A weakening of independence in judgment aggregation: agenda separability

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Abstract. One of the better studied properties for operators in judgment aggregation is *independence*, which essentially dictates that the collective judgment on one issue should not depend on the individual judgments given on some other issue(s) in the same agenda. Independence is a desirable property for various reasons, but unfortunately it is too strong, as, together with mild additional conditions, it implies dictatorship. We propose here a weakening of independence, named *agenda separability* and show that this property is discriminant, *i.e.*, some judgment aggregation rules satisfy it, others do not.

1 Introduction

Judgment aggregation studies the problem of finding collective judgments that are representative of a collection of individual judgments on a given set of logically interrelated issues, the agenda. Judgment aggregation problems originate in the domains of political theory and public choice, however they also occur in various areas of artificial intelligence.

The main research focus of judgment aggregation is the development and analysis of judgement aggregation operators. One of the better studied properties for operators in judgment aggregation is the *independence* property, which essentially dictates that the collective judgment on one issue should not depend on the individual judgments given on some other issues in the same agenda. Independence is a desirable property for various reasons: it is a necessary condition for strategyproofness [2], and it leads to rules that are both conceptually simple and easy to compute. On the other hand, independence is too strong, as it is not consistent with other more desirable properties; in particular, together with mild additional conditions, it implies dictatorship [4]. Possible relaxations on independence have been proposed by Mongin [5]; however, this version of the property has not circumvented the problems we outlined. We propose here a weakening of independence, named *agenda separability*.

A judgment aggregation rule satisfies agenda separability if, whenever the agenda is composed of several sub-agendas that are syntactically unrelated to each other, the resulting collective judgment sets can be computed separately for each sub-agenda and then put together. This property is very intuitive and motivations for it can be easily found.

2 Preliminaries

Let $\mathcal{L}$ be a set of well-formed propositional logical formulas, including $\top$ (tautology) and $\bot$ (contradiction). Let $S \subset \mathcal{L}$. We define $\texttt{Atoms}(S)$ as the set of all propositional variables appearing in S. For example, $\texttt{Atoms}(\{p, q \wedge r, \neg s \rightarrow \neg\neg p\}) = \{p, q, r, s\}$.

An *issue* is a pair of formulas $\varphi, \neg\varphi$ where $\varphi \in \mathcal{L}$ and φ is neither a tautology nor a contradiction. An *agenda* $\mathcal{A}$ is a finite set of issues, and has the form $\mathcal{A} = \{\varphi_1, \neg\varphi_1, \ldots, \varphi_m, \neg\varphi_m\}$. The *preagenda* $[\mathcal{A}]$ associated with $\mathcal{A}$ is $[\mathcal{A}] = \{\varphi_1, \ldots, \varphi_m\}$. A *sub-agenda* is a subset of issues from $\mathcal{A}$. A *subpreagenda* is a subset of $[\mathcal{A}]$.

A *judgment* on $\varphi \in [\mathcal{A}]$ is one of φ or $\neg\varphi$. A *judgment set* J is a subset of $\mathcal{A}$. J is *complete* iff for each $\varphi \in [\mathcal{A}]$, either $\varphi \in J$ or $\neg\varphi \in J$. A judgment set J (and more generally, a set of propositional formulas) is *consistent* if and only if $J \not\models \bot$. Let $\mathcal{D}(\mathcal{A})$ be the set of all consistent judgment sets (for agenda $\mathcal{A}$) and $\mathbb{D}(\mathcal{A}) \subset \mathcal{D}(\mathcal{A})$ be the set of all judgment sets that are also *complete*.

A *profile* $P = \langle J_1, \ldots, J_n \rangle \in \mathbb{D}^n(\mathcal{A})$ is a collection of complete and consistent individual judgment sets. We further define $N(P, \varphi) = |\{i \mid \varphi \in J_i\}|$ to be the number of all agents in P whose judgment set includes φ.

A *resolute judgment aggregation rule*, for n voters, is a function $F : \mathbb{D}^n(\mathcal{A}) \rightarrow \mathbb{D}(\mathcal{A})$, *i.e.*, F maps a profile of complete and consistent judgment sets to a complete and consistent judgment set. An *irresolute judgment aggregation rule*, for n voters, is a function $R : \mathbb{D}^n \rightarrow 2^{\mathcal{D}} \setminus \{\emptyset\}$, *i.e.*, R maps a judgement profile to a nonempty set of consistent, but possibly incomplete, judgment sets.

