Preemption Operators

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Abstract. We introduce a family of operators for belief change that aim at making a new piece of information to be preemptive so that any former belief subsuming it is given up. That is, the current belief base is to be altered even in the case that it is logically consistent with the new piece of information. Existing operators for belief revision are inadequate for this purpose because they amount to set-theoretic union in a contradiction-free case. We propose a series of postulates for such preemption operators. We show that a preemption operator can be defined as a multiple contraction followed by an expansion, drawing on operators from belief revision.

1 INTRODUCTION

Formalizing belief change is a major topic in Artificial Intelligence. Belief revision is dedicated to the special case that a new piece of information must be taken into account, as a statement to be inserted in the belief base. Should the current belief base be contradicted by the new piece of information, then the current belief base must undergo some modifications before it can simply be unioned with the new piece of information, resulting in a new current belief base.

The AGM setting gives a logic-based characterization of revision operators via a list of postulates that a "rational" revision operator is meant to satisfy [1, 5]. Two of the postulates, *vacuity* and *inclusion*, when taken together, enforce the property that the belief base is simply supplemented with the new piece of information in the case that the latter is logically consistent with the belief base: in such a case, no information is to be expelled from the belief base (see Appendix). However, should the new piece of information be preemptive in a belief base that it can be deduced from, then some information must be taken out –this may happen to be necessary even though the current belief base is logically consistent with the new piece of information.

An illustration is as follows. Assume that the current belief base expresses "Paul is in his office or at home". Consider the situation that the information "Paul is in his office or at home or at his club" is then provided. In some respects, if the information "Paul is in his office or at home or at his club" is now at hand, it presumably should take precedence over the former information. That is, "Paul is in his office or at home" should no longer be deduced from the belief base. The new piece of information "Paul is in his office or at home or at his club" conveys some uncertainty that Paul's office or home are where he is right now. Yet, from a purely logical viewpoint, the new piece of information "Paul is in his office or at home or at his club" does not contradict the current belief base. Moreover, "Paul is in his office or at home or at his club" provides per se no means to obviate "Paul is in his office or at home" from which it can be deduced.

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Here is another illustration. Assume that "If Dana agrees then we begin tomorrow" is in the current belief base. Presumably, a new, incoming, belief stating that "If Dana and Alexander agree then we begin tomorrow" is meant to rule out the former belief although they do not form a contradiction in terms of classical logic.

There recently has been some work dealing with this, in a classical logic setting [2] and in a non-monotonic logic setting [3]. It is shown there that expelling from the belief base every piece of information f entailing (possibly through other information from the belief base) the preemptive information g is not enough. The way the problem is addressed in [2, 3] is to apply contraction of the current belief base by $g \Rightarrow f$, for prime implicants f of g, then add g.

This paper is a first attempt at providing postulates for such preemption operators. It is also shown that a preemption operator can be alternatively defined as multiple contraction [4] (of appropriate formulas) followed by expansion.

We consider classical logic throughout. We assume a propositional language $\mathcal{L}$ built via the connectives $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), and $\Rightarrow$ (material implication). A literal is a propositional variable or its negation. A *clausal formula* (called a *clause*) is a finite disjunction of literals. Lowercase letters denote formulas of $\mathcal{L}$ whereas uppercase letters denote sets of formulas, these being called *belief bases*. $\vdash$ denotes deduction, i.e., $A \vdash p$ denotes that p is a logical consequence of A. A *theory* A is a deductively closed set of formulas, $A = \{p \mid A \vdash p\}$. It is assumed throughout that belief bases are deductively closed. Two formulas p and q are logically equivalent, written $p \equiv q$, iff $p \vdash q$ and $q \vdash p$. $\vdash p$ means that p is tautologous and $\vdash \neg p$ that p is a contradiction. $\top$ stands for a tautology and $\bot$ stands for a contradiction. As usual, a set of formulas A is consistent iff $A \nvdash \bot$. $K_\bot$ is the trivial belief base, i.e., it consists of all formulas of $\mathcal{L}$. $K_\top$ is the tautologous belief base, i.e., it consists of the tautologous formulas of $\mathcal{L}$. The concept of prime implicant is central in this paper : f is a strict implicant of g iff $f \vdash g$ and $g \nvdash f$.

2 POSTULATES

Let K be a consistent belief base and g a clause. Let $+$ be an AGM expansion operator [1, 5] (see Appendix). Preemption by g over K is denoted $K \circledast g$. Here is a tentative list of postulates for $\circledast$.

