Implementing a Constraint Solver: A Case Study

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Cork Constraint Computation Centre & University College Cork

Road-map

- Goal
- Blueprint
- Data Structures
- Propagation
- Search
- Code Optimization
- Competition

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- ...if given enough time!

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Model: Golomb ruler

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int main(int argc, char *argv[])
{
  // input
  int nbMarks = (argc > 1 ? atoi(argv[1]) : 8);
  int rulerSize = ( 2 << (nbMarks-1) );

  // declare model and variables
  CSP model;
  VarArray mark(nbMarks, 0, rulerSize-1);
  VarArray distance(nbMarks*(nbMarks-1)/2, 1, rulerSize-1);

  // post constraints
  int i, j, k=0;
  for(i=1; i<nbMarks; ++i) {
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  }
  model.add( BoundAllDifferent(distance) );

  // post objective function
  model.add( Minimise( Max(mark) ) );

  // solve
  Solver s( model, mark );
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  - Automatic choices of the best representation/algorithm
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    - ✓ Constraints (Specific algorithm, Decomposition, Generic algorithms, ...)
    - ✓ Heuristics
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    - ✓ Heuristics
  - Automatic rewritting?
- Robustness
  - Worst case principle

Road-map

- Goal
- Blueprint
- Data Structures
- Propagation
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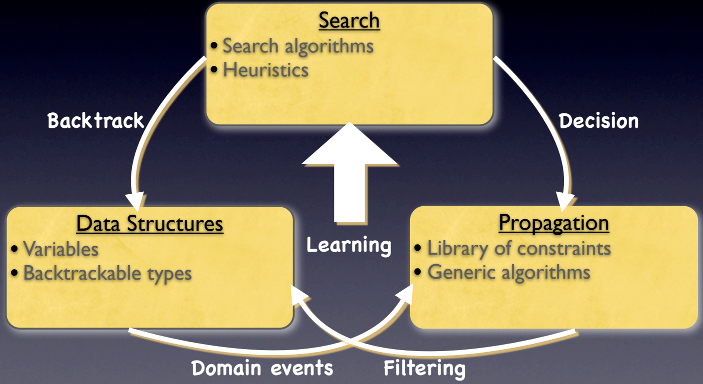
A little bit of structure

Search
- Search algorithms
- Heuristics

Data Structures
- Variables
- Backtrackable types

Propagation
- Library of constraints
- Generic algorithms

A little bit of structure



Road-map

- Goal
- Blueprint
- Data Structures
  - Variables (Baktracks)
- Propagation
- Search
- Code Optimization
- Competition

Backtrackable Data-Structures

- Copying/Trailing
  - See Shulte's papers and PhD Thesis
  - Copying
    - ✓ Easier to implement data structures
    - ✓ Easier to implement search strategies
    - ✓ Easier to parallelize
  - Trailing
    - ✓ Do only necessary work
    - ✓ Memory efficient

Copying

Trailing

Domain as a Bitset

- One 32 bits word for every value in [min(D)..max(D)]

X in
{0,1,2,5,7,18,19,21}

11100101	00000000	00110100

Domain as a Bitset

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- For every word, we allocate as many word as values in that word:
  - O((max-min+1) + 32*|D|) bits

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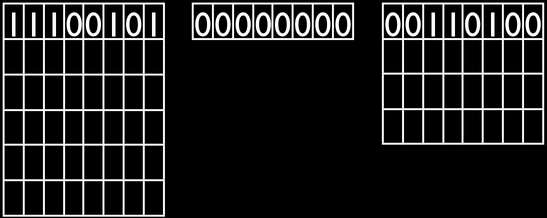

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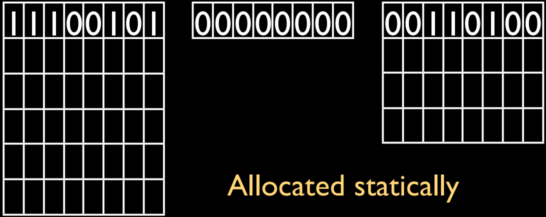