We consider eight judgment aggregation rules: the definitions of $\mathrm{MSA}, \mathrm{MCSA}, \mathrm{MWA}, \mathrm{RA}, \mathrm{Y}, \mathrm{MNAC}$ can be found in the literature [3]. The distance-based rule $\mathrm{R}^{d_H,\mathrm{MAX}}$ is defined as $\mathrm{R}^{d_H,\mathrm{MAX}}(P) = \operatorname{argmin}_{J \in \mathbb{D}(\mathcal{A})} \max_{i=1}^{n} d_H(J_i, J)$. With the exception of $\mathrm{R}^{d_H,\mathrm{MAX}}$ and Y these rules also appear with different names in other work. We also consider the class of scoring rules R_S introduced by Dietrich [1]:
$R_S(P) = \operatorname{argmax}_{J \in \mathbb{D}(\mathcal{A})} \sum_{i \in [1,n]} \sum_{\varphi \in J \cap J_i} s(J_i, \varphi)$, where s is a *scoring function* $s : \mathbb{D}(\mathcal{A}) \times \mathcal{A} \rightarrow \mathbb{R}$ and $J_i \in P$.

All of the rules defined here are irresolute, but similarly as in voting theory, can be made resolute by composing them with a tie-breaking mechanism.

3 Relaxing independence

The most common of properties in judgment aggregation are universal domain, collective rationality, anonymity, unanimity preservation and various independence properties. The universal domain states that the aggregation rule is defined for all $\mathcal{A}$ and profiles from $\mathbb{D}^n(\mathcal{A})$, while the collective rationality stipulates that the collective outcome is a consistent judgment set. In our rules, both resolute and irresolute, these two properties are built into the definition of the rule itself. Anonymity is the requirement that the order of the judgment

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sets in the profile does not influence the collective judgment set selection. Unanimity preservation requires that if all agents support the same judgment on an issue, the unanimously supported judgment is selected as collective for that issue. There are various versions of the independence property, all defined only for the resolute judgment aggregation rules.

The first version of the independence requirement is called *systematicity* and it combines the *neutrality* requirement, that requires the individual judgments on each issue to be aggregated in the same manner across all issues, with the requirement that for every two profiles $P, P' \in \mathbb{D}^n(\mathcal{A})$, and every $\varphi \in \mathcal{A}$, if $P_{\downarrow\{\varphi,\neg\varphi\}} = P'_{\downarrow\{\varphi,\neg\varphi\}}$, then $\varphi \in F(P)$ iff $\varphi \in F(P')$. The relaxation of systematicity consists in dropping the neutrality requirement, and is known as *Independence of Irrelevant Alternatives*, or IIA. The IIA property is necessary for F to be non manipulable [2]. Mongin [5] proposed further relaxation, called *Independence of Irrelevant Propositional Alternatives*(IIPA). IIPA is the requirement that for every $P, P' \in \mathbb{D}^n(\mathcal{A})$, and every $\varphi \in \mathcal{A}$ *that is either an atom or a negation of an atom*, if $P_{\downarrow\{\varphi,\neg\varphi\}} = P'_{\downarrow\{\varphi,\neg\varphi\}}$, then $\varphi \in F(P)$ iff $\varphi \in F(P')$. However it can be shown [5] that IIPA, modulo some conditions on the agenda, is not consistent with the unanimity preservation requirement.

Rules that satisfy systematicity, anonymity and collective rationality, but do not satisfy universal domain, do exist, and one example of these are the quota rules [4]. However, without controlling the profile, the independence properties defined in the form of systematicity, IIA and IIPA, are unfortunately *too strong*. Under variations in the conditions put on the agenda, IIA, as well as systematicity, together with universal domain, anonymity and collective rationality imply that the rule is a dictatorship. This does not come as a surprise: given that judgment aggregation studies the aggregation of judgments on logically related issues, it is unintuitive to require that the aggregation of the judgments on one issue should be fully independent from judgments on other syntactically related issues. We address this problem by proposing an independence property that relaxes the requirement to account for the logic relations that may exist among issues. Moreover, our independence property is applicable not only to resolute rules but also to irresolute rules (unlike classical independence properties).

