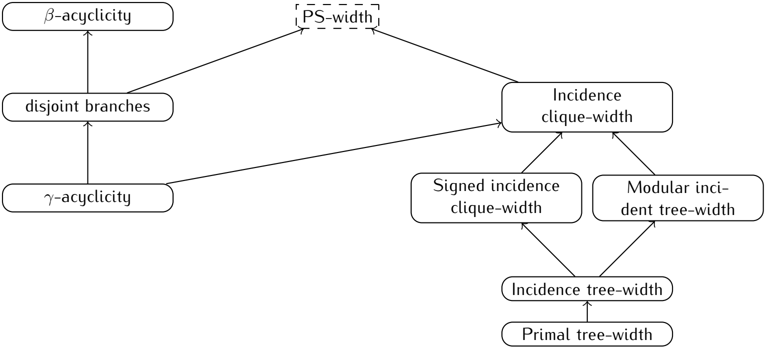

Parameter of the hypergraph **Parameter of the incidence graph**

- all algorithms dynamic programming

(Samer, Szeider '10; Paulusma, Slivovsky, Szeider '13; Fisher, Makowsky, Ravve '08; Slivovsky, Szeider '14; Capelli, Durand, M '14; Brault-Baron, Capelli, M '15; Sæther, Telle, Vatshelle '14)

Treewidth vs. Cliquewidth

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- runtimes for formula F
  - treewidth k FPT: $2^k|F|$
  - cliquewidth k not FPT: $|F|^k$
- FPT runtime far preferable

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Can runtime for cliquewidth be improved to FPT?

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  - treewidth k FPT: $2^k|F|$
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- FPT runtime far preferable

Question

Can runtime for cliquewidth be improved to FPT?

- NO, unless strange things happen, i.e. FPT = W[1] (Paulusma, Slivovsky, Szeider '13)
- but can we say something unconditionally?

Exhaustive DPLL and its traces

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- Exhaustive DPLL
  - backtracking search to find/count solutions of CNF
  - caching (reuse counts of subformulas already solved)
  - components (independent formulas treated independently)
- basis of state of the art model counters

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Theorem (Huang, Darwiche '05)

Traces of runs of exhaustive DPLL are DNNF.

- DNNF
  - restricted boolean circuits
  - extensively studied in knowledge compilation
- so model counters can also compile: sharpSAT $\rightsquigarrow$ Dsharp

Compilation and Traces of the Algorithms

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Theorem (Bova, Capelli, M, Slivovsky '15, Capelli '16)

Consider any of the known graph based model counting algorithms.
Then its traces are also DNNF.

- consequence: essentially all known counting algorithms lead to DNNF (except linear algebra...)

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- to show that known counting algorithms do not solve $\#SAT$, "only" need to show DNNF lower bounds
- ...but that was an open problem for 15 years

Lower Bounds for DNNF

11

Connection (Bova, Capelli, M, Slivovsky '16)

Functions with high multipartition communication complexity have no small DNNF.

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Functions with high multipartition communication complexity have no small DNNF.

Theorem (essentially Duris et al. '04)

There are CNF with high multipartition communication complexity.

- lets us show lower bounds for DNNF and thus for counting "easily"

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Theorem (roughly, M '16)

Cliquewidth k formulas generally require DNNF size n^k
Thus known counting algorithms require time n^k.

Conclusion

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- model counting is hard
- graph restrictions help to a certain extent
- connection to compilation (DNNF)
- lower bounds with communication complexity
- cliquewidth harder than treewidth

Model Counting in Practice

- Many model counters (both exact and approximate) have been developed so far
  - **search-based model counters**: `Cachet`, `SharpSAT`, etc.,
  - **compilation-based:** `C2D`, `SDD`, etc.