Allocated statically

Domain as a Bitset

X in {0,1,2,5,7,18,19,21}

Domain as a Bitset

1	1	1	0	0	1	0	1

0	0	0	0	0	0	0	0

0	0	1	1	0	1	0	0

X in {0,1,2,5,7,18,19,21}

Domain as a Bitset

decision 1 → {0,5,7,18,19,21}

1	1	1	0	0	1	0	1

0	0	0	0	0	0	0	0

0	0	1	1	0	1	0	0

X in {0,1,2,5,7,18,19,21}

Domain as a Bitset

decision 1 ⇒ {0,5,7,18,19,21} w1

1	1	1	0	0	1	0	1
1	0	0	0	0	1	0	1

0	0	0	0	0	0	0	0

0	0	1	1	0	1	0	0

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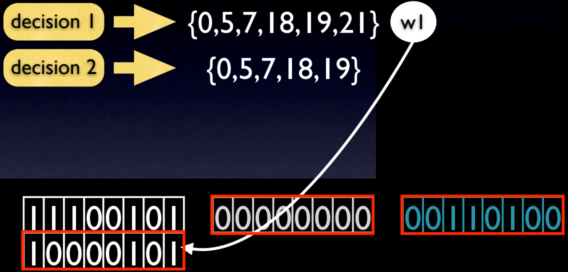

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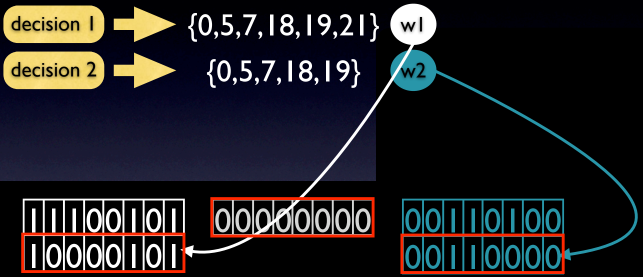

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Domain as a Bitset

decision 1 → {0,5,7,18,19,21} w1

decision 2 → {0,5,7,18,19} w2

decision 3 → {0,5,18}

11100101 00000000 00110100

10000101 00110000

X in {0,1,2,5,7,18,19,21}

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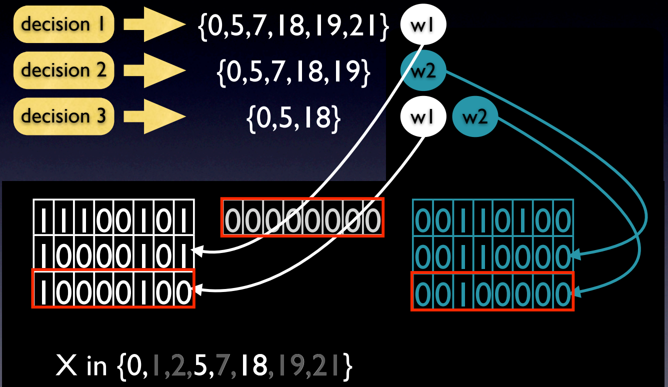

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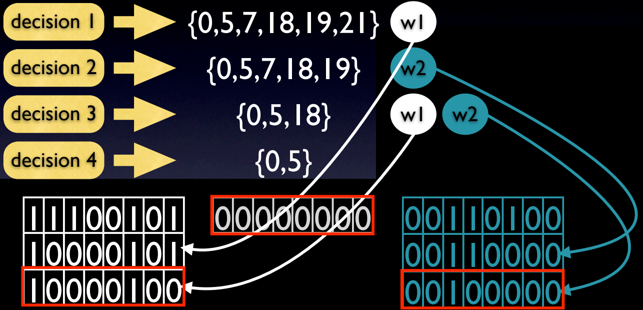

