Revising option status in argument-based decision systems ⋆

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Abstract

Decision making is usually based on the comparative evaluation of different options by means of a decision criterion. Recently, the qualitative pessimistic criterion was articulated in terms of a four-step argumentation process: i) to build arguments in favor/against each option, ii) to compare and evaluate those arguments, iii) to assign a status for each option, and iv) to rank-order the options on the basis of their status. Thus, the argumentative counter-part of the pessimistic criterion provides not only the "best" option to the user but also the reasons justifying this recommendation.

The aim of this paper is to study the dynamics of this argumentation model. The idea is to study how the ordering on options changes in light of a new argument. For this purpose, we study under which conditions an option may change its status, and under which conditions the new argument has no impact on the status of options, and consequently, on the ordering. This amounts to study how the acceptability of arguments evolves when the decision system is extended by new arguments. In the paper, we focus on two acceptability semantics: the skeptical grounded semantics, and the credulous preferred semantics.

Key words: Argumentation, Decision making, Revision.

⋆ This paper extensively develops and extends the content of the conference paper [8]. It adds more results on revision under grounded semantics. The paper also explores revision under preferred semantics. Finally, it simplifies the results on revising complete decision systems.

1 Introduction

Decision making, often viewed as a form of reasoning toward action, has raised the interest of many scholars including economists, psychologists, and computer scientists for a long time. A decision problem amounts to selecting the "best" or sufficiently "good" option(s) among different alternatives, given some available information about the current state of the world and the benefits of options. Available information may be incomplete or pervaded with uncertainty. Besides, the goodness of an option is judged by estimating how much its possible benefits fit the preferences of the decision maker.

Decision making relies on the comparative evaluation of options on the basis of a decision criterion, which can be usually justified by means of a set of postulates. This is, for example, the Savage view of decision under uncertainty based on expected utility [26]. Thus, standard approaches for decision making consist in defining decision criteria in terms of analytical expressions that summarize the whole decision process. With such an approach, it is hard for a person who is not familiar with the abstract decision methodology, to understand why a proposed alternative is good, or better than another. It is thus important to have an approach in which one can better understand the underpinnings of the evaluation. Argumentation is the most appropriate way to advocate a choice thanks to its explanatory power.

Argumentation is a reasoning process in which interacting arguments are built and evaluated. An argument gives a reason for choosing an option, believing a statement, adopting a goal, etc. Argumentation is an important component of autonomous agents' reasoning, namely for handling inconsistency in knowledge bases (e.g. [2,10,16,27]), making decisions (e.g. [1,7,12,18,20,24]), or doing practical reasoning (e.g. [3,9,21]). Argumentation is also gaining increasing interest in multi-agent systems research community, in particular for modeling multi-agent interactions such as negotiation (e.g. [6,22,23]).

In a decision making context, argumentation has obvious benefits. Indeed, in everyday life, decision is often based on arguments and counter-arguments. Argumentation can also be useful for explaining a choice already made. Another great advantage of argumentation is that it is a powerful approach for handling inconsistency in knowledge bases. Thus, not only it can rank order options in a decision problem, but it can do that under inconsistent information. It is worth mentioning that most classical approaches for decision making assume that, in a decision problem, the available information is consistent.

Recently, in [5], the qualitative pessimistic decision criterion that was proposed by Dubois and Prade in [14], was articulated in terms of an argumentation process. The proposed model is an instantiation of Dung's abstract framework [15]. It takes as input a set of options, two sets of arguments (supporting respectively options and beliefs), and a defeat relation among them. The defeat relation is defined from conflicts between arguments and the intrinsic strengths of these arguments. The model evaluates the arguments using Dung's acceptability semantics [15], and assigns a status for each option on the basis of the result of this evaluation. Finally, the model rank-orders the options on the basis of their respective status. Thus, this argumentative counter-part of the pessimistic criterion provides not only the "best" option to the user but also the reasons justifying this recommendation.

In this paper, we are interested by this particular decision model. Our choice is mainly motivated by the fact that this model encodes a well-known decision criterion (i.e. pessimistic) in decision making research community. Indeed, this criterion was axiomatically justified in [19], which means that the ordering returned by this criterion obeys postulates that are supposedly meaningful.

The aim of the paper is to analyze the *dynamics* of this model. The idea is to study how the ordering on options changes in light of a new argument, and what is the impact of a new argument on the ordering without having to re-compute this latter. This issue is very important, especially in negotiation dialogues in which agents use argument-based decision making models for rank-ordering the possible values of the negotiation object, and for generating and evaluating arguments. From a strategical point of view, it is important for an agent to know what will be the impact of a given argument on the ordering of the receiving agent. This avoids sending useless arguments.
In the paper, we assume that the new argument concerns an option. This means that new information about an option is received. Moreover, the original set of options remains the same. Thus, the new argument is about an existing option. We investigate under which conditions this option changes its status, and under which conditions the new argument does not influence neither positively nor negatively the quality of this option. Similarly, we investigate the impact of the new argument on the status of other arguments. For that purpose, we study how the acceptability of arguments evolves when the decision system is extended by new arguments. We particularly focus on the skeptical grounded semantics, and the credulous preferred semantics.

This paper is organized as follows: Section 2 recalls briefly the decision model proposed in [5]. Section 3 studies the revision of option status when a new argument is received. In section 4 we study the revision of option status under some assumptions on the decision model. The last section concludes. All the proofs are put in an appendix at the end of the paper.

2 An argumentation framework for decision making

This section recalls briefly the argument-based system for decision making that has been proposed in [5]. Recall that this system is an argumentative counter-part of the qualitative pessimistic decision criterion proposed in [14] and axiomatized in [19]. We present also some new results on the impact of the choice of an acceptability semantics on the output of this system.

In what follows, $\mathcal{L}$ will denote a logical language. From $\mathcal{L}$, a finite set $\mathcal{O} = \{o_1, \ldots, o_n\}$ of n distinct *options* is identified; the decision maker has to choose only one of them. Note that an option o may be the conjunction of other options in $\mathcal{O}$. Let us consider the following example borrowed from [5].

> *Assume that Carla wants a drink and has to choose between tea, milk or both. Thus, there are three options: o_1: tea, o_2: milk, and o_3: tea and milk.*

Two kinds of arguments are distinguished: arguments supporting options, called *practical* arguments and gathered in a set $\mathcal{A}_o$, and arguments supporting beliefs, called *epistemic* arguments and gathered in a set $\mathcal{A}_b$, such that $\mathcal{A}_o \cap A_b = \emptyset$. The structure of these arguments is not specified in the paper. For instance, in [7], an epistemic argument involves beliefs while a practical argument involves beliefs and benefits/goals that may be reached/violated if the option supported by that argument is chosen. In [5], practical arguments are assumed to highlight positive features of their conclusions. This means that in order to encode the pessimistic criterion, only arguments in favor of options are required. In [7], it has been shown that the argumentative counterpart of the *optimistic* decision criterion proposed in [14] requires arguments against options. More complex decision criteria that take into account both types of arguments (in favor and against options) have been proposed in [7].

Practical arguments are linked to the options they support by a function $\mathcal{H}$ defined as follows:

$$\mathcal{H}\colon \mathcal{O} \to 2^{\mathcal{A}_o} \text{ such that } \forall i, j \text{ if } i \neq j \text{ then } \mathcal{H}(o_i) \cap \mathcal{H}(o_j) = \emptyset \text{ and } \mathcal{A}_o = \bigcup_{i=1}^{n} \mathcal{H}(o_i) \text{ with } \mathcal{O} = \{o_1, \ldots, o_n\}.$$

Each practical argument a supports only one option o. We say that o is the conclusion of the practical argument a, and we write $\texttt{Conc}(a) = o$. Note that there may exist options that do not have arguments in their favor (i.e. $\mathcal{H}(o) = \emptyset$).

Example 1 *Let $\mathcal{O} = \{o_1, o_2, o_3\}$, $\mathcal{A}_b = \{b_1, b_2, b_3\}$, and $\mathcal{A}_o = \{a_1, a_2, a_3\}$. The arguments supporting the three options are summarized in the table below.*

$\mathcal{H}(o_1)$	$= \{a_1\}$
$\mathcal{H}(o_2)$	$= \{a_2, a_3\}$
$\mathcal{H}(o_3)$	$= \emptyset$

Three preference relations between arguments are considered. They express the fact that some arguments may be stronger than others. The first preference relation, denoted by $\succeq_b$, is a partial preorder [1] on the set $\mathcal{A}_b$. In order to capture the pessimistic criterion, this relation should be based on the certainty degree of the information used in the arguments. The idea is that an argument which is built from more certain information is stronger than any argument based on less certain information. The second relation, denoted by $\succeq_o$, is a partial preorder on the set $\mathcal{A}_o$. It should be based both on the certainty degrees of the information involved in the arguments and on the importance of the benefits of the options (see [7] for formal definitions). Finally, a third preorder, denoted by $\succeq_m$ (m for *mixed* relation), captures the idea that any epistemic argument is stronger than any practical argument. The role of epistemic arguments in a decision problem is to validate or to undermine the beliefs on which practical arguments are built. Indeed, decisions should be made under certain information. Thus, $(\forall a \in \mathcal{A}_b)(\forall a' \in \mathcal{A}_o)\ (a, a') \in \succeq_m \wedge\ (a', a) \notin \succeq_m$. Note that $(a, a') \in \succeq_x$ (with $x \in \{b, o, m\}$) means that a is *at least as good as* a'. In what follows, $\succ_x$ denotes the strict relation associated with $\succeq_x$. It is defined as follows: $(a, a') \in \succ_x$ iff $(a, a') \in \succeq_x$ and $(a', a) \notin \succeq_x$.

Three conflict relations among arguments are also distinguished. The first one, denoted by $\mathcal{R}_b$, captures the conflicts that may hold between epistemic arguments. In [5], the structure of this relation is not specified. The second relation, denoted $\mathcal{R}_o$, captures the conflicts among practical arguments. Two practical arguments are conflicting if they support distinct options. This is mainly due to the fact that the options are mutually exclusive and competitive. Formally, for all $a, b \in \mathcal{A}_o$, $(a, b) \in \mathcal{R}_o$ iff $\texttt{Conc}(a) \neq \texttt{Conc}(b)$. Finally, practical arguments may be attacked by epistemic ones. The idea is that an epistemic argument may undermine the belief part of a practical argument. However, practical arguments are not allowed to attack epistemic ones. This avoids wishful thinking, i.e., avoids making decisions according to what might be pleasing to imagine instead of by appealing to evidence. This relation, denoted by $\mathcal{R}_m$, contains pairs (a, a') where $a \in \mathcal{A}_b$ and $a' \in \mathcal{A}_o$.

In [5], each conflict relation $\mathcal{R}_x$ (with $x \in \{b, o, m\}$) is combined with the preference relation $\succeq_x$ into a unique relation between arguments, called defeat and denoted by $\texttt{Def}_x$, as follows: For all $a, b \in \mathcal{A}_b \cup \mathcal{A}_o$, $(a, b) \in \texttt{Def}_x$ iff (a, b)

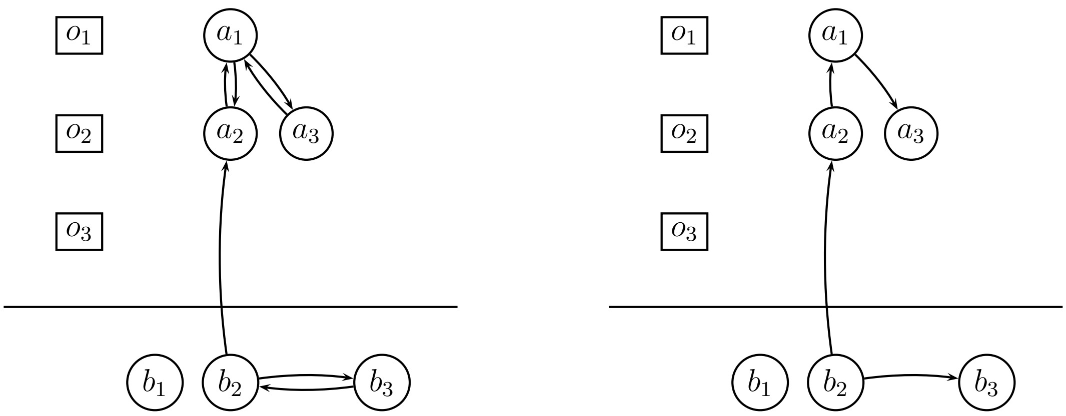
[1] Recall that a relation is a preorder iff it is *reflexive* and *transitive*.

$\in \mathcal{R}_x$ and $(b, a) \notin \succeq_x$. Let $\mathtt{Def}_b$, $\mathtt{Def}_o$ and $\mathtt{Def}_m$ denote the three defeat relations corresponding to the three conflict relations. Since arguments in favor of beliefs are always preferred (in the sense of $\succeq_m$) to arguments in favor of options, it holds that $\mathcal{R}_m = \mathtt{Def}_m$.

Throughout the paper, we use the following convention when depicting decision systems. Options, put in squares, are on the same line as their arguments. Epistemic arguments are separated from practical ones by a horizontal line.

Example 2 *(Example 1 cont.) The graph on the left of Fig. 1 depicts the conflicts (wrt $\mathcal{R}_x$) among arguments. Assume that $(b_2, b_3) \in\succeq_b$, $(a_2, a_1) \in\succeq_o$ and $(a_1, a_3) \in\succeq_o$. The graph of* Def *is depicted on the right of the same figure.*

Fig. 1.



Definition 1 *(Decision system) A* decision system *is a tuple $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o, \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m \rangle$.*

Let $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o$ and $\mathtt{Def} = \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m$. The arguments of $\mathcal{A}$ are evaluated using any acceptability semantics among the ones proposed by Dung in [15]. Recall that an acceptability semantics amounts to defining sets of arguments, called *extensions*, that satisfy two minimal requirements:

- *Conflict-freeness*: Each extension $\mathcal{B} \subseteq \mathcal{A}$ should be free of conflicts, i.e. there is no element of $\mathcal{B}$ that defeats another element of $\mathcal{B}$,
- *Self-defense*: Each extension $\mathcal{B} \subseteq \mathcal{A}$ should defend its elements, i.e. for every argument $a \in \mathcal{A}$, if a defeats (wrt $\mathtt{Def}$) an argument in $\mathcal{B}$, then there exists an argument in $\mathcal{B}$ that defeats a.

The main semantics introduced in [15] are recalled here. Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \mathtt{Def} \rangle$

be a decision system, and $\mathcal{B} \subseteq \mathcal{A}$ such that $\mathcal{B}$ is conflict-free.

- $\mathcal{B}$ is an *admissible extension* iff it defends all its elements.
- $\mathcal{B}$ is a *complete extension* iff it is admissible and contains all the arguments it defends.
- $\mathcal{B}$ is a *grounded extension* iff it is a minimal (wrt set $\subseteq$) complete extension.
- $\mathcal{B}$ is a *preferred extension* iff it is a maximal (wrt set $\subseteq$) admissible extension.
- $\mathcal{B}$ is a *stable extension* iff it defeats any argument in $\mathcal{A} \setminus \mathcal{B}$.

Using an acceptability semantics, the status of each argument can be defined.

Definition 2 *(Status of arguments) Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ be a decision system, and $(\mathcal{E}_i)_{i \in \mathcal{I}}$ its extensions under a given semantics. Let $a \in \mathcal{A}$.*

- *a is* skeptically accepted *iff $a \in \bigcap_{i \in \mathcal{I}} \mathcal{E}_i$.*
- *a is* credulously accepted *iff $\exists i, j \in \mathcal{I}$ s.t. $a \in \mathcal{E}_i$ and $a \notin \mathcal{E}_j$.*
- *a is* rejected *iff $\nexists i \in \mathcal{I}$ s.t. $a \in \mathcal{E}_i$.*

From the above definition, it follows that an argument is skeptically accepted iff it belongs to the intersection of all the extensions, while it is rejected iff it does not belong to their union. It is also worth mentioning that the definition of credulous arguments is not the usual one. In the literature, an argument is credulously accepted if it is supported by an argument in at least one of the extensions. Thus, each argument that is skeptically accepted is also credulously accepted. In [5], this definition was slightly modified. The reason is that in a decision making context, one looks for a preference relation on the set of options. Thus, it is important to distinguish between options that are supported by arguments in all the extensions, and those supported by arguments in only some extensions.

Example 3 *(Example 1 cont.) The decision system of Fig. 1 (graph on the right) has one preferred extension, which is also the grounded one, $\{a_1, b_1, b_2\}$. Thus, the three arguments a_1, b_1, and b_2 are skeptically accepted while a_2, a_3 and b_3 are rejected.*

Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ be a decision system. The functions $\texttt{Sc}(\mathcal{AF})$, $\texttt{Cr}(\mathcal{AF})$ and $\texttt{Rej}(\mathcal{AF})$ return respectively the sets of skeptically accepted arguments, credulously accepted arguments and rejected arguments. It is easy to show that these three sets are disjoint. Moreover, their union is the set $\mathcal{A}$ of arguments.

Property 1 *Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ be a decision system.*

(1) The sets $\texttt{Sc}(\mathcal{AF}), \texttt{Cr}(\mathcal{AF}), \texttt{Rej}(\mathcal{AF})$ are pairwise disjoint.
(2) $\texttt{Sc}(\mathcal{AF}) \cup \texttt{Cr}(\mathcal{AF}) \cup \texttt{Rej}(\mathcal{AF}) = \mathcal{A}$.

From the status of arguments, a status is assigned to each option of the set

$\mathcal{O}$. Four cases are distinguished: an option may be:

- *acceptable* if it is supported by at least one skeptically accepted argument,
- *negotiable* if it has no skeptically accepted arguments, but it is supported by at least one credulously accepted argument,
- *non-supported* if it is not supported at all by arguments,
- *rejected* if all its arguments are rejected.

Definition 3 *(Status of options) Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ be a decision system and $o \in \mathcal{O}$.*

- *o is* acceptable *iff $\exists a \in \mathcal{H}(o)$ such that $a \in \texttt{Sc}(\mathcal{AF})$.*
- *o is* negotiable *iff $\nexists a \in \mathcal{H}(o)$ s.t $a \in \texttt{Sc}(\mathcal{AF})$ and $\exists a' \in \mathcal{H}(o)$ s.t. $a' \in \texttt{Cr}(\mathcal{AF})$.*
- *o is* non-supported *iff $\mathcal{H}(o) = \emptyset$.*
- *o is* rejected *iff $\mathcal{H}(o) \neq \emptyset$ and $\forall a \in \mathcal{H}(o)$, $a \in \texttt{Rej}(\mathcal{AF})$.*

Let $\mathcal{O}^x_y(\mathcal{AF})$ denote the set of options having status y under semantics x in the decision system $\mathcal{AF}$. We assume that $x \in \{ad, p, g, s, c\}$, where ad stands for admissible, p for preferred, g for grounded, s for stable and c for complete. Similarly, $y \in \{a, n, ns, r\}$, where a stands for acceptable, n for negotiable, ns for non-supported and r for rejected. For example, $\mathcal{O}^p_a(\mathcal{AF})$ is the set of acceptable options under preferred semantics in the system $\mathcal{AF}$. When there is no ambiguity on the acceptability semantics, we will use the simplified notation $\mathcal{O}_y(\mathcal{AF})$.

Example 4 *(Example 1 cont.) Option o_1 is acceptable, o_2 is rejected and o_3 is non-supported.*

It can be checked that an option has only one status. This status may change in light of new arguments as we will see in next sections. The following property compares the sets of acceptable options under the different semantics proposed in [15]. As expected, since the empty set is an admissible extension for any argumentation system, then there is no acceptable option under this semantics. Consequently, this semantics is not interesting in our application. Due to the fact that the grounded extension is exactly the intersection of the complete extensions, the acceptable options are the same under the two semantics. The property shows also that there may be more acceptable options under preferred semantics than under grounded semantics. This result is not surprising as grounded semantics is skeptical while preferred semantics is credulous. Finally, when stable extensions exist, this semantics may accept more acceptable options than any other semantics.

Property 2 *Let $\mathcal{AF}$ be a decision system.*

- $\mathcal{O}^{ad}_a(\mathcal{AF}) = \emptyset$

- $\mathcal{O}_a^g(\mathcal{AF}) = \mathcal{O}_a^c(\mathcal{AF}) \subseteq \mathcal{O}_a^p(\mathcal{AF})$
- *If $\mathcal{AF}$ has no stable extension, then $\mathcal{O}_a^s(\mathcal{AF}) = \emptyset$*
- *If $\mathcal{AF}$ has at least one stable extension, then $\mathcal{O}_a^p(\mathcal{AF}) \subseteq \mathcal{O}_a^s(\mathcal{AF})$*

Preferred semantics is the most suitable acceptability semantics since preferred extensions always exist. Moreover, it guarantees more acceptable options than the other semantics. Grounded semantics is the most skeptical one, it accepts less options than the other semantics. In the remainder of this document, we will focus only on these two semantics.

The last output of the decision proposed in [5] is a complete pre-ordering on the set $\mathcal{O}$. Indeed, it has been argued that an acceptable option is preferred to any negotiable option. A negotiable option is preferred to a non-supported one, which is itself preferred to a rejected option.

3 Revising decision systems

Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o, \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m \rangle$ be a decision system. The function $\mathcal{H}$ relates options of $\mathcal{O}$ with the arguments that support them (i.e. $\mathcal{H} : \mathcal{O} \longmapsto 2^{\mathcal{A}_o}$).

