Introducing Preference-Based Argumentation to Inconsistent Ontological Knowledge Bases – long version –

Madalina Croitoru[1], Rallou Thomopoulos[2], Srdjan Vesic[3]

[1]INRIA, LIRMM, Univ. Montpellier 2, France
[2] INRA GraphIK, France
[3] CRIL, CNRS - Univ. Artois, France

Abstract

Handling inconsistency is an inherent part of decision making in traditional agri-food chains – due to the various concerns involved. In order to explain the source of inconsistency and represent the existing conflicts in the ontological knowledge base, argumentation theory can be used. However, the current state of art methodology does not allow to take into account the level of significance of the knowledge expressed by the various ontological knowledge sources. We propose to use preferences in order to model those differences between formulas and evaluate our proposal practically by implementing it within the INRA platform and showing a use case using this formalism in a bread making decision support system.

1 Introduction

Querying several heterogeneous data sources while taking into account the ontological information (Lenzerini, 2002) recently received growing interest both from academia and from industry. In many societal oriented programs, the need for logic reasoning becomes imperative in order to provide sound scientific recommendations and capitalise on expert knowledge.

Let us consider the platform developed in the French Institute for Research in Agronomy (INRA) to link agronomy insights with socio-economic developments and behaviour of various stakeholders involved (farmers, consumers, biologists, industrial partners). It aims at identifying ways and solutions to maintain the quality of production and satisfy the needs of the users, while limiting the environmental impact (see e.g. the MEANS initiative[1]). The long-term ambition is to homogeneously integrate information from different sources, namely the regional production practices, market organization at local, national and international levels, and along the agri-food chains.

[1]http://www6.inra.fr/means_eng/

In practical applications such as the one described above, the knowledge obtained by the union of several sources is inconsistent. Different inconsistency methods have been devised in order to reason with such knowledge. The approaches by Bienvenu (2012) and Lembo et al. (2010) investigate inconsistency tolerant semantics. *Argumentation theory* (Dung, 1995) is another well-known method for dealing with inconsistent knowledge (Benferhat et al., 1993; Amgoud and Cayrol, 2002; Modgil and Prakken, 2013). It not only allows to resolve inconsistency; furthermore, the reasons why certain formulas are not compatible can be highlighted and presented to a user in form of arguments. This intuitive knowledge representation is of great interest when the output of the system is to be explained to end-users. Logic-based argumentation (Besnard and Hunter, 2008) considers constructing arguments from inconsistent knowledge bases, identifying attacks between them and selecting acceptable arguments and their conclusions. The logic-based argumentation ontological instantiation using the Datalog+/- family of languages has already shown the practical interest in using argumentation for query answering explanation in OBDA (Arioua et al., 2014a,b).

While argumentation-based techniques have already been successfully applied in agronomy, for instance in traditional agri-food chains (Thomopoulos et al., 2014) or packaging conception (Tamani et al., 2014), the current state of art methodology does not allow to take into account the degree of significance of the knowledge expressed by the various knowledge sources. In the INRA platform handling preferences is fundamental, since not all participants provide information of equal importance, regarding the scope, priority and urgency of the issues considered. Such handling needs to be generic: presupposing a total order (or any property) of the preference relation would induce some loss of generality that will limit the practical applicability.

The research task of this paper is to define the first *preference-based argumentation system that works with inconsistent ontologies* and *apply it and evaluate it in a bread conception scenario within INRA*. We demonstrate the expressivity gain of our approach and provide a *preliminary evaluation* relying on domain experts. We also *evaluate our proposal theoretically* by showing that there is a full correspondence between the results obtained by using the newly proposed argumentation formalism and those obtained by applying existing works in ontological base query answering.

2 Practical Scenario

The case of study considered in this paper relates to the debate that followed a recommendation of the French Ministry for Health within the framework of the PNNS program ("National Program for Nutrition and Health"). This recommendation concerns the ash content (mineral matter rate) in flour used for common French bread. Various actors of the agronomy sector are concerned, in particular the millers, the bakers, the nutritionists and the consumers.

The PNNS recommends to privilege the whole-grain cereal products and in particular to pass to a common bread of T80 type, i.e made with flour containing an ash content of 0.80%, instead of the type T65 (0.65%) currently used. Increasing the ash content comes down to using a more complete flour, since mineral matter is concentrated in the peripheral layers of the wheat grain, as well as a good amount of components of nutritional interest (vitamins, fibers). However, the peripheral layers of the grain are also exposed to the phytosanitary products, which does not make them advisable from a health point of view, unless one uses organic flour.

Other arguments, of various nature, are in favour or discredit whole-grain bread. From an organoleptic point of view for example, the bread loses out in its property of "being crusty". From a nutritional point of view, the argument according to which the fibers are beneficial for health is discussed, since some fibers could irritate the digestive system. From an economic point of view, the bakers fear selling less bread, because whole-grain bread increases satiety – which is beneficial from a nutritional point of view, for the regulation of the appetite and the fight against food imbalances and pathologies. However whole-grain bread requires also less flour and more water for its production, thus reducing the cost. The millers also fear a decrease in the technicity of the processing methods used in the flour production.

In this paper we will explain how the level of significance of the available information, expressed using the preference relation, can be usefully exploited to provide a priorization of possible decisions, an essential feature of decision support, which was not the case before.

3 Knowledge Representation

We consider the well known rulebased Tuple-Generating Dependencies (Datalog+/-) family of languages that generalise certain subsets of Description Logics (Baader et al., 2005; Calvanese et al., 2007). Here we restrict ourselves to Datalog+/- classes where the skolemised chase is finite (Finite Expansion Sets).

We consider *the positive existential* conjunctive fragment of first-order logic, denoted by FOL(∧,∃), which is composed of formulas built with the connectors $(\wedge, \rightarrow)$ and the quantifiers $(\exists, \forall)$. We consider first-order vocabularies with constants but no other function symbols. A term t is a constant or a variable, different constants represent different values (unique name assumption), an atomic formula (or atom) is of the form $p(t_1, ..., t_n)$ where p is an n-ary predicate, and $t_1, ..., t_n$ are terms. A *ground* atom is an atom with no variables. A variable in a formula is free if it is not in the scope of a quantifier. A formula is closed if it has no free variable. We denote by $\mathbf{X}$ (with a bold font) a sequence of variables $X_1, .., X_k$ with $k \geq 1$. A *conjunct* $C[\mathbf{X}]$ is a finite conjunction of atoms, where $\mathbf{X}$ is the sequence of variables occurring in C. Given an atom or a set of atoms A, $vars(A)$, $consts(A)$ and $terms(A)$ denote its set of variables, constants and terms, respectively.

An *existential rule* (rule) is a first-order formula of the form $r = \forall \mathbf{X} \forall \mathbf{Y} (H[\mathbf{X}, \mathbf{Y}]) \rightarrow \exists \mathbf{Z} C[\mathbf{Z}, \mathbf{Y}]$, with $vars(H) = \mathbf{X} \cup \mathbf{Y}$, and $vars(C) = \mathbf{Z} \cup \mathbf{Y}$ where H and C are *conjuncts* called the hypothesis and conclusion of R, respectively. We denote by $R = (H, C)$ a contracted form of a rule R. An existential rule with an empty hypothesis is called a *fact*. A fact is an existentially closed (with no free variable) conjunct.

We recall that a *homomorphism* π from a set of atoms A_1 to a set of atoms A_2 is a substitution of $vars(A_1)$ by $terms(A_2)$ such that $\pi(A_1) \subseteq A_2$. Given two facts f and f' we have $f \models f'$ iff there is a homomorphism from f' to f, where $\models$ is the first-order semantic entailment.