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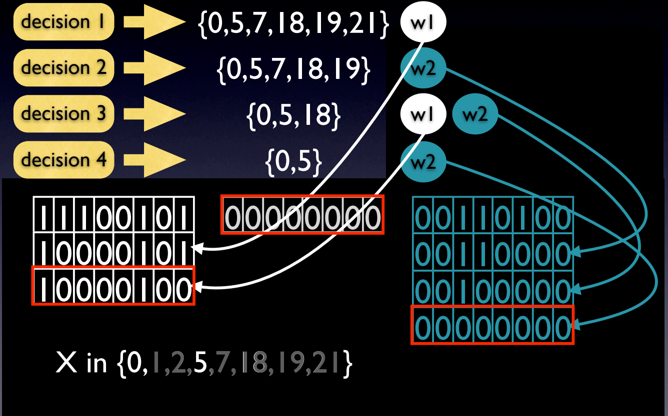

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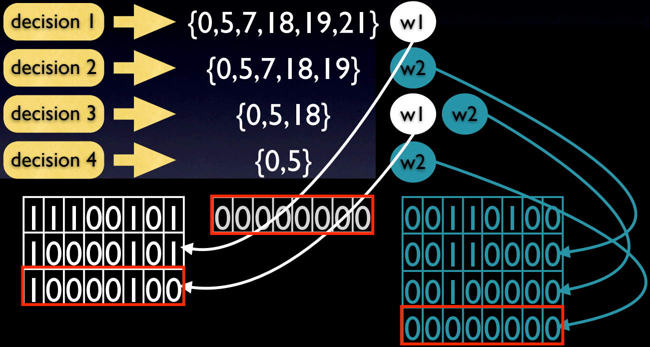

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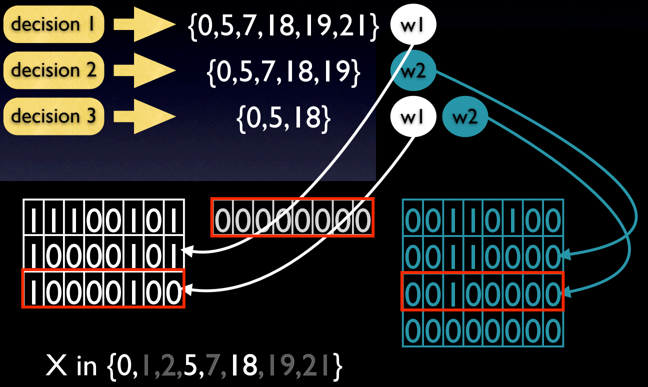

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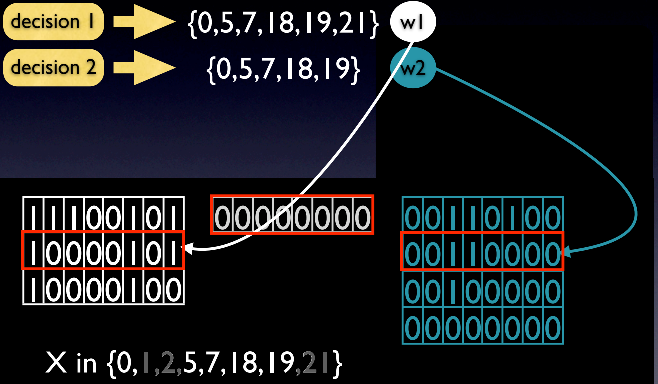

Domain as a List

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9	1	5	12	2	14	6	4	list

Domain as a List

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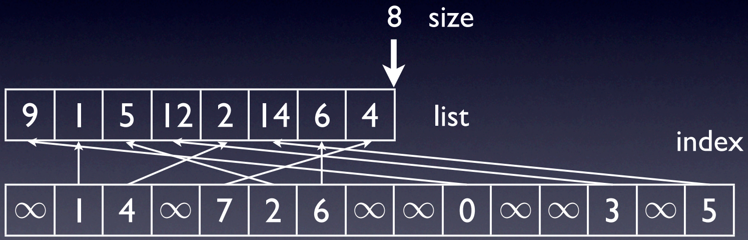

