Belief Change Tools for Negotiation in Multi-Agent Systems

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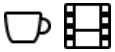
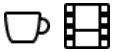
Belief change for solving conflicts

Ally, Brian and Charles have to decide what they will do this night. Brian and Ally want to go to the restaurant and to the cinema. Charles does not want to go out this night and so he does not want to go nor to the restaurant nor to the cinema.

Ally  Brian  Charles 

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Ideal setting

- Aggregation
  - Merging

Real setting

- Negotiation
  - Conciliation

Merging

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$a, b \rightarrow c$	a, b	$\neg a$

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Merging

- Applications :
  - Distributed information systems
    - Databases
    - Multi-agent systems
- Propositional bases can encode different types of information :
  - knowledge
  - beliefs
  - goals
  - rules / laws
  - . . .

Definitions

- A base φ is a finite set of propositional formulae.
- A profile E is a multi-set of bases : $E = \{\varphi_1, \ldots, \varphi_n\}$.
- $\bigwedge E$ denotes the conjunction of the bases of E.
- A profile E is consistent if and only if $\bigwedge E$ is consistent.
  We will note $mod(E)$ instead of $mod(\bigwedge E)$.

Merging

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Merging

Profile

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$$\mu$$

Merging

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Integrity Constraints

Merging

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Merged base

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(IC7) $\triangle_{\mu_1}(E) \wedge \mu_2 \vdash \triangle_{\mu_1 \wedge \mu_2}(E)$

(IC8) If $\triangle_{\mu_1}(E) \wedge \mu_2$ is consistent, then $\triangle_{\mu_1 \wedge \mu_2}(E) \vdash \triangle_{\mu_1}(E)$

Majority vs Arbitration

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Majority   restaurant and cinema

Ally	+ +
Brian	+ +
Charles	– –

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Ally Brian Charles

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Arbitration restaurant xor cinema

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Majority - Arbitration

(Maj) $\exists n \ \triangle_\mu(E_1 \sqcup E_2{}^n) \vdash \triangle_\mu(E_2)$

- An IC merging operator is a majority operator if it satisfies (*Maj*).

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(Arb)

$$\left.\begin{array}{l} \triangle_{\mu_1}(\varphi_1) \leftrightarrow \triangle_{\mu_2}(\varphi_2) \\ \triangle_{\mu_1 \leftrightarrow \neg\mu_2}(\varphi_1 \sqcup \varphi_2) \leftrightarrow (\mu_1 \leftrightarrow \neg\mu_2) \\ \mu_1 \not\vdash \mu_2 \\ \mu_2 \not\vdash \mu_1 \end{array}\right\} \Rightarrow \triangle_{\mu_1 \vee \mu_2}(\varphi_1 \sqcup \varphi_2) \leftrightarrow \triangle_{\mu_1}(\varphi_1)$$

▷ An IC merging operator is an arbitration operator if it satifies (*Arb*).

Model-Based Merging

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- Distance between an interpretation and a profile
  - $d_{d,f}(\omega, E) = f(d(\omega, \varphi_1), \ldots d(\omega, \varphi_n))$

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  - max, *leximax*, Σ, Σ^n, *leximin*, ...

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- Examples of aggregation function :
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- Let d be any distance between interpretations.
  - $\triangle^{d,\max}$ operators satisfy (IC0-IC5), (IC7), (IC8) and (Arb).
  - $\triangle^{d,\textsc{Gmin}}$ operators are IC merging operators.
  - $\triangle^{d,\textsc{Gmax}}$ operators are arbitration operators.
  - $\triangle^{d,\Sigma}$ and $\triangle^{d,\Sigma^n}$ operators are majority operators.