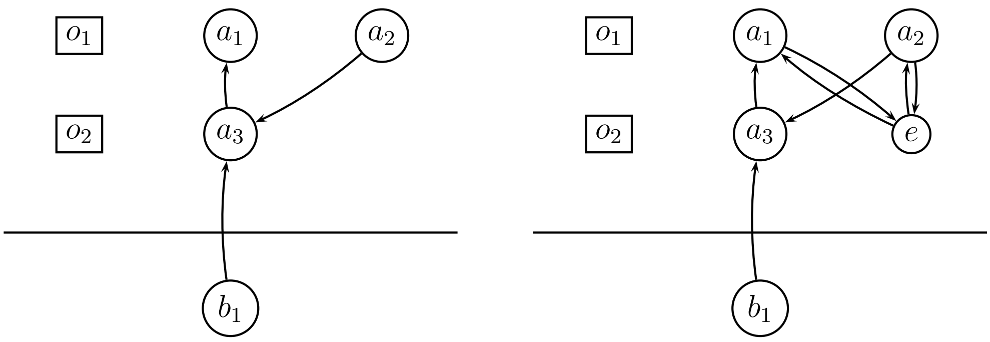
Assume that a new argument, say e, is received (for instance, from another agent). Thus, the decision system $\mathcal{AF}$ is extended by this argument and by new defeats. Let $\mathcal{AF} \oplus e = \langle \mathcal{O}', \mathcal{A}', \mathtt{Def}' \rangle$ denote the new system. It is clear that when $e \in \mathcal{A}$, then $\mathcal{O}' = \mathcal{O}$, $\mathcal{A}' = \mathcal{A}$ and $\mathtt{Def}' = \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m$. The more interesting case is when $e \notin \mathcal{A}$, thus $\mathcal{A}' = \mathcal{A} \cup \{e\}$. In this paper, we assume that the argument e is practical, meaning that it supports an option. Moreover, we assume this option is already in the set $\mathcal{O}$. Thus, $\mathcal{O}' = \mathcal{O}$ and $\exists o \in \mathcal{O}$ such that $\mathtt{Conc}(e) = o$.
Regarding the relation $\mathtt{Def}'$, it contains all the elements of $\mathtt{Def}$, all the defeats between e and the arguments of $\mathcal{A}_o$ that support other options than $\mathtt{Conc}(e)$, and all the defeats emanating from epistemic arguments in $\mathcal{A}_b$ towards the argument e. Recall that practical arguments are not allowed to attack an epistemic one. The question now is how to recognize an attack from an epistemic argument towards e? This is done by checking the formal definition of the attack relation that is used. For instance, if $\mathcal{R}_m$ is the well-known assumption attack defined in [17], then an argument $x \in \mathcal{A}_b$ attacks e if the conclusion of x undermines a premise in e. For our purpose, we assume that $\mathcal{R}_m^{\mathcal{L}}$ contains all the conflicts that may exist between all the epistemic arguments and the practical arguments that may be built from the logical language $\mathcal{L}$. Thus, $\mathcal{R}_m \subseteq \mathcal{R}_m^{\mathcal{L}}$.
Defeats between practical arguments of $\mathcal{A}_o$ and the new argument e are based

on i) the conflicts between arguments, and these capture the idea that two arguments support different options, and ii) a preference relation between the arguments. The new argument needs then to be compared to the other arguments of $\mathcal{A}_o$. The question is how this can be done? Here again by applying the formal definition of the preference relation that is used in the decision system. For instance, if $\succeq_o$ privileges the argument that is based on most certain information and most important benefit, then the new argument e is compared to any argument in $\mathcal{A}_o$ using these criteria. At an abstract level, we assume that this is captured by a new preference relation, denoted by $\succeq'_o$, on the set $\mathcal{A}'_o$. The definition of $\mathtt{Def}'$ of the extended system $\mathcal{AF} \oplus e$ is summarized below.

$$\mathtt{Def}' = \mathtt{Def} \cup \{(x,e) | x \in \mathcal{A}_b \text{ and } (x,e) \in \mathcal{R}^{\mathcal{L}}_m\} \cup$$
$$\{(e,y) | y \in \mathcal{A}_o \text{ and } \mathtt{Conc}(y) \neq \mathtt{Conc}(e) \text{ and } (y,e) \notin \succeq'_o\} \cup$$
$$\{(y,e) | y \in \mathcal{A}_o \text{ and } \mathtt{Conc}(y) \neq \mathtt{Conc}(e) \text{ and } (e,y) \notin \succeq'_o\}.$$

Extending a decision system by a new argument may have an impact on the output of the original system, namely on the status of the arguments, the status of options, and on the ordering on options. This is illustrated by the following example.

Fig. 2.



Example 5 *Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o, \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m \rangle$ be a decision system such that $\mathcal{O} = \{o_1, o_2\}$, $\mathcal{A}_o = \{a_1, a_2, a_3\}$, $\mathcal{A}_b = \{b_1\}$, $\mathcal{H}(o_1) = \{a_1, a_2\}$, $\mathcal{H}(o_2) = \{a_3\}$, $\mathcal{R}_b = \emptyset$, and $\mathcal{R}_m = \{(b_1, a_3)\}$. Assume that $(a_3, a_1) \in \succeq_o$ and $(a_2, a_3) \in \succeq_o$. The graph of defeat is depicted on the left side of Fig. 2. It can be checked that the grounded extension of this system is $\mathtt{GE} = \{a_1, a_2, b_1\}$. Thus, $\mathtt{Sc}(\mathcal{AF}) = \{a_1, a_2, b_1\}$ and $\mathtt{Rej}(\mathcal{AF}) = \{a_3\}$. Consequently, the option o_1 is acceptable while o_2 is rejected, and o_1 is strictly preferred to o_2.*

Assume now that the system is extended by a new practical argument e in favor of option o_2 (i.e. $\mathtt{Conc}(e) = o_2$), and that this argument is incomparable with the other practical arguments. The new graph of defeat is depicted on the right side of Fig. 2. The grounded extension of the extended system is $\mathtt{GE} = \{b_1\}$. Thus, $\mathtt{Sc}(\mathcal{AF} \oplus e) = \{b_1\}$ and $\mathtt{Rej}(\mathcal{AF} \oplus e) = \{a_1, a_2, a_3, e\}$. Consequently, *the two options o_1 and o_2 are rejected, and are thus equally preferred.*

The aim of this section is to study the impact of a new practical argument e on the result of a decision system. In particular, we show under which conditions:

(1) an accepted argument in $\mathcal{AF}$ remains accepted (resp. becomes rejected) in $\mathcal{AF} \oplus e$. In other words, we show which arguments are strengthened by the new argument, and which arguments are rather weakened.
(2) an option in $\mathcal{O}_x(\mathcal{AF})$ moves to $\mathcal{O}_y(\mathcal{AF} \oplus e)$ with $x \neq y$

The study is undertaken under two acceptability semantics: the skeptical grounded semantics and the credulous preferred semantics.

3.1 Revision under grounded semantics

Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o, \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m \rangle$ be a decision system, and $\mathcal{AF} \oplus e$ $= \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o \cup \{e\}, \mathtt{Def}_b \cup \mathtt{Def}'_o \cup \mathtt{Def}'_m \rangle$ its extension by a practical argument e. In this section, we assume that arguments in both $\mathcal{AF}$ and $\mathcal{AF} \oplus e$ are evaluated under grounded semantics. In this case, an argument is either skeptically accepted or rejected. The set of credulously accepted arguments is empty since there exists exactly one extension under this semantics. Consequently, an option may be either non-supported, or acceptable or rejected.

The following property shows that a new practical argument will never influence the status of existing epistemic arguments. This means that the status of any epistemic argument in the system $\mathcal{AF}$ remains the same in $\mathcal{AF} \oplus e$. This is mainly due to the fact that practical arguments are not allowed to attack epistemic ones. Let $\mathtt{Status}(a, \mathcal{AF})$ [2] be the function that returns the status of an argument a in the decision system $\mathcal{AF}$.

Proposition 1 *Let $\mathcal{AF}$ be a decision system. For all $a \in \mathcal{A}_b$, $\mathtt{Status}(a, \mathcal{AF})$ $= \mathtt{Status}(a, \mathcal{AF} \oplus e)$.*

Example 5 shows that this result is not always true for the practical arguments of the set $\mathcal{A}_o$. However, it holds in case the new argument is defeated by a skeptically accepted epistemic argument. In this case, the argument e has clearly no impact on the results of the original system $\mathcal{AF}$.

Proposition 2 *Let $\mathcal{AF}$ be a decision system. If $\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ such that $(a, e) \in \mathtt{Def}'_m$, then*

- *$e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$,*
- *for all $a \in \mathcal{A}_o$, $\mathtt{Status}(a, \mathcal{AF}) = \mathtt{Status}(a, \mathcal{AF} \oplus e)$.*

[2] Recall that $\mathtt{Status}(a, \mathcal{AF}) \in$ {skeptically accepted, rejected}.

In case the new argument e is not defeated by an accepted epistemic argument, we show that the status of practical arguments in $\mathcal{A}_o$ which are in favor of $\mathtt{Conc}(e)$ may either be the same as in the original system or improved, moving thus from a rejection to an acceptance. However, things are different with the practical arguments that support other options than $\mathtt{Conc}(e)$. Indeed, the status of these arguments may either remain the same or be worsened. This means that the new argument can *improve* only the status of the other arguments supporting its own option.

Proposition 3 *Let $\mathcal{AF}$ be a decision system.*

- *For all $a \in \mathcal{H}(\mathtt{Conc}(e))$, if $a \in \mathtt{Sc}(\mathcal{AF})$ then $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.*
- *For all $a \in \mathcal{A}_o$, if $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$, then $e \in \mathcal{H}(\mathtt{Conc}(a))$.*

The following table summarizes the different situations that may hold when a decision system $\mathcal{AF}$ is extended by a new practical argument e. Before analyzing the table, let us first introduce some notations. The symbol $\times$ means that the status of the argument does not change in the new system, the symbol $-$ denotes the fact the argument moves from an acceptance to a rejection, while $+$ means that the status of the argument is improved (i.e. the argument moves from a rejection to an acceptance).

$a \in \mathcal{A}_o$ s.t. $\mathtt{Conc}(a) = \mathtt{Conc}(e)$	$\times$	$\times$	$+$	$+$
$a' \in \mathcal{A}_o$ s.t. $\mathtt{Conc}(a') \neq \mathtt{Conc}(e)$	$\times$	$-$	$\times$	$-$

There are four possible situations (corresponding to the four columns of the table). In the first situation, both arguments supporting $\mathtt{Conc}(e)$ and those supporting the other options keep their original status. In the second situation, the arguments in favor of $\mathtt{Conc}(e)$ do not change their status while the arguments supporting the other options are weakened. In the two remaining situations, the argument in favor of $\mathtt{Conc}(e)$ improve their status while the arguments supporting the other options either do not change their status or are weakened.

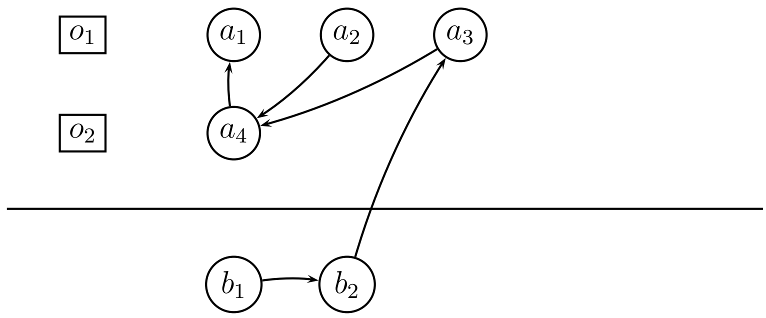
The previous results make it possible to characterize the situations in which the status of an option is revised. Before continuing our analysis, we present a definition which is useful for the remainder of the section. Let us define the set of arguments defended by epistemic arguments in $\mathcal{AF}$.

Definition 4 *Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o, \mathtt{Def} \rangle$ be a decision system and $a \in \mathcal{A}$. We say that a is defended by epistemic arguments in $\mathcal{AF}$, and we write $a \in \mathtt{Dbe}(\mathcal{AF})$, iff $\forall x \in \mathcal{A}$, if $(x, a) \in \mathtt{Def}$ then $\exists b \in \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b$ such that $(b, x) \in \mathtt{Def}$.*

Let us illustrate this definition through the following example.

Example 6 *Let $\mathcal{AF}$ be a decision system such that $\mathcal{O} = \{o_1, o_2\}$, $\mathcal{A}_b = \{b_1, b_2\}$, $\mathcal{A}_o = \{a_1, a_2, a_3, a_4\}$, $\mathcal{H}(o_1) = \{a_1, a_2, a_3\}$ and $\mathcal{H}(o_2) = \{a_4\}$. The defeat relations* $\texttt{Def}_x$ *are depicted in Fig. 3 below.*

Fig. 3.



The grounded extension of this system is $\texttt{GE} = \texttt{Sc}(\mathcal{AF}) = \{a_1, a_2, a_3, b_1\}$. *It can be checked that* $\texttt{Dbe}(\mathcal{AF}) = \{b_1, a_2, a_3\}$. *Note that* $a_1 \notin \texttt{Dbe}(\mathcal{AF})$ *even if it is indirectly defended by argument* b_1. *In fact, definition of* $\texttt{Dbe}$ *uses only direct defense.*

It is worth noticing that non-defeated arguments are defended by epistemic arguments. Consequently, the arguments that are defended by epistemic arguments are skeptically accepted.

Let us now come back to the status of options. Recall that, under grounded semantics, an option may be either acceptable, or rejected or non-supported. We are interested by i) the case where an option is rejected in the system $\mathcal{AF}$ and becomes acceptable in $\mathcal{AF} \oplus e$, and ii) the case where an option is acceptable in $\mathcal{AF}$ and becomes rejected in $\mathcal{AF} \oplus e$. From the previous results, it is clear that the first case holds only for the option that is supported by the new argument. Indeed, the new argument may improve the status of its own conclusion. However, it never improves the status of the other options in the system. This is formally shown by the following result.

Proposition 4 *Let $\mathcal{AF}$ be a decision system and $o \in \mathcal{O}_r(\mathcal{AF})$. It holds that $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ iff $e \in \mathcal{H}(o)$ and* $e \in \texttt{Sc}(\mathcal{AF} \oplus e)$.

Note that the above result depends on the status of the new argument in the extended system. The following result characterizes when this argument is skeptically accepted in $\mathcal{AF} \oplus e$ without computing the grounded extension of this system.The idea is that for every attack from an argument $x \in \mathcal{A}_b \cup \mathcal{A}_o$ to e, there exists an argument a which either supports $\texttt{Conc}(e)$ or is epistemic, such that a is in the grounded extension of $\mathcal{AF}$ and that it defeats x.

Proposition 5 *Let* $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ *be a decision system, and* $\mathcal{AF} \oplus e = \langle \mathcal{O}, \mathcal{A}', \texttt{Def}' \rangle$ *its extension with argument* e*. It holds that* $e \in \texttt{Sc}(\mathcal{AF} \oplus e)$ *iff for all* $a \in \mathcal{A}$*, if* $(a, e) \in \texttt{Def}'$*, then* $\exists b \in \texttt{Sc}(\mathcal{AF}) \cap (\mathcal{A}_b \cup \mathcal{H}(\texttt{Conc}(e)))$ *s.t.* $(b, a) \in \texttt{Def}$*.*

Let us now analyze the case where an option is acceptable in $\mathcal{AF}$ and becomes rejected in $\mathcal{AF} \oplus e$. This case concerns only the options that are not supported by the new argument e. Indeed, since practical arguments supporting other options than $\texttt{Conc}(e)$ may be weakened by the new argument, then their conclusions may be weakened as well. The following result shows the conditions under which this is possible.

Proposition 6 *Let* $\mathcal{AF}$ *be a decision system and* $o \in \mathcal{O}_a(\mathcal{AF})$*. It holds that* $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ *iff*

(1) $e \notin \mathcal{H}(o)$*, and*
(2) $\nexists a \in \mathcal{A}_b \cap \texttt{Sc}(\mathcal{AF})$ *s.t.* $(a, e) \in \texttt{Def}'_m$*, and*
(3) $\forall a \in \texttt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o)$*,* $(e, a) \in \texttt{Def}'_o$*.*

The first condition says that the new argument should not support the rejected option. The second condition states that the new argument should not be defeated by an epistemic argument which is accepted in the original decision system $\mathcal{AF}$. This is important because otherwise the new argument is rejected in $\mathcal{AF} \oplus e$ and has thus no impact on the results of $\mathcal{AF}$. The last condition says that all the practical arguments that defend the arguments supporting the rejected option are themselves defeated by the new argument.

3.2 Revision under preferred semantics

In this section, the arguments of a decision system $\mathcal{AF}$ and those of its extension $\mathcal{AF} \oplus e$ are evaluated under preferred semantics. Thus, an argument may be either skeptically accepted, or credulously accepted or rejected. Consequently, an option may have one of the corresponding statuses (acceptable, negotiable or rejected).

Like in the case of grounded semantics, epistemic arguments will not change their status when a new practical argument is received. This shows that the system is protected against wishful thinking.

Proposition 7 *Let* $\mathcal{AF}$ *be a decision system. For all* $a \in \mathcal{A}_b$*,* $\texttt{Status}(a, \mathcal{AF}) = \texttt{Status}(a, \mathcal{AF} \oplus e)$*.*

We now prove that if the new practical argument is attacked by a skeptically accepted epistemic argument in $\mathcal{AF}$, then the preferred extensions of $\mathcal{AF}$

and $\mathcal{AF} \oplus e$ coincide. As a consequence, all the existing arguments keep their status. Moreover, the new argument e is rejected. This means that such an argument does not influence the output of the decision system.

Proposition 8 *Let $\mathcal{AF}$ be a decision system. If $\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ such that $(a, e) \in \mathtt{Def}'_m$, then*

- *$\mathcal{E} \subseteq \mathcal{A}_b \cup \mathcal{A}_o$ is a preferred extension of $\mathcal{AF}$ iff $\mathcal{E}$ is a preferred extension of $\mathcal{AF} \oplus e$,*
- *$e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$,*
- *for all $a \in \mathcal{A}_o$, $\mathtt{Status}(\mathcal{AF}, a) = \mathtt{Status}(\mathcal{AF} \oplus e, a)$.*

Like with grounded semantics, the status of the arguments supporting $\mathtt{Conc}(e)$ in $\mathcal{AF}$ can be improved and can never be weakened in $\mathcal{AF} \oplus e$.

Proposition 9 *Let $\mathcal{AF}$ be a decision system. For all $a \in \mathcal{A}_o$ such that $\mathtt{Conc}(a) = \mathtt{Conc}(e)$, it holds that:*

- *If $a \in \mathtt{Sc}(\mathcal{AF})$ then $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$*
- *If $a \in \mathtt{Cr}(\mathcal{AF})$ then $a \in \mathtt{Sc}(\mathcal{AF} \oplus e) \cup \mathtt{Cr}(\mathcal{AF} \oplus e)$*

Things are different for the arguments supporting other options than $\mathtt{Conc}(e)$ in $\mathcal{AF}$. Indeed, again like under grounded semantics, the status of these arguments cannot be improved in $\mathcal{AF} \oplus e$.

Proposition 10 *Let $\mathcal{AF}$ be a decision system, and $a \in \mathcal{A}_o$. If $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \in \mathtt{Sc}(\mathcal{AF} \oplus e) \cup \mathtt{Cr}(\mathcal{AF} \oplus e)$ then $\mathtt{Conc}(a) = \mathtt{Conc}(e)$.*

From the above results on the status of arguments, we can show under which conditions a given option may change its status in the extended decision system $\mathcal{AF} \oplus e$. We have seen that the quality of the arguments of $\mathcal{A}_o$ that support $\mathtt{Conc}(e)$ may be improved. Thus, it is expected that the status of $\mathtt{Conc}(e)$ may be improved as well. The following result shows, in particular, when $\mathtt{Conc}(e)$ moves from a rejection to a better status (i.e. becomes either negotiable or acceptable).

Proposition 11 *Let $\mathcal{AF}$ be a decision system and $o \in \mathcal{O}_r(\mathcal{AF})$. Then $o \in \mathcal{O}_a(\mathcal{AF} \oplus e) \cup \mathcal{O}_n(\mathcal{AF} \oplus e)$ iff $e \in \mathcal{H}(o) \wedge e \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$.*

The above result shows also that the status of the remaining options cannot be improved. This is due to the fact that the quality of their arguments cannot be improved in the extended system $\mathcal{AF} \oplus e$. The following result characterizes when e is not rejected in the extended system $\mathcal{AF} \oplus e$.

Proposition 12 *Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o, \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m \rangle$ be a decision system. It holds that $e \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$ iff $\exists \mathcal{E} \subseteq \mathcal{A}_b$ and $\exists \mathcal{E}' \subseteq \mathcal{H}(\mathtt{Conc}(e))$*

such that:

(1) $\mathcal{E} \cup \mathcal{E}'$ *is conflict-free, and*
(2) $\mathcal{E}$ *is a preferred extension of the argumentation system* $\langle \mathcal{A}_b, \texttt{Def}_b \rangle$*, and*
(3) $\forall a \in \mathcal{E}' \cup \{e\}$*, if* $\exists x \in \mathcal{A}_b \cup \mathcal{A}_o$ *s.t.* $(x, a) \in \texttt{Def}_o \cup \texttt{Def}_m$*, then* $\exists a' \in \mathcal{E} \cup \mathcal{E}' \cup \{e\}$ *s.t.* $(a', x) \in \texttt{Def}_b \cup \texttt{Def}_o \cup \texttt{Def}_m$.

We have previously shown that the status of arguments that do not support $\texttt{Conc}(e)$ may be weakened. It may thus be possible that the status of their conclusions may be weakened as well. The following result summarizes under which conditions an option may become rejected in the extended decision system.

Proposition 13 *Let* $\mathcal{AF}$ *be a decision system and* $o \in \mathcal{O}_a(\mathcal{AF}) \cup \mathcal{O}_n(\mathcal{AF})$*. Then* $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ *iff*

(1) $e \notin \mathcal{H}(o)$*, and*
(2) there does not exist a preferred extension $\mathcal{E}$ *of* $\mathcal{AF}$ *s.t.* $\mathcal{E} \cap \mathcal{H}(o) \neq \emptyset$ *and* $\exists a \in \mathcal{E} \cap \mathcal{A}_b$ *s.t.* $(a, e) \in \texttt{Def}'_m$*, and*
(3) there does not exist a preferred extension $\mathcal{E}$ *of* $\mathcal{AF}$ *s.t. there exists an admissible set* $\mathcal{E}''$ *of* $\mathcal{AF}$ *with* $\mathcal{E}'' \cap \mathcal{A}_o \subseteq \mathcal{E} \cap \mathcal{H}(o)$ *and* $\mathcal{E}'' \cap \mathcal{A}_b = \mathcal{E} \cap \mathcal{A}_b$ *and* $\forall a \in \mathcal{E}'' \cap \mathcal{H}(o), (a, e) \in \succeq'_o$ *or* $\exists a' \in \mathcal{E}'' \cap \mathcal{H}(o)$ *s.t.* $(e, a) \notin \succeq'_o$.

The first condition says that for an option to become rejected, it should not be supported by the new argument. The second condition says that the new argument should not be attacked by an epistemic argument which is in a preferred extension that contains arguments in favor of this option. The last condition claims that the new argument should be preferred to some arguments in favor of the option.

4 Complete decision systems

In [4], another variant of the decision system presented in Section 2 has been defined. This variant is used by agents for selecting the "best" offers to utter during a negotiation, and also the "strongest" argument. For that purpose, it was assumed that not only arguments in favor of distinct options are conflicting, but also those supporting the same option. This particular class of decision systems is called "complete" systems. The conflict relation $\mathcal{R}_o$ is thus defined as follows: $\forall a, a' \in \mathcal{A}_o$, if $a \neq a'$, then $(a, a') \in \mathcal{R}_o$.

In this section, we assume that the set $\mathcal{A}_b$ of epistemic arguments is empty. Note that this assumption is not too strong since epistemic arguments are only useful for handling inconsistency of the available information, and as already explained in the introduction, most classical approaches for decision making focus only on the decision process and assume that the information is consistent. The decision system that we will study is thus $\mathcal{AF}_o = \langle \mathcal{O}, \mathcal{A}_o, \mathtt{Def}_o \rangle$, and arguments of $\mathcal{A}_o$ are evaluated under preferred semantics.