Model Counting in Practice

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- Many model counters (both exact and approximate) have been developed so far
  - **search-based model counters**: `Cachet`, `SharpSAT`, etc.,
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- **Objective at CRIL:** Improving model counters by preprocessing the input (a `CNF` formula)

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Off-line approaches for improving the model counting task

- **knowledge compilation**

$\texttt{CNF}\ \Sigma \longrightarrow \boxed{\text{compilation}} \longrightarrow \texttt{ADT}\ \Psi \longrightarrow \boxed{\text{model counting}} \longrightarrow \|\Sigma\|$

[IJCAI'13 - joint work with F. Koriche, J.-M. Lagniez and S. Thomas]

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- **preprocessing**

  $\texttt{CNF } \Sigma \longrightarrow \boxed{\text{preprocessing}} \longrightarrow \texttt{CNF } \Phi \longrightarrow \boxed{\text{model counting}} \longrightarrow \|\Sigma\|$

  [AAAI'14 – joint work with J.-M. Lagniez]
  [IJCAI'16 – joint work with J.-M. Lagniez and E. Lonca]

Gate Identification and Replacement

15

Gate identification and replacement is a valuable preprocessing for model counting [AAAI'14 - joint work with J.-M. Lagniez]

$$\Sigma = \begin{array}{l} \overline{x} \vee u \vee v \\ \overline{x} \vee \overline{y} \vee u \\ \overline{x} \vee \overline{z} \vee u \\ x \vee \overline{u} \\ y \vee z \vee \overline{u} \end{array}$$

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$$\Sigma = \begin{array}{l} \overline{x} \lor u \lor v \\ \overline{x} \lor \overline{y} \lor u \\ \overline{x} \lor \overline{z} \lor u \\ x \lor \overline{u} \\ y \lor z \lor \overline{u} \end{array} \qquad u \leftrightarrow (x \land (y \lor z))$$

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$$(\overline{x} \vee u \vee v) \wedge (u \leftrightarrow (x \wedge (y \vee z)))$$

identification

$$\Sigma \equiv$$

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$$\Sigma \equiv \begin{array}{ll} (\overline{x} \vee u \vee v) \wedge (u \leftrightarrow (x \wedge (y \vee z))) & \text{identification} \\ (\overline{x} \vee (x \wedge (y \vee z)) \vee v) \wedge (u \leftrightarrow (x \wedge (y \vee z))) & \text{replacement} \end{array}$$

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$$\|\Sigma\| = \|\Sigma[u \leftarrow (x \wedge (y \vee z))]\| = \|\overline{x} \vee y \vee z \vee v\| = 15$$

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Limitations

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- The replacement phase **requires gates to be identified**
  - The search space for gates is **huge**: if $|Var(\Sigma)| = n$, then for each variable x of Σ, there exist
    $$2^{2^{n-1}} \text{ gates}$$
    with output x
  - The size of a gate can be **huge** as well: not polynomially bounded in $|\Sigma| + |X|$ unless $\text{NP} \cap \text{coNP} \subseteq \text{P/poly}$

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- Identifying "complex gates" is incompatible with the efficiency expected for a preprocessing:
  only "simple" gates are targeted

literal equivalences	$y \leftrightarrow x_1$
AND/OR gates	$y \leftrightarrow (x_1 \wedge \overline{x_2} \wedge x_3)$
XOR gates	$y \leftrightarrow (x_1 \oplus \overline{x_2})$

Overcoming the Limitations

- The (explicit) identification phase can be replaced by an **implicit identification phase**
- There is **no need to identify** f to determine that a gate of the form $y \leftrightarrow f(x_1, \ldots, x_n)$ exists in Σ

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- There is **no need to identify** f to determine that a gate of the form $y \leftrightarrow f(x_1, \ldots, x_n)$ exists in Σ
- The replacement phase can be replaced by an **output variable elimination phase**
- There is **no need to identify** f to compute $\Sigma[y \leftarrow f(x_1, \ldots, x_n)]$

Evert Willem Beth (1908–1964)

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- Σ **explicitly defines** y in terms of $X = \{x_1, \ldots, x_n\}$ iff there exists a formula $f(x_1, \ldots, x_n)$ over X such that