Model-Based Merging

An aggregation function f is a function that associates a positive number to any tuple of positive numbers such that :

- If $x \leq y$, then $f(x_1, \ldots, x, \ldots, x_n) \leq f(x_1, \ldots, y, \ldots, x_n)$ *(monotony)*
- $f(x_1, \ldots, x_n) = 0$ if and only if $x_1 = \ldots = x_n = 0$ *(minimality)*
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Theorem The operateur $\triangle^{d,f}$ satisfies properties (IC0-IC8) if and only if f satisfies :

- For any permutation σ, $f(x_1, \ldots, x_n) = f(\sigma(x_1, \ldots, x_n))$ *(symmetry)*
- If $f(x_1, \ldots, x_n) \leq f(y_1, \ldots, y_n)$, then $f(x_1, \ldots, x_n, z) \leq f(y_1, \ldots, y_n, z)$ *(composition)*
- If $f(x_1, \ldots, x_n, z) \leq f(y_1, \ldots, y_n, z)$, then $f(x_1, \ldots, x_n) \leq f(y_1, \ldots, y_n)$ *(decomposition)*

Example

$\mu = ((S \wedge T) \vee (S \wedge P) \vee (T \wedge P)) \rightarrow I$

$\varphi_1 = \varphi_2 = S \wedge T \wedge P$
$\varphi_3 = \neg S \wedge \neg T \wedge \neg P \wedge \neg I$
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$(0,0,0,0)$	3	3	0	2				

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$(0,0,0,1)$	3	3	1	3	3	10	28	(3,3,3,1)
$(0,0,1,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,0,1,1)$	2	2	2	2	2	8	16	(2,2,2,2)
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$(0,1,1,1)$	1	1	3	1	3	6	12	(3,1,1,1)
$(1,0,0,0)$	2	2	1	2	2	7	13	(2,2,2,1)
$(1,0,0,1)$	2	2	2	3	3	9	21	(3,2,2,2)
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$(1,1,1,1)$	0	0	4	1	4	5	17	(4,1,0,0)

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$(1,1,1,1)$	0	0	4	1	4	5	17	(4,1,0,0)

Example

$\mu = ((S \wedge T) \vee (S \wedge P) \vee (T \wedge P)) \rightarrow I$

$\varphi_1 = \varphi_2 = S \wedge T \wedge P$ $\quad mod(\varphi_1) = \{(1,1,1,1),(1,1,1,0)\}$

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	φ_1	φ_2	φ_3	φ_4	$d_{d_H,Max}$	$d_{d_H,\Sigma}$	d_{d_H,Σ^2}	$d_{d_H,GMax}$
$(0,0,0,0)$	3	3	0	2	3	8	22	(3,3,2,0)
$(0,0,0,1)$	3	3	1	3	3	10	28	(3,3,3,1)
$(0,0,1,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,0,1,1)$	2	2	2	2	2	8	16	(2,2,2,2)
$(0,1,0,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,1,0,1)$	2	2	2	2	2	8	16	(2,2,2,2)
$(0,1,1,1)$	1	1	3	1	3	6	12	(3,1,1,1)
$(1,0,0,0)$	2	2	1	2	2	7	13	(2,2,2,1)
$(1,0,0,1)$	2	2	2	3	3	9	21	(3,2,2,2)
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Merging in other frameworks

- Merging of weighted formulae
  - Benferhat-Dubois-Kaci-Prade [2000,2002,2003]
  - Meyer [2001]
- First order logic
  - Gorogiannis-Hunter [2008]
- Logic programs
  - Delgrande-Schaub-Tompits-Woltran [2009]
  - Hué-Papini-Würbel [2009]
- Constraints Networks
  - Condotta-Kaci-Marquis-Schwind [2009]
- Argumentation systems
  - Dung : arguments + attack relation [AAAI'05, AIJ-07]
    - Partial argumentation frameworks (PAF)
    - Edition distances
  - Weighted Argumentation Frameworks [IJCAI'15]
  - Merging Extensions [KR'16]