Before studying how such systems are revised, let us first present some interesting properties. The first one characterizes the three possible statuses of arguments without referring to the extensions. Indeed, an argument is skeptically accepted if it is preferred wrt $\succeq_o$ to the other arguments. It is rejected if there exists at least one argument which is strictly preferred to it. Finally, an argument is credulously accepted if it is not preferred to all the other arguments, but there is no argument which is strictly preferred to it. Formally:

Property 3 *Let $\mathcal{AF}_o$ be a complete decision system, and $a \in \mathcal{A}_o$.*

- *a is skeptically accepted iff $\forall x \in \mathcal{A}_o$, $(a, x) \in \succeq_o$.*
- *a is rejected iff $\exists x \in \mathcal{A}_o$ s.t. $(x, a) \in \succ_o$.*
- *a is credulously accepted iff $\exists x' \in \mathcal{A}_o$ s.t. $(a, x') \notin \succeq_o$, and $\forall x \in \mathcal{A}_o$, if $(a, x) \notin \succeq_o$, then $(x, a) \notin \succeq_o$.*

It can be checked that the skeptically accepted arguments of a complete decision system are indifferent wrt the preference relation $\succeq_o$.

Property 4 *Let $\mathcal{AF}_o$ be a complete decision system. For all $a, b \in \mathtt{Sc}(\mathcal{AF}_o)$, it holds that $(a, b) \in \succeq_o$ and $(b, a) \in \succeq_o$.*

Similarly, we show that the credulously accepted arguments are either incomparable or indifferent with respect to $\succeq_o$.

Property 5 *Let $\mathcal{AF}_o$ be a complete decision system. For all $a, b \in \mathtt{Cr}(\mathcal{AF}_o)$, it holds that $(a, b) \in \succeq_o$ and $(b, a) \in \succeq_o$, or $(a, b) \notin \succeq_o$ and $(b, a) \notin \succeq_o$.*

We will now prove that in this particular system, there are two possible cases: the case where there exists at least one skeptically accepted argument but there are no credulously accepted arguments, and the case where there are no skeptically accepted arguments but there is "at least" one credulously accepted argument. This means that one cannot have a state with both skeptically accepted and credulously accepted arguments. Moreover, it cannot be the case that all the arguments are rejected.

Property 6 *Let $\mathcal{AF}_o$ be a complete decision system.*

(1) If $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$ then $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$.
(2) If $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$ then $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$.

As a consequence of the above properties, the following result shows that negotiable options and acceptable ones cannot exist at the same time.

Property 7 *Let $\mathcal{AF}_o$ be a complete decision system. It holds that $\mathcal{O}_a(\mathcal{AF}_o) \neq \emptyset \Leftrightarrow \mathcal{O}_n(\mathcal{AF}_o) = \emptyset$.*

4.1 Revising the status of arguments

Like in the previous section, we assume that a complete decision system $\mathcal{AF}_o = \langle \mathcal{O}, \mathcal{A}_o, \texttt{Def}_o \rangle$ is extended by a new practical argument e with $\texttt{Conc}(e) \in \mathcal{O}$. Thus, $\mathcal{AF}_o \oplus e = \langle \mathcal{O}, \mathcal{A}_o \cup \{e\}, \texttt{Def}'_o \rangle$ where $\texttt{Def}'_o$ is a combination of $\mathcal{R}'_o$ and a preference relation $\succeq'_o$ between the arguments. The relation $\mathcal{R}'_o = \mathcal{R}_o \cup \{(x, e), (e, x) | x \in \mathcal{A}_o\}$ and $\succeq'_o$ is the relation $\succeq_o$ extended by the pairs comparing the new argument e to the elements of the set $\mathcal{A}_o$. Throughout this section, we will characterize the status of the arguments and the options in the extended system without computing its preferred extensions.

Let us start by characterizing the status of the new argument e in $\mathcal{AF} \oplus e$. From Property 3, it is easy to guess the status of e. For instance, it is skeptically accepted if is preferred wrt $\succeq'_o$ to any argument in $\mathcal{A}_o$. It is rejected if it is strictly weaker wrt $\succeq'_o$ to at least one argument in $\mathcal{A}_o$.

Regarding the existing practical arguments, the following result summarizes under which condition a skeptically accepted argument moves to another status. Indeed, the argument remains skeptically accepted in $\mathcal{AF} \oplus e$ if is preferred wrt $\succeq'_o$ to the new argument. It becomes rejected if it is weaker than e. Finally, it becomes credulously accepted if it is incomparable with the new argument. It is clear that the status of such arguments can be weakened.

Proposition 14 *Let $\mathcal{AF}_o$ be a complete decision system. Let $a \in \mathcal{A}_o$ such that $a \in \texttt{Sc}(\mathcal{AF}_o)$.*

- *$a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in \succeq'_o$.*
- *$a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in \succ'_o$.*
- *$a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \notin \succeq'_o$ and $(e, a) \notin \succeq'_o$.*

Similarly, the following result characterizes when a credulously accepted argument shifts to another status. We show that such an argument can never become skeptically accepted. However, depending on whether it is weaker or not than the new argument, it can either become rejected or remain credulously accepted.

Proposition 15 *Let $\mathcal{AF}_o$ be a complete decision system. Let $a \in \mathcal{A}_o$ such* *that* $a \in \texttt{Cr}(\mathcal{AF}_o)$.

- $a \in \texttt{Cr}(\mathcal{AF}_o) \wedge a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ *is not possible.*
- $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$ *iff* $(e, a) \notin \succ'_o$.
- $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ *iff* $(e, a) \in \succ'_o$.

Finally, we show that any rejected argument in $\mathcal{AF}_o$ remains rejected in the system $\mathcal{AF}_o \oplus e$. This means that such arguments cannot be saved.

Proposition 16 *Let* $\mathcal{AF}_o$ *be a complete decision system. If* $a \in \texttt{Rej}(\mathcal{AF}_o)$, *then* $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.

4.2 Revising the status of options

We will now show under which conditions an option can change its status. We start by studying acceptable options. The next result shows that an acceptable option o may remain acceptable if either existing skeptically accepted arguments are preferred to the new argument e (in which case e induces no change) or e is in favor of o and it is strictly preferred to existing skeptically accepted arguments (in which case e will be skeptically accepted in the new system). It can become negotiable if it is not possible to compare existing skeptically accepted arguments in favor of o with the new argument. Informally, the fact that they are incomparable implies that there will be several extensions, thus arguments will be credulously accepted. Finally, it may become rejected if e is strictly stronger than existing arguments since they will become rejected.

Proposition 17 *Let* $\mathcal{AF}_o$ *be a complete decision system, and* $o \in \mathcal{O}_a(\mathcal{AF}_o)$.

- $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff* $(\forall a \in \texttt{Sc}(\mathcal{AF}_o))\ ((a, e) \in \succeq'_o) \vee (e \in \mathcal{H}(o)) \wedge ((e, a) \in \succ'_o)$
- $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff* $(\forall a \in \texttt{Sc}(\mathcal{AF}_o))\ ((a, e) \notin \succeq_o) \wedge ((e, a) \notin \succeq'_o))$
- $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $(\forall a \in \texttt{Sc}(\mathcal{AF}_o))\ (e \notin \mathcal{H}(o)) \wedge (e, a) \in \succ'_o)$

A similar characterization is given bellow for negotiable options.

Proposition 18 *Let* $\mathcal{AF}_o$ *be a complete decision system, and* $o \in \mathcal{O}_n\mathcal{AF}$.

- $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff* $(e \in \mathcal{H}(o)) \wedge (\forall a \in \texttt{Cr}(\mathcal{AF}_o),\ (e, a) \in \succ'_o)$.
- $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff* $((e \in \mathcal{H}(o)) \wedge (\exists a' \in \texttt{Cr}(\mathcal{AF}_o))\ (e, a') \notin \succ'_o \wedge (\nexists a'' \in \texttt{Cr}(\mathcal{AF}_o))\ (a'', e) \in \succ'_o) \vee ((\exists a' \in \texttt{Cr}(\mathcal{AF}_o))\ (a' \in \mathcal{H}(o) \wedge (e, a') \notin \succ'_o))$
- $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $((e \notin \mathcal{H}(o)) \wedge ((\forall a \in \texttt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow (e, a) \in \succ'_o))$.

A negotiable option o becomes acceptable if the new argument is in its favor and if it is strictly stronger than all existing skeptically accepted arguments.

It remains negotiable if either the new argument is in favor of o and it is incomparable with existing credulously accepted arguments (so, it will also be credulously accepted), or there exists a credulously acceptable argument in favor of o such that e is not strictly preferred to it (in this case that argument will certainly stay credulously accepted, and the option will stay negotiable). Finally, it becomes rejected if e is not in favor of o and e is strictly preferred to all credulously accepted arguments supporting o.

Let us now analyze when a rejected option o in $\mathcal{AF}_o$ may change its status in $\mathcal{AF} \oplus e$.

Proposition 19 *Let $\mathcal{AF}_o$ be a complete decision system, and $o \in \mathcal{O}_r(\mathcal{AF})$.*

- $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff* $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (e, a) \in \succeq'_o)$
- $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff* $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (a, e) \notin \succ'_o) \wedge ((\exists a \in \mathcal{A}_o)\ (e, a) \notin \succ'_o)$
- $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in \succ'_o)$

A rejected option o may become acceptable if the new argument e is in its favor and e is (strictly or not) preferred to all accepted arguments. It becomes negotiable if the new argument is in favor of o and no skeptically accepted argument is strictly preferred to it and there exists a skeptically accepted argument which is not comparable with e. Finally, it remains rejected in case the new argument is not in its favor or it is in its favor but there is an argument which is strictly preferred to it.

5 Related work

The notion of dynamics in argumentation systems is gaining an increasing interest. In [13], the authors have addressed the problem of revision in abstract argumentation systems. They mainly studied under which conditions an extension of the original argumentation system remains an extension in the extended system. In [11], the authors focused on grounded semantics. A great limitation of both works is that they assume that only one attack is added to the extended system. We have shown, in the particular argumentation system studied in our paper, that the new argument can attack and be attacked by more than one argument. In our paper, we are more interested by the evolution of the status of a given argument without having to compute the extensions of the new argumentation system. Moreover, we have studied a more general case, since the new argument may attack and be attacked by an arbitrary number of arguments of the initial argumentation system. Finally, we have studied two acceptability semantics: grounded semantics and preferred one.

Another interesting work on revision is that proposed in [25]. In that work, the authors considered a particular argumentation system in which the origin and the structure of arguments are specified. The main goal was to update the knowledge base, from which arguments are built, in such a way to guarantee the acceptance of the new argument. Thus, this work focuses on very particular systems rather than abstract ones. It studies mainly the revision of knowledge instead of the revision of arguments' status. Finally, arguments are evaluated using a skeptical semantics which is not among the ones proposed by Dung in [15].

6 Conclusion

This paper has tackled the problem of revision in argument-based decision systems. Such systems aims at rank-ordering a set of possible options on the basis of arguments supporting or attacking the options. Three outputs are mainly returned by these systems: a status for each argument stating whether the argument is good or not, a status for each option, and finally an ordering on the options. The aim of this paper is to investigate how these three input evolve when a new argument is received. This should be done without making all the necessary computations.

To the best of our knowledge, in this paper we have proposed the first investigation on the impact of a new argument on a decision system. We have studied two particular decision systems. The first one, proposed in [5], encodes the well-known pessimistic qualitative decision making under uncertainty. The second one, proposed in [4], is a slightly different version of the first one. In both cases, we have provided a full characterization of acceptable options that become rejected, and of rejected options that become acceptable in the extended system. A full characterization of the evolution of the status of arguments is also provided. Our study is undertaken under two acceptability semantics: grounded semantics and preferred one.

These results may be used in negotiation dialogues, namely to determine strategies. Indeed, at a given step of a dialog, an agent may choose which argument to send to another agent in order to change the status of an option. Our results may help to understand which arguments are useful and which ones are useless in a given situation.

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Appendix

Property 1 Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ be a decision system.

(1) The sets $\texttt{Sc}(\mathcal{AF}), \texttt{Cr}(\mathcal{AF}), \texttt{Rej}(\mathcal{AF})$ are pairwise disjoint.
(2) $\texttt{Sc}(\mathcal{AF}) \cup \texttt{Cr}(\mathcal{AF}) \cup \texttt{Rej}(\mathcal{AF}) = \mathcal{A}$.

Proof *This property is straightforward. It follows from Definition 2.* ■

Property 2 Let $\mathcal{AF}$ be a decision system.

- $\mathcal{O}_a^{ad}(\mathcal{AF}) = \emptyset$
- $\mathcal{O}_a^{g}(\mathcal{AF}) = \mathcal{O}_a^{c}(\mathcal{AF}) \subseteq \mathcal{O}_a^{p}(\mathcal{AF})$
- If $\mathcal{AF}$ has no stable extension, then $\mathcal{O}_a^{s}(\mathcal{AF}) = \emptyset$
- If $\mathcal{AF}$ has at least one stable extension, then $\mathcal{O}_a^{p}(\mathcal{AF}) \subseteq \mathcal{O}_a^{s}(\mathcal{AF})$

Proof
- $\mathcal{O}_a^{ad}(\mathcal{AF}) = \emptyset$ *follows from the definition of an acceptable option and the fact that the empty set is an admissible set of any argumentation system.*
- $\mathcal{O}_a^{g}(\mathcal{AF}) = \mathcal{O}_a^{c}(\mathcal{AF})$ *follows immediately from the fact that the grounded extension of* $\mathcal{AF}$ *is exactly the intersection of all complete extensions of* $\mathcal{AF}$*.*
  $\mathcal{O}_a^{c}(\mathcal{AF}) \subseteq \mathcal{O}_a^{p}(\mathcal{AF})$ *follows from the fact that the grounded extension is subset of the intersection of all preferred extensions of* $\mathcal{AF}$*.*
- $\mathcal{O}_a^{s}(\mathcal{AF}) = \emptyset$ *if no extensions is obvious since there are no skeptically accepted arguments.*
- $\mathcal{O}_a^{p}(\mathcal{AF}) \subseteq \mathcal{O}_a^{s}(\mathcal{AF})$ *follows from the fact that each stable extension of* $\mathcal{AF}$ *is also a preferred one. Thus, a system may have more preferred extensions than stable ones.*

■

Proposition 1 Let $\mathcal{AF}$ be a decision system. For all $a \in \mathcal{A}_b$, $\texttt{Status}(a, \mathcal{AF}) = \texttt{Status}(a, \mathcal{AF} \oplus e)$.

Proof *Let* $a \in \mathcal{A}_b$*. Since under grounded semantics, an argument can be either skeptically accepted or rejected, it is sufficient to show that* $a \in \texttt{Sc}(\mathcal{AF}) \Rightarrow a \in \texttt{Sc}(\mathcal{AF} \oplus e)$ *and* $a \in \texttt{Rej}(\mathcal{AF}) \Rightarrow a \in \texttt{Rej}(\mathcal{AF} \oplus e)$*.*

- *Assume that* $a \in \texttt{Sc}(\mathcal{AF})$ *and* $a \in \texttt{Rej}(\mathcal{AF} \oplus e)$*. This means that*
  (1) $(\exists i \in \{1, 2, 3, \ldots\})$ $(\exists a_i \in \texttt{Sc}^i(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$.
  Let us now prove that:
  (2) *if* $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in \texttt{Sc}^i(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$ *then* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \texttt{Sc}^j(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$.

Suppose that $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF} \oplus e)$ *then* $(\exists x \in \mathcal{A} \cup \{e\})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. *Note that from* $a_i \in \mathcal{A}_b$ *and* $(x, a_i) \in \mathtt{Def}$ *we conclude that* $x \in \mathcal{A}_b$. *Since e is practical, then* $x \neq e$. *Thus, x has already existed before the agent has received the argument e. This implies* $(\exists x \in \mathcal{A}_b)$ $(x, a_i) \in \mathtt{Def}$. *From* $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *we conclude that some skeptically accepted argument defends argument* a_i, *i.e.,* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_b)$. *Since* $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$ *it must be that* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *From* 1 *and* 2 *we get:* $\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b$. *Hence,* a_1 *is not defeated in* $\mathcal{AF}$ *and it is defeated in* $\mathcal{AF} \oplus e$. *So,* $(e, a_1) \in \mathtt{Def}$. *Contradiction, since e is practical and a is epistemic.*

- *Let* $a \in \mathcal{A}_b$ *be an epistemic argument such that* $a \in \mathtt{Rej}(\mathcal{AF})$. *Let us suppose that* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *This means that*
  (1) $(\exists i \in \{1,2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$.
  Let us now prove that:
  (2) *if* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$.

  Suppose that $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF})$ *then* $(\exists x \in \mathcal{AF})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF})$ $(b, x) \in \mathtt{Def}$. *Since* $(x, a_i) \in \mathtt{Def}$ *and* $a_i \in \mathcal{A}_b$ *then* $x \in \mathcal{A}_b$. *But* $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *implies that* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathcal{A}_b) \wedge (a_j, x) \in \mathtt{Def}$. *From* $(a_j, x) \in \mathtt{Def}$ *and* $x \in \mathcal{A}_b$ *we have that* a_j *is also epistemic (since practical arguments cannot attack epistemic ones). The fact that* $a_j \in \mathcal{A}_b$ *and e is practical implies that* $a_j \neq e$. *Thus,* a_j *existed before agent has received the new argument e. Since* $(\nexists b \in \mathtt{Sc}(\mathcal{AF}))$ $(b, x) \in \mathtt{Def}$ *then* $a_j \in \mathtt{Rej}(\mathcal{AF})$. *Now we have proved* 1 *and* 2. *From* 1 *and* 2 *we have directly the following:* $(\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$. *From* $a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e)$ *we have* $(\nexists y \in \mathcal{A} \cup \{e\})$ $(y, a_1) \in \mathtt{Def}$ *and from* $a_1 \in \mathtt{Rej}(\mathcal{AF})$ *we have* $(\exists y \in \mathcal{AF})$ $(y, a_1) \in \mathtt{Def}$. *Contradiction.*

■

Proposition 2 Let $\mathcal{AF}$ be a decision system. If $\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ such that $(a, e) \in \mathtt{Def}'_m$, then

- $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$,
- for all $a \in \mathcal{A}_o$, $\mathtt{Status}(a, \mathcal{AF}) = \mathtt{Status}(a, \mathcal{AF} \oplus e)$.

Proof *Let* $a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$. *From Proposition 1,* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Thus,* $e \notin \mathtt{GE}$ *since* $\mathtt{GE}$ *is conflict-free. Consequently,* $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$.

We now prove that $\mathtt{Sc}(\mathcal{AF}) \subseteq \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Suppose not. Then* $(\exists b \in \mathcal{A})$ $b \in \mathtt{Sc}(\mathcal{AF}) \wedge b \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *We will prove that:*

(1) $(\exists i \in \{1,2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$

(2) *if* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$.

Note that the 1 *is already proved. Let us now prove* 2. *Suppose that* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF} \oplus e)$ *then* $(\exists x \in \mathcal{A} \cup \{e\})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. *Suppose now that* $e = x$. *But* $(\exists a \in \mathcal{A}_b \cup \mathtt{Sc}(\mathcal{AF}))$ $(a, e) \in \mathtt{Def}$. *Contradiction with* $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. *Thus,* $x \neq e$, *and* x *was present in the system* $\mathcal{AF}$. *Since* $x \in \mathcal{A}$ *and* $(x, a_i) \in \mathtt{Def}$, *from* $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *we conclude that some skeptically accepted argument defends argument* a_i *in* $\mathcal{AF}$, *i.e.,* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_b) \wedge (a_j, x) \in \mathtt{Def}$. *Since* $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$ *it must be that* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *So, we proved* 2. *As the consequence of* 1 *and* 2 *together, it holds that:* $\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e)$. *This means that* $(\nexists b \in \mathcal{AF})$ $(b, a_1) \in \mathtt{Def}$ *and* $(\exists b \in \ \cup \{e\})$ $(b, x) \in \mathtt{Def}$. *So,* $b = e$, *i.e.,* $(e, a_1) \in \mathtt{Def}$. *Note that* e *is the* only *argument that defeats* a_1 *in* $\mathcal{AF} \oplus e$. *But* $(\exists a \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(a, e) \in \mathtt{Def}$. *Hence,* a_1 *is defended against all defeaters and, consequently,* $a_1 \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Contradiction.*

We now prove that $\mathtt{Sc}(\mathcal{AF} \oplus e) \subseteq \mathtt{Sc}(\mathcal{AF})$. *Suppose not. Then* $(\exists a_i \in \mathcal{A})$ $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e) \wedge a_i \in \mathtt{Rej}(\mathcal{AF})$. *We will prove that:*

(1) $(\exists i \in \{1,2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$

(2) *if* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$.

Note that the 1 *is already proved. Let us now prove* 2. *Suppose that* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF})$ *then* $(\exists x \in \mathcal{AF})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF})$ $(b, x) \in \mathtt{Def}$. *Since* $(x, a_i) \in \mathtt{Def}$ *and* $a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e)$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. *From* $(\nexists b \in \mathtt{Sc}(\mathcal{AF})$ $(b, x) \in \mathtt{Def}$ *we obtain that* $a_j \in \mathtt{Rej}(\mathcal{AF})$. *Now we have proved* 1 *and* 2. *From* 1 *and* 2 *we have directly the following:* $(\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. *From* $a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e)$ *we have* $(\nexists y \in \mathcal{A} \cup \{e\})$ $(y, a_1) \in \mathtt{Def}$ *and from* $a_1 \in \mathtt{Rej}(\mathcal{AF})$ *we have* $(\exists y \in \mathcal{A})$ $(y, a_1) \in \mathtt{Def}$. *Contradiction.* ∎

Lemma 1 *Let* $o \in \mathcal{O}$, $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *and* $x \in \mathcal{A}$ *such that* $(x, a_i) \in \mathtt{Def}$.

(1) *If* $x \in \mathcal{A}_b$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{A}_b \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$,

(2) *If* $x \in \mathcal{A}_o$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$.

Proof *We first prove that if* $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$, $x \in \mathcal{A}$ *and* $(x, a_i) \in \mathtt{Def}$, *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$.