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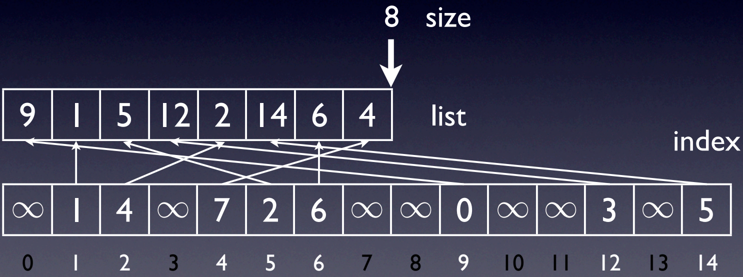
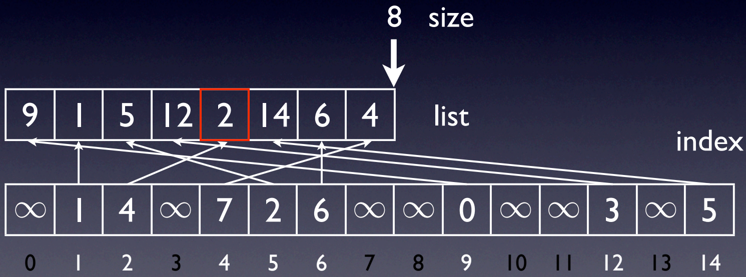


membership: *index[v] < size*

Domain as a List



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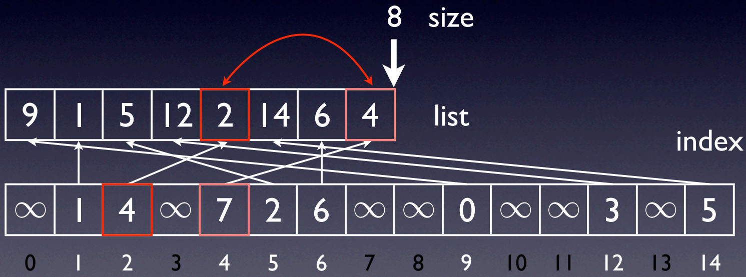

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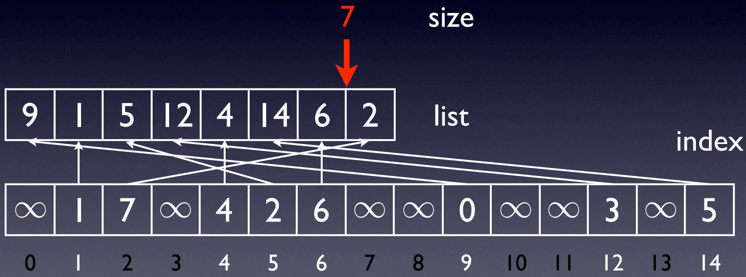

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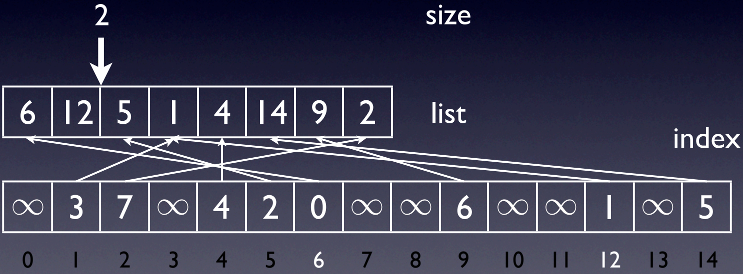