Iterated Merging - Conciliation

- Iterated Merging Operators

$(\varphi_1^0, \ldots, \varphi_n^0)$

Iterated Merging - Conciliation

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$$(\varphi_1^0, \ldots, \varphi_n^0) \xrightarrow{\text{Merging}} \varphi^{\Delta_0}$$

Iterated Merging - Conciliation

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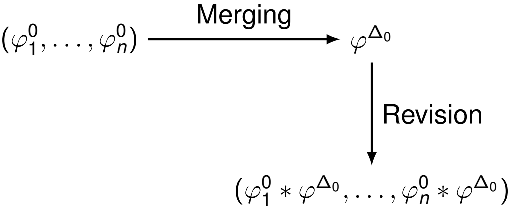

Iterated Merging - Conciliation

- Iterated Merging Operators

$$(\varphi_1^0, \ldots, \varphi_n^0) \xrightarrow{\text{Merging}} \varphi^{\Delta_0}$$

$$\varphi^{\Delta_0} \xrightarrow{\text{Revision}} (\varphi_1^0 * \varphi^{\Delta_0}, \ldots, \varphi_n^0 * \varphi^{\Delta_0})$$

$$(\varphi_1^0 * \varphi^{\Delta_0}, \ldots, \varphi_n^0 * \varphi^{\Delta_0}) \;\Big\|\; (\varphi_1^1, \ldots, \varphi_n^1)$$

Iterated Merging - Conciliation

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$$(\varphi_1^0, \ldots, \varphi_n^0) \xrightarrow{\text{Merging}} \varphi^{\Delta_0}$$

$$\downarrow \text{Revision}$$

$$(\varphi_1^0 * \varphi^{\Delta_0}, \ldots, \varphi_n^0 * \varphi^{\Delta_0})$$

$$\|$$

$$(\varphi_1^1, \ldots, \varphi_n^1) \longrightarrow \varphi^{\Delta_1}$$

Iterated Merging - Conciliation

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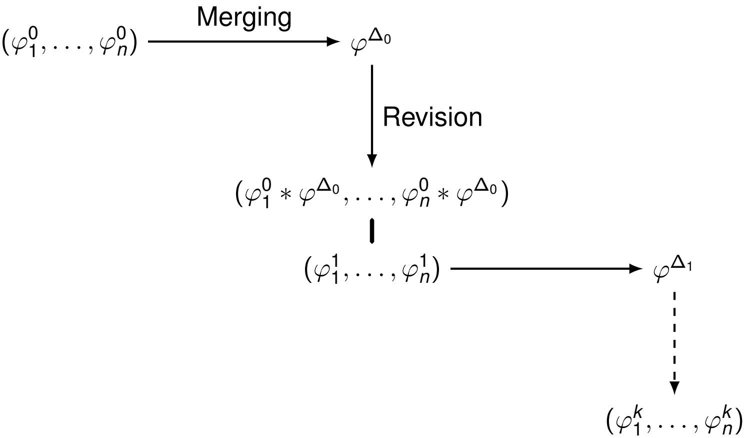

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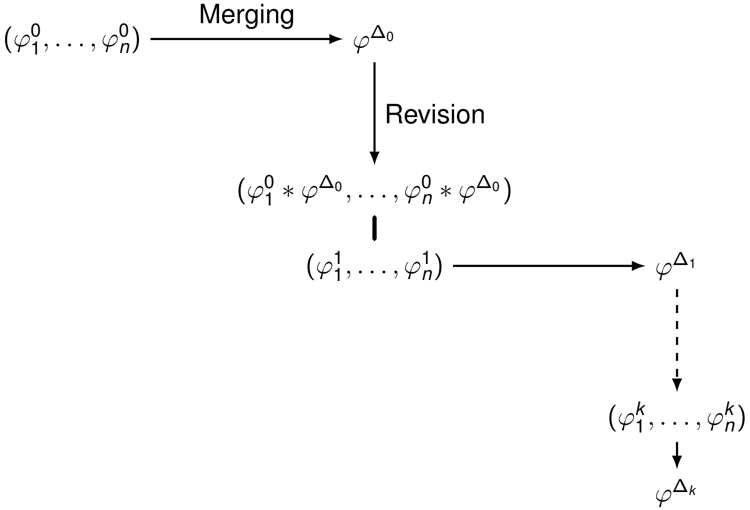