Assume that $(\nexists j \in \{1,2,3,\ldots\})\ (j < i) \wedge (\exists a_j \in \mathcal{H}(o) \cup \mathcal{A}_b)\ a_j \in \texttt{Sc}^j(\mathcal{AF}) \wedge (a_j, x) \in \texttt{Def}$. *Since* a_i *is skeptically accepted and defeated, then it is defended, so* $(\exists j \in \{1,2,3,\ldots\})\ (j < i) \wedge (\exists a_j \in \mathcal{A}_o \setminus \mathcal{H}(o))\ a_j \in \texttt{Sc}^j(\mathcal{AF}) \wedge (a_j, x) \in \texttt{Def}$. *Hence,* $(\exists o' \in \mathcal{O})\ (o' \neq o)$ *and* $a_j \in \mathcal{H}(o')$. *Since both* a_i *and* a_j *are in the grounded extension, there is no attack between them. Since* $a_i \in \mathcal{H}(o)$ *and* $a_j \in \mathcal{H}(o')$, *with* $o' \neq o$, *then* $(a_i, a_j) \in \mathcal{R}_o$ *and* $(a_j, a_i) \in \mathcal{R}_o$. *So,* $(a_i, a_j) \in \succeq_o$ *and* $(a_j, a_i) \in \succeq_o$. *Suppose that* $(a_j, x) \in \succ_o$. *Then, using the transitivity of preference relation, one can easily prove that* $(a_i, x) \in \succ_o$. *Contradiction, since* $(x, a_i) \in \texttt{Def}$. *Hence,* $(x, a_j) \in \texttt{Def}$. *We will prove that:*

(1) $(\exists j \in \{1,2,3,\ldots\})\ (\exists a_j \in (\texttt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o)) \wedge (a_j, x) \in \texttt{Def} \wedge (x, a_j) \in \texttt{Def}$

(2) if $(\exists j \in \{2,3,\ldots\})\ (\exists a_j \in (\texttt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o)) \wedge (a_j, x) \in \texttt{Def} \wedge (x, a_j) \in \texttt{Def}$ *then* $(\exists k \in \{1,2,3,\ldots\})\ (k < j) \wedge (\exists a_k \in (\texttt{Sc}^k(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o)) \wedge (a_k, x) \in \texttt{Def} \wedge (x, a_k) \in \texttt{Def}$

Note that we have already proved 1. Let us prove 2. Suppose that $(\exists j \in \{2,3,\ldots\})\ (\exists a_j \in (\texttt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o))$. *Since* a_j *is skeptically accepted and defeated, then it is defended, so* $(\exists k \in \{1,2,3,\ldots\})\ (k < j) \wedge (\exists a_k \in \mathcal{A}_o \setminus \mathcal{H}(o))\ a_k \in \texttt{Sc}^k(\mathcal{AF}) \wedge (a_k, x) \in \texttt{Def}$. *Hence,* $(\exists o'' \in \mathcal{O})\ (o'' \neq o)$ *and* $a_k \in \mathcal{H}(o'')$. *Recall that* $o'' \neq o$ *since we have supposed that* $(\nexists m \in \{1,2,3,\ldots\})\ (m < i) \wedge (\exists a_m \in \mathcal{H}(o) \cup \mathcal{A}_b)\ a_m \in \texttt{Sc}^m(\mathcal{AF}) \wedge (a_m, x) \in \texttt{Def}$. *Since both* a_j *and* a_k *are in the grounded extension, there is no defeat between them. Since* $a_j \in \mathcal{H}(o)$ *and* $a_k \in \mathcal{H}(o'')$, *with* $o'' \neq o$, *then* $(a_j, a_k) \in \mathcal{R}$ *and* $(a_k, a_j) \in \mathcal{R}$. *So,* $(a_j, a_k) \in \succeq_o$ *and* $(a_k, a_j) \in \succeq_o$. *Suppose that* $(a_k, x) \in \succ_o$. *Then, using the transitivity of preference relation, one can easily prove that* $(a_j, x) \in \succ_o$. *Contradiction, since* $(x, a_j) \in \texttt{Def}$. *Hence,* $(x, a_k) \in \texttt{Def}$. *Since we have proved 1 and 2, we conclude that* $(\exists a_1 \in (\texttt{Sc}^1(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o)) \wedge (a_1, x) \in \texttt{Def} \wedge (x, a_1) \in \texttt{Def}$. *Contradiction since* a_1 *is defeated by* x *and at the same time* $a_1 \in \texttt{Sc}^1(\mathcal{AF})$. *Suppose now that* $x \in \mathcal{A}_b$. *We have proved that* $(\exists j \in \{1,2,3,\ldots\})\ (j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \texttt{Sc}^j(\mathcal{AF}))\ (a_j, x) \in \texttt{Def}$. *Suppose that* $a_j \in \mathcal{H}(o)$. *This means that a practical argument attacks an epistemic one. Contradiction. So,* $a_j \in \mathcal{A}_b$. ■

Proposition 3 Let $\mathcal{AF}$ be a decision system.

- For all $a \in \mathcal{H}(\texttt{Conc}(e))$, if $a \in \texttt{Sc}(\mathcal{AF})$ then $a \in \texttt{Sc}(\mathcal{AF} \oplus e)$.
- For all $a \in \mathcal{A}_o$, if $a \in \texttt{Rej}(\mathcal{AF})$ and $a \in \texttt{Sc}(\mathcal{AF} \oplus e)$, then $e \in \mathcal{H}(\texttt{Conc}(a))$.

Proof *Let* $o \in \mathcal{O}$ *such that* $e \in \mathcal{H}(o)$.

- *Suppose that* $\exists a \in \texttt{Sc}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e)$. *We will prove that:*

(1) $(\exists i \in \{1,2,3,\ldots\})\ (\exists a_i \in (\texttt{Sc}^i(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$

(2) if $(\exists i \in \{2,3,\ldots\})\ (\exists a_i \in (\texttt{Sc}^i(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *then* $(\exists j \in \{1,2,3,\ldots\})\ (j < i) \wedge$
$(\exists a_j \in (\texttt{Sc}^j(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$

Note that we have already proved 1*. Let us now prove* 2*. Suppose that* $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. Since argument* a_i *is rejected in the new system, then* $(\exists x \in \mathcal{A} \cup \{e\})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists y \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(y, x) \in \mathtt{Def}$*. Note that* $x \neq e$*, because* $e \in \mathcal{H}(o)$ *and arguments in favor of same option do not attack each other. Since* $(a_i \in \mathtt{Sc}(\mathcal{AF}))$ *and* $(x, a_i) \in \mathtt{Def}$*, then according to Lemma 1,* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j \in \mathcal{H}(o) \cup \mathcal{A}_b) \wedge (a_j, x) \in \mathtt{Def}$*. Note that* $a_j \neq e$*, because* $a_j \in \mathtt{Sc}^j(\mathcal{AF})$ *and* $e \notin \mathtt{Sc}(\mathcal{AF})$*. Since* $(\nexists y \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(y, x) \in \mathtt{Def}$*, then* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$*. Argument* a_j *is practical, since* $a_j \in \mathcal{A}_b$*, according to Proposition 1, implies* $a_j \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ *which is in contradiction with the fact that* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$*. So,* $a_j \in \mathcal{H}(o)$*. Now that we see that* 1 *and* 2 *are true, we may conclude that* $(\exists a_1 \in (\mathtt{Sc}^1(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. Since* a_1 *was not defeated in* $\mathcal{AF}$ *and it is defeated in* $\mathcal{AF} \oplus e$*, it holds that* $(e, a_1) \in \mathtt{Def}$*. Contradiction, since* $a_1 \in \mathcal{H}(o)$ *and* $e \in \mathcal{H}(o)$*, and arguments in favor of same option do not defeat each other.*

- *Suppose the contrary. Then,* $(\exists a \in \mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *and* $e \notin \mathcal{H}(o)$*. Since* e *is practical, it holds that* $(\exists o' \in \mathcal{O})$ $o' \neq o \wedge e \in \mathcal{H}(o')$*. We will prove that:*

(1) $(\exists i \in \{1, 2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$

(2) if $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *then* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{H}(o) \cap \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap (\mathtt{Rej}(\mathcal{AF}))$

Since $a \in \mathcal{H}(o)$*,* $a \in \mathtt{Rej}(\mathcal{AF})$ *and* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$*, we see that* 1 *is true. So, let us prove the* 2*. Suppose* $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. Since* a_i *was rejected,* $a_i \in \mathtt{Rej}(\mathcal{AF})$*, then* $(\exists x \in \mathcal{A})$ $(x, a_i) \in \mathtt{Def}$ $\wedge$ $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$*. Since* $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ *then, according to Lemma 1,* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap (\mathcal{H}(o) \cup \mathcal{A}_b)) \wedge (a_j, x) \in \mathtt{Def}$*. We have* $a_j \neq e$ *because* $a_j \in \mathcal{H}(o)$ *and* $e \notin \mathcal{H}(o)$*. So,* $a_j \in \mathcal{A}$*. If* $a_j \in \mathcal{A}_b$*, then, according to Proposition 1,* $a_j \in \mathtt{Sc}(\mathcal{AF})$*. Contradiction with the fact* $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$*. So,* $a_j \in \mathcal{H}(o)$*. On the other hand, since* $a_i \in \mathtt{Rej}(\mathcal{AF})$ *then* $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$*. Hence, since* $a_j \in \mathcal{A}$*, then, it must be the case that* $a_j \in \mathtt{Rej}(\mathcal{AF})$*. With* 1 *and* 2 *we have the following:* $(\exists a_1 \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. So,* a_1 *is not defeated in* $\mathcal{AF} \oplus e$ *and* a_1 *is defeated in* $\mathcal{AF}$*. Contradiction.*

■

Lemma 2 *It holds that under grounded semantics* $\mathtt{Dbe}(\mathcal{AF}) \subseteq \mathtt{Sc}(\mathcal{AF})$*.*

Proof *Let* $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ *and* $a \in \mathtt{Dbe}(\mathcal{AF})$*. Let* $Att(a) = \{x_i \in \mathcal{A} \mid (x_i, a) \in \mathtt{Def}\}$*. Since the set* $\mathcal{A}$ *of arguments is finite,* $Att(a) = \{x_1, \ldots, x_n\}$*. From* $a \in \mathtt{Dbe}(\mathcal{AF})$*, we obtain* $(\forall x_i \in \mathcal{A})$ *if* $(x_i, a) \in \mathtt{Def}$ *then* $(\exists \alpha \in \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b)$ *such that* $(\alpha, x_i) \in \mathtt{Def}$*. Let* $Defends(a) = \{\alpha_1, \ldots, \alpha_k\}$ *be a set such that* $Defends(a) \subseteq \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ *and* $(\forall x_i \in Att(a))$ $(\exists \alpha_j \in Defends(a))$ $(\alpha_j, x_j) \in \mathtt{Def}$*. Since* $Defends(a) \subseteq \mathtt{Sc}(\mathcal{AF})$ *then* $(\forall \alpha_i \in Defends(a))$ $(\exists m_i \in \{1, 2, 3, \ldots\})$ *s.t.* $\alpha_i \in \mathtt{Sc}^{m_i}(\mathcal{AF})$*. Let* $m = max\{m_1, \ldots, m_k\}$*. It holds* *that $Defends(a) \subseteq \mathcal{F}^m(\emptyset)$. Then, according to the definition of grounded semantics, it holds that $a \in \mathcal{F}^{m+1}(\emptyset)$* [3] *, since argument a is defended by arguments of $\mathcal{F}^m(\emptyset)$against all attacks. From $a \in \mathcal{F}^m(\emptyset)$, we have $a \in \mathtt{Sc}(\mathcal{AF})$.* ■

Proposition 4 Let $\mathcal{AF}$ be a decision system and $o \in \mathcal{O}_r(\mathcal{AF})$. It holds that $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ iff $e \in \mathcal{H}(o)$ and $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.

Proof $\Rightarrow$ *Let $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$.*

(1) Let us prove that $e \in \mathcal{H}(o)$. Suppose not. Then $(\exists o' \in \mathcal{O})\ o \neq o' \wedge e \in \mathcal{H}(o')$. But, according to Proposition 3., all rejected arguments in favor of o will remain rejected, i.e. $\mathcal{H}(o) \subseteq \mathtt{Rej}(\mathcal{AF} \oplus e)$. This means that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$.

(2) Let us now prove that $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Suppose not. So, $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. Since $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ then $(\exists a \in \mathcal{H}(o))\ a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Note that $a \neq e$ because $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. This is equivalent to
(a) $(\exists i \in \{1, 2, 3, \ldots\})\ (\exists a_i \in \mathcal{H}(o))\ (a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$.
Let us prove that:
(b) if $(\exists i \in \{2, 3, \ldots\})\ (\exists a_i \in \mathcal{H}(o))\ (a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$ then $(\exists j \in \{1, 2, 3, \ldots\})\ (j < i) \wedge (\exists a_j \in \mathcal{H}(o))\ (a_j \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$.

Suppose $(\exists i \in \{2, 3, \ldots\})\ (\exists a_i \in \mathcal{H}(o))\ (a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. Since $a_i \in \mathtt{Rej}(\mathcal{AF})$ then $(\exists x \in \mathcal{A})\ (x, a_i) \in \mathtt{Def} \wedge (\nexists y \in \mathtt{Sc}(\mathcal{AF}))\ (y, x) \in \mathtt{Def}$. Since $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ then, according to Lemma 1, $(\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e))\ (a_j \in \mathcal{H}(o) \cup \mathcal{A}_b) \wedge (a_j, x) \in \mathtt{Def}$. Here, we have $a_j \neq e$ because $a_j \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $e \notin \mathtt{Sc}(\mathcal{AF} \oplus e)$. So, a_j was already present before the agent has received the new argument e. Since $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))\ (y, x) \in \mathtt{Def}$ then $a_j \in \mathtt{Rej}(\mathcal{AF})$. Suppose that $a_j \in \mathcal{A}_b$. Then, according to Proposition 1, $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$, contradiction. So, $a_j \in \mathcal{H}(o)$. Now, when we have proved both (a) and (b), we conclude that $(\exists a_1 \in \mathcal{H}(o))\ (a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. Since a_1 is not defeated in $\mathcal{AF} \oplus e$, than it is not defeated in $\mathcal{AF}$. Contradiction with $a_1 \in \mathtt{Rej}(\mathcal{AF})$.

$\Leftarrow$ *If $e \in \mathcal{H}(o)$ and $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$, then $\mathtt{Conc}(e)$ Then, the option o is acceptable according to Definition 3.* ■

Proposition 5 Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}, \mathtt{Def} \rangle$ be a decision system, and $\mathcal{AF} \oplus e = \langle \mathcal{O}, \mathcal{A}', \mathtt{Def}' \rangle$ its extension with argument e. It holds that $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ iff for all $a \in \mathcal{A}$, if $(a, e) \in \mathtt{Def}'$, then $\exists b \in \mathtt{Sc}(\mathcal{AF}) \cap (\mathcal{A}_b \cup \mathcal{H}(\mathtt{Conc}(e)))$ s.t. $(b, a) \in \mathtt{Def}$.

Proof *Let $o \in \mathcal{O}$ be an option such that $\mathtt{Conc}(e) = o$.*
$\Rightarrow$ *Since $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$, then $(\exists i \in \mathbb{N})$ s.t. $e \in \mathtt{Sc}^i(\mathcal{AF} \oplus e)$. Let us now*

[3] Let $S \subseteq \mathcal{A}$. $\mathcal{F}(S) = \{\alpha \in \mathcal{A} | S \text{ defends } \alpha\}$.

suppose that this property does not hold, i.e. that:

$$(\exists x \in \mathcal{A})((x,e) \in \texttt{Def} \wedge (\forall a \in \mathcal{A})(a,x) \in \texttt{Def} \Rightarrow a \notin \texttt{Sc}(\mathcal{AF}) \cap (\mathcal{A}_b \cup \mathcal{H}(o))) \quad (1)$$

Suppose $x \in \mathcal{A}_b$. *Then, since* $e \in \texttt{Sc}(\mathcal{AF} \oplus e)$ *it holds that* $(\exists \alpha \in \mathcal{A}_b \cap \texttt{Sc}(\mathcal{AF} \oplus e))$ *s.t.* $(\alpha, x) \in \texttt{Def}$. *From Proposition 1,* $\alpha \in \texttt{Sc}(\mathcal{AF})$*, which ends the proof. We will now study the case when* $x \in \mathcal{A}_o$. *Since* $e \in \texttt{Sc}(\mathcal{AF} \oplus e)$ *then from Property 1* $(\exists y \in \texttt{Sc}(\mathcal{AF} \oplus e))$ *s.t.* $(y,x) \in \texttt{Def} \wedge y \in \mathcal{A}_b \cup \mathcal{H}(o)$. *From this fact and from* (1)*, we obtain:* $(\exists x \in \mathcal{A}_o)$ *s.t.* $(x,e) \in \texttt{Def}$ *and if* $(\forall x \in \mathcal{A})(x,e) \in \texttt{Def} \Rightarrow (\exists \alpha \in \texttt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$ *s.t.* $(\alpha, x) \in \texttt{Def}$ *then, since for* $\alpha \in \mathcal{A}_b$ *it holds that* $\alpha \in \texttt{Sc}(\mathcal{AF})$ *iff* $\alpha \in \texttt{Sc}(\mathcal{AF} \oplus e)$*, the proof is over. Else, if* $(\exists x \in \mathcal{A})(x,e) \in \texttt{Def} \wedge (\nexists \alpha \in \texttt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$ *s.t.* $(\alpha, x) \in \texttt{Def}$ *then* $(\exists a_{i-1} \in (\bigcup_{l=1}^{i-1} \texttt{Sc}^l(\mathcal{AF} \oplus e)) \cap \mathcal{H}(o))$ *s.t.* $(a_{i-1}, x) \in \texttt{Def}$.

We will prove by induction on k *that:*

$$(\forall k \in \mathbb{N})(1 \leq k \leq i-1) \Rightarrow (\exists a_k \in (\bigcup_{l=1}^{i} \texttt{Sc}^l(\mathcal{AF} \oplus e)) \cap \mathcal{H}(o)) a_k \notin \texttt{Sc}(\mathcal{AF}) \quad (2)$$

- *Base. We have already seen that* $(\exists a_{i-1} \in (\bigcup_{l=1}^{i-1} \texttt{Sc}^l(\mathcal{AF} \oplus e)))$ *s.t.* $(a_{i-1}, x) \in \texttt{Def}$. *We have also shown that* $a_{i-1} \in \mathcal{H}(o)$. *From* (1)*, we have that* $a_{i-1} \notin \texttt{Sc}(\mathcal{AF})$.
- *Step. Let* (2) *be true for* $k \in \mathbb{N}$, $2 \leq k \leq i-1$*, and let us prove that it is true for* $k-1$. *From the hypothesis, we have that* $(\exists a_k \in (\bigcup_{l=1}^{k} \texttt{Sc}^l(\mathcal{AF} \oplus e)) \cap \mathcal{H}(o))$ *s.t.* $a_k \notin \texttt{Sc}(\mathcal{AF})$. *Since* $a_k \notin \texttt{Sc}(\mathcal{AF})$ *then* $(\exists b \in \mathcal{A})(b, a_k) \in \texttt{Def}$. *It is impossible that for all such* $b \in \mathcal{A}$ $(\exists \alpha \in \texttt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)(\alpha, b) \in \texttt{Def}$ *since that would mean* $\alpha \in \texttt{Sc}(\mathcal{AF}) \wedge (\alpha, b) \in \texttt{Def}$ *so* $a_k \in \texttt{Sc}(\mathcal{AF})$*, contradiction. Thus, from this fact and by using Property 1, we obtain that* $(\exists b \in \mathcal{A})(b, a_k) \in \texttt{Def}$ *and* $(\exists a_{k-1} \in \bigcup_{l=1}^{k-1} \texttt{Sc}^l(\mathcal{AF} \oplus e) \cap \mathcal{H}(o)))$ *s.t.* $(a_{k-1}, b) \in \texttt{Def}$. *If for every such an argument* $b \in \mathcal{A}$ *s.t.* $(b, a_k) \in \texttt{Def} \wedge (\nexists \alpha \in \texttt{Sc}(\mathcal{AF}))$ *s.t.* $(\alpha, b) \in \texttt{Def}$ *it holds that* $(\exists a_{k-1} \in \texttt{Sc}(\mathcal{AF}))$ *then we have that* $a_k \in \texttt{Sc}(\mathcal{AF})$*, contradiction. Thus, it must be that* $(\exists a_{k-1} \in \bigcup_{l=1}^{k-1} \texttt{Sc}^l(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *s.t.* $a_{k-1} \notin \texttt{Sc}(\mathcal{AF})$.

By induction, (2) *holds. For* $k = 1$*, we obtain* $\exists a_1 \in \texttt{Sc}^1(\mathcal{AF} \oplus e)$ *s.t.* $a_1 \notin \texttt{Sc}(\mathcal{AF})$. *From* $a_1 \in \texttt{Sc}^1(\mathcal{AF} \oplus e)$ *we have that* a_1 *is not attacked in* $\mathcal{AF} \oplus e$*, i.e.* $(\nexists c \in \mathcal{A} \cup \{e\})$ *s.t.* $(c, a) \in \texttt{Def}$. *So, it is not attacked in* $\mathcal{AF}$ *neither. Contradiction with* $a_1 \notin \texttt{Sc}(\mathcal{AF})$.

$\Leftarrow$ *Let us suppose that* e *is defended from all attacks in* $\mathcal{AF} \oplus e$ *by arguments of* $\texttt{Sc}(\mathcal{AF}) \cap (\mathcal{H}(o) \cup \mathcal{A}_b)$. *From Proposition 3 and Proposition 1 we have that*

$$\texttt{Sc}(\mathcal{AF}) \cap (\mathcal{H}(o) \cup \mathcal{A}_b) \subseteq \texttt{Sc}(\mathcal{AF} \oplus e) \cap (\mathcal{H}(o) \cup \mathcal{A}_b).$$

This means that e is defended from all attacks in $\mathcal{AF} \oplus e$ by arguments of $\mathtt{Sc}(\mathcal{AF} \oplus e)$. Consequently, $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. ■

Proposition 6 Let $\mathcal{AF}$ be a decision system and $o \in \mathcal{O}_a(\mathcal{AF})$. It holds that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ iff

(1) $e \notin \mathcal{H}(o)$, and
(2) $\nexists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ s.t. $(a, e) \in \mathtt{Def}'_m$, and
(3) $\forall a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o)$, $(e, a) \in \mathtt{Def}'_o$.

Proof $\Rightarrow$ *Since $o \in \mathcal{O}_a(\mathcal{AF})$, then $(\exists a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF})$. Let $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$.*

(1) Suppose $e \in \mathcal{H}(o)$. Then, according to Proposition 3, $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Consequently, $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$, contradiction.
(2) Suppose that $(\exists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ $(x, e) \in \mathtt{Def}$. According to Proposition 2, $\mathtt{Sc}(\mathcal{AF} \oplus e) = \mathtt{Sc}(\mathcal{AF})$ and $\mathtt{Rej}(\mathcal{AF} \oplus e) = \mathtt{Rej}(\mathcal{AF}) \cup \{e\}$. So, $a \in \mathtt{Sc}(\mathcal{AF})$ implies $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Contradiction with the fact that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$.
(3) Suppose that $(\exists a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))$ $(e, a) \notin \mathtt{Def}$. Since $a \in \mathtt{Dbe}(\mathcal{AF})$, Lemma 2 implies that $a \in \mathtt{Sc}(\mathcal{AF})$. From $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ we obtain $a \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. So, $(\exists x \in \mathcal{A})$ $(x, a) \in \mathtt{Def}$ $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. Note that $x \neq e$ because $(x, a) \in \mathtt{Def}$ and $(e, a) \notin \mathtt{Def}$. So, $x \in \mathcal{A}$. From $a \in \mathtt{Dbe}(\mathcal{AF})$ we have $(\exists \alpha \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ $(\alpha, x) \in \mathtt{Def}$. From Proposition 1, we have $\alpha \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Contradiction with $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$.