A *rule* $r = (H, C)$ is *applicable* to a set of facts F iff there exists $F' \subseteq F$ such that there is a homomorphism π from H to the conjunction of elements of F'. If a rule r is applicable to a set F, its application according to π produces a set $F \cup \{\pi(C)\}$. The new set $F \cup \{\pi(C)\}$, denoted also by $r(F)$, is called *immediate derivation* of F by r.

A *negative constraint* is a first-order formula $n = \forall \mathbf{X}\ H[\mathbf{X}] \rightarrow \perp$ where $H[\mathbf{X}]$ is a conjunct called hypothesis of n and **X** the sequence of variables appearing in the hypothesis.

Knowledge base. A knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ is composed of a finite set of facts $\mathcal{F}$, a finite set of existential rules $\mathcal{R}$ and a finite set of negative constraints $\mathcal{N}$.

$\mathcal{R}$-derivation. Let $F \subseteq \mathcal{F}$ be a set of facts and $\mathcal{R}$ be a set of rules. An $\mathcal{R}$-derivation of F in $\mathcal{K}$ is a finite sequence $\langle F_0, ..., F_n \rangle$ of sets of facts s.t $F_0 = F$, and for all $i \in \{0, ..., n\}$ there is a rule $r_i = (H_i, C_i) \in \mathcal{R}$ and a homomorphism π_i from H_i to F_i s.t $F_{i+1} = F_i \cup \{\pi(C_i)\}$. For a set of facts $F \subseteq \mathcal{F}$ and a query Q and a set of rules $\mathcal{R}$, we say $F, \mathcal{R} \models Q$ iff there exists an $\mathcal{R}$-derivation $\langle F_0, ..., F_n \rangle$ such that $F_n \models Q$.

Closure. Given a set of facts $F \subseteq \mathcal{F}$ and a set of rules $\mathcal{R}$, the closure of F with respect to $\mathcal{R}$, denoted by $\mathtt{Cl}_{\mathcal{R}}(F)$, is defined as the smallest set (with respect to $\subseteq$) which contains F and is closed under $\mathcal{R}$-derivation. The tractability conditions of the considered rule-based language **rely on different saturation (chase)** methods. For **algorithmic considerations**, there are well known studied tractable fragments of Datalog+/- (such as weakly-acyclic rule sets) that function with respect to the skolemised chase. By considering the skolemised chase and the finite fragments of Datalog+/- $\mathtt{Cl}_{\mathcal{R}}(F)$ is unique (i.e. universal model).

Finally, we say that a set of facts $F \subseteq \mathcal{F}$ and a set of rules $\mathcal{R}$ *entail* a fact f (and we write $F, \mathcal{R} \models f$) iff the closure of F by all the rules entails f (i.e. $\mathtt{Cl}_{\mathcal{R}}(F) \models f$).

Given a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, a set $F \subseteq \mathcal{F}$ is said to be *inconsistent* iff there exists a constraint $n \in \mathcal{N}$ such that $F \models H_n$, where H_n is the hypothesis of the constraint n. A set of facts is consistent iff it is not inconsistent. A set $F \subseteq \mathcal{F}$ is $\mathcal{R}$-*inconsistent* iff there exists a constraint $n \in \mathcal{N}$ such that $\mathtt{Cl}_{\mathcal{R}}(F) \models H_n$. A set of facts is said to be $\mathcal{R}$-*inconsistent* iff it is not $\mathcal{R}$-*consistent*. A knowledge base $(\mathcal{F}, \mathcal{R}, \mathcal{N})$ is said to be *inconsistent* iff $\mathcal{F}$ is $\mathcal{R}$-inconsistent.

Example 1 *Following the scenario use case of the paper, let us consider* $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ *where:*

- $\mathcal{F}$ *contains the following facts:*
  - $F_1 =$ *Bread(bleuette)* $\wedge$ *ContaminantFree(bleuette)*
  - $F_2 = \exists$ *e ExtractionRate(e,bleuette)*
  - $F_3 = \exists$ *f (FiberContent(f,bleuette)* $\wedge$ *High(f))*
- $\mathcal{R}$ *consists of the following rules:*
  - $R_1 = \forall$ *x,y (Bread(x)* $\wedge$ *ExtractionRate(y,x)* $\wedge$ *PesticideFree(x)* $\rightarrow$ *Moderate(y))*
  - $R_2 = \forall$ *x,y,z (Bread(x)* $\wedge$ *ExtractionRate(y,x)* $\wedge$ *FiberContent(z,x)* $\wedge$ *High(z)* $\rightarrow$ *Intensive(y))*

– $R_3 = \forall\, x$ *(Bread(x)* $\wedge$ *ContaminantFree(x)*

$\rightarrow$ *PesticideFree(x)* $\wedge$ *MycotoxinFree(x))*

- $\mathcal{N}$ *contains the following negative constraint:*
  – $N = \forall\, x$ *(Intensive(x)* $\wedge$ *Moderate(x))* $\rightarrow \perp$

$\mathcal{K}$ *is inconsistent since* $(\mathcal{F}, \mathcal{R}) \models N$. *Indeed,* F_1 *and* R_3 *allow to deduce PesticideFree(bleuette). Combined to* F_2 *and* R_1 *we obtain Moderate(e).* F_1*,* F_2*,* F_3 *and* R_2 *deduce Intensive(e), violating the negative constraint* N.

Note that (like in classical logic), if a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ is inconsistent, then everything is entailed from it. A common solution for knowledge bases where preferences are not considered (Bienvenu, 2012; Lembo et al., 2010) is to construct maximal (with respect to set inclusion) consistent subsets of facts. In this finite chase case there is a finite number of such sets. They are called *repairs* and denoted by $\mathcal{R}epair(\mathcal{K})$ defined: $\mathcal{R}epair(\mathcal{K}) = \{\mathcal{F}' \subseteq \mathcal{F} \mid \mathcal{F}'$ is maximal for $\subseteq$ $\mathcal{R}$-consistent set$\}$.

In Example 1, we have the following repairs: $\{F_1, F_2\}$, $\{F_2, F_3\}$ and $\{F_1, F_3\}$.

3.1 Argumentation

We first define the notion of an argument. For a set of formulae $\mathcal{G} = \{G_1, \ldots, G_n\}$, notation $\bigwedge G$ is used as an abbreviation for $G_1 \wedge \ldots \wedge G_n$.

Definition 1 *Given a knowledge base* $\mathcal{K}$*, an argument* a *is a tuple* $a = (F_0, F_1, \ldots, F_n)$ *where:*

- $(F_0, \ldots, F_{n-1})$ *is a derivation sequence with respect to* $\mathcal{K}$
- F_n *is an atom, a conjunction of atoms, the existential closure of an atom or the existential closure of a conjunction of atoms such that* $F_{n-1} \models F_n$.

Example 2 (Example 1 Cont.) *As an example of an argument, built with* F_1 *and* R_3*, consider* $a = (\{Bread(bleuette) \wedge ContaminantFree(bleuette)\},$
$\{Bread(bleuette) \wedge ContaminantFree(bleuette),$
$PesticideFree(bleuette) \wedge MycotoxinFree(bleuette)\},$
$PesticideFree(bleuette))$.

This is a natural way to define an argument when dealing with ontological rule-based languages, since this way, an *argument* corresponds to a *derivation*.