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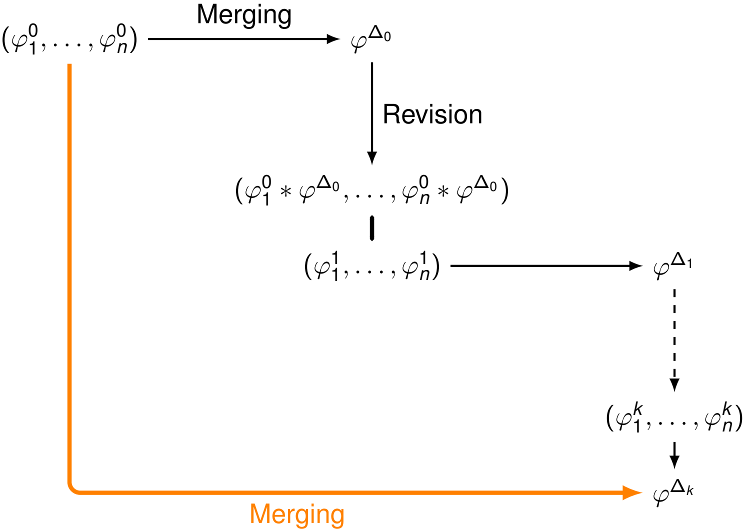

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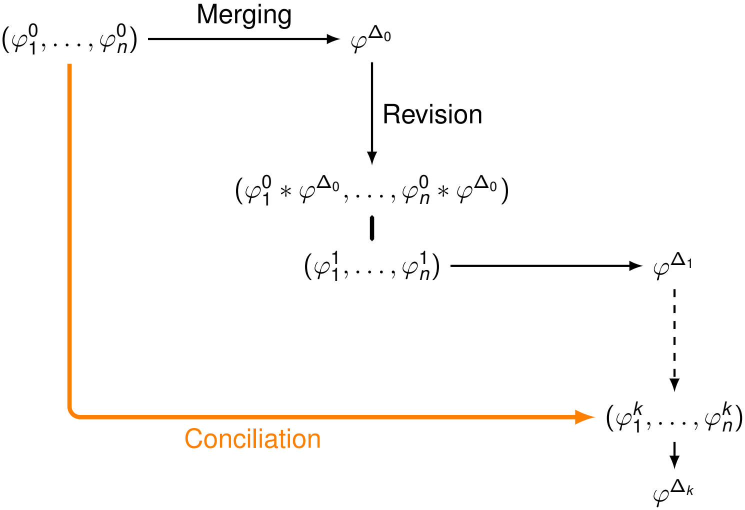

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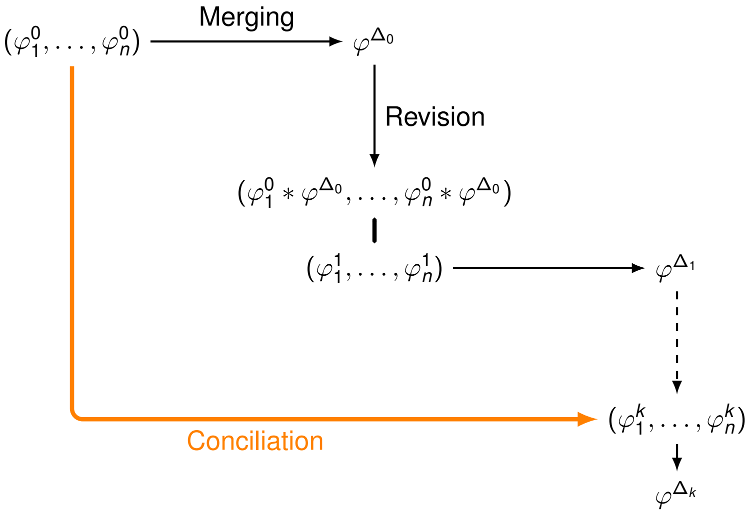