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- Σ **implicitly defines** y in terms of $X = \{x_1, \ldots, x_n\}$ iff for every canonical term γ_X over X, we have $\Sigma \wedge \gamma_X \models y$ or $\Sigma \wedge \gamma_X \models \overline{y}$

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- Beth's theorem: Σ **explicitly defines** y in terms of X iff Σ **implicitly defines** y in terms of X

Padoa's theorem:

Let Σ'_X be equal to Σ where each variable but those of X have been renamed in a uniform way

If $y \not\in X$, then Σ (implicitly) defines y in terms of X iff $\Sigma \wedge \Sigma'_X \wedge y \wedge \overline{y'}$ is inconsistent

Alessandro Padoa (1868–1937)

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If $y \notin X$, then Σ (implicitly) defines y in terms of X iff $\Sigma \wedge \Sigma'_X \wedge y \wedge \overline{y'}$ is inconsistent

Deciding whether Σ (implicitly) defines y in terms of X is "only" coNP-complete

From Replacement to Variable Elimination

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The replacement phase can be replaced by an **output variable elimination phase:** if $y \leftrightarrow f(x_1, \ldots, x_n)$ is a gate of Σ, then

$$\Sigma[y \leftarrow f(x_1, \ldots, x_n)] \equiv \exists y.\Sigma$$

The $\mathtt{B}+\mathtt{E}$ Preprocessor

A two-step preprocessing

- "Identification = Bipartition":
  compute a **definability bipartition** $\langle I, O \rangle$ of Σ such that $I \cup O = Var(\Sigma)$, $I \cap O = \emptyset$, and Σ defines every variable $o \in O$ in terms of I

The B + E Preprocessor

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- "Replacement = Elimination":
  compute $\exists E.\Sigma$ for $E \subseteq O$

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Identifying u as an Output Variable and Eliminating it

Identification:

$\Sigma \wedge \Sigma'_{\{x,y,z\}} \wedge u \wedge \overline{u'}$ is inconsistent

$\overline{x} \vee u \vee v$

$\overline{x} \vee \overline{y} \vee u$

$\overline{x} \vee \overline{z} \vee u$

$x \vee \overline{u}$

$y \vee z \vee \overline{u}$

$\overline{x} \vee u' \vee v'$

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u

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$\overline{u'}$

Elimination:

computing resolvents over u

$\overline{x} \vee v \vee x$	valid
$\overline{x} \vee v \vee y \vee z$	
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$\|\Sigma\| = \|\overline{x} \vee v \vee y \vee z\| = 15$

Tuning the Computational Effort

Both steps B and E of B + E can be tuned in order to keep the preprocessing phase **light from a computational standpoint**

- It is not necessary to determine a definability bipartition $\langle I, O \rangle$ with $|I|$ minimal
  $\Rightarrow$ B is a **greedy algorithm** (one definability test per variable)
  $\Rightarrow$ Only the minimality of I for $\subseteq$ is guaranteed

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  $\Rightarrow$ Only the minimality of I for $\subseteq$ is guaranteed

- It is not necessary to eliminate in Σ every variable of O but focusing on a subset $E \subseteq O$ is enough
  $\Rightarrow$ Eliminating every output variable could lead to an **exponential blow up**
  $\Rightarrow$ The elimination of $y \in O$ is committed only if $|\Sigma|$ after the elimination step and some additional preprocessing (occurrence simplification and vivification) remains **small enough**

Experiments

Objectives:

- Evaluating the computational benefits offered by $\mathtt{B} + \mathtt{E}$ when used upstream to state-of-the-art model counters:
  - the search-based model counter `Cachet`
  - the search-based model counter `SharpSAT`
  - the compilation-based model counter `C2D` (used with `-count -in_memory -smooth_all`)

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- Comparing the benefits offered by $\mathtt{B}+\mathtt{E}$ with those offered by our previous preprocessor `pmc` (based on gate identification and replacement) or with no preprocessing