To simplify the notation, from now on, we suppose that we are given a fixed knowledge base $\mathcal{K}$ and do not explicitly mention $\mathcal{F}$, $\mathcal{R}$, $\mathcal{N}$ if not necessary. Let $a = (F_0, ..., F_n)$ be an argument. Then, we denote $\texttt{Supp}(a) = F_0$ and $\texttt{Conc}(a) = F_n$. Let $S \subseteq \mathcal{F}$ a set of facts, $\texttt{Arg}(S)$ is defined as the set of all arguments a such that $\texttt{Supp}(a) \subseteq S$. Note that the set $\texttt{Arg}(S)$ is also dependent on the set of rules and the set of constraints, but for simplicity reasons, we do not write $\texttt{Arg}(S, \mathcal{R}, \mathcal{N})$ when it is clear to which $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ we refer. Finally, let $\mathcal{E}$ be a set of arguments. The base of $\mathcal{E}$ is defined as the union of the argument supports: $\texttt{Base}(\mathcal{E}) = \bigcup_{a \in \mathcal{E}} \texttt{Supp}(a)$.

Arguments may attack each other, which is captured by a binary attack relation $\mathtt{Att} \subseteq \mathtt{Arg}(\mathcal{F}) \times \mathtt{Arg}(\mathcal{F})$. Recall that the repairs are the subsets of $\mathcal{F}$ while the set $\mathcal{R}$ is always taken as a whole. This means that the authors of the semantics used to deal with an inconsistent ontological KB envisage the set of facts as inconsistent and the set of rules as consistent. When it comes to the attack relation, this means that we only need the so called "assumption attack" since, roughly speaking, all the inconsistency "comes from the facts".

Definition 2 *Let $\mathcal{K}$ be a knowledge base and let a and b be two arguments. The argument a attacks argument b, denoted $(a, b) \in Att$, if and only if there exists $\varphi \in \mathtt{Supp}(b)$ such that the set $\{Conc(a), \varphi\}$ is $\mathcal{R}$-inconsistent.*

Please note that this attack relation is not symmetric.

Definition 3 (Dung, 1995) *Given a knowledge base $\mathcal{K}$, the* corresponding argumentation framework *$\mathcal{AF}_K$ is $(\mathcal{A} = \mathtt{Arg}(\mathcal{F}), \mathtt{Att})$ where $\mathcal{A}$ is the set of arguments that can be constructed from $\mathcal{F}$ and $\mathtt{Att}$ is the corresponding attack relation as specified in Definition 2. Let $\mathcal{E} \subseteq \mathcal{A}$ and $a \in \mathcal{A}$. We say that $\mathcal{E}$ is* conflict free *iff there exists no arguments $a, b \in \mathcal{E}$ such that $(a, b) \in Att$. $\mathcal{E}$* defends *a iff for every argument $b \in \mathcal{A}$, if we have $(b, a) \in Att$ then there exists $c \in \mathcal{E}$ such that $(c, b) \in Att$. $\mathcal{E}$ is* admissible *iff it is conflict free and defends all its arguments. $\mathcal{E}$ is a* complete extension *iff $\mathcal{E}$ is an admissible set which contains all the arguments it defends. $\mathcal{E}$ is a* preferred extension *iff it is maximal (with respect to set inclusion) admissible set. $\mathcal{E}$ is a* stable extension *iff it is conflict-free and for all $a \in \mathcal{A} \setminus \mathcal{E}$, there exists an argument $b \in \mathcal{E}$ such that $(b, a) \in Att$. $\mathcal{E}$ is a* grounded extension *iff $\mathcal{E}$ is a minimal (for set inclusion) complete extension. For an argumentation framework $AS = (\mathcal{A}, Att)$ we denote by $Ext_x(AS)$ (or by $Ext_x(\mathcal{A}, Att)$) the set of its extensions with respect to semantics σ. We use the abbreviations c, p, s, and g for respectively complete, preferred, stable and grounded semantics.*

4 Preference Handling

A preference-based knowledge base is a 4-tuple $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ composed of four finite sets of formulae: a set $\mathcal{F}$ of facts, a set $\mathcal{R}$ of rules, a set $\mathcal{N}$ of constraints and a set $\geq$ of preferences. The preference relation $\geq$ is defined over the facts $\mathcal{F}$ ($\geq \subseteq \mathcal{F} \times \mathcal{F}$). We put no constraints on the preference relation except that it has to be reflexive and transitive.

Example 3 *Let us consider the following preference-based knowledge base from the scenario use case:*

- *$\mathcal{F}$ contains the following facts:*
  - F_1 = *ExtractionRate(T65,bleuette)*
  - F_2 = *Bread(p) $\wedge$ ExtractionRate(τ,p) $\wedge$ Moderate(τ)*
  - F_3 = *Bread(p) $\wedge$ SaltAdjunction(s,p) $\wedge$ Reduced(s)*

– F_4 = *Bread(p)* $\wedge$ *ExtractionRate(τ,p)* $\wedge$ *Intensive(τ)*

– F_5 = *Bread(p)* $\wedge$ *Crusty(p)* $\wedge$ *SaltAdjunction(s,p)* $\wedge$ *Maintained(s)*

- *$\mathcal{R}$ consists of the following rules:*

  – $R_1 = \forall$ *x,y (Bread(x)* $\wedge$ *ExtractionRate(y,x)* $\wedge$ *Moderate(y)* $\rightarrow$ *Digestible(x))*

  – $R_2 = \forall$ *x,z (Bread(x)* $\wedge$ *SaltAdjunction(z,x)* $\wedge$ *Reduced(z)* $\rightarrow$ *LowSalt(x))*

  – $R_3 = \forall$ *x,y (Bread(x)* $\wedge$ *ExtractionRate(y,x)* $\wedge$ *Moderate(y)* $\rightarrow$ *PesticideFree(x))*

  – $R_4 = \forall$ *x,y (Bread(x)* $\wedge$ *ExtractionRate(y,x)* $\wedge$ *Intensive(y)* $\rightarrow$ *HighFiber(x))*

  – $R_5 = \forall$ *x,z (Bread(x)* $\wedge$ *Crusty(x)* $\wedge$ *SaltAdjunction(z,x)* $\wedge$ *Maintained(z)* $\rightarrow$ *ConsumerFriendly(x))*

- *$\mathcal{N}$ contains the following negative constraints:*

  – $N_1 = \forall$ *x (Intensive(x)* $\wedge$ *Moderate(x))* $\rightarrow\perp$

  – $N_2 = \forall$ *x (Reduced(x)* $\wedge$ *Maintained(x))* $\rightarrow\perp$

- *$\geq$ is defined by the experts as follows:*

  F_3 and F_4 express nutritional concerns, F_2 expresses a sanitary concern, F_5 a sensorial concern, while F_1 is neutral.

  In the PNNS programme recommendation, nutritional concerns take priority over sanitary ones, which take priority over sensorial ones.

  The preference relation $\geq$ is thus defined by:
  $F_3 \sim F_4 > F_2 > F_5$, while F_1 is incomparable with the other facts.

The preferences on the facts are used to refine the set of repairs of an inconsistent knowledge base. They specify different criteria allowing to select only some of the repairs. We consider here three notions introduced by Staworko et al. (2006): locally optimal, Pareto optimal and global optimal.

The *locally optimal* notions ensures that it is not possible to obtain a better set by exchanging one of its formulae. We denote the set of LO repairs by $\mathcal{R}epair^{lo}(\mathcal{K})$.