$\Leftarrow$ *Let $e \notin \mathcal{H}(o) \wedge (\nexists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ $(x, e) \in \mathtt{Def} \wedge (\forall a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))$ $(e, a) \in \mathtt{Def}$. Suppose that $o \notin \mathcal{O}_r(\mathcal{AF} \oplus e)$. Thus, $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$. This means that $(\exists a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. We will prove the following:*

(1) $(\exists i \in \{1, 2, 3, \ldots\})$ $(\exists a_i \in \mathcal{H}(o))$ $(a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$.
(2) if $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in \mathcal{H}(o))$ $(a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$ then $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{H}(o))$ $(a_j \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$.

Note that we have already proved 1, since $(\exists a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Let us prove 2. Suppose that $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in \mathcal{H}(o))$ $(a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$. Let us explore two possibilities: $a \in \mathtt{Dbe}(\mathcal{AF})$ and $a \notin \mathtt{Dbe}(\mathcal{AF})$. Suppose that $a_i \in \mathtt{Dbe}(\mathcal{AF})$. Since $a_i \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o)$ then $(e, a_i) \in \mathtt{Def}$. Since $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $(e, a) \in \mathtt{Def}$ then, according to Lemma 1, $(\exists j \in \{1, 2, 3, \ldots\})$ $j < i \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e))$ $(a_j \in \mathcal{A}_b \cup \mathcal{H}(o)) \wedge (a_j, e) \in \mathtt{Def}$. We will now show that $a_j \in \mathcal{H}(o)$. Suppose that $a_j \in \mathcal{A}_b$. According to Proposition 1, $a_j \in \mathtt{Sc}(\mathcal{AF})$. Contradiction with $(\nexists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ $(x, e) \in \mathtt{Def}$. Let us now explore the case when $a_i \notin \mathtt{Dbe}(\mathcal{AF})$. From Definition 4, we have $(\exists x \in \mathcal{A})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists a_j \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(a_j, x) \in \mathtt{Def}$. Since $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $(x, a_i) \in \mathtt{Def}$, Lemma 1 implies that $(\exists j \in \{1, 2, 3, \ldots\})$

$j < i \wedge (\exists a_j \in \texttt{Sc}^j(\mathcal{AF} \oplus e))\ (a_j \in \mathcal{A}_b \cup \mathcal{H}(o)) \wedge (a_j, e) \in \texttt{Def}$. *Since* $(\nexists a_j \in \mathcal{A}_b \cap \texttt{Sc}(\mathcal{AF} \oplus e))\ (a_j, x) \in \texttt{Def}$ *then* $a_j \in \mathcal{H}(o)$.

Now, we have proved that 1 *and* 2. *As the consequence, we have that:* $(\exists a_1 \in \mathcal{H}(o))\ (a_1 \in \texttt{Sc}^1(\mathcal{AF} \oplus e))$. *This means that* a_1 *is not defeated by any argument in* $\mathcal{AF} \oplus e$. *This implies that* a_1 *is not defeated by any argument in* $\mathcal{AF}$, *i.e.,* $a_1 \in \texttt{Sc}^1(\mathcal{AF})$. *Consequently,* $a_1 \in \texttt{Dbe}(\mathcal{AF})$. *So,* $(e, a_1) \in \texttt{Def}$. *Contradiction with the fact that* a_1 *is not defeated in* $\mathcal{AF} \oplus e$. ■

Lemma 3 *Let* $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ *be an argumentation framework, with* $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o$, $\mathcal{R} = \mathcal{R}_b \cup \mathcal{R}_m \cup \mathcal{R}_o$ *and* $\succeq = \succeq_b \cup \succeq_m \cup \succeq_o$, *and let* $\mathcal{AF}_b = \langle \mathcal{A}_b, \mathcal{R}_b, \succeq_b \rangle$ *be its epistemic part.*

(1) If $\mathcal{E}$ *is a preferred extension of* $\mathcal{AF}$, *then* $\mathcal{E} \cap \mathcal{A}_b$ *is a preferred extension of* $\mathcal{AF}_b$.

(2) If $\mathcal{E}$ *is a preferred extension of* $\mathcal{AF}_b$, *then* $\exists \mathcal{E}' \subseteq \mathcal{A}_o$ *s.t.* $\mathcal{E} \cup \mathcal{E}'$ *is a preferred extension of* $\mathcal{AF}$.

Proof

(1) Let $\mathcal{E}$ *be a preferred extension of* $\mathcal{AF}$ *and let* $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$. *It is trivial that* $\mathcal{E}'$ *is conflict-free. If* $(\exists x \in \mathcal{A}_b)(\exists y \in \mathcal{E}')(x, y) \in \texttt{Def}$ *and* $(\nexists z \in \mathcal{E}')(z, y) \in \texttt{Def}$ *then* $\mathcal{E}$ *is not admissible in* $\mathcal{AF}$ *because of the same attack of* x *in* $\mathcal{AF}$. *So, it must be that* $\mathcal{E}'$ *is admissible in* $\mathcal{AF}_b$. *If* $\mathcal{E}'$ *is not preferred in* $\mathcal{AF}_b$ *then there exists* $\mathcal{E}'' \subseteq \mathcal{A}_b$ *s.t.* $\mathcal{E}''$ *is preferred in* $\mathcal{AF}_b$. *In this case,* $\mathcal{E} \cup \mathcal{E}''$ *admissible in* $\mathcal{AF}$, *contradiction.*

(2) Let $\mathcal{E}$ *be a preferred extension of* $\mathcal{AF}_b$. *It is conflict-free and admissible in* $\mathcal{AF}$. *If it is not preferred, then exists* $\mathcal{E}'' \subseteq \mathcal{A}$ *such that* $\mathcal{E} \subseteq \mathcal{E}''$ *and* $\mathcal{E}''$ *is preferred extension of* $\mathcal{AF}$. *If* $\mathcal{E}'' \cup \mathcal{A}_b = \mathcal{E}$, *the proof is over. Else, from the first part of this property, we have that* $(\mathcal{E}'' \cap \mathcal{A}_b)$ *is a preferred extension of* $\mathcal{AF}_b$. *Contradiction with the fact that* $\mathcal{E}$ *is a preferred extension, since there exists a proper superset of* $\mathcal{E}$ *which is admissible, contradiction.*

■

Proposition 7 Let $\mathcal{AF}$ be a decision system. For all $a \in \mathcal{A}_b$, $\texttt{Status}(a, \mathcal{AF}) = \texttt{Status}(a, \mathcal{AF} \oplus e)$.

Proof *Let* $a \in \mathcal{A}_b$.

(1) Suppose that $a \in \texttt{Sc}(\mathcal{AF})$ *and* $a \notin \texttt{Sc}(\mathcal{AF} \oplus e)$. *This means that exists an extension* $\mathcal{E}$ *in* $\mathcal{AF} \oplus e$ *s.t.* $a \notin \mathcal{E}$. *Let* $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$. *Note that the argumentation system* $\mathcal{AF}_b$ *does not change when a new practical argument is received. From Lemma 3,* $\mathcal{E}'$ *is a preferred extension of* $\mathcal{AF}_b$. *From the same lemma, then exists* $\mathcal{E}'' \subseteq \mathcal{A}_o$ *s.t.* $\mathcal{E}' \cup \mathcal{E}''$ *is a preferred extension of* $\mathcal{AF}$. *Thus, there exists a preferred extension* $\mathcal{E}' \cup \mathcal{E}''$ *such that* $a \notin \mathcal{E}' \cup \mathcal{E}''$. *Contradiction with the fact that* $a \in \texttt{Sc}(\mathcal{AF})$.

(2) *Suppose that* $a \in \mathtt{Cr}(\mathcal{AF})$ *and* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *This means that there exists an extension* $\mathcal{E}$ *in* $\mathcal{AF}$ *such that* $a \notin \mathcal{E}$. *Let* $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$. *From Lemma 3,* $\mathcal{E}'$ *is a preferred extension of* $\mathcal{AF}_b$. *From the same lemma, exists* $\mathcal{E}'' \subseteq \mathcal{A}_o \cup \{e\}$ *s.t.* $\mathcal{E}' \cup \mathcal{E}''$ *is a preferred extension of* $\mathcal{AF} \oplus e$. *Thus, there exists a preferred extension* $\mathcal{E}' \cup \mathcal{E}''$ *such that* $a \notin \mathcal{E}' \cup \mathcal{E}''$. *Contradiction with the fact that* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.
Assume now that $a \in \mathtt{Cr}(\mathcal{AF})$ *and* $a \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *This means that exists an extension* $\mathcal{E}$ *in* $\mathcal{AF}$ *such that* $a \in \mathcal{E}$. *Let* $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$. *from Lemma 3,* $\mathcal{E}'$ *is a preferred extension of* $\mathcal{AF}_b$. *From the same lemma, exists* $\mathcal{E}'' \subseteq \mathcal{A}_o \cup \{e\}$ *s.t.* $\mathcal{E}' \cup \mathcal{E}''$ *is a preferred extension of* $\mathcal{AF} \oplus e$. *Thus, there exists a preferred extension* $\mathcal{E}' \cup \mathcal{E}''$ *of such that* $a \in \mathcal{E}' \cup \mathcal{E}''$. *Contradiction with the fact that* $a \in \mathtt{Rej}(\mathcal{AF} \oplus e)$.

(3) *Suppose that* $a \in \mathtt{Rej}(\mathcal{AF})$ *and* $a \notin \mathtt{Rej}(\mathcal{AF})$. *This means that then exists an extension* $\mathcal{E}$ *in* $\mathcal{AF} \oplus e$ *such that* $a \in \mathcal{E}$. *Let* $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$. *From Lemma 3,* $\mathcal{E}'$ *is a preferred extension of* $\mathcal{AF}_b$. *From the same lemma, exists* $\mathcal{E}'' \subseteq \mathcal{A}_o$ *s.t.* $\mathcal{E}' \cup \mathcal{E}''$ *is a preferred extension of* $\mathcal{AF}$. *Thus, there exists a preferred extension* $\mathcal{E}' \cup \mathcal{E}''$ *of* $\mathcal{AF}$ *such that* $a \in \mathcal{E}' \cup \mathcal{E}''$. *Contradiction with the fact that* $a \in \mathtt{Rej}(\mathcal{AF})$.

■

Proposition 8 Let $\mathcal{AF}$ be a decision system. If $\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ such that $(a, e) \in \mathtt{Def}'_m$, then

- $\mathcal{E} \subseteq \mathcal{A}_b \cup \mathcal{A}_o$ is a preferred extension of $\mathcal{AF}$ iff $\mathcal{E}$ is a preferred extension of $\mathcal{AF} \oplus e$,
- $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$,
- for all $a \in \mathcal{A}_o$, $\mathtt{Status}(\mathcal{AF}, a) = \mathtt{Status}(\mathcal{AF} \oplus e, a)$.

Proof

(1) $\Rightarrow$ *Let* $\mathcal{E}$ *be a preferred extension of* $\mathcal{AF}$. *It is obvious that it is conflict-free. It is admissible in* $\mathcal{AF} \oplus e$ *since it defends all its elements in* $\mathcal{AF}$. *So, it trivially defends the arguments in* $\mathcal{AF} \oplus e$ *from all attacks except from attacks of e. Since skeptically accepted arguments are in all extensions,* $a \in \mathcal{E}$. *So, a defends* $\mathcal{E}$ *from attacks of e in* $\mathcal{AF} \oplus e$. *Thus,* $\mathcal{E}$ *is admissible in* $\mathcal{AF} \oplus e$. *Suppose now that* $\mathcal{E}$ *is not a preferred extension of* $\mathcal{AF} \oplus e$. *Then, there exists* $\mathcal{E}' \subseteq \mathcal{A} \cup \{e\}$ *such that* $\mathcal{E}'$ *is preferred extension in* $\mathcal{AF} \oplus e$ *and* $\mathcal{E} \subsetneq \mathcal{E}'$. *Since e is rejected then* $e \notin \mathcal{E}'$. *But it is now easy to see that* $\mathcal{E}'$ *admissible in* $\mathcal{AF}$, *thus* $\mathcal{E}$ *is not a preferred extension of* $\mathcal{AF}$.

$\Leftarrow$ *Let* $\mathcal{E}$ *be a preferred extension in* $\mathcal{AF} \oplus e$. *Since e is rejected then* $e \notin \mathcal{E}$. *It is clear that* $\mathcal{E}$ *is conflict-free. Since* $\mathcal{E}$ *is admissible in* $\mathcal{AF} \oplus e$, *i.e. it defends all its elements, then it is easy to conclude that it defends all its elements in* $\mathcal{AF}$. *We will now see that* $\mathcal{E}$ *is preferred in* $\mathcal{AF}$. *Let us suppose the contrary. Then, there exists* $\mathcal{E}' \subseteq \mathcal{A}$ *such that* $\mathcal{E}'$ *is preferred* *in $\mathcal{AF}$ and $\mathcal{E} \subsetneq \mathcal{E}'$. As shown above, this means that $\mathcal{E}'$ is admissible in $\mathcal{AF} \oplus e$.*

(2) By Lemma 3, from $\alpha \in \texttt{Sc}(\mathcal{AF})$, we have that $\alpha \in \texttt{Sc}(\mathcal{AF} \oplus e)$. Since e is attacked by a skeptically accepted argument, it must be rejected since every extension contains α and every extension is conflict-free, thus no extension can contain argument e.

(3) Since extensions do not change, status of arguments do not change.

■

Lemma 4 *Let $\langle \mathcal{O}, \mathcal{A}, \texttt{Def} \rangle$ be a decision system, and $\mathcal{E}$ one its preferred extension. Let $a \in \mathcal{E} \cap \mathcal{A}_b$ and $x \in \mathcal{A}$ s.t. $(x, a) \in \texttt{Def}$. Then:*

$$(\exists a_i \in \mathcal{E} \cap (\mathcal{A}_b \cup \mathcal{H}(\texttt{Conc}(a)))) \quad (a_i, x) \in \texttt{Def}$$

Proof *Let $o \in \mathcal{O}$ be such that $a \in \mathcal{H}(o)$ and let $(x, a) \in \texttt{Def}$. Since $a \in \mathcal{E}$ then $(\exists a_i \in \mathcal{E})(a_i, x) \in \texttt{Def}$. If $(\exists a_i \in \mathcal{E})(a_i, x) \in \texttt{Def} \wedge a_i \in \mathcal{E} \cap (\mathcal{A}_b \cup \mathcal{H}(o))$ the proof is over. Else, we have that $(\forall a_i \in \mathcal{E})(a_i, x) \in \texttt{Def} \Rightarrow a_i \in \mathcal{A}_o \setminus \mathcal{H}(o)$. From this fact, $(a, x) \notin \texttt{Def}$. Consequently, $(x, a) \in \succ_o$. Since $(a_i, x) \in \texttt{Def}$ then $(x, a_i) \notin \succeq_o$. If $(a_i, x) \in \succ_o$ then $(a_i, a) \in \succ_o$ and $(a_i, a) \in \texttt{Def}$. Contradiction with $a, a_i \in \mathcal{E}$. Else, since $(a_i, x) \notin \succeq_o$ and $(a_i, x) \notin \succ_o$, then $(a_i, x) \notin \succeq_o \wedge (x, a_i) \notin \succeq_o$. From $a, a_i \in \mathcal{E}$ we have $(a, a_i) \in \succeq_o \wedge (a_i, a) \in \succeq_o$. From $(x, a) \in \succ_o$ and $(a, a_i) \in \succeq_o$, it holds that $(x, a_i) \in \succ_o$. Contradiction since it would mean that $(x, a_i) \in \texttt{Def}$ and $(a_i, x) \notin \texttt{Def}$ which is not possible.* ■

Proposition 9 Let $\mathcal{AF}$ be a decision system. For all $a \in \mathcal{A}_o$ such that $\texttt{Conc}(a) = \texttt{Conc}(e)$, it holds that:

- If $a \in \texttt{Sc}(\mathcal{AF})$ then $a \in \texttt{Sc}(\mathcal{AF} \oplus e)$
- If $a \in \texttt{Cr}(\mathcal{AF})$ then $a \in \texttt{Sc}(\mathcal{AF} \oplus e) \cup \texttt{Cr}(\mathcal{AF} \oplus e)$

Proof *Let $o \in \mathcal{O}$, $a \in \mathcal{A}_o$ and let $a, e \in \mathcal{H}(o)$.*

- *Assume that $a \in \texttt{Sc}(\mathcal{AF})$ and $a \notin \texttt{Sc}(\mathcal{AF} \oplus e)$. This means that there exists a preferred extension of $\mathcal{AF} \oplus e$, $\mathcal{E}'$, such that $a \notin \mathcal{E}'$. It is easy to see that $\mathcal{E}' \setminus \{e\} \setminus \mathcal{H}(o)$ is admissible in $\mathcal{AF}$: it is trivial that it is conflict-free, and from Lemma 4 we see that it defends all its elements since every practical argument can be defended either by an epistemic argument or by a practical argument having the same conclusion. Since practical arguments do not attack epistemic ones, we see that $\mathcal{A}_b \cap (\mathcal{E}' \setminus \{e\} \setminus \mathcal{H}(o))$ is also admissible in $\mathcal{AF}$. So, there exists $\mathcal{E}'' \subseteq \mathcal{A}$ s.t. $(\mathcal{E}' \setminus \{e\} \setminus \mathcal{H}(o)) \subseteq \mathcal{E}''$ and $\mathcal{E}''$ is preferred extension of $\mathcal{AF}$. Since $a \in \texttt{Sc}(\mathcal{AF})$ it must be that $a \in \mathcal{E}''$. Set $\mathcal{E}' \cup (\mathcal{E}'' \cap \mathcal{H}(o))$ is admissible in $\mathcal{AF} \oplus e$:*
  - *it is conflict-free as union of two conflict-free sets which do not attack each another since arguments in $\mathcal{H}(o)$ do not attack other arguments in $\mathcal{H}(o)$ and arguments in $\mathcal{H}(o) \cap \mathcal{E}''$ are not in conflict with other arguments in*

$\mathcal{E}'$ since $\mathcal{E}''$ is conflict-free

· *it defends its elements since $\mathcal{E}'$ is admissible in $\mathcal{AF} \oplus e$ and $\mathcal{E}'' \cap (\mathcal{H}(o) \cup \mathcal{A}_b)$ is admissible in $\mathcal{AF} \oplus e$ and union of two admissible sets which do not attack each another is an admissible set.*

Contradiction, since $\mathcal{E}'$ is a preferred extension in $\mathcal{AF} \oplus e$ and there exists its strict superset $\mathcal{E}' \cap (\mathcal{E}'' \cup \mathcal{H}(o))$ which is admissible in $\mathcal{AF} \oplus e$.

- *Since $a \in \mathtt{Cr}(\mathcal{AF})$ then $(\exists \mathcal{E} \subseteq \mathcal{A})$ s.t. $\mathcal{E}$ is a preferred extension in $\mathcal{AF}$ and $a \in \mathcal{E}$. As a consequence of Lemma 4, $\mathcal{E}' = (\mathcal{H}(o) \cup \mathcal{A}_b) \cap \mathcal{E}$ is admissible in $\mathcal{AF} \oplus e$. Thus, $(\exists \mathcal{E}'' \subseteq \mathcal{A} \cup \{e\})$ s.t. $\mathcal{E}' \subseteq \mathcal{E}''$ and $\mathcal{E}''$ is a preferred extension of $\mathcal{AF} \oplus e$. This proves that $a \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$.*

■

Proposition 10 Let $\mathcal{AF}$ be a decision system, and $a \in \mathcal{A}_o$. If $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \in \mathtt{Sc}(\mathcal{AF} \oplus e) \cup \mathtt{Cr}(\mathcal{AF} \oplus e)$ then $\mathtt{Conc}(a) = \mathtt{Conc}(e)$.

Proof *Let $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$. Since $a \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$ then there exists $\mathcal{E} \subseteq \mathcal{A} \cup \{e\}$ s.t. $a \in \mathcal{E}$ and $\mathcal{E}$ is a preferred extension of $\mathcal{AF} \oplus e$. Let $\mathtt{Conc}(e) = o'$, with $o' \in \mathcal{O}$. Set $\mathcal{E}' \setminus \mathcal{H}(o')$ is admissible in $\mathcal{AF}$: it is conflict-free (since it is conflict-free in $\mathcal{AF} \oplus e$) and from Lemma 4, it defends all its elements. Since $a \in \mathtt{Rej}(\mathcal{AF})$ then a cannot be in any admissible set of $\mathcal{AF}$ since for every admissible set there exists its superset which is a preferred extension, thus a would be in at least one preferred extension which could not be the case. Consequently, $a \notin \mathcal{E} \setminus \mathcal{H}(o)'$. From $a \in \mathcal{E}$ and $a \notin \mathcal{E} \setminus \mathcal{H}(o')$ it follows that $a \in \mathcal{H}(o')$, i.e. $o = o'$. In other words, this means that $e \in \mathcal{H}(o)$.*
■

Proposition 11 Let $\mathcal{AF}$ be a decision system and $o \in \mathcal{O}_r(\mathcal{AF})$. Then $o \in \mathcal{O}_a(\mathcal{AF} \oplus e) \cup \mathcal{O}_n(\mathcal{AF} \oplus e)$ iff $e \in \mathcal{H}(o) \wedge e \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$.