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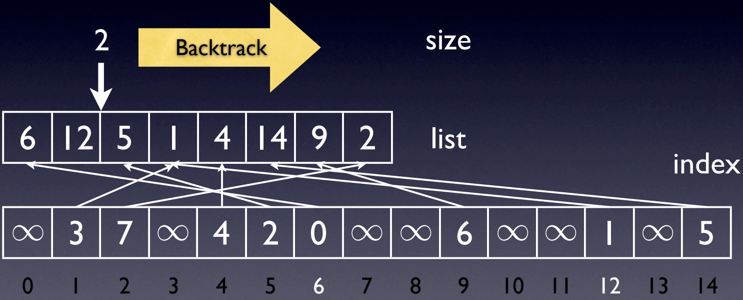

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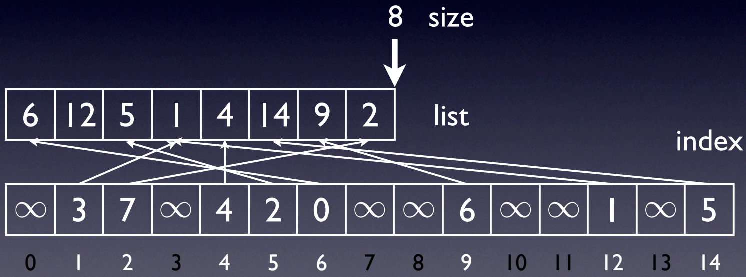

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Pigeon holes

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- Domain as a Bitset:
  - ✓ Space complexity in O(max-min)
  - ✓ Restore up to 32 values at a time
  - ✓ 600,000 Bts/second

Pigeon holes

- Domain as a Bitset:
  - ✓ Space complexity in O(max-min)
  - ✓ Restore up to 32 values at a time
  - ✓ 600,000 Bts/second
- Domain as a List:
  - ✓ Similar space complexity
  - ✓ Restore any number of values in one operation
  - ✓ 900,000 Bts/second
  - ✓ However, operations are much slower on the list (interval reasoning, set operations)

Road-map

- Goal
- Blueprint
- Data Structures
- Propagation
  - Nested predicates
  - GAC valid v allowed
- Search
- Code Optimization
- Competition

Constraint Propagation

- Variable/Constraint Queue
- Specific Propagators
- Nested Predicates
- Generic AC algorithms
  - Binary: AC3Bitset
  - Tight: GAC2001Allowed
  - Loose: GAC3rValid

Pruning:

Nested Predicates

Example: open-shop scheduling:

```
<predicate name="P0">
  <parameters>int X0 int X1  int X2 int X3 int X4 int X5</parameters>
  <expression>
    <functional>or(le(add(X0,X1),X2),le(add(X3,X4),X5))<functional>
  </expression>
</predicate>

 <constraint name="C0" arity="2" scope="V0 V1" reference="P0">
   <parameters>V0 85 V1 V1  64 V0</parameters>
 </constraint>
```

Nested Predicates: Decomposition

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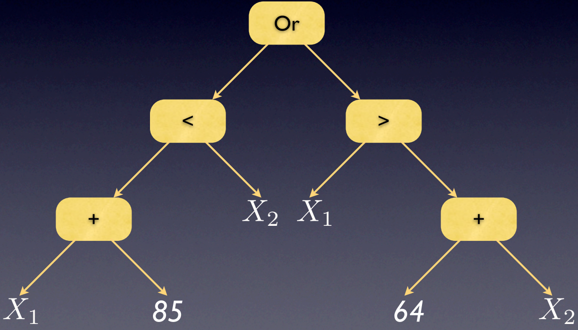