The next notion is that of a Pareto optimal repair; the idea is that a subset X of a given repair cannot be exchanged by a formula ψ strictly preferred to all the formulae of X. We denote the set of PO repairs by $\mathcal{R}epair^{po}(\mathcal{K})$.

The third notion is that of globally optimal repair. Here one considers exchanging a set with another set. We denote the set of GO repairs by $\mathcal{R}epair^{go}(\mathcal{K})$. All three notions are defined below. Note that $\mathcal{R}epair^{go}(\mathcal{K}) \subseteq \mathcal{R}epair^{po}(\mathcal{K}) \subseteq \mathcal{R}epair^{lo}(\mathcal{K})$.

Definition 4 (Staworko et al., 2006) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base and $A' \in \mathcal{R}epair(\mathcal{K})$ one of its repairs. We say that A' is a locally optimal (LO) repair iff there exist no $\varphi \in A'$ and $\psi \in \mathcal{F} \setminus A'$ such that $\psi > \varphi$ and $(A' \setminus \{\varphi\}) \cup \{\psi\}$ is an $\mathcal{R}$-consistent set.*

Definition 5 (Staworko et al., 2006) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base and $A' \in \mathcal{R}epair(\mathcal{K})$ one of its repairs. We say that A' is a Pareto optimal (PO) repair iff there exist no $X \subseteq A'$ and $\psi \in \mathcal{F} \setminus A'$ such that*

- $X \neq \emptyset$
- *for every $\varphi \in X$ we have $\psi > \varphi$*
- *$(A' \setminus X) \cup \{\psi\}$ is an $\mathcal{R}$-consistent set.*

Definition 6 (Staworko et al., 2006) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base and $A' \in \mathcal{R}epair(\mathcal{K})$ one of its repairs. We say that A' is a globally optimal (GO) repair iff there exist no $X \subseteq A'$ and $Y \subseteq \mathcal{F} \setminus A'$ such that*

- $X \neq \emptyset$
- *for every $\varphi \in X$ there exists $\psi \in Y$ such that $\psi > \varphi$*
- *$(A' \setminus X) \cup Y$ is an $\mathcal{R}$-consistent set.*

5 Preference ranking

First note that the attack relation considered in this paper does not depend on the preference relation $\geq$. Its goal is to underline the conflicts between the arguments coming from conflicts from the knowledge base. Those conflicts still exist even if some piece of information is preferred to another one. So in our framework we suppose that all attacks always succeed.

This is the reason why Definition 3 does not take preferences into account. It just allows to resolve the conflicts between different sets of arguments and to obtain the extensions of the system. The preferences will allow to select only the best extensions. This is showed in the next definition.

Definition 7 *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework, let σ be a semantics and $\mathcal{E}$ an extension with respect to σ.*

- *$\mathcal{E}$ is a locally optimal (LO) extension iff there exists no $\varphi \in \texttt{Base}(\mathcal{E})$ and $\psi \in \mathcal{F} \setminus \texttt{Base}(\mathcal{E})$ such that $\texttt{Arg}((\texttt{Base}(\mathcal{E}) \setminus \{\varphi\}) \cup \{\psi\})$ is a conflict-free set and $\psi > \varphi$.*
- *$\mathcal{E}$ is a Pareto optimal (PO) extension iff there exists no $X \subseteq \texttt{Base}(\mathcal{E})$ and $\psi \in \mathcal{F} \setminus \texttt{Base}(\mathcal{E})$ such that $X \neq \emptyset$ and $\texttt{Arg}((\texttt{Base}(\mathcal{E}) \setminus X) \cup \{\psi\})$ is a conflict-free set and for every $\varphi \in X$ we have $\psi > \varphi$.*
- *$\mathcal{E}$ is a globally optimal (GO) extension iff there exists no $X \subseteq \texttt{Base}(\mathcal{E})$ and $Y \subseteq \mathcal{F}$ such that $X \neq \emptyset$ and $\texttt{Arg}((\texttt{Base}(\mathcal{E}) \setminus X) \cup Y)$ is a conflict-free set and for every $\varphi \in X$ there exists $\psi \in Y$ such that $\psi > \varphi$.*

We denote by $\texttt{Ext}^{lo}_{\sigma}(\mathcal{AF}_K)$ (respectively $\texttt{Ext}^{po}_{\sigma}(\mathcal{AF}_K)$, $\texttt{Ext}^{go}_{\sigma}(\mathcal{AF}_K)$) the sets of locally (resp. Pareto, globally) optimal extensions under semantics σ.

The output of an argumentation framework is usually defined (Caminada and Amgoud, 2007, Definition 12) as the set of conclusions that appear in all the extensions (under a given semantics). In our case, thanks to preferences, we have more information so we can restrict the number of extensions to be used in reasoning. This allows for the intersection of extensions' conclusions to be larger; consequently, we draw more sceptical conclusions.

Definition 8 (Output of an arg. framework) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base and $\mathcal{AF}_K$ the corresponding preference-based argumentation framework. The output of $\mathcal{AF}_K$ under semantics σ is defined as:*

$$\texttt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K) = \bigcap_{\mathcal{E} \in \texttt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)} \texttt{Concs}(\mathcal{E})$$

where $\beta \in \{lo, po, go\}$.

When $\texttt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K) = \emptyset$, we define $\texttt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K) = \emptyset$ by convention.

Example 4 (Example 3 Cont.) *From the argumentation graph of the knowledge base we obtain the stable / preferred extensions :*
$ext_1 = \texttt{Arg}(\{F_1, F_2, F_3\})$
$ext_2 = \texttt{Arg}(\{F_1, F_2, F_5\})$
$ext_3 = \texttt{Arg}(\{F_1, F_4, F_3\})$
$ext_4 = \texttt{Arg}(\{F_1, F_4, F_5\})$.

In this example, extension ext_3 is the best according to all criteria (LO, PO, GO). Selecting this extension yields the following conclusions:
ExtractionRate(T65,bleuette),
Bread(p),
SaltAdjunction(s,p),
Reduced(s),
ExtractionRate(τ,p),
Intensive(τ),
LowSalt(p)),
HighFiber(p)).

6 Theoretical Evaluation

This section contains the theoretical evaluation of our proposal. It shows that the result returned by the argumentation system is equivalent to that returned by using repairs. We also show that our argumentation framework satisfies the postulates for instantiated argumentation systems. Finally, we study the properties of our system in case when the preference relation is total.

6.1 Equivalence Results

This section shows that there is a full correspondence between the result obtained by using our preference-based argumentation system and the result obtained by using the repairs of the given inconsistent ontological knowledge base.

We show that if stable or preferred semantics is used to calculate extensions of the argumentation system, its LO (resp. PO, GO) extensions correspond exactly to LO (resp. PO, GO) repairs of the given knowledge base[2].

Proposition 1 *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base and $\mathcal{AF}_K$ the corresponding preference-based argumentation system. Let $\sigma \in \{p, s\}$ and $\beta \in \{lo, po, go\}$. Then:*

$$\mathtt{Ext}_\sigma^\beta(\mathcal{AF}_K) = \{\mathtt{Arg}(A') \mid A' \textit{ is a } \beta \textit{ repair of } \mathcal{K}\}.$$

The previous result makes it possible to build a bridge between argumentation theory and inconsistent ontological knowledge bases. Namely, it shows that a repair corresponds to an extension (under preferred or stable semantics). We see this result as an opportunity to import the existing results from argumentation theory to inconsistent ontological knowledge bases and vice versa.