Proof $\Rightarrow$ *Let us suppose that option $o \in \mathcal{O}$ was rejected before the argument e was received, i.e. $o \in \mathtt{Rej}(\mathcal{AF})$ and that its status was improved, formally $o \in \mathcal{O}_a(\mathcal{AF} \oplus e) \cup \mathcal{O}_n(\mathcal{AF} \oplus e)$. This means that all the arguments in $\mathcal{H}(o)$ were rejected, and that in system $\mathcal{AF} \oplus e$ there exists at least one argument in favor of o which is not rejected. We see that $e \notin \mathcal{H}(o)$ is not possible since, that would mean that some of arguments in $\mathcal{H}(o)$ improved its status, and according to Proposition 10 that $e \in \mathcal{H}(o)$. So, we proved that $e \in \mathcal{H}(o)$. Let us now prove that $e \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$. Suppose the contrary, i.e. let $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. This means that $\exists \mathcal{E} \subseteq \mathcal{A}$ s.t. $\mathcal{E}$ is a preferred extension in $\mathcal{AF} \oplus e$ and that $(\exists a \in \mathcal{H}(o) \cap \mathcal{E})$. In other words, there exists a non-rejected argument in favor of o. From Lemma 4 we see that set $\mathcal{E} \cap (\mathcal{A}_b \cup \mathcal{H}(o))$ is admissible in $\mathcal{AF} \oplus e$. It must also be admissible in $\mathcal{AF}$. This means that $a \notin \mathtt{Rej}(\mathcal{AF})$ and, consequently $o \notin \mathtt{Rej}(\mathcal{AF})$. Contradiction.*
$\Leftarrow$ *This part of proof is trivial, since it follows directly from Definition 3.* ■

Proposition 12 Let $\mathcal{AF} = \langle \mathcal{O}, \mathcal{A}_b \cup \mathcal{A}_o, \texttt{Def}_b \cup \texttt{Def}_o \cup \texttt{Def}_m \rangle$ be a decision system. It holds that $e \notin \texttt{Rej}(\mathcal{AF} \oplus e)$ iff $\exists \mathcal{E} \subseteq \mathcal{A}_b$ and $\exists \mathcal{E}' \subseteq \mathcal{H}(\texttt{Conc}(e))$ such that:

(1) $\mathcal{E} \cup \mathcal{E}'$ is conflict-free, and
(2) $\mathcal{E}$ is a preferred extension of the argumentation system $\langle \mathcal{A}_b, \texttt{Def}_b \rangle$, and
(3) $\forall a \in \mathcal{E}' \cup \{e\}$, if $\exists x \in \mathcal{A}_b \cup \mathcal{A}_o$ s.t. $(x, a) \in \texttt{Def}_o \cup \texttt{Def}_m$, then $\exists a' \in \mathcal{E} \cup \mathcal{E}' \cup \{e\}$ s.t. $(a', x) \in \texttt{Def}_b \cup \texttt{Def}_o \cup \texttt{Def}_m$.

Proof *Let $o \in \mathcal{O}$ such that $o = \texttt{Conc}(e)$.*
$\Rightarrow$ *Let $e \notin \texttt{Rej}(\mathcal{AF} \oplus e)$. In other words, $\exists \mathcal{E}' \subseteq \mathcal{A} \cup \{e\}$ s.t. $\mathcal{E}'$ is a preferred extension in $\mathcal{AF} \oplus e$ and $e \in \mathcal{E}'$. Let $\mathcal{E}_b = \mathcal{E}' \cap \mathcal{A}_b$ and $\mathcal{E}_o = \mathcal{E}' \cap \mathcal{H}(o)$. From Lemma 4, we obtain that $\mathcal{E}'$ is admissible in $\mathcal{AF} \oplus e$.*

(1) It is obvious that $\mathcal{E}_b \cup \mathcal{E}_o$ is conflict-free.
(2) Since $\mathcal{E}'$ is a preferred extension in $\mathcal{AF} \oplus e$, then from Lemma 3 we have that $\mathcal{E}_b$ is a preferred extension of system $\mathcal{AF}_b = \langle \mathcal{A}_b, \mathcal{R}_b, \succeq_b \rangle$.
(3) Let $a \in \mathcal{E}_o \cup \{e\}$ and let $(x, a) \in \texttt{Def}$. Since $\mathcal{E}'$ is a preferred extension in $\mathcal{AF} \oplus e$, then $\exists a' \in \mathcal{E}'$ s.t. $(a', x) \in \texttt{Def}$.

$\Leftarrow$ *Let us suppose that the three conditions are satisfied and let us prove that $e \notin \texttt{Rej}(\mathcal{AF} \oplus e)$. We define $\mathcal{E}'$ as follows: $\mathcal{E}' = \mathcal{E}_b \cup \mathcal{E}_o \cup \{e\}$. Recall that $\mathcal{E}_b \cup \mathcal{E}_o$ is conflict-free. Since $\mathcal{E}_o \subseteq \mathcal{H}(o)$ then $\mathcal{E}_o \cup \{e\}$ is conflict-free. Argument e being practical, it cannot attack the arguments in $\mathcal{E}_b$. Suppose now that $\mathcal{E}_b$ attacks e, i.e. $(\exists \alpha \in \mathcal{E}_b)(\alpha, e) \in \texttt{Def}$. In that case, from the third item, $(\exists \beta \in \mathcal{E}_b)(\beta, \alpha) \in \texttt{Def}$, contradiction with the fact $\mathcal{E}_b$ is conflict-free. Thus, $\mathcal{E}'$ is conflict-free. The set $\mathcal{E}_b$ is a preferred extension in $\mathcal{AF}_b$, and since practical arguments cannot attack epistemic ones, then it is a preferred extension in epistemic part $(\mathcal{AF} \oplus e)_b$ of system $\mathcal{AF} \oplus e$. Consequently, it defends its arguments. From the third item, $\mathcal{E}'$ defends arguments of $\mathcal{E}_o \cup \{e\}$. Thus, $\mathcal{E}'$ is an admissible extension of argumentation system $\mathcal{AF} \oplus e$. Then, $\exists \mathcal{E} \subseteq \mathcal{A} \cup \{e\}$ s.t. $\mathcal{E}' \subseteq \mathcal{E}$, $e \in \mathcal{E}$ and $\mathcal{E}$ is a preferred extension of $\mathcal{AF} \oplus e$. So, $e \notin \texttt{Rej}(\mathcal{AF} \oplus e)$.* ■

Proposition 13 Let $\mathcal{AF}$ be a decision system and $o \in \mathcal{O}_a(\mathcal{AF}) \cup \mathcal{O}_n(\mathcal{AF})$. Then $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ iff

(1) $e \notin \mathcal{H}(o)$, and
(2) there does not exist a preferred extension $\mathcal{E}$ of $\mathcal{AF}$ s.t. $\mathcal{E} \cap \mathcal{H}(o) \neq \emptyset$ and $\exists a \in \mathcal{E} \cap \mathcal{A}_b$ s.t. $(a, e) \in \texttt{Def}'_m$, and
(3) there does not exist a preferred extension $\mathcal{E}$ of $\mathcal{AF}$ s.t. there exists an admissible set $\mathcal{E}''$ of $\mathcal{AF}$ with $\mathcal{E}'' \cap \mathcal{A}_o \subseteq \mathcal{E} \cap \mathcal{H}(o)$ and $\mathcal{E}'' \cap \mathcal{A}_b = \mathcal{E} \cap \mathcal{A}_b$ and $\forall a \in \mathcal{E}'' \cap \mathcal{H}(o), (a, e) \in \succeq'_o$ or $\exists a' \in \mathcal{E}'' \cap \mathcal{H}(o)$ s.t. $(e, a) \notin \succeq'_o$.

Proof $\Rightarrow$ *Let $o \in \mathcal{O}_a(\mathcal{AF}) \cup \mathcal{O}_n(\mathcal{AF})$ and let us suppose that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$. We prove that the three conditions stated in the proposition are satisfied.*

(1) From Proposition 9 we have that in the case when $e \in \mathcal{H}(o)$, all skeptically accepted arguments in $\mathcal{H}(o)$ will stay skeptically accepted and that all credulously accepted arguments in $\mathcal{H}(o)$ will either stay credulously accepted or become skeptically accepted. So, it must be that $e \notin \mathcal{H}(o)$.

(2) Let us suppose that there exists a preferred extension of $\mathcal{AF}$, denoted $\mathcal{E}$, s.t. $a \in \mathcal{E} \cap \mathcal{H}(o)$ and $(\exists \alpha \in \mathcal{E} \cap \mathcal{A}_b)(\alpha, e) \in \mathcal{R}$. In that case, set $\mathcal{E}$ is admissible in $\mathcal{AF} \oplus e$, since it is conflict-free (trivial) and it defends all its elements: this come from the fact that $\mathcal{E}$ is admissible in $\mathcal{AF}$ and that it attacks e. So, there exists $\mathcal{E}' \subseteq \mathcal{A} \cup \{e\}$ which is a preferred extension in $\mathcal{AF} \oplus e$, such that $\mathcal{E} \subseteq \mathcal{E}'$. Hence, $a \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$. Consequently, $o \notin \mathcal{O}_r(\mathcal{AF} \oplus e)$. Contradiction.

(3) Let us suppose that the third condition of proposition is not satisfied, and let $\mathcal{E}'' \subseteq \mathcal{A}$ s.t. $\mathcal{E}'' \cap \mathcal{A}_o \subseteq \mathcal{E} \cap \mathcal{H}(o)$ and $\mathcal{E}'' \cap \mathcal{A}_b = \mathcal{E} \cap \mathcal{A}_b$ and $\mathcal{E}''$ is admissible in $\mathcal{AF}$ and $((\forall a \in \mathcal{E}'' \cap \mathcal{H}(o))(a, e) \in \succeq_o$ or $(\exists a' \in \mathcal{E}'' \cap \mathcal{H}(o)$ s.t. $\neg(e, a) \in \succeq_o))$. Since $\mathcal{E}''$ is admissible in $\mathcal{AF}$, then it is conflict-free and it defends all its arguments from all attacks in $\mathcal{AF}$. To check whether or not it is admissible in $\mathcal{AF} \oplus e$, it is sufficient to see that it defends itself also from attacks of e: in the case when $(\forall a \in \mathcal{E}'' \cap \mathcal{H}(o))(a, e) \in \succeq_o$ then it is not defeated by e at all, in the case when $(\exists a' \in \mathcal{E}'' \cap \mathcal{H}(o))\neg(e, a) \in \succeq_o$ we have $\neg(e, a) \in \succeq_o$ which means that $(a, e) \in \mathtt{Def}$ so in this case also we have that $\mathcal{E}''$ is admissible. This means that $o \notin \mathtt{Rej}(\mathcal{AF} \oplus e)$, contradiction.

$\Leftarrow$ *Let us suppose that $o \in \mathcal{O}_a(\mathcal{AF}) \cup \mathcal{O}_n(\mathcal{AF})$ and that three conditions of the proposition are satisfied. We prove that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$. Suppose the contrary. This would mean that $(\exists \mathcal{E} \subseteq \mathcal{A} \cup \{e\})$ s.t. $\mathcal{E}$ is a preferred extension of $\mathcal{AF} \oplus e$ and $\mathcal{E} \cap \mathcal{H}(o) \neq \emptyset$. Let $\mathcal{E}' = \mathcal{E} \cap (\mathcal{H}(o) \cup \mathcal{A}_b)$. From Proposition 9, $\mathcal{E}'$ is admissible in $\mathcal{AF} \oplus e$. Since $e \notin \mathcal{H}(o)$ then $(\forall a \in \mathcal{E}' \cap \mathcal{H}(o))(e, a) \in \mathcal{R} \wedge (a, e) \in \mathcal{R}$. Let us suppose that $(\exists \alpha \in \mathcal{E} \cap \mathcal{A}_b)(\alpha, e) \in \mathcal{R}$. Since $\mathcal{E}'$ is admissible in $\mathcal{AF} \oplus e$ then $\mathcal{E}'$ is admissible in $\mathcal{AF}$. This is in contradiction with the second condition of the proposition. Thus, $(\nexists \alpha \in \mathcal{E} \cap \mathcal{A}_b)(\alpha, e) \in \mathcal{R}$. Since $\mathcal{E}'$ is admissible in $\mathcal{AF}$ and $(\nexists \alpha \in \mathcal{E} \cap \mathcal{A}_b)(\alpha, e) \in \mathcal{R}$ then either e does not defeat any of arguments in $\mathcal{E}' \cap \mathcal{H}(o)$, formally $(\forall a \in \mathcal{E}'' \cap \mathcal{H}(o))(a, e) \in \succeq_o$ or $\mathcal{E}' \cap \mathcal{H}(o)$ defeats e, formally $(\exists a' \in \mathcal{E}'' \cap \mathcal{H}(o)$ s.t. $\neg(e, a) \in \succeq_o)$. This is in contradiction with the third condition of the proposition since from the fact that $\mathcal{E}'$ is admissible in $\mathcal{AF}$ it holds that there exists a preferred extension of $\mathcal{AF}$, denoted $\mathcal{E}''$, such that $\mathcal{E}' \subseteq \mathcal{E}''$. Note that it must be $\mathcal{E}' \cap \mathcal{A}_b = \mathcal{E}'' \cap \mathcal{A}_b$ since in the case when $\mathcal{E}' \cap \mathcal{A}_b \subsetneq \mathcal{E}'' \cap \mathcal{A}_b$, according to Lemma 3, $\mathcal{E}'' \cap \mathcal{A}_b$ would have been a preferred extension of $\mathcal{AF}_b$, which is not possible since $\mathcal{E}' \cap \mathcal{A}_b$ is preferred extension of $\mathcal{AF}_b$. Thus, the hypothesis that $o \notin \mathcal{O}_r(\mathcal{AF} \oplus e)$ was false.* ■

Property 3 Let $\mathcal{AF}_o$ be a complete decision system, and $a \in \mathcal{A}_o$.

- a is skeptically accepted iff $\forall x \in \mathcal{A}_o$, $(a, x) \in \succeq_o$.
- a is rejected iff $\exists x \in \mathcal{A}_o$ s.t. $(x, a) \in \succ_o$.

- a is credulously accepted iff $\exists x' \in \mathcal{A}_o$ s.t. $(a, x') \notin \succeq_o$, and $\forall x \in \mathcal{A}_o$, if $(a, x) \notin \succeq_o$, then $(x, a) \notin \succeq_o$.

Proof

(1) $\Rightarrow$ *Suppose that a is skeptically accepted. It can be shown that in this system an argument a is skeptically accepted iff* $(\nexists x \in \mathcal{A}_o)\ (x, a) \in \mathtt{Def}_o$. *Suppose that there is an argument* x' *such that* $(a, x') \notin \succeq_o$. *Since* $\mathcal{R}_o$ *is complete, then* $(x', a) \in \mathcal{R}_o$. *Thus, according to definition of* $\mathtt{Def}_o$, *we have* $(x', a) \in \mathtt{Def}_o$. *Contradiction with the fact* $(\nexists x \in \mathcal{A}_o)\ (x, a) \in \mathtt{Def}_o$.

$\Leftarrow$ *Let us now suppose that* $(\forall x \in \mathcal{A}_o)\ (a, x) \in \succeq_o$ *and that a is not skeptically accepted. It can be shown that in this system an argument a is skeptically accepted iff* $(\nexists x \in \mathcal{A}_o)\ (x, a) \in \mathtt{Def}_o$. *Since a is not skeptically accepted,* $(\exists x')\ (x, a) \in \mathtt{Def}_o$. *Since* $(x', a) \in \mathtt{Def}_o$ *then, according to definition of* $\mathtt{Def}_o$, $(a, x') \notin \succeq_o$. *Contradiction with the fact* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \succeq_o$.

(2) $\Rightarrow$ *Suppose that a is rejected. Then, there is no extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$. *It can be shown that for an arbitrary argument* $a \in \mathcal{A}$, *there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending. This means that a is not self-defending. So,* $(\exists x' \in \mathcal{A}_o)\ ((x', a) \in \mathtt{Def}_o \wedge (a, x') \notin \mathtt{Def}_o)$. *Since* $(x', a) \in \mathcal{R}_o$ *and* $(x', a) \in \mathtt{Def}_o$ *then, according to definition of* $\mathtt{Def}_o$, *we have* $(a, x') \notin \succeq_o$. *Since* $(a, x') \in \mathcal{R}_o$ *and* $(a, x') \notin \mathtt{Def}_o$ *then, according to definition of* $\mathtt{Def}_o$, *we have* $(x', a) \in \succeq_o$. *According to the definition of* $\succ_o$, $(a, x') \notin \mathtt{Def}_o$ *and* $(x', a) \in \succeq_o$ *give* $(x', a) \in \succ_o$.

$\Leftarrow$ *Suppose now that* $(\exists x' \in \mathcal{A}_o)\ (x', a) \in \succ_o$. *Since the relation* $\mathcal{R}_o$ *is complete, we have* $(x', a) \in \mathcal{R}_o$. *According to definition of* $\succ_o$, *we have* $(a, x') \notin \succeq_o$. *These two facts, together with the the definition of* $\mathtt{Def}_o$ *imply* $(x', a) \in \mathtt{Def}_o$. *The fact that* $(x', a) \in \succeq_o$ *implies that, according to definition of* $\mathtt{Def}_o$, $(a, x') \notin \mathtt{Def}_o$. *So,* $(x', a) \in \mathtt{Def}_o$ *and* $(a, x') \notin \mathtt{Def}_o$ *which means that a is not self-defending. Since it can be shown that for an arbitrary argument* $a \in \mathcal{A}$, *there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending, then there is no extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$. *So, a is rejected.*

(3) $\Rightarrow$ *Let us suppose that a is credulously accepted. According to Definition 2, there is at least one extension* $\mathcal{E}_i$ *such that* $a \in \mathcal{E}_i$. *Since it can be shown that for an arbitrary argument* $a \in \mathcal{A}$, *there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending and since a is in* $\mathcal{E}_i$ *then a is self-defending. Suppose now that* $(a, x') \notin \succeq_o$. *So,* $(x', a) \in \mathtt{Def}_o$. *Since a is self-defending, we have* $(a, x') \in \mathtt{Def}_o$. *So,* $(x', a) \notin \succeq_o$. *Hence,* $((\forall x \in \mathcal{A})\ ((a, x) \notin \succeq_o) \Rightarrow (x, a) \notin \succeq_o))$. *We will now prove that* $((\exists x' \in \mathcal{A})$ $(a, x') \notin \succeq_o)$. *Since a is not skeptically accepted, and it can be shown that an argument* $a \in \mathcal{A}_l$ *is skeptically accepted iff it is not attacked, then* $(\exists y' \in \mathcal{A}_o)\ (y', a) \in \mathtt{Def}_o$. *This means that* $(a, y') \notin \succeq_o$. *So, we proved that* $((\exists x' \in \mathcal{A})\ (a, x') \notin \succeq_o) \wedge ((\forall x \in \mathcal{A})\ ((a, x) \notin \succeq_o) \Rightarrow (x, a) \notin \succeq_o))$.

$\Leftarrow$ *Let us now suppose that* $((\exists x' \in \mathcal{A})\ (a, x') \notin \succeq_o) \wedge ((\forall x \in \mathcal{A})$ $((a, x) \notin \succeq_o) \Rightarrow (x, a) \notin \succeq_o))$. *We have* $((\exists x' \in \mathcal{A})\ (a, x') \notin \succeq_o)$, *so*

$(x', a) \in \mathtt{Def}_o$. It can be shown that an argument $a \in \mathcal{A}_\wr$ is skeptically accepted iff it is not attacked, so, a is not skeptically accepted. Suppose now that a is rejected. That means that $((\exists x' \in \mathcal{A})\ ((x', a) \in \mathtt{Def}_o)$ and $((a, x') \notin \mathtt{Def}_o)$. The fact $((x', a) \in \mathtt{Def}_o)$ implies $((a, x') \notin \succeq_o)$. According to the assumption $((\forall x \in \mathcal{A})\ ((a, x) \notin \succeq_o) \Rightarrow (x, a) \notin \succeq_o))$, we have $((x', a) \notin \succeq_o)$. Thus, $((a, x') \in \mathtt{Def}_o)$. Contradiction. Since a is neither skeptically accepted nor rejected, it is credulously accepted.

■

Property 4 Let $a, b \in \mathtt{Sc}(\mathcal{AF}_o)$, where $\mathcal{AF}_o$ is a complete decision system. Then $(a, b) \in \succeq_o$ and $(b, a) \in \succeq_o$.

Proof *It can be shown that the framework has an accepted argument iff it has exactly one extension. Since the system has a skeptically accepted argument, there is exactly one extension $\mathcal{E}$. Since both a and b are accepted, then $a, b \in \mathcal{E}$. Since $\mathcal{E}$ is conflict-free, $(a, b) \notin \mathtt{Def}_o$ and $(b, a) \notin \mathtt{Def}_o$. The fact $(a, b) \notin \mathtt{Def}_o$ implies $(b, a) \in \succeq_o$ and, similarly, $(b, a) \notin \mathtt{Def}_o$ implies $(a, b) \in \succeq_o$.* ■

Lemma 5 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making and $a \in \mathcal{A}_o$. If $a \in \mathtt{Rej}(\mathcal{AF}_o)$ then $(\exists x' \in \mathcal{A}_o)$ such that $x' \notin \mathtt{Rej}(\mathcal{AF}_o) \wedge (x', a) \in \succ_o$.*

Proof *Let $a \in \mathcal{A}_o$. Assume that a is rejected. Thus, from Property 3, there is at least one argument z_0 such that $(z_0, a) \in \succ_o$. Since z_0 is rejected, there exists at least one argument z_1 such that $(z_1, z_0) \in \succ_o$. Now, we can construct the sequence of arguments $z_0, \ldots, z_k$ such that $(\forall i \in \{1, \ldots, k\})\ (z_i, z_{i-1}) \in \succ_o$. Let $z_0, \ldots, z_n$ be a maximal such a sequence. We will now prove that that all the arguments in this sequence are different. Suppose that $(\exists i, j \in \{0, \ldots, n\})$ $z_i = z_j$. Without loss of generality, suppose that $i > j$. Then, because of transitivity of the relation $\succ_o$, we have $(z_i, z_j) \in \succ_o$. On the other hand, $z_i = z_j$, so $(z_i, z_i) \in \succ_o$. This implies that $(z_i, z_i) \in \succeq_o$ and $(z_i, z_i) \notin \succeq_o$. Contradiction. Hence, all the arguments in this sequence are different. Moreover, since there is a finite number of arguments and all the arguments in the sequence are different, the sequence is finite as well. So, let z_n be the last argument in this sequence. Note that, because of the transitivity of relation $\succ_o$, it holds that $(z_n, x) \in \succ_o$. The argument z_n can be rejected or not. Suppose that it is rejected. This implies that $(\exists z_{n+1})\ (z_{n+1}, z_n) \in \succ_o$. Contradiction with the fact that the sequence which ends with z_n is maximal. Suppose that z_n is not rejected. So, $(z_n, x) \in \succ_o$ and z_n is not rejected. Contradiction with the fact $(\forall x \in \mathcal{A}_o)\ (x, a) \in \succ_o \Rightarrow x \in \mathtt{Rej}(\mathcal{AF}_o)$. Hence, $(\exists x' \in \mathcal{A}_o)\ (x', a) \in \succ_o \wedge$ $x' \notin \mathtt{Rej}(\mathcal{AF}_o)$.* ■

Lemma 6 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making and $a \in \mathcal{A}_o$. If $\mathcal{A}_o \setminus \{a\} \subseteq \mathtt{Rej}(\mathcal{AF}_o)$ then $a \in \mathtt{Sc}(\mathcal{AF}_o)$.*

Proof *Suppose that e is not skeptically accepted. Then, e is credulously accepted or rejected.*

(1) Suppose that e is rejected. According to Lemma 5, $(\exists x' \in \mathcal{A}_o)$ $x' \notin \texttt{Rej}(\mathcal{AF}_o)$ *and* $(x', e) \in \succ_o$*. Contradiction with the fact that all the arguments are rejected.*

(2) Suppose that $e \in \texttt{Cr}(\mathcal{AF}_o)$*. According to Property 3,* $((\exists x' \in \mathcal{A})\ (e, x') \notin \succeq_o) \wedge ((\forall x \in \mathcal{A})\ ((e, x) \notin \succeq_o \Rightarrow (x, e) \notin \succeq_o))$*. Since there are no self-attacking arguments, we have* $x' \neq e$*. Since* $x' \neq e$ *and all the arguments except e are rejected, then* x' *is rejected. According to Lemma 5,* $(\exists y' \in \mathcal{A}_o)$ *such that* y' *is not rejected and* $(y', x') \in \succ_o$*. Since* y' *is not rejected and all the arguments except e are rejected, then* $y' = e$*. Since* $(y', x') \in \succ_o$ *and* $y' = e$*, then* $(e, x') \in \succ_o$*. Since* $(e, x') \in \succ_o$ *then* $(e, x') \in \succeq_o$*. Contradiction with the fact* $(e, x') \notin \succeq_o$*.*

■

Property 5 Let $\mathcal{AF}_o$ be a complete decision system. For all $a, b \in \texttt{Cr}(\mathcal{AF}_o)$, it holds that $(a, b) \in \succeq_o$ and $(b, a) \in \succeq_o$, or $(a, b) \notin \succeq_o$ and $(b, a) \notin \succeq_o$.