4 Agenda separability

Following the idea that only judgments on logically related issues should influence the collective judgment on each issue, we define agenda separability as the property requiring that when two agendas can be split into sub-agendas that are irrelevant to each other, the output judgment sets can be obtained by first applying the rule on each sub-agenda separately and then taking the pairwise unions of judgment sets from the two resulting sets.

Definition 1 (Agenda separability) *We say that rule R satisfies* agenda separability *if for all agendas $\mathcal{A}$, for all profiles $P \in \mathbb{D}^n(\mathcal{A})$, for all $\mathcal{A}_1, \mathcal{A}_2 \subseteq \mathcal{A}$, if $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$ and $\texttt{Atoms}(\mathcal{A}_1) \cap \texttt{Atoms}(\mathcal{A}_2) = \emptyset$, then $R(P) = \{J_1 \cup J_2 \mid J_1 \in R(P_{\downarrow\mathcal{A}_1})$ and $J_2 \in R(P_{\downarrow\mathcal{A}_2})\}$.*

Note that if R is a resolute rule, then the last line of the definition simplifies into $R(P) = R(P_{\downarrow\mathcal{A}_1}) \cup R(P_{\downarrow\mathcal{A}_2})$. Note also that by associativity of $\cup$, it immediately generalises to agendas that can be partitioned into a collection $\{\mathcal{A}_1, \ldots, \mathcal{A}_k\}$ such that $\texttt{Atoms}(\mathcal{A}_i) \cap \texttt{Atoms}(\mathcal{A}_j) = \emptyset$ for all i, j such that $i \neq j$. In that case, $R(P) = \{\cup_{i=1}^k J_i | J_1 \in R(P_{\downarrow\mathcal{A}_1}), \ldots, J_k \in R(P_{\downarrow\mathcal{A}_k})\}$.

Observe that the independence property is not applied on individual issues (as with IIA) but on sets of issues. Since IIA is usually defined for resolute rules, we show below that agenda separability restricted to resolute rules is a weakening of IIA. We omit the proofs due to space restrictions.

Proposition 1 *Any resolute judgment aggregation rule that satisfies IIA is agenda separable.*

As we shall see now, the reverse implication does not hold. We now show that there exist (resolute and irresolute) judgment aggregation rules that satisfy agenda separability (but not IIA).

Proposition 2

- MSA, MCSA, MWA, RA , MNAC, *and* R_S *(for every scoring function s) are agenda separable.*
- Y *and* $\mathrm{R}^{d_H,\mathrm{MAX}}$ *do not satisfy agenda separability.*

Note that the fact that the (irresolute) rules MSA, MCSA, MWA, RA, MNAC and R_S satisfy agenda separability immediately carries on to their resolute versions (obtained from the corresponding irresolute rule by a tie-breaking mechanism). Since they satisfy universal domain, anonymity and collective rationality, then they do not satisfy IIA [4]. Hence, the implication stated in Proposition 1 is strict.

5 Concluding remarks

When a rule satisfies agenda separability, not only it is conceptually simpler, but it also becomes computationally simpler when the agenda can be decomposed in an efficient way into a set of 'small' independent sub-agendas. Namely, let K be a constant and say that agenda $\mathcal{A}$ is K-decomposable if $\mathcal{A}$ can be partitioned into p syntactically unrelated agendas $\mathcal{A}_1, \ldots, \mathcal{A}_p$ such that for all i $|\texttt{Atoms}(\mathcal{A}_i)| \leq K$. Then, for most 'reasonable' rules (including all those considered here), agenda separability implies that the output can be computed in time $O(n.2^K)$ whenever the agenda is K-decomposable, that is, in polynomial time. Thus, MSA, MCSA, MWA, RA and MNAC are polynomial-time computable for K-decomposable agendas.

Agenda separability also offers a weak form of strategyproofness: if $\mathcal{A}$ can be partitioned into p syntactically unrelated agendas $\mathcal{A}_1, \ldots, \mathcal{A}_p$, then no voter is able to influence the outcome on some issue in $\mathcal{A}_i$ by reporting strategic judgments about issues of $\mathcal{A}_j$ for $j \neq i$.

Unlike other properties considered in judgment aggregation, agenda separability does not appear to have a natural counterpart in voting theory.

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