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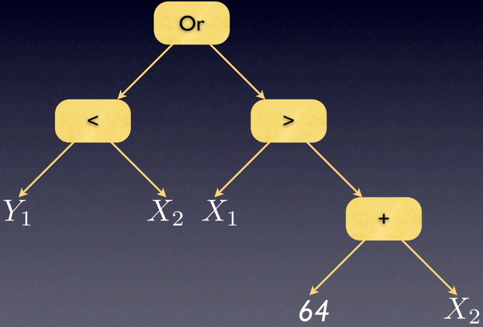
Example: open-shop scheduling:

$$Y_1 = X_1 + 85$$



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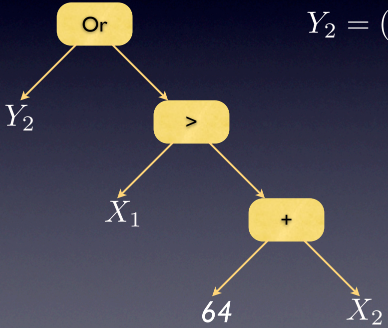
Example: open-shop scheduling:

$$Y_1 = X_1 + 85$$
$$Y_2 = (Y_1 < X_2)$$



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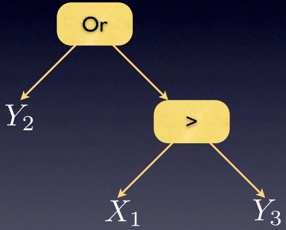

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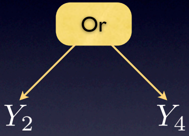

$$Y_1 = X_1 + 85$$
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$$Y_4 = (Y_3 < X_1)$$

Nested Predicates: Decomposition

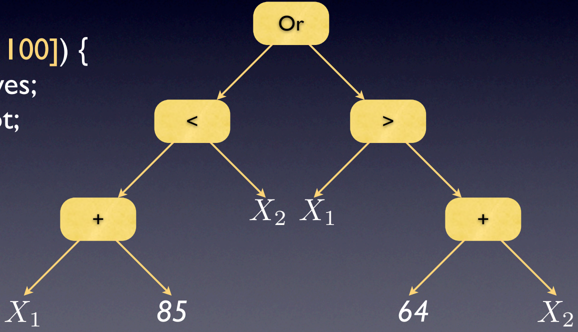
Example: open-shop scheduling:

$$Y_1 = X_1 + 85$$
$$Y_2 = (Y_1 < X_2)$$
$$Y_3 = X_2 + 64$$
$$Y_4 = (Y_3 < X_1)$$
$$(Y_2 \vee Y_4)$$

Nested Predicates: GAC-Checker

Example: open-shop scheduling:

```
check ([30, 100]) {
  assign leaves;
  query root;
}
```



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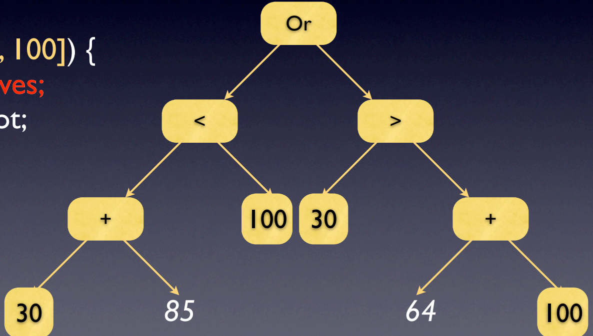

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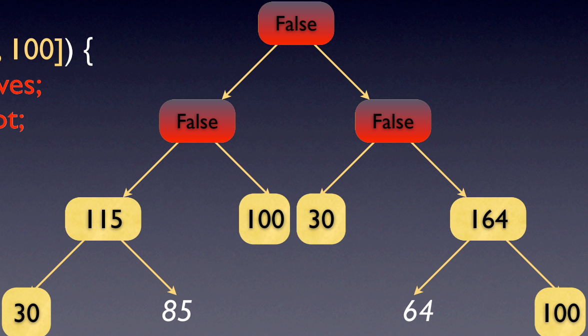

Golomb ruler / FAPP

Instance:

Decomposition

GAC-Checker

Golomb ruler / FAPP

Instance:	Golomb Ruler
Decomposition	128 nodes 0.18 seconds
GAC-Checker	87 nodes 38.22 seconds