Proof *Suppose that the* $(\exists a \in \texttt{Cr}(\mathcal{AF}_o))(\exists b \in \texttt{Cr}(\mathcal{AF}_o))\ \neg((a, b) \in \succeq_o \wedge (a, b) \in \succeq_o) \wedge \neg((a, b) \notin \succeq_o \wedge (a, b) \notin \succeq_o)$*. Then, either* $(a, b) \in \succ_o$ *or* $(b, a) \in \succ_o$ *Without loss of generality, we can suppose that* $(a, b) \in \succ_o$*. Then, with* $(a, b) \in \mathcal{R}_o$*, we have* $(a, b) \in \texttt{Def}_o$ *and* $(b, a) \notin \texttt{Def}_o$*. So, the argument b is not self-defending. Since it can be shown that for an arbitrary argument* $a \in \mathcal{A}$*, there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending, then there is no extension* $\mathcal{E}$ *such that* $b \in \mathcal{E}$*. Consequently, b is not credulously accepted. Contradiction with the fact* $b \in \texttt{Cr}(\mathcal{AF}_o)$*.* ■

Property 6 Let $\mathcal{AF}_o$ be a complete decision system.

(1) If $\texttt{Sc}(\mathcal{AF}_o) \neq \emptyset$ then $\texttt{Cr}(\mathcal{AF}_o) = \emptyset$.
(2) If $\texttt{Cr}(\mathcal{AF}_o) = \emptyset$ then $\texttt{Sc}(\mathcal{AF}_o) \neq \emptyset$.

Proof

(1) It can be shown that the framework has an accepted argument iff it has exactly one extension. Since the framework has a skeptically accepted argument, then it has only one extension, say $\mathcal{E}$*. Suppose that* $(\exists a \in \mathcal{A}_o)$ *a is credulously accepted. According to Definition 2, there are two different extensions* $\mathcal{E}_1$ *and* $\mathcal{E}_2$ *such that* $a \in \mathcal{E}_1$ *and* $a \notin \mathcal{E}_2$*. Contradiction with the fact that there is exactly one extension.*

(2) It can be shown that the argumentation framework $\mathcal{AF}_o$ *always has at least one non-empty extension* $\mathcal{E}_1$*. Let* $a \in \mathcal{E}_1$ *be an arbitrary argument which belongs to this extension. Since* $a \in \mathcal{E}_1$*, according to Definition 2, a is skeptically accepted or credulously accepted. Since we have supposed that there are no credulously accepted arguments, then a is skeptically*

accepted.

■

Lemma 7 *The following equivalences hold.*

(1) There is at least one skeptically accepted argument iff there is at least one acceptable option.
(2) There is at least one credulously accepted argument iff there is at least one negotiable option.

Proof

(1) $\Rightarrow$ *Suppose that there is at least one skeptically accepted argument a. Since all the arguments are practical arguments, a is in favor of some option o. Then, according to Definition 3, o is acceptable.*

$\Leftarrow$ *Let us now suppose that there is at least one acceptable option o. Then, according to Definition 3, there is at least one skeptically accepted argument a such that* $a \in \mathcal{H}(o)$.

(2) $\Rightarrow$ *Suppose that there is at least one credulously accepted argument a. Then, according to Property 6, there are no skeptically accepted arguments. Since all the arguments are practical arguments, a is in favor of some option o. Since there are no skeptically accepted arguments at all, there are no skeptically accepted arguments in favor of option o. So, there is at least one credulously accepted argument a in favor of option o and there are no skeptically accepted arguments in favor of option o. According to Definition 3, o is negotiable.*

$\Leftarrow$ *According to Definition 3, there is at least one credulously accepted argument a in favor of a negotiable option.*

■

Lemma 8 *Let* $\mathcal{AF}_o = \langle \mathcal{O}, \mathcal{A}_o, \mathtt{Def} \rangle$ *be a complete decision system and* $x \in \mathcal{A}_o$. *If* $(\exists a \in \mathtt{Sc}(\mathcal{AF}_o))$ *such that* $(a, x) \in \odot$[4] *then* $(\forall a' \in \mathtt{Sc}(\mathcal{AF}_o))\ (a', x) \in \odot$.

Proof *Let us suppose that* $((\exists a \in \mathcal{A}_o)\ a \in \mathtt{Sc}(\mathcal{AF}_o) \wedge (a, x) \in \odot)$. *Let b be an arbitrary accepted argument. According to Property 4,* $(a, b) \in \succeq_o$ *and* $(b, a) \in \succeq_o$. *Now, using the transitivity of preference relation, it can easily be shown that* $(a, x) \in \odot$ *implies* $(b, x) \in \odot$. ■

[4] We use the symbol $\odot$ to refer to any of four possible situations regarding the preference between two arguments. For example, this lemma says that if $\exists a \in \mathtt{Sc}(\mathcal{AF}_o)(a, x) \in \succ_o$ then $\forall a \in \mathtt{Sc}(\mathcal{AF}_o)(a, x) \in \succ_o$ and that if $\exists a \in \mathtt{Sc}(\mathcal{AF}_o)(x, a) \in \succ_o$ then $\forall a \in \mathtt{Sc}(\mathcal{AF}_o)(x, a) \in \succ_o$ and that if $\exists a \in \mathtt{Sc}(\mathcal{AF}_o)(x, a) \notin \succeq_o \wedge (a, x) \notin \succeq_o$ then $\forall a \in \mathtt{Sc}(\mathcal{AF}_o)(x, a) \notin \succeq_o \wedge (a, x) \notin \succeq_o$ and that if $\exists a \in \mathtt{Sc}(\mathcal{AF}_o)(x, a) \in \succeq_o \wedge (a, x) \notin \succeq_o$ then $\forall a \in \mathtt{Sc}(\mathcal{AF}_o)(x, a) \in \succeq_o \wedge (a, x) \in \succeq_o$.

Lemma 9 *Let $\mathcal{AF}_o$ be a complete decision system and $\texttt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Then it holds that: $(\forall a' \in \texttt{Cr}(\mathcal{AF}_o))\ (\exists a'' \in \texttt{Cr}(\mathcal{AF}_o))\ (a', a'') \notin \succeq_o \wedge\ (a'', a') \notin \succeq_o$.*

Proof *Suppose the converse. Then $(\exists a' \in \texttt{Cr}(\mathcal{AF}_o))\ (\forall a \in \texttt{Cr}(\mathcal{AF}_o))\ \neg ((a, a') \notin \succeq_o \wedge\ (a', a) \notin \succeq_o)$. Recall the result of the Property 5 which states that $(\forall a \in \texttt{Cr}(\mathcal{AF}_o))(\forall b \in \texttt{Cr}(\mathcal{AF}_o))\ ((a, b) \in \succeq_o \wedge\ (b, a) \in \succeq_o) \vee ((a, b) \notin \succeq_o \wedge (b, a) \notin \succeq_o)$. So, if for two credulously accepted arguments a and a' it holds that $\neg\ ((a, a') \notin \succeq_o \wedge\ (a', a) \notin \succeq_o)$, then it must be the case that $((a, a') \in \succeq_o \wedge\ (a', a) \in \succeq_o)$. So, $(\exists a' \in \texttt{Cr}(\mathcal{AF}_o))\ (\forall a \in \texttt{Cr}(\mathcal{AF}_o))\ ((a, a') \in \succeq_o \wedge\ (a', a) \in \succeq_o)$. Let $b, c \in \texttt{Cr}(\mathcal{AF}_o)$. Since $(b, a') \in \succeq_o$ and $(a', c) \in \succeq_o$, then, because of the transitivity of the preference relation, $(b, c) \in \succeq_o$. Similarly, since $(c, a') \in \succeq_o$ and $(a', b) \in \succeq_o$, then $(c, b) \in \succeq_o$. So, all the credulously accepted arguments are in the same class of equivalence with respect to $\succeq_o$. This means that there is no attack in the sense of $\texttt{Def}_o$ between arguments of $\texttt{Cr}(\mathcal{AF}_o)$. So, $\texttt{Cr}(\mathcal{AF}_o)$ is admissible. Since there are some credulously accepted arguments, according to Definition 2, there are at least two different non-empty preferred extensions $\mathcal{E}_1$ and $\mathcal{E}_2$. Since there are some credulously accepted arguments, then, according to Property 6, there are no skeptically accepted arguments. Since all the arguments in $\mathcal{E}_1$ and $\mathcal{E}_2$ are in some extension, they are not rejected. Since there are no skeptically accepted arguments, they are credulously accepted. Since it can be shown that all the extensions are pairwise disjoint, then $\mathcal{E}_1 \cap \mathcal{E}_2 = \emptyset$. All the arguments that are not in $\texttt{Cr}(\mathcal{AF}_o)$ are not credulously accepted. Since there are no skeptically accepted arguments, they are rejected. Let us prove that $\mathcal{E}_1, \mathcal{E}_2 \subseteq \texttt{Cr}(\mathcal{AF}_o)$. If $\neg(\mathcal{E}_1 \subseteq \texttt{Cr}(\mathcal{AF}_o))$ then there is some argument which is credulously accepted (since it is in $\mathcal{E}_1$) and in the same time it is rejected (since it is not in $\texttt{Cr}(\mathcal{AF}_o)$). Contradiction. So, $\mathcal{E}_1 \subseteq \texttt{Cr}(\mathcal{AF}_o)$. The same proof for $\mathcal{E}_2$. So, $\mathcal{E}_1$ and $\mathcal{E}_2$ are preferred extensions and $\mathcal{E}_1 \cap \mathcal{E}_2 = \emptyset$ and $\mathcal{E}_1 \neq \emptyset$ and $\mathcal{E}_2 \neq \emptyset$. Since $\mathcal{E}_2 \neq \emptyset$, then $\mathcal{E}_1 \neq \texttt{Cr}(\mathcal{AF}_o)$. So, $\mathcal{E}_1$ is preferred and $\texttt{Cr}(\mathcal{AF}_o)$ is admissible and $\mathcal{E}_1 \subseteq \texttt{Cr}(\mathcal{AF}_o)$ and $\mathcal{E}_1 \neq \texttt{Cr}(\mathcal{AF}_o)$. Contradiction, because a preferred extension is a maximal admissible extension.* ■

Lemma 10 *Let $\mathcal{AF}_o = \langle \mathcal{O}, \mathcal{A}_o, \texttt{Def} \rangle$ be a complete decision system and let $o \in \mathcal{O}$. The option o is negotiable iff there is at least one credulously accepted argument in its favor in $\mathcal{AF}_o$.*

Proof $\Rightarrow$ *Trivial, according to Definition 3.*

$\Leftarrow$ *Let a be an credulously accepted argument in favor of o. Since there exists at least one credulously accepted argument, Property 6 implies that there are no skeptically accepted arguments. In particular, there are no skeptically accepted arguments in favor of o. According to Definition 3, o is negotiable.* ■

Property 7 Let $\mathcal{AF}_o$ be a complete decision system. It holds that $\mathcal{O}_a \neq \emptyset \Leftrightarrow \mathcal{O}_n = \emptyset$.

Proof $\Rightarrow$ *Let $\mathcal{O}_a \neq \emptyset$. According to Lemma 7, there is at least one skeptically* *accepted argument. Then, according to Property 6, there are no credulously accepted arguments. Using Lemma 7, we conclude that there are no negotiable options.*

$\Leftarrow$ *Let* $\mathcal{O}_n = \emptyset$. *According to Lemma 7, there are no credulously accepted arguments. Then, according to Property 6, there is at least one skeptically accepted argument. The Lemma 7 implies that there is at least one acceptable option.* ■

Lemma 11 *Let* $\mathcal{AF}_o$ *be a complete decision system. If* $a \in \mathtt{Cr}(\mathcal{AF}_o)$*, then* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.

Proof *Assume that* a *is credulously accepted in* $\mathcal{AF}_o$. *Thus, according to Property 3,* $\exists x \in \mathcal{A}_o$ *such that* $(a, x) \notin \succeq_o$. *It is clear that* $a, x \in \mathcal{A}_o \cup \{e\}$. *Assume that* a *is skeptically accepted in the system* $\mathcal{AF}_o \oplus e$. *According to Property 3,* $(\forall x \in \mathcal{A}_o \cup \{e\})$ $(a, x) \in \succeq_o$. *Contradiction with the fact* $(a, x) \notin \succeq_o$ ■

Lemma 12 *Let* $\mathcal{AF}_o$ *be a complete decision system.*

(1) If $a \in \mathtt{Sc}(\mathcal{AF}_o)$ *then* $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *iff* $(a, e) \in \succeq_o$.
(2) If $a \notin \mathtt{Rej}(\mathcal{AF}_o)$ *then* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ *iff* $(e, a) \in \succ_o$.

Proof

(1) Let $a \in \mathtt{Sc}(\mathcal{AF}_o)$.

$\Rightarrow$ *Suppose that* $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *and* $(a, e) \notin \succeq_o$. *Since the attack relation* $\mathcal{R}_o$ *is complete, then* $(a, e) \in \mathcal{R}_o$ *and* $(e, a) \in \mathcal{R}_o$. *With* $(a, e) \notin \succeq_o$, *we have* $(e, a) \in \mathtt{Def}_o$. *Since* $(e, a) \in \mathtt{Def}_o$, *according to Property 3, we have that* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Contradiction.*

$\Leftarrow$ *Let* $(a, e) \in \succeq_o$. *Since* $a \in \mathtt{Sc}(\mathcal{AF}_o)$, *according to Property 3,* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \succeq_o$. *Suppose that* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Then, according to Property 3,* $(\exists x' \in \mathcal{A}_o \cup \{e\})$ $(a, x') \notin \succeq_o$. *We will prove that* $x' \notin \mathcal{A}_o$. *Suppose the converse, i.e., suppose that* $x' \in \mathcal{A}_o$. *Since* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \succeq_o$, *then* $(a, x') \in \succeq_o$. *Contradiction, so it must be the case that* $x' \notin \mathcal{A}_o$. *With* $x' \in \mathcal{A}_o \cup \{e\}$ *and* $x' \notin \mathcal{A}_o$ *we have* $x' = e$, *and, consequently,* $(a, e) \notin \succeq_o$. *Contradiction.*

(2) Let $a \in A_o \setminus \mathtt{Rej}(\mathcal{AF}_o)$.

$\Rightarrow$ *Let* a *become rejected. Since* $a \notin \mathtt{Rej}(\mathcal{AF}_o)$, *then, according to Property 3,* $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in \succ_o$. *Since* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, *then, according to Property 3,* $(\exists y \in \mathcal{A}_o \cup \{e\})$ $(y, a) \in \succ_o$. *We will prove that* $y = e$. *Suppose not. Then,* $y \in \mathcal{A}_o$ *and* $(y, a) \in \succ_o$. *Contradiction with the fact* $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in \succ_o$. *So,* $y = e$ *and, consequently,* $(e, a) \in \succ_o$.

$\Leftarrow$ *Let* $(e, a) \in \succ_o$. *Since* $(e, a) \in \succ_o$, *then, according to Property 3,* a *is rejected.*

■

Lemma 13 *Let $\mathcal{AF}_o$ be a complete decision system and $a, b \in \texttt{Sc}(\mathcal{AF}_o)$. Let $e \notin \mathcal{A}_o$.*

(1) If $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.
(2) If $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$.
(3) If $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.

Proof

(1) Since $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$, then, according to Lemma 12, $(a, e) \in \succeq_o$. According to Lemma 8, $(b, e) \in \succeq_o$. According to Lemma 12, $b \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.

(2) Since $a \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$, then, according to Lemma 12, $(a, e) \notin \succeq_o$. Since $a \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Lemma 12, $(e, a) \notin \succ_o$. According to Lemma 8, $(b, e) \notin \succeq_o$ and $(e, b) \notin \succ_o$. Since $(b, e) \notin \succeq_o$, then, according to Lemma 12, $b \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Since $(e, b) \notin \succ_o$, then, according to Lemma 12, we have $b \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$. Hence, according to Property 1, $b \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$.

(3) Since $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Lemma 12, $(e, a) \in \succ_o$. According to Lemma 8, $(e, b) \in \succ_o$. According to Lemma 12, $b \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.

■

Lemma 14 *Let $\mathcal{AF}_o$ be a complete decision system, $a \in \texttt{Sc}(\mathcal{AF}_o)$ and $e \notin \mathcal{A}_o$. The following holds:*

(1) $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \in \succeq_o) \wedge ((e, a) \in \succeq_o)$
(2) $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in \succ_o$
(3) $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in \succ_o$
(4) $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \notin \succeq_o) \wedge ((a, e) \notin \succeq_o)$

Proof

(1) Let $((a, e) \in \succeq_o) \wedge ((e, a) \in \succeq_o)$. Let us prove $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. So, a changed its status. According to Lemma 12, $(a, e) \notin \succeq_o$. Contradiction. Thus, $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.

We will now prove that $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. Then, according to Property 3, $(\exists x' \in \mathcal{A}_o \cup \{e\})\ (e, x) \notin \succeq_o$. Since we proved that $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$, then, according to Property 3, $(\forall x \in \mathcal{A}_o \cup \{e\})\ (a, x) \in \succeq_o$. In particular, $(a, x') \in \succeq_o$. Since $(e, a) \in \succeq_o$ and $(a, x') \in \succeq_o$, the transitivity of the preference relation $\succeq_o$ implies that $(e, x') \in \succeq_o$. Contradiction. So, $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.

(2) Let $(e, a) \in \succ_o$. According to Property 3, it holds that $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$, since there is now at least one argument which is strictly preferred to it.

Let us now prove that $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. Then, according to Property 3, $(\exists x' \in \mathcal{A}_o \cup \{e\})\ (e, x') \notin \succeq_o$. Since there are no self-attacking *arguments, we have $x' \neq e$. So, $x' \in \mathcal{A}_o$. Since $a \in \texttt{Sc}(\mathcal{AF}_o)$, it holds that $(\forall x \in \mathcal{A}_o)\ (a, x) \in \succeq_o$. In particular, $(a, x') \in \succeq_o$. So, $(e, a) \in \succ_o$ and $(a, x') \in \succeq_o$. One can easily see that $(e, x') \in \succ_o$. Consequently, we have $(e, x') \in \succeq_o$. Contradiction with the fact $(e, x') \notin \succeq_o$. Hence, $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.*

(3) $(a, e) \in \succ_o$. Suppose that $a \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. So, a changes its status. According to Lemma 12, $(a, e) \notin \succeq_o$. Contradiction with the fact $(a, e) \in \succ_o$. So, $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.

We will now prove that $e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$. Since $(a, e) \in \succ_o$, then, according to Property 3, it holds that $e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.

(4) Let $((a, e) \notin \succeq_o) \wedge ((a, e) \notin \succeq_o)$. We will prove that $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. Suppose that $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. So, according to Property 3, $(\forall x \in \mathcal{A}_o \cup \{e\})\ (a, x) \in \succeq_o$. But, $(a, e) \notin \succeq_o$. Contradiction. So, $a \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose that $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$. Then, according to Property 3, $(\exists x' \in \mathcal{A}_o \cup \{e\})\ (x', a) \in \succ_o$. $a \in \texttt{Sc}(\mathcal{AF}_o)$. So, according to Property 3, $(\forall x \in \mathcal{A}_o)\ (a, x) \in \succeq_o$. Suppose that $x' \in \mathcal{A}_o$. Then, $(x', a) \in \succ_o$ and $(a, x') \in \succeq_o$. Contradiction, so $x' \notin \mathcal{A}_o$. The fact that $x' \in \mathcal{A}_o \cup \{e\}$ and $x' \notin \mathcal{A}_o$ implies that $x' = e$. So, $(e, a) \in \succ_o$. Contradiction. Hence, $a \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$. Since we proved that $a \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$ and $a \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Property 1 $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$.

Let us now prove that $e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. Suppose that $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. According to Property 3, $(\forall x \in \mathcal{A}_o)\ (e, x) \in \succeq_o$. But, $(e, a) \notin \succeq_o$. Contradiction. So, $e \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose now that $e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$. Then, according to Property 3, $(\exists y' \in \mathcal{A})\ (y', e) \in \succ_o$. Since $(y', e) \in \succ_o$ then $(e, y') \notin \succeq_o$. Since $\succeq_o$ is reflexive, then $y' \neq e$. So, $y' \in \mathcal{A}_o$. $a \in \texttt{Sc}(\mathcal{AF}_o)$. So, according to Property 3, $(\forall x \in \mathcal{A}_o)\ (a, x) \in \succeq_o$. Since $y' \in \mathcal{A}_o$, then $(a, y') \in \succeq_o$. So, we have $(a, y') \in \succeq_o$ and $(y', e) \in \succ_o$. Now, it is easy to see that $(a, e) \in \succ_o$. Contradiction. Since we proved that $e \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$ and $e \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Property 1, it must be the case that e is credulously accepted.