Empirical Setting

- 703 `CNF` instances from the SAT LIBrary
- 8 data sets: BN (Bayesian networks) (192), BMC (Bounded Model Checking) (18), Circuit (41), Configuration (35), Handmade (58), Planning (248), Random (104), Qif (7) (Quantitative Information Flow analysis - security)
- Cluster of Intel Xeon E5-2643 (3.30 GHz) processors with 32 GiB RAM on Linux CentOS
- Time-out =1h
- Memory-out = 7.6 GiB

Empirical Results

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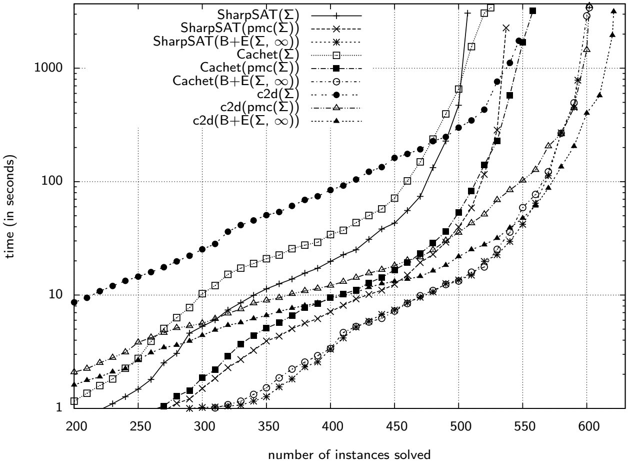
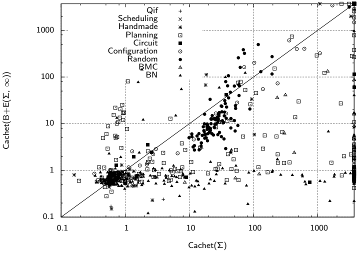
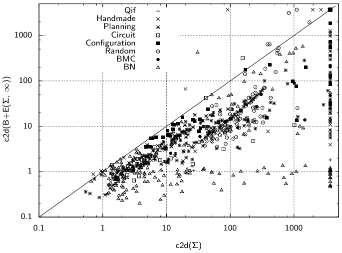


Empirical Results

B + E vs. no preprocessing



(a) B + E+Cachet vs. Cachet



(b) B + E+C2D vs. C2D

Empirical Results

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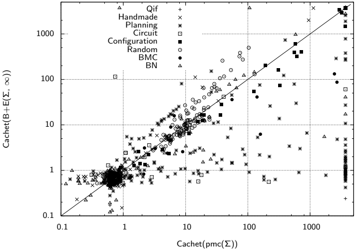
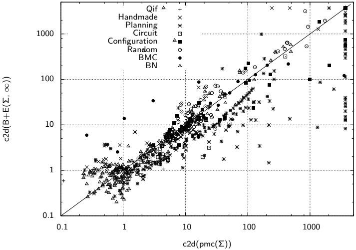
B + E vs. pmc



(c) B + E+Cachet vs. pmc+Cachet



(d) B + E+C2D vs. pmc+C2D

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Conclusion

- Design and implementation of the $\texttt{B} + \texttt{E}$ preprocessor
- Empirical evaluation of $\texttt{B} + \texttt{E}$: for several model counters $\texttt{mc}$, $\texttt{mc}(\texttt{B} + \texttt{E}(.))$ proves computationally more efficient than $\texttt{mc}(.)$

Conclusion and Perspectives

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- Design and implementation of the $\texttt{B} + \texttt{E}$ preprocessor
- Empirical evaluation of $\texttt{B} + \texttt{E}$: for several model counters $\texttt{mc}$, $\texttt{mc}(\texttt{B} + \texttt{E}(.))$ proves computationally more efficient than $\texttt{mc}(.)$

Perspectives

- Developing other ordering heuristics for $\texttt{B}$
- Connection to **projected model counting:** computing $\|\exists E.\Sigma\|$ given a set E of variables and a formula Σ
  - compute $\|\Sigma\|$ as $\exists mc(\Sigma, O)$ where $B(\Sigma) = \langle I, O \rangle$
  - compute $\exists mc(\Sigma, E)$ as $\texttt{mc}(\texttt{E}(\Sigma))$