- Merging $(\varphi_1, \ldots, \varphi_n) \longrightarrow \varphi_\Delta$
- Conciliation $(\varphi_1, \ldots, \varphi_n) \longrightarrow (\varphi_1^*, \ldots, \varphi_n^*)$

Negotiation - Conciliation

Let $E = (\varphi_1, \ldots, \varphi_n)$ be a profile of belief/goal bases.

Two questions :

- What are the beliefs/goals of the group of agents ?
  - Merging (vote, social choice, MCDM, . . .)
- Can the agents find a consensual position ?
  - Conciliation (negotiation, bargaining, . . .)

A Game between Sources

- Negotiation :
    - Some sources have to concede to solve the conflicts

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- The idea :
  - Each source gives her base
  - Contest between the bases :
    - The weakest ones loose
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  - Ends when a compromise is reached

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Definition A Belief Game Model is a pair $\mathcal{N} = \langle g, \blacktriangledown \rangle$ where g is a choice function and $\blacktriangledown$ is a weakening function.
The solution to a belief profile E for a Belief Game Model $\mathcal{N} = \langle g, \blacktriangledown \rangle$, noted $\mathcal{N}(E)$, is the belief profile $E_{\mathcal{N}}$, defined as :

- $E_0 = E$
- $E_{i+1} = \blacktriangledown_{g(E_i)}(E_i)$
- $E_{\mathcal{N}}$ is the first E_i that is consistent

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Belief Game Model

A choice function is a function $g : \mathcal{E} \to \mathcal{E}$ such that :

- $g(E) \subseteq E$
- If $\bigwedge E \not\equiv \top$, then $\exists \varphi \in g(E)$ s.t. $\varphi \not\equiv \top$
- If $E \leftrightarrow E'$, then $g(E) \leftrightarrow g(E')$

A weakening function is a function $\blacktriangledown : \mathcal{K} \to \mathcal{K}$ such that :

- $\varphi \vdash \blacktriangledown(\varphi)$
- If $\varphi = \blacktriangledown(\varphi)$, then $\varphi \leftrightarrow \top$
- If $\varphi \leftrightarrow \varphi'$, then $\blacktriangledown(\varphi) \leftrightarrow \blacktriangledown(\varphi')$

Example : Database Class [Revesz, 1994]

- $g = d_D^{\Sigma}, \blacktriangledown = \delta$

$\varphi_1 = \{100, 001, 101\}$ $\varphi_2 = \{010, 001\}$ $\varphi_3 = \{111\}$

$$mod(\varphi_1 \wedge \varphi_2 \wedge \varphi_3) = \emptyset$$

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	φ_1	φ_2	φ_3	Σ	g
φ_1		0	1	1	
φ_2	0		1	1	
φ_3	1	1		2	•

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$\varphi_3 = \{111, 011, 101, 110\}$

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	φ_1	φ_2	φ_3	Σ	g
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φ_1		0	0	0	
φ_2	0		1	1	•
φ_3	0	1		1	•

Merging - Conciliation

- Merging
  - Postulates - Representation Theorems [JLC'02] [AIJ'04]
  - Manipulability issues [JAIR'07]
  - Truth-tracking issues [ECAI'10]
  - Egalitarism issues [KR'14]
  - Judgement Aggregation [AAMAS'14, AAMAS'15]
- Conciliation
  - Iterated Belief Merging [JLC'07]
  - Belief Negotiation Games [JANCL'04]
  - Confluence operators [AMAI'13]
  - Belief Revision Games [AAAI'15, IJCAI'16]