Golomb ruler / FAPP

Instance:	Golomb Ruler	FAPP
Decomposition	128 nodes 0.18 seconds	60181 nodes 55.18 seconds
GAC-Checker	87 nodes 38.22 seconds	374 nodes 1.18 seconds

Making the Right Choice

Making the Right Choice

Feature	GAC	Decomp	OSP

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Constraint Arity	-	+	binary

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Ratio node/leaves	+	-	5/2

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Making the Right Choice

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Cartesian product cardinality	-	+	>60,000

Making the Right Choice

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Constraint Arity	-	+	binary
Ratio node/leaves	+	-	5/2
Domain continuity	-	+	no holes
Cartesian product cardinality	-	+	>60,000
Boolean domains	-	+	no

Making the Right Choice

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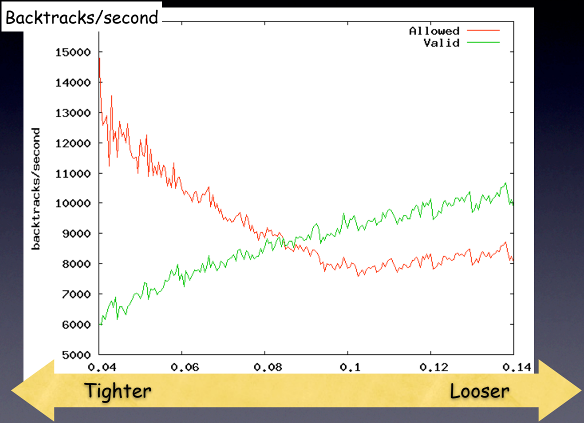
Ratio node/leaves	+	-	5/2
Domain continuity	-	+	no holes
Cartesian product cardinality	-	+	>60,000
Boolean domains	-	+	no
Total number of constraints	+	-	small

Making the Right Choice

Feature	GAC	Decomp	OSP
Constraint Arity	-	+	binary
Ratio node/leaves		-	5/2
Domain continuit		+	no holes
Cartesian product cardinality	-	+	>60,000
Boolean domains	-	+	no
Total number of constraints	+	-	small

Decomposition!

GAC Allowed v. GAC Valid



Road-map

- Goal
- Blueprint
- Data Structures
- Propagation
- Search
  - Heuristics
- Code Optimization
- Competition

Search Strategies

- Depth-first, Breadth-first, LDS,...
- Branching Choices
  - Domain Splitting
  - Arbitrary Constraint
- Variable/Value Ordering
  - "Learning" heuristics
    - ✓ Weighted Degree, Impact

Weighted Heuristics

- The best general purpose orderings are based on some kind of learning (or weighting)
  - Weighted Degree [Boussemart, Hemery, Lecoutre, Sais 2004]
  - Impact [Refalo 2004]
- A "Weighter" can suscribe for different types of event
  - Weighted degree: failures
  - Impact: Decisions, success, failures
- This architecture allows easy development of variations around these models
  - Why isn't Impact/Weighted Degree any good?

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Optimisation: Binary Extensional

- Standard algorithms:
  - AC3-bitset, Variable queue (fifo), revision condition
- Profiling, what does take time?
  - Propagation:.. 68%
    - “&” operation:.................................... 25.9%
    - Domain iteration:................................ 20.6%
    - AC3 (queuing/dequeuing):................ 11.4%
    - Revision condition + virtual call:.. 10.0%
  - Data structure modification:........... 19%
    - Domain modification:......................... 19.0%
  - Search:.. 12%
    - Trailing:.. 9.7%
    - Variable choice + Branching:........... 2.2%

Intersection on Bitsets (25%)

```
bool VariableList::wordIntersect(const MistralSet& s) const
{
  return values.wordIntersect(s);
}

inline bool MistralSet::wordIntersect(const MistralSet& s) const
{
  return ( table[neg_words] & s.table[neg_words] ) ;
}
```

Values Iteration (20%)

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- Domain as a Bitset:
  - 6,500 Bts/second

Values Iteration (20%)