Proof *It is known (Croitoru and Vesic, 2013, Theorem 1) that $\mathtt{Ext}_\sigma(\mathcal{AF}_K) = \{\mathtt{Arg}(A') \mid A'$ is a repair of $\mathcal{K}\}$. So it is sufficient to show that for every $A' \in \mathcal{R}epair(\mathcal{K})$, we have that A' is a β repair of $\mathcal{K}$ if and only if $\mathtt{Arg}(A')$ is a β extension of $\mathcal{AF}_K$ under semantics σ.*

Case $\beta = LO$. Let $A' \in \mathcal{R}epair(\mathcal{K})$. We know (Croitoru and Vesic, 2013, Theorem 1) that $\mathcal{E} = \mathtt{Arg}(A')$ is a stable and preferred extension. Let us prove that A' is not *a β repair if and only if $\mathcal{E} = \mathtt{Arg}(A')$ is* not *a β extension (during the proof, we suppose the semantics σ with $\sigma \in \{p, s\}$). A' is not a β repair if and only if there exist $\varphi \in A'$ and $\psi \in \mathcal{F} \setminus A'$ such that $((A' \setminus \{\varphi\}) \cup \{\psi\})$ is $\mathcal{R}$-consistent and $\psi > \varphi$. Note that $\mathcal{E}$ is not a β extension if and only if there exist $\varphi \in \mathtt{Base}(\mathcal{E})$ and $\psi \in \mathcal{F} \setminus \mathtt{Base}(\mathcal{E})$ such that $((\mathtt{Arg}(\mathtt{Base}(\mathcal{E})) \setminus \{\varphi\}) \cup \{\psi\})$ is a conflict-free set and $\psi > \varphi$. Since $\mathcal{E} = \mathtt{Arg}(A')$ then the previous condition is equivalent to: there exist $\varphi \in A'$ and $\psi \in \mathcal{F} \setminus A'$ such that $\mathtt{Arg}((A' \setminus \{\varphi\}) \cup \{\psi\})$ is conflict-free and $\psi > \varphi$. We know that for every set $S \subseteq K$, we have that S is $\mathcal{R}$-consistent if and only if $\mathtt{Arg}(S)$ is conflict-free. Thus, $\mathcal{E}$ is not a locally optimal extension if and only if A' is not a locally optimal repair. $\mathtt{Ext}_\sigma^{lo}(\mathcal{AF}_K) = \{\mathtt{Arg}(A') \mid A'$ is a LO repair of $\mathcal{K}\}$.*

Case $\beta = PO$. Let $A' \in \mathcal{R}epair(\mathcal{K})$. Note that A' is not a PO repair if and only if there exist $X \subseteq A'$ and $\psi \in \mathcal{F} \setminus A'$ such that $X \neq \emptyset$ and $\psi > \varphi$ and $(A' \setminus X) \cup \{\psi\}$ is $\mathcal{R}$-consistent. This is equivalent with: there exist $X \subseteq \mathtt{Base}(\mathcal{E})$ and $\psi \in \mathcal{F} \setminus \mathtt{Base}(\mathcal{E})$ such that $X \neq \emptyset$ and $\psi > \varphi$ and $\mathtt{Arg}((\mathtt{Base}(\mathcal{E}) \setminus X) \cup \{\psi\})$ is conflict-free. We conclude that $\mathtt{Ext}_\sigma^{po}(\mathcal{AF}_K) = \{\mathtt{Arg}(A') \mid A'$ is a PO repair of $\mathcal{K}\}$.

Case $\beta = GO$. We proceed as in the previous two cases. It is sufficient to note that A' is not a GO repair if and only if there exist $X \subseteq A'$ and $Y \subseteq \mathcal{F}$ such that $X \neq \emptyset$ and $((A' \setminus X) \cup Y)$ is $\mathcal{R}$-consistent and for every $\varphi \in X$ there exists $\psi \in Y$

[2] Note that in the extreme cases when there are no preferences between formulae or when all the formulae are equally preferred, all the repairs are LO, PO and GO. In that case, all the preferred / stable extensions are LO, PO and GO as well.

such that $\psi > \varphi$. As in previous cases, we conclude that this is equivalent with: there exist $X \subseteq \mathtt{Base}(\mathcal{E})$ and $Y \subseteq \mathcal{F}$ such that $X \neq \emptyset$ and $((\mathtt{Base}(\mathcal{E}) \setminus X) \cup Y)$ is conflict-free and for every $\varphi \in X$ there exists $\psi \in Y$ such that $\psi > \varphi$. Thus, $\mathtt{Ext}^{go}_{\sigma}(\mathcal{AF}_K) = \{\mathtt{Arg}(A') \mid A'$ is a GO repair of $\mathcal{K}\}$. ■

6.2 Postulate Compliance

This section shows that the preference-based argumentation framework we propose in this paper satisfies the rationality postulates for instantiated argumentation frameworks Caminada and Amgoud (2007). We first prove the indirect consistency postulate which says that the closure of the set of conclusions of every extension is a consistent set. It also specifies that the closure of the output of an argumentation system must be consistent.

Proposition 2 (Indirect consistency) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework, $\sigma \in \{s, p\}$ and $\beta \in \{lo, po, go\}$. Then:*

- *for every $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$, $\mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i))$ is a consistent set*
- *$\mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K))$ is a consistent set.*

Proof *Let $\sigma \in \{s, p\}$. Regarding the first item: We know that $\mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i))$ is a consistent set for every extension $\mathcal{E}_i$ under semantics σ (Croitoru and Vesic, 2013, Prop.1). Since $\mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K) \subseteq \mathtt{Ext}_{\sigma}(\mathcal{AF}_K)$, the postulate holds in case of the preference-based argumentation system as well.*

Let us now show why the second item holds. We know (Croitoru and Vesic, 2013, Theorem 1) that there is the same number of stable / preferred extensions of $\mathcal{AF}_K$ and repairs of $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$. Since there is at least one repair of $\mathcal{K}$, then there is at least one stable / preferred extension of $\mathcal{AF}_K$. Thus there exists at least one β extension of $\mathcal{AF}_K$. Denote this extension $\mathcal{E}$. Now, from Definition 8, we see that $\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K) \subseteq \mathtt{Concs}(\mathcal{E})$. Since $\mathtt{Concs}(\mathcal{E})$ is $\mathcal{R}$-consistent (as we proved in the first item) then $\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K)$ is $\mathcal{R}$-consistent. In other words, $\mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K))$ is consistent. ■

Since our instantiation satisfies indirect consistency then it also satisfies direct consistency. This comes from $\mathcal{R}$-consistency definition; namely, if a set is $\mathcal{R}$-consistent, then it is necessarily consistent. Thus, we obtain the following corollary.

Corollary 1 (Direct consistency) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework and $\sigma \in \{s, p\}$. Then:*

- *for every $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$, $\mathtt{Concs}(\mathcal{E}_i)$ is a consistent set*
- *$\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K)$ is a consistent set.*

We can now also show that the present argumentation formalism also satisfies the closure postulate. This means that the set of conclusions of every extension is closed with respect to the set of rules. The output of the argumentation system is closed with respect to $\mathcal{R}$ as well.

Proposition 3 (Closure) *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework and $\sigma \in \{s, p\}$. Then:*

- *for every $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$, $\mathtt{Concs}(\mathcal{E}_i) = \mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i))$.*
- $\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K) = \mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K))$.