■

Lemma 15 *Let $\mathcal{AF}_o$ be a complete decision system such that $\texttt{Cr}(\mathcal{AF}_o) \neq \emptyset$. The following result holds: $((\forall a \in \texttt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succ_o)$ iff $((\forall a \in \texttt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succeq_o)$.*

Proof $\Rightarrow$ *Trivial, according to definition of $\succ_o$.*

$\Leftarrow$ *Let us suppose that $(\exists a' \in \texttt{Cr}(\mathcal{AF}_o))\ ((e, a') \notin \succ_o \wedge (e, a') \in \succeq_o)$. So, according to definition of $\succ_o$, $(a', e) \in \succeq_o$. According to Lemma 9, $(\exists a'' \in \texttt{Cr}(\mathcal{AF}_o))\ ((a', a'') \notin \succeq_o \wedge (a'', a') \notin \succeq_o)$. Since $(\forall a \in \texttt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succeq_o$, then, in particular, $(e, a'') \in \succeq_o$. With $(a', e) \in \succeq_o$ and $(e, a'') \in \succeq_o$ we have $(a', a'') \in \succeq_o$. Contradiction.* ■

Lemma 16 *Let $\mathcal{AF}_o$ be a complete decision system such that* $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. *The following holds:* $((\forall a \in \mathtt{Cr}(A_o))\ a \in \mathtt{Rej}(A_o \oplus e))$ *iff* $((\forall a \in \mathtt{Cr}(A_o))\ (e, a) \in \succ_o)$.

Proof $\Rightarrow$ *Let all the credulously accepted arguments become rejected. Suppose that* $a' \in \mathtt{Cr}(\mathcal{AF}_o)$. *According to Lemma 12, since* $a' \in \mathtt{Cr}(\mathcal{AF}_o)$ *and* $a' \in \mathtt{Rej}(\mathcal{A}_o \oplus e)$, *it holds that* $(e, a') \in \succ_o$.

$\Leftarrow$ *Let* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succ_o$. *Suppose that* $a' \in \mathtt{Cr}(\mathcal{AF}_o)$. *According to Property 3, since* $(e, a') \in \succ_o$ *then* $a' \in \mathtt{Rej}(\mathcal{A}_o \oplus e)$. ■

Proposition 14 Let $\mathcal{AF}_o$ be a complete decision system. Let $a \in \mathcal{A}_o$ such that $a \in \mathtt{Sc}(\mathcal{AF}_o)$.

- $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in \succeq'_o$.
- $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in \succ'_o$.
- $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \notin \succeq'_o$ and $(e, a) \notin \succeq'_o$.

Proof *This proposition is a direct consequence of Lemma 14.* ■

Proposition 15 Let $\mathcal{AF}_o$ be a complete decision system. Let $a \in \mathcal{A}_o$ such that $a \in \mathtt{Cr}(\mathcal{AF}_o)$.

- $a \in \mathtt{Cr}(\mathcal{AF}_o) \wedge a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ is not possible.
- $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in \succ'_o$.
- $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \notin \succ'_o$.

Proof
- *Follows from Lemma 11.*
- $\Rightarrow$ *Since* $a \in \mathtt{Cr}(\mathcal{AF})$, *from Property 3 we have that* $\forall x \in \mathcal{A}_o$ *if* $(x, a) \in \mathtt{Def}_o$ *then* $(a, x) \in \mathtt{Def}_o$. *Suppose* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *From the same property, it holds that* $\exists x \in \mathcal{A}_o \cup \{e\}$ *s.t.* $(x, a) \in \succ_o$. *It is obvious that* $(e, a) \in \mathtt{Def}_o$ *and* $(a, e) \notin \mathtt{Def}_o$. *Thus,* $(e, a) \in \succ'_o$.
  $\Leftarrow$ *Let* $(e, a) \in \succ'_o$. *As a consequence of Property 3,* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.
- *From the first item of this proposition, we see that* $a \notin \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Since* $a \in \mathtt{Rej}(\mathcal{AF} \oplus e)$ *iff* $(e, a) \in \succ'_o$, *then* $a \in \mathtt{Cr}(\mathcal{AF} \oplus e)$ *iff* $(e, a) \notin \succ'_o$.

■

Proposition 16 Let $\mathcal{AF}_o$ be a complete decision system. If $a \in \mathtt{Rej}(\mathcal{AF}_o)$, then $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.

Proof *Let* $a \in \mathcal{A}_o$. *Assume that* a *is rejected in* $\mathcal{AF}_o$. *According to Property 3,* $\exists x \in \mathcal{A}_o$ *such that* $(x, a) \in \succ_o$. *Let* $e \notin \mathcal{A}_o$. *So,* $a, x \in \mathcal{A}_o \cup \{e\}$, *which (according to Property 3) means that* a *is rejected in* $\mathcal{AF}_o \oplus e$. ■

Lemma 17 *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be an argumentation framework such that*

$\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. *Then, the following holds:*

(1) $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succ_o$ *iff* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \wedge \mathcal{A}_o = \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.
(2) $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succ_o \wedge (\nexists a' \in \mathtt{Cr}(\mathcal{AF}_o))$
$(a',e) \in \succ_o$ *iff* $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$
(3) $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a,e) \in \succ_o$ *iff* $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e) \wedge \mathcal{A}_o = \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.

Proof *During the proof, we will sometimes use the following fact. Since, according to Lemma 15,* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succ_o$ *is equivalent to* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succeq_o$*, then the negation of* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succ_o$ *is equivalent to negation of* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succeq_o$*. So,* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succ_o$ *is equivalent to* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succeq_o$*.*

(1) $\Rightarrow$ *Let* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succ_o$*. Let* $a \in \mathtt{Cr}(\mathcal{AF}_o)$*. Since* $(e,a) \in \succ_o$*, then, Property 3 implies that* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. So,* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. Since, according to Proposition 16, rejected arguments cannot change their status, then* $\mathcal{A}_o \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. So, as the consequence of Lemma 6, we have that e is skeptically accepted.*

$\Leftarrow$ *Let* $a \in \mathtt{Cr}(\mathcal{AF}_o)$*. Since* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*, then, according to Lemma 12, it holds that* $(e,a) \in \succ_o$*. Since* $a \in \mathtt{Cr}(\mathcal{AF}_o)$ *was arbitrary, we have* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \succ_o$*.*

(2) $\Rightarrow$ *Since* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succ_o$ *then we have* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e,a) \notin \succeq_o$*. Since it holds that* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succeq_o$*, then, according to Property 3, e is not skeptically accepted. Since* $(\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(a'',e) \in \succ_o$*, then, according to the same property, e is not rejected. Since e is neither skeptically accepted nor rejected, according to Property 1, it is credulously accepted.*

$\Leftarrow$ *Let e be credulously accepted. Since e is credulously accepted, according to Property 1, it is neither skeptically accepted, nor rejected. Since e is not rejected, then, according to Property 3, it holds that* $(\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'',e) \in \succ_o$*. Since e is not skeptically accepted, then, according to the same property,* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succeq_o$*. Since* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e,a) \notin \succeq_o$ *then* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \notin \succ_o$*.*

(3) $\Rightarrow$ *Let* $(\exists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'',e) \in \succ_o$*. According to Property 3, e is rejected. Let us prove that* $\mathtt{Cr}(\mathcal{AF}_o) \subseteq \mathtt{Cr}(\mathcal{AF}_o \oplus e)$*. Suppose not. So,* $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))$ *such that a′ changes its status. Since, according to Lemma 11, no argument can become skeptically accepted, then a′ becomes rejected. According to Lemma 12, it holds that* $(e,a') \in \succ_o$*. Since* $(a'',e) \in \succ_o$ *and* $(e,a') \in \succ_o$ *then* $(a'',a') \in \succ_o$*. Since the preference relation between the arguments does not change, this means that* $(a'',a') \in \succ_o$ *was true in the moment when a′ and a″ were both credulously accepted. Contradiction with Property 5. So, we proved that e is rejected and that no other argument changes its status.*

$\Leftarrow$ *Let e be rejected. So, according to Lemma 5,* $(\exists a' \in \mathcal{A}_o)$ *such that* $(a',e) \in \succ_o$ *and* $a' \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. Since* $a' \neq e$ *then* $a' \in \mathcal{A}_o$*. So,*

$a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. Since $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$, then, according to Proposition 16, $a \notin \texttt{Rej}(\mathcal{AF}_o)$. Since $\texttt{Sc}(\mathcal{AF}_o) = \emptyset$, then $a \in \texttt{Cr}(\mathcal{AF}_o)$. So, $(\exists a' \in \texttt{Cr}(\mathcal{AF}_o))\ (a', e) \in \succ_o$.

■

Proposition 17 Let $\mathcal{AF}_o$ be a complete decision system, and $o \in \mathcal{O}_a(\mathcal{AF}_o)$.

- $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ iff $(\forall a \in \texttt{Sc}(\mathcal{AF}_o))\ ((a, e) \in \succeq'_o) \vee (e \in \mathcal{H}(o)) \wedge ((e, a) \in \succ'_o)$
- $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ iff $(\forall a \in \texttt{Sc}(\mathcal{AF}_o))\ ((a, e) \notin \succeq_o) \wedge ((e, a) \notin \succeq'_o))$
- $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ iff $(\forall a \in \texttt{Sc}(\mathcal{AF}_o))\ (e \notin \mathcal{H}(o)) \wedge (e, a) \in \succ'_o)$

Proof

(1) $\Rightarrow$ According to Definition 3, option o was acceptable, so there was already at least one skeptically accepted argument a' in its favor before receiving the new argument e. Suppose that the option o remains acceptable. According to Proposition 16 and Lemma 11, no argument can become skeptically accepted, then either some skeptically accepted argument in favor of o remained skeptically accepted or e is skeptically accepted and e is in favor of o. Let us explore the first possibility. So, $\exists a'' \in \mathcal{H}(o) \cap \texttt{Sc}(\mathcal{AF}_o \oplus e)$. The argument a'' remained skeptically accepted, so, according to Lemma 13, a will remain skeptically accepted as well. Since $(a'', e) \in \succeq_o$ and, according to Lemma 8, all the skeptically accepted arguments are in the same relation with e, then $(a, e) \in \succeq_o$. Suppose now that $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e) \cap \mathcal{H}(o)$. Since e is skeptically accepted, according to Lemma 14, we have $(e, a) \in \succeq_o$. If $(a, e) \in \succeq_o$ then the first part of the disjunction is true, i.e., $(a, e) \in \succeq_o$. If $(a, e) \notin \succeq_o$ then $(e, a) \in \succ_o$. So, the second part of the disjunction is true, i.e., $(e, a) \in \succ_o \wedge e \in \mathcal{H}(o)$.

$\Leftarrow$ Suppose now that $(a, e) \in \succeq_o \vee ((e, a) \in \succ_o \wedge e \in \mathcal{H}(o))$. Suppose that the first part of the disjunction is true, i.e., $(a, e) \in \succeq_o$. According to Lemma 14, $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Consequently, o remains acceptable. Suppose now that the second part of the disjunction is true, i.e., $(e, a) \in \succ_o \wedge e \in \mathcal{H}(o)$. Since $(e, a) \in \succ_o$, then, according to Lemma 14, $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Since $e \in \mathcal{H}(o)$ then o is acceptable.

(2) $\Rightarrow$ Since the option o becomes negotiable, according to the Definition 3, there is at least one credulously accepted argument in its favor. Proposition 16 states that rejected arguments cannot become credulously accepted. So, either a skeptically accepted argument a' in favor of o becomes credulously accepted or e is credulously accepted and e is in favor of o. The first possibility, with respect to Lemma 14, implies that $(a, e) \notin \succeq_o$ and $(e, a) \notin \succeq_o$. The second possibility, according to the same lemma, leads to the same conclusion: $(a, e) \notin \succeq_o$ and $(e, a) \notin \succeq_o$.

$\Leftarrow$ Let $(a, e) \notin \succeq_o \wedge (e, a) \notin \succeq_o$. Lemma 14 together with the fact that $(a, e) \notin \succeq_o \wedge (e, a) \notin \succeq_o$ lead to the conclusion that $a, e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Since we have $\texttt{Cr}(\mathcal{AF}_o \oplus e) \neq \emptyset$, according to Lemma 7, $\texttt{Sc}(\mathcal{AF}_o \oplus e) = \emptyset$.

So, there will be no skeptically accepted arguments in favor of o*, and there will be at least one credulously accepted argument in its favor. According to Definition 3,* o *becomes negotiable.*

(3) $\Rightarrow$ *Let* o *be an acceptable option that becomes rejected. The option* o *was acceptable, so, according to Definition 3, there was at least one skeptically accepted argument* a' *in its favor. Since* o *has become rejected, according to the same definition,* $\mathcal{H}(o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*, so* a' *must have become rejected. So,* a' *was not rejected but it is rejected now. Let* a'' *be an arbitrary skeptically accepted argument.* $a' \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*, so, according to Lemma 13,* $a'' \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. Since* a'' *has become rejected, the Lemma 12 implies that* $(e, a'') \in \succ_o$*. Let us now prove that* $e \notin \mathcal{H}(o)$*. Suppose that the converse,* $e \in \mathcal{H}(o)$*, is true. The fact* $(e, a) \in \succ_o$*, according to Lemma 14, implies that* e *is skeptically accepted. Since* $e \in \mathcal{H}(o)$*, then there is at least one skeptically accepted argument in favor of the option* o*, which, according to Definition 3, contradicts the fact that* o *became rejected. So, the assumption* $e \in \mathcal{H}(o)$ *is false. Hence,* $e \notin \mathcal{H}(o)$*.*

$\Leftarrow$ *Let* $(e, a) \in \succ_o \wedge e \notin \mathcal{H}(o)$*. The fact* $(e, a) \in \succ_o$*, according to Lemma 14, implies that* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *and* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. Let* a' *be the arbitrary skeptically accepted argument. According to Lemma 13,* a' *will become rejected, too. So, an arbitrary skeptically accepted argument becomes rejected. This means that all skeptically accepted arguments will become rejected,* $\mathtt{Sc}(\mathcal{AF}_o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. Since* $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$*, according to Property 6,* $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$*. According to Proposition 16, rejected arguments cannot change their status. Since there were no credulously accepted arguments and all skeptically accepted arguments became rejected and all the rejected arguments remain rejected, we conclude that all the arguments except* e *are rejected,* $\mathcal{A}_o \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$*. Recall that* $e \notin \mathcal{H}(o)$*. All the arguments in favor of* o *are rejected. Since there is at least one argument in favor of* o *and all the arguments in its favor are rejected, according to Definition 3,* o *is rejected.*

■

Proposition 18 Let $\mathcal{AF}_o$ be a complete decision system, and $o \in \mathcal{O}_n\mathcal{AF}$.

- $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ iff $(e \in \mathcal{H}(o)) \wedge (\forall a \in \mathtt{Cr}(\mathcal{AF}_o), (e, a) \in \succ'_o)$.
- $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ iff $((e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin \succ'_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in \succ'_o) \vee ((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a' \in \mathcal{H}(o) \wedge (e, a') \notin \succ'_o))$
- $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ iff $((e \notin \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow (e, a) \in \succ'_o))$

Proof

(1) $\Rightarrow$ *Let* o *become acceptable. According to Definition 3, this means that there will be at least one skeptically accepted argument in its favor. According to Proposition 16 and Lemma 11, no argument can become skep-*

tically accepted. So, in order to make o become acceptable, the new argument should be in favor of o. Hence, $e \in \mathcal{H}(o)$ *and* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Since e is skeptically accepted, then, according to Property 6,* $\mathtt{Cr}(\mathcal{AF}_o \oplus e) = \emptyset$. *So, all the credulously accepted arguments have changed their status. With respect to Lemma 11, they are all rejected. So, all arguments in* $\mathcal{A}_o \setminus \{e\}$ *are rejected. Lemma 6 states that in this case, e must be skeptically accepted. Since* $\mathtt{Cr}(\mathcal{AF}_o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, *then, according to Lemma 16,* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succ_o$.

$\Leftarrow$ *Let* $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succ_o)$. *The fact* $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \succ_o)$ *is, according to Lemma 16, equivalent to* $\mathtt{Cr}(\mathcal{AF}_o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *So, all the credulously accepted arguments have become rejected. There were no skeptically accepted arguments. According to the Proposition 16, all the rejected arguments remain rejected. So, all the arguments except e are rejected. According to the Lemma 6,* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Since* $(e \in \mathcal{H}(o))$, *then there is exactly one accepted argument in favor of the option o. According to Definition 3, o is acceptable.*

(2) $\Rightarrow$ *Let o stay negotiable. According to Lemma 10, this means that there is at least one credulously accepted argument in favor of o. If* $((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ a' \in \mathcal{H}(o) \wedge (e, a') \notin \succ_o)$ *the proof is over. Suppose that* $((\nexists a \in \mathtt{Cr}(\mathcal{AF}_o))\ a \in \mathcal{H}(o) \wedge (e, a) \notin \succ_o)$. *According to Proposition 16, all the rejected arguments remain rejected. Since* $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ a \in \mathcal{H}(o) \Rightarrow (e, a) \in \succ_o)$, *this means that for all the credulously accepted arguments in favor of o, it holds that* $(e, a) \in \succ_o$. *According to Property 3, this means that all the credulously accepted arguments in favor of o will become rejected. Since o remains negotiable, according to Lemma 10, this means that there is at least one credulously accepted argument in its favor. So, it must be the case that* $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ *and* $e \in \mathcal{H}(o)$. *According to Lemma 17, since e is credulously accepted then* $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin \succ_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in \succ_o$.

$\Leftarrow$ *Let* $(e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin \succ_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in \succ_o)$ *or* $((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ a' \in \mathcal{H}(o) \wedge (e, a') \notin \succ_o)$. *Suppose that* $(e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin \succ_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in \succ_o$). *According to Lemma 17,* $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$. *Since* $e \in \mathcal{H}(o)$, *according to the Lemma 10, o is negotiable. Let us now suppose that* $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ a' \in \mathcal{H}(o) \wedge (e, a') \notin \succ_o$ *is true. The fact* $(e, a') \notin \succ_o$ *and Lemma 12 imply that* $a' \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Since, according to Proposition 16 and Lemma 11, no argument cannot become skeptically accepted,* a' *is neither rejected nor skeptically accepted. According to Property 1, it is credulously accepted. Lemma 10 implies that o is negotiable.*

(3) $\Rightarrow$ *Since o becomes rejected, according to Definition 3, this means that* $\mathcal{H}(o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Suppose that* $(\exists a' \in \mathcal{H}(o) \cap \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin \succ_o$. *According to Lemma 12,* $a \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *So, there is at least one argument in favor of o which is not rejected. According to Definition 3, o is not rejected. Contradiction. Suppose now that* $e \in \mathcal{H}(o)$. *Since o is rejected, then* $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Since e is rejected, according to*

Lemma 5, $(\exists x' \in \mathcal{A}_o)$ $x' \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$ and $(x', e) \in \succ_o$. Since o was negotiable, $\mathcal{H}(o) \cap \texttt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Let $a'' \in \mathcal{H}(o) \cap \texttt{Cr}(\mathcal{AF}_o)$. It holds that $(\forall a \in \texttt{Cr}(\mathcal{AF}_o))$ $(a \in \mathcal{H}(o)) \Rightarrow ((e, a) \in \succ_o)$. In particular, $(e, a'') \in \succ_o$. It also holds that $(x', e) \in \succ_o$. From the transitivity of preference relation one can easily conclude that $(x', a'') \in \succ_o$. So, a'' was not self-defending in $\mathcal{AF}_o$ (before the agent has received the argument e), so $a'' \in \texttt{Rej}(\mathcal{AF}_o)$. Contradiction. So, $e \notin \mathcal{H}(o)$.

$\Leftarrow$ Since $(\forall a \in \texttt{Cr}(\mathcal{AF}_o))$ $(a \in \mathcal{H}(o) \Rightarrow (e, a) \in \succ_o)$, then, as a consequence of Lemma 12, $(\forall a \in \texttt{Cr}(\mathcal{AF}_o))$ $(a \in \mathcal{H}(o)) \Rightarrow a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$. So, $\texttt{Cr}(\mathcal{AF}_o) \cap \mathcal{H}(o) \subseteq \texttt{Rej}(\mathcal{AF}_o \oplus e)$ and, according to the Proposition 16, $\texttt{Rej}(\mathcal{AF}_o) \subseteq \texttt{Rej}(\mathcal{AF}_o \oplus e)$. So, since $e \notin \mathcal{H}(o)$, all the arguments in favor of o are rejected. Since o was negotiable, then $\mathcal{H}(o) \neq \emptyset$. So, according to Definition 3, o becomes rejected.

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Proposition 19 Let $\mathcal{AF}_o$ be a complete decision system, and $o \in \mathcal{O}_r(\mathcal{AF})$.

- $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ iff $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)$ $(e, a) \in \succeq'_o)$
- $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ iff $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)$ $(a, e) \notin \succ'_o) \wedge ((\exists a \in \mathcal{A}_o)$ $(e, a) \notin \succ'_o)$
- $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ iff $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in \succ'_o)$

Proof

(1) *$\Rightarrow$ Suppose that option o becomes acceptable. This means that there is at least one skeptically accepted argument in its favor. Since it was rejected, and, according to Proposition 16, all rejected arguments remain rejected, it must be that $e \in \mathcal{H}(o)$ and $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Property 3 now implies that $(\forall a \in \mathcal{A}_o)$ $(e, a) \in \succeq_o$.*

$\Leftarrow$ Suppose that $e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)$ $(e, a) \in \succeq_o$. According to Property 3, $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Since $e \in \mathcal{H}(o)$, we have one skeptically accepted argument in favor of option o, hence it is acceptable.

(2) *$\Rightarrow$ Suppose that option o becomes negotiable. According to Lemma 10, there is at least one credulously accepted argument in its favor. Since it was rejected, and, according to Proposition 16, all rejected arguments remain rejected, it must be that $e \in \mathcal{H}(o)$ and $e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. From Property 3, we have $((\forall a \in \mathcal{A}_o)$ $(a, e) \notin \succ_o) \wedge ((\exists a \in \mathcal{A}_o)$ $(e, a) \notin \succ_o)$.*

$\Leftarrow$ Suppose that $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)$ $(a, e) \notin \succ_o) \wedge ((\exists a \in \mathcal{A}_o)$ $(e, a) \notin \succ_o)$. According to Property 3, $e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. Since $e \in \mathcal{H}(o)$, we have one credulously accepted argument in favor of option o, which together with Lemma 10 means that o is negotiable.

(3) *$\Rightarrow$ Suppose that option o stays rejected. This means that all arguments in its favor are rejected. If $e \notin \mathcal{H}(o)$ the proof is over. Let us suppose that $e \in \mathcal{H}(o)$. Since $e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ then Property 3 implies that $(\exists a \in \mathcal{A}_o)(a, e) \in \succ_o$.*

$\Leftarrow$ *Let* $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in \succ_o)$. *If* $e \notin \mathcal{H}(o)$, *then, according to Proposition 16, all rejected arguments remain rejected, so the option remains rejected. Else, if* $e \in \mathcal{H}(o)$ *then* $(\exists a \in \mathcal{A}_o)(a, e) \in \succ_o$. *Property 3 implies that* e *is rejected, so with* $\mathcal{H}(o) \subseteq \texttt{Rej}(\mathcal{AF}_o)$, o *is rejected.*

■