- Random binary CSP
- Domain as a Bitset:
  - 6,500 Bts/second
- Domain as a List (hybrid bitset/list):
  - 10,000 Bts/second
  - Values are stored contiguously in an array
  - The order does not matter

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Quick Comparison
(#instances)

Abscon Choco Mistral

Quick Comparison
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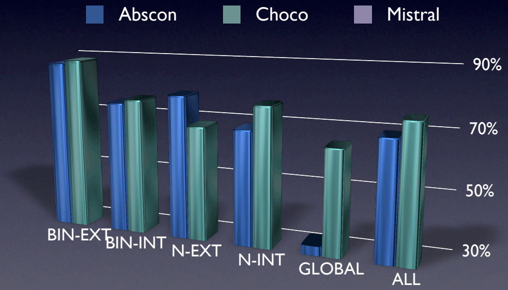

Quick Comparison
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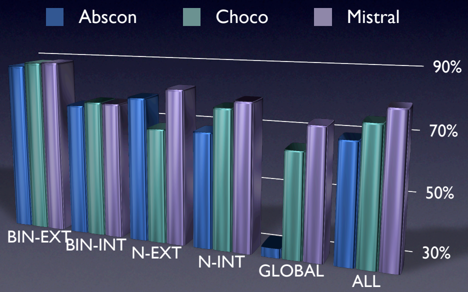
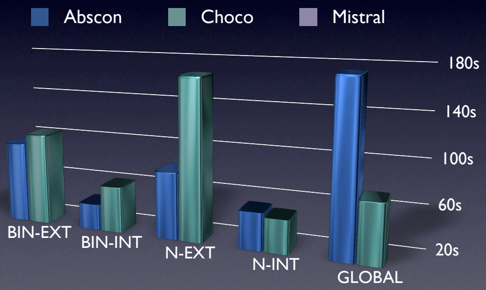


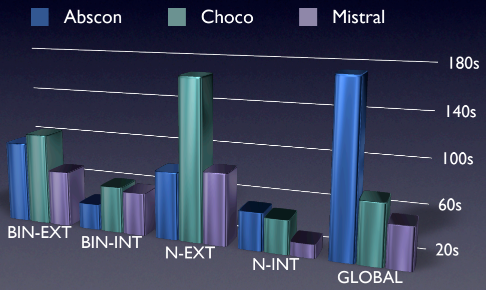
Quick Comparison
(cpu time)

Abscon Choco Mistral

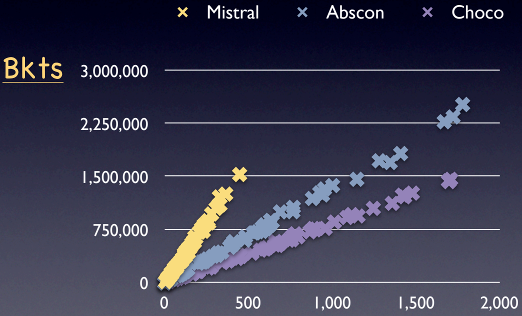
Quick Comparison
(cpu time)



Quick Comparison
(cpu time)

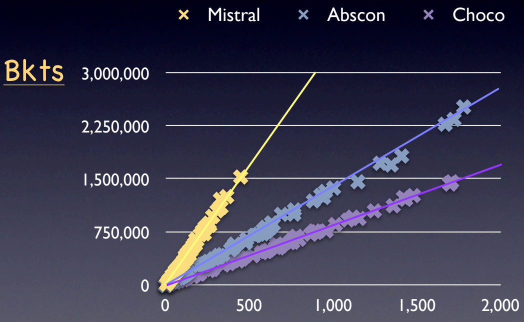


Backtracks v CPU-time



rand-2-40-19-443-230-* and frb40-19

Backtracks v CPU-time



rand-2-40-19-443-230-* and frb40-19

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- Adaptability
- Attention to details
- Robustness
  - Weaknesses are always more obvious to a user than strengths