Proof *Consider the first item; we know that* $\mathtt{Concs}(\mathcal{E}_i) = \mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i))$ *holds for every* $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$ *(Croitoru and Vesic, 2013, Prop. 3). Since* $\mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K) \subseteq \mathtt{Ext}_{\sigma}(\mathcal{AF}_K)$*, the postulate holds in case of the preference-based argumentation system as well.*

Let us now prove the second item. From the definition of $\mathtt{Cl}_{\mathcal{R}}$, $\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K) \subseteq \mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K))$. *So we only need to prove that* $\mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K)) \subseteq \mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K)$. *Let* $\alpha \in \mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K))$. *Then there exist* $\alpha_1, \ldots, \alpha_k \in \mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K)$ *such that there is a derivation sequence* $F_0, \ldots, F_n$ *such that* $F_0 = \{\alpha_1, \ldots, \alpha_k\}$ *and* $\alpha \in F_n$. *For every* $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$*, we have* $\alpha_1, \ldots, \alpha_k \in \mathcal{E}_i$. *Therefore for every* $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$, $\alpha \in \mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i))$. *From the first part of the proof,* $\mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i)) = \mathtt{Concs}(\mathcal{E}_i)$. *Thus, for every* $\mathcal{E}_i \in \mathtt{Ext}^{\beta}_{\sigma}(\mathcal{AF}_K)$, $\alpha \in \mathtt{Concs}(\mathcal{E}_i)$. *This means that* $\alpha \in \mathtt{Output}^{\beta}_{\sigma}(\mathcal{AF}_K)$. ■

6.3 The particular case of a total preference relation

This section studies the case when $\geq$ is total. We say that $\geq$ is total if and only if for every $\varphi, \psi \in \mathcal{F}$, we have $\varphi \geq \psi$ or $\psi \geq \varphi$ (or both). We show that in the case when $\geq$ is total, Pareto optimal and globally optimal repairs coincide. Furthermore, we show that PO repairs (and GO repairs) coincide with preferred subtheories of $\mathcal{K}$. However, using LO repairs may still yield a different result.

Since for every $\varphi, \psi \in \mathcal{F}$, we have $\varphi \geq \psi$ or $\psi \geq \varphi$ (or both) then we can stratify $\mathcal{F}$ (with respect to $\geq$) in $\mathcal{F}_1, \ldots, \mathcal{F}_n$ such that:

- $\mathcal{F} = \mathcal{F}_1 \cup \ldots \cup \mathcal{F}_n$
- for every i, j such that $i \neq j$, we have $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset$
- for every i, j, for every $\varphi \in \mathcal{F}_i$, for every $\psi \in \mathcal{F}_j$ we have that $\varphi \geq \psi$ if and only if $i \leq j$.

For a formula φ, we define $\mathtt{Level}(\varphi) = \{i \mid \varphi \in \mathcal{F}_i\}$. For a set $A' \subseteq \mathcal{F}$, we define $\mathtt{Level}(A') = max\{\mathtt{Level}(\varphi) \mid \varphi \in A'\}$.

Let us recall the definition of a preferred subtheory. The original definition (Brewka, 1989) supposes classical logic; we present a version adopted to the case of an inconsistent ontological knowledge base.

Definition 9 (Preferred subtheory) *Suppose we are given a knowledge base* $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ *such that* $\geq$ *is total. Let* $\mathcal{F}$ *be stratified with respect to* $\geq$ *into* $\mathcal{F}_1 \cup \ldots \cup \mathcal{F}_n$. *A* preferred subtheory *is a set* $A = A_1 \cup \ldots \cup A_n$ *such that* $\forall k \in [1, n]$, $A_1 \cup \ldots \cup A_k$ *is a maximal (for set inclusion)* $\mathcal{R}$*-consistent subset of* $\mathcal{F}_1 \cup \ldots \cup \mathcal{F}_k$.

It can easily be checked that every preferred subtheory is a maximal for set inclusion $\mathcal{R}$-consistent set.

The next proposition shows that in the case when preference relation $\geq$ is total, GO repairs coincide with PO repairs.

Proposition 4 *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base such that $\geq$ is a total order. Then, the set of Pareto optimal repairs coincides with the set of globally optimal repairs.*

Proof *It is obvious to see that (even in the case when $\geq$ is not total) $\mathcal{R}epair^{go}(\mathcal{K}) \subseteq \mathcal{R}epair^{po}(\mathcal{K})$. Hence, $A' \notin \mathcal{R}epair^{po}(\mathcal{K})$ implies $A' \notin \mathcal{R}epair^{go}(\mathcal{K})$. In the rest of the proof, we show that $A' \notin \mathcal{R}epair^{go}(\mathcal{K})$ implies $A' \notin \mathcal{R}epair^{po}(\mathcal{K})$. The case $A' \notin \mathcal{R}epair(\mathcal{K})$ is trivial. Thus, suppose that $A' \in \mathcal{R}epair(\mathcal{K})$ and $A' \notin \mathcal{R}epair^{go}(\mathcal{K})$. This means that there exist $X \subseteq A'$ and $Y \subseteq \mathcal{F} \setminus A'$ such that:*

- $X \neq \emptyset$
- *for every $\varphi \in X$ there exists $\psi \in Y$ such that $\psi > \varphi$*
- *$(A' \setminus X) \cup Y$ is an $\mathcal{R}$-consistent set.*

Since $\geq$ is a total order then there exists $\psi' \in Y$ such that for every $\psi \in Y$, $\psi' \geq \psi$. This means that A' is not a Pareto optimal repair since for every $\varphi \in X$, $\psi' > \varphi$ and $(A' \setminus X) \cup \{\psi'\}$ is an $\mathcal{R}$-consistent set.

Thus $A' \notin \mathcal{R}epair^{po}(\mathcal{K})$ implies $A' \notin \mathcal{R}epair^{go}(\mathcal{K})$ and $A' \notin \mathcal{R}epair^{go}(\mathcal{K})$ implies $A' \notin \mathcal{R}epair^{po}(\mathcal{K})$. In other words, $\mathcal{R}epair^{po}(\mathcal{K}) = \mathcal{R}epair^{go}(\mathcal{K})$. ■

The next proposition shows that in the case when preference relation $\geq$ is total, PO repairs (and GO repairs) coincide with preferred sub-theories.

Proposition 5 *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ be a knowledge base such that $\geq$ is a total order. Then, the set of preferred sub-theories of $\mathcal{K}$ coincides with the set of Pareto optimal repairs of $\mathcal{K}$.*

Proof *The first part of the proof shows that if A' is not a Pareto optimal repair then A' is not a preferred subtheory. The second part shows that if A' is not a preferred subtheory then A' is not a Pareto optimal repair. Let us start by supposing the $A' \subseteq \mathcal{K}$ is not a Pareto optimal repair. If A' is not a repair, then A' is not a maximal with respect to set inclusion $\mathcal{R}$-consistent set, thus A' is not a preferred subtheory. Let us consider the case when A' is a maximal $\mathcal{R}$-consistent subset of $\mathcal{K}$. This means that there exist $\psi' \in \mathcal{K} \setminus A'$ and X such that $\emptyset \neq X \subseteq A'$ and for every $\varphi \in X$, $\psi' > \varphi$ and $(A' \setminus X) \cup \{\psi'\}$ is an $\mathcal{R}$-consistent set. From the fact that for every $\varphi \in X$, $\psi' > \varphi$ we conclude that $\texttt{Level}(X) > \texttt{Level}(\psi')$. Let $A'' = \{\varphi \in \mathcal{A}' \mid \texttt{Level}(\varphi) \leq \texttt{Level}(\psi')\}$. Observe that A'' is a proper subset of A'. Note also that $A'' \cup \{\psi'\} \subseteq (A' \setminus X) \cup \{\psi'\}$. Since $(A' \setminus X) \cup \{\psi'\}$ is $\mathcal{R}$-consistent then $A'' \cup \{\psi'\}$ is $\mathcal{R}$-consistent. This means that A' is not a preferred subtheory since A'' is not a maximal $\mathcal{R}$-consistent set in $\{\varphi \in \mathcal{K} \mid \texttt{Level}(\varphi) \leq \texttt{Level}(A'')\}$.*

Let us now prove that if A' is not a preferred subtheory then A' is not a Pareto optimal repair. Note that if A' is not a maximal $\mathcal{R}$-consistent subset of $\mathcal{K}$ then A' is not *a repair, thus A' is not a PO repair. Thus, in the rest of the proof, we consider the case when A' is a maximal $\mathcal{R}$-consistent set, but it is not a preferred subtheory. This means that there exists $\psi' \in \mathcal{K} \setminus A'$ such that $\{\varphi \in \mathcal{A} \mid \texttt{Level}(\varphi) \leq \texttt{Level}(\psi')\} \cup \{\psi'\}$ is $\mathcal{R}$-consistent. Consider ψ' with a minimal level satisfying the previous condition, i.e. let $\psi' \in \mathcal{K} \setminus A'$ be such that $\{\varphi \in \mathcal{A} \mid \texttt{Level}(\varphi) \leq \texttt{Level}(\psi')\} \cup \{\psi'\}$ is $\mathcal{R}$-consistent and there exists no $\psi'' \in \mathcal{K} \setminus \mathcal{A}$ such that $\{\varphi \in \mathcal{A} \mid \texttt{Level}(\varphi) \leq \texttt{Level}(\psi'')\} \cup \{\psi''\}$ is $\mathcal{R}$-consistent and $\texttt{Level}(\psi'') < \texttt{Level}(\psi')$. Now, note that the case $\texttt{Level}(\psi') \geq \texttt{Level}(A')$ is not possible since that would imply that A' is not a maximal $\mathcal{R}$-consistent set. Thus $\texttt{Level}(\psi') < \texttt{Level}(A')$. Let $A'' = \{\varphi \in A' \mid \texttt{Level}(\varphi) \leq \texttt{Level}(\psi')\}$. Let $X = A' \setminus A''$. We have that $X \neq \emptyset$, that for every $\varphi \in X$, $\psi' > \varphi$ and that $(A' \setminus X) \cup \{\psi'\}$ is $\mathcal{R}$-consistent. Hence, A' is not a Pareto optimal repair.*

We conclude that: (1) if A' is not a Pareto optimal repair then A' is not a preferred subtheory and (2) if A' is not a preferred subtheory then A' is not a Pareto optimal repair. From (1) and (2) we conclude that the set of Pareto optimal repairs is equal to the set of preferred subtheories. ■

The next example shows that LO repairs do not coincide with PO repairs even in the case when $\geq$ is a total order.

Example 5 *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N}, \geq)$ with $\mathcal{F} = \{$whiteBread(B), wholeWheatBread(B), organicWholeWheatBread(B)$\}$, $\mathcal{R} = \emptyset$, $\mathcal{N} = \{\forall x$ (whiteBread(x) $\wedge$ wholeWheatBread(x) $\rightarrow \perp$), $\forall x$ (whiteBread (x) $\wedge$ organicWholeWheatBread(x) $\rightarrow \perp$)$\}$, and let whiteBread(B) $\geq$ wholeWheatBread(B) $\geq$ organicWholeWheatBread(B).*

Set $A' = \{$wholeWheatBread(B), organicWholeWheatBread(B)$\}$ is a locally optimal repair but it not a Pareto optimal repair.

7 Qualitative Evaluation

The evaluation of the implemented system was done via a series of interviews with domain experts. The first meeting dealt with the delimitation of the project objectives and addressed fundamental questions such as: Is it possible to uniquely define "good" bread? Which scenarios of "good bread" should be considered? How could they be defined from a nutritional, sanitary, sensorial and economic point of view? Which are the main known ways to achieve them?

The first point to highlight is that our initial approach with experts included no preference expression. The experts themselves raised the question of the importance attached to the different pieces of knowledge modeled in the system. Moreover, in some cases experts hesitated on the relevance of some facts or rules. From that first step of the project, the need to take into account different levels of importance among arguments became obvious. Preferences were introduced from that point.

Then a series of individual interviews constituted the elicitation phase. Each expert gave several arguments which were complementing one each other. In parallel, the writing of specifications for the demonstrator and the definition of the knowledge base structure were conducted.

The knowledge and reasoning procedures were implemented using the COGUI knowledge representation tool, with an extension of 2000 lines of supplemental code.

Three experts have validated our approach: two researchers in food science and cereal technologies of the French national institute of agronomic research, specialists respectively of the grain-to-flour transformation process and of the bread-making process, and one industrial expert - the Director of the French National Institute of Bread and Pastry.

In the next plenary meeting the real potential of the approach was shown. The experts were formulating goals and viewpoints they were interested in and the used system together with the argumentation extension was yielding and ranking the propositions.

More specifically, four scenarios were evaluated. These scenarios concern four kinds of consumers: obese (fiber preference), people with iron deficiency (micronutrient preference), people with cardiovascular disease (decreased salt preference) and vegetarians (limited phytic acid), which produces different sets of goals. For each scenario, the system proposes several output recommendations. The audience for decreasing salt tips the balance in favour of a recommendation for the T80 bread, while the audience for decreasing phytic acid pushes to specify recommendations towards a natural sourdough bread or a conservative T65 bread. The results were considered as explainable by experts, but not obvious, since many considerations had to be taken into account.

Let us focus on the case of vegetarians. Phytic acid, which is contained in the outer layers of the wheat grain, is known to limit the bio-availability of cations, including essential minerals such as copper, zinc or iron, which must be preserved especially for vegetarians. Therefore the conservative solution of T65 bread can be explained by the fact that the current T65 bread contains few outer grain layers, thus limiting the phytic acid risks. Furthermore, natural sourdough bread has a lower pH level than T65. This acidity interferes with the activity of phytic acid, thus avoiding the decrease of mineral bio-availability. Now why chose one solution rather than another one? This point could be highlighted by the system. Indeed, the choice depends on the ordering of consumer preferences. Favouring organoleptic aspects of bread (e.g. crusty, white, honeycombed bread) leads to chose the T65 solution, whereas favouring nutritional aspects (e.g. fibers, vitamins, satiety) leads to the natural sourdough solution.

More particularly, two interests of the approach were highlighted. They concern cognitive considerations. Firstly, experts were conscious that the elicitation procedure was done according to their thought processes, that is, in the order of the production chain, which is more natural and intuitive. The system was thus able to restitute the knowledge in a different manner than the experts usually do, that is, combining information from different steps of the chain, different disciplines and different objectives. Secondly, from a problem that could initially seem simple, the experts realized that it covered a huge complexity that a human mind could hardly address alone. The tool is currently available to them under restricted access.

8 Conclusion and Related Work

This paper studied the problem of handling inconsistency in decision making in agri-food chains. In this scenario, the ontological knowledge base can be inconsistent, due to the various concerns involved. Argumentation theory can be used not only to deal with inconsistency but also to explain the decision made by the system to a user. However, despite the fact that the different pieces of knowledge can be of different importance for a decision maker, existing argumentation-based systems for inconsistent ontology handling cannot take this information into account. In this paper, we present the first preference-based argumentation system that works with inconsistent ontological knowledge bases and apply it to an agronomy scenario. We illustrate the scenario on examples and present an overview of its evaluation by the domain experts. We also formally prove that it has desirable theoretical properties.

In order to position our work let us discuss the related papers. A two-step approach for preference-based argumentation was proposed recently by Amgoud and Vesic (2014). In that work, the authors propose a general argumentation framework that can be instantiated in different ways. They propose to take into account both attacks and preferences in the first phase; the second phase uses only preferences to refine the result. The main difference is that we show it is possible to define an instantiation in which taking attacks and preferences into account is done in completely separated phases: namely, in our approach the first phase (inconsistency resolution) is done without looking at the preferences.

The links between argumentation semantics (stable, preferred, grounded) and different semantics in inconsistent ontological knowledge bases, such as AR, IAR or ICR were recently studied by Croitoru and Vesic (2013). The present paper is more general since it also takes into account preferences. We also show the significance of the approach by showing the practical added value of our framework.

We now summarize other approaches that are more or less related to our work. The ASPIC+ system (Modgil and Prakken, 2013) has also recently studied using preferences and structured argumentation. This approach imposes restrictions on the preference relation and, of course, does not consider equivalence results with the inconsistent ontology query answering semantics or preference-based repair selection. Another related contribution comprises constructing an argumentation framework with ontological knowledge allowing two agents to discuss the answer to queries concerning their knowledge without one agent having to copy all of their ontology to the other (Black et al., 2009). However the authors do not consider preferences. Let us also mention the work of Kaci (2010) that only considers symmetrical attack relations.

Binas and McIlraith (2008) use argumentation in order to answer inconsistent queries. The authors use the similar definitions of argument and attack as in this paper but only consider propositional logic. Benferhat et al. (1999) consider that a formula should be deduced if no stronger reasons for deducing its negation exist. Recently, in OBDA, preference handling methods have been extended to Datalog+/- families and DL-Lite knowledge bases (Lukasiewicz et al., 2012, 2013; Bienvenu et al., 2014).

References

Amgoud, L. and C. Cayrol (2002), "Inferring from inconsistency in preference-based argumentation frameworks." *Journal of Automated Reasoning*, 29 (2), 125–169.

Amgoud, L. and S. Vesic (2014), "Rich preference-based argumentation frameworks." *International Journal of Approximate Reasoning*, 55, 585–606.

Arioua, Abdallah, Nouredine Tamani, and Madalina Croitoru (2014a), "On conceptual graphs and explanation of query answering under inconsistency." In *Proceedings of the 21st International Conference on Conceptual Structures (ICCS'14)*, volume LNAI 8577, 51–64, Springer.

Arioua, Abdallah, Nouredine Tamani, Madalina Croitoru, and Patrice Buche (2014b), "Query failure explanation in inconsistent knowledge bases using argumentation." In *Proceedings of the Fifth International Conference on Computational Models of Argument (COMMA'14)*, 101–108.

Baader, F., S. Brandt, and C. Lutz (2005), "Pushing the EL envelope." In *Proceedings of the 19th International Joint Conferences on Artificial Intelligence (IJCAI'05)*, 364–369.

Benferhat, S., D. Dubois, and H. Prade (1993), "Argumentative inference in uncertain and inconsistent knowledge bases." In *Proceedings of the 9th Conference on Uncertainty in Artificial intelligence (UAI'93)*, 411–419.

Benferhat, S., D. Dubois, and H. Prade (1999), "Some syntactic approaches to the handling of inconsistent knowledge bases: a comparative study part 2: the prioritized case." In *Logic at Work: Essays Dedicated to the Memory of Helen Rasiowa*, volume 24, 437–511, Physica-Verlag.

Besnard, Ph. and A. Hunter (2008), *Elements of Argumentation*. MIT Press.

Bienvenu, Meghyn (2012), "On the complexity of consistent query answering in the presence of simple ontologies." In *In Proceedings of the the 26th Conference on Artificial Intelligence (AAAI'12)*, 705–711.

Bienvenu, Meghyn, Camille Bourgaux, and François Goasdoué (2014), "Querying inconsistent description logic knowledge bases under preferred repair semantics." In *Proceedings of the 28th Conference on Artificial Intelligence (AAAI'14)*, 996–1002.

Binas, A. and S. A. McIlraith (2008), "Peer-to-peer query answering with inconsistent knowledge." In *Proceedings of the 11th International Conference on Principles of Knowledge Representation and Reasoning (KR'08)*, 329–339.

Black, E., A. Hunter, and J. Z. Pan (2009), "An argument-based approach to using multiple ontologies." In *Proceedings of the 3rd International Conference on Scalable Uncertainty Management (SUM'09)*, 68–79, Springer-Verlag.

Brewka, G. (1989), "Preferred subtheories: An extended logical framework for default reasoning." In *Proceedings of the 11th International Joint Conference on Artificial Intelligence (IJCAI'89)*, 1043–1048.

Calvanese, D., G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati (2007), "Tractable reasoning and efficient query answering in description logics: The DL-lite family." *Journal of Automated Reasoning*, 39, 385–429.

Caminada, M. and L. Amgoud (2007), “On the evaluation of argumentation formalisms.” *Artificial Intelligence Journal*, 171 (5-6), 286–310.

Croitoru, M. and S. Vesic (2013), “What can argumentation do for inconsistent ontology query answering?” In *Proceedings of the 7th International Conference on Scalable Uncertainty Management, (SUM'13)*, 15–29, Springer.

Dung, P M (1995), “On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.” *Artificial Intelligence Journal*, 77, 321–357.

Kaci, Souhila (2010), “Refined preference-based argumentation frameworks.” In *In Proceedings of the Third International Conference on Computational Models of Argument (COMMA'10)*, 299–310.

Lembo, Domenico, Maurizio Lenzerini, Riccardo Rosati, Marco Ruzzi, and Domenico Fabio Savo (2010), “Inconsistency-tolerant semantics for description logics.” In *Proceedings of the Fourth International Conference on Web Reasoning and Rule Systems (RR'10)*, 103–117.

Lenzerini, M. (2002), “Data integration: A theoretical perspective.” In *Proceedings of the 21st Symposium on Principles of Database Systems (PODS'02)*, 233–246.

Lukasiewicz, Thomas, Maria Vanina Martinez, and Gerardo I. Simari (2012), “Inconsistency handling in Datalog+/- ontologies.” In *Proceedings of the 20th European Conference on Artificial Intelligence (ECAI'12)*, 558–563.

Lukasiewicz, Thomas, Maria Vanina Martinez, and Gerardo I. Simari (2013), “Complexity of inconsistency-tolerant query answering in Datalog+/-.” In *Informal Proceedings of the 26th International Workshop on Description Logics*, 791–803.

Modgil, S. and H. Prakken (2013), “A general account of argumentation and preferences.” *Artificial Intelligence Journal*, 195, 361–397.

Staworko, S., J. Chomicki, and J. Marcinkowski (2006), “Preference-driven querying of inconsistent relational databases.” In *Proceedings of EDBT Workshops*, volume 4254 of *Lecture Notes in Computer Science*, 318–335, Springer.

Tamani, Nouredine, Madalina Croitoru, and Patrice Buche (2014), “Conflicting viewpoint relational database querying: An argumentation approach.” In *Proceedings of the 13th International Conference on Autonomous Agents and Multi-agent Systems (AAMAS'14)*, 1553–1554.

Thomopoulos, Rallou, Madalina Croitoru, and Nouredine Tamani (2014), “Decision support for agri-food chains: A reverse engineering argumentation-based approach.” *Ecological Informatics*.