($\mathbf{K} \circledast \mathbf{1}$) $K \circledast g$ is a theory. **(closure)**

($\mathbf{K} \circledast \mathbf{2}$) $g \in K \circledast g$. **(success of insertion)**

($\mathbf{K} \circledast \mathbf{3}$) $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**

($\mathbf{K} \circledast \mathbf{4}$) $K \circledast g \subseteq K + g$. **(inclusion)**

($\mathbf{K} \circledast \mathbf{5}$) If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**

($\mathbf{K} \circledast \mathbf{6}$) If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Similarly with revision, $(\mathbf{K} \circledast \mathbf{1})$ shows that the output of preemption is required to be deductived closed. $(\mathbf{K} \circledast \mathbf{2})$ shows that the new piece of information is meant to be part of the resulting belief base. $(\mathbf{K} \circledast \mathbf{3})$ is in contrast with revision. Here, no clausal strict implicant f of g is allowed in the resulting belief base. If g is a contradiction, no formula of $\mathcal{L}$ is a strict implicant of g according to classical logic, hence the postulate vacuously holds despite Property 1 below. Observe that $(\mathbf{K} \circledast \mathbf{3})$ cannot be extended to all strict implicants of g because, together with $(\mathbf{K} \circledast \mathbf{1})$ and $(\mathbf{K} \circledast \mathbf{2})$, it would entail that $K \circledast g$ be logically equivalent with g (see Property 3). $(\mathbf{K} \circledast \mathbf{4})$ shows that preempting never introduces beliefs beyond those in (the deductive closure of) the expansion of K by g. $(\mathbf{K} \circledast \mathbf{5})$ shows that if no $g \Rightarrow f$ is in K, whatever f clausal strict implicant of g, then preempting amounts to expanding K by g. Finally, $(\mathbf{K} \circledast \mathbf{6})$, similarly with revision, shows that the outcome of preempting does not depend on the "syntax" of g.

Property 1. *Let $\circledast$ satisfy $(\mathbf{K} \circledast \mathbf{1})$-$(\mathbf{K} \circledast \mathbf{3})$. Then, $K \circledast g = K_\perp$ iff $\vdash \neg g$.*

Property 2. *Let $\circledast$ satisfy $(\mathbf{K} \circledast \mathbf{2})$-$(\mathbf{K} \circledast \mathbf{3})$. Then, if $\vdash g$ then $K \circledast g = K_\top$.*

Property 3. *Let $\circledast$ satisfy $(\mathbf{K} \circledast \mathbf{1})$-$(\mathbf{K} \circledast \mathbf{2})$. Then, $f \notin K \circledast g$ for all strict implicants f of g iff $K \circledast g$ is logically equivalent with g.*

Property 4. *If $g \Rightarrow f \notin K$ for all clausal strict implicants f of g then $\neg g \notin K$.*

Property 5. *Let K and g be such that $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g. Then, there exist no clausal strict implicants i and j of g such that $(i \vee j) \not\equiv g$ and $(g \Rightarrow i) \vee (g \Rightarrow j) \in K$.*

Property 1 shows that the only way a trivial belief base results from preempting is by means of preempting by a contradiction. Property 2 states that if g is tautologous, then the outcome of preempting is a tautologous belief base. Property 3 formally states the case mentioned in the comment concerning $(\mathbf{K} \circledast \mathbf{3})$. Property 4 shows why $(\mathbf{K} \circledast \mathbf{5})$ does not require a proviso about the negation of g not to be in K (please observe that such a proviso occurs in the corresponding postulate for revision operators). Property 5 shows that it is otiose to check in $(\mathbf{K} \circledast \mathbf{5})$ that disjunctions of $g \Rightarrow f_i$ (for distinct clausal strict implicants f_i's of g) are not in K.

3 CHARACTERIZATION

According to [2, 3], similarly to Levi's identity [5] defining revision as contraction followed by expansion, a preemption operation could be captured as multiple contraction followed by expansion. As given by Fuhrmann and Hansson [4], multiple contraction permits to contract a belief base K by a set of information Λ, written $K \ominus \Lambda$, so that no information of Λ can be inferred from $K \ominus \Lambda$ (see Appendix).

Definition 1 ($|||$ operator)**.** *Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.*

$$K \mathbin{|||} g = (K \ominus \{g \Rightarrow f_i\}_{i=1,2,..}) + g.$$

Theorem 1. *If $\ominus$ satisfies $(K \ominus 1) - (K \ominus 4)$ and $(K \ominus 6)$, and if $+$ satisfies $(K + 1) - (K + 6)$, then $|||$ satisfies $(K \circledast 1) - (K \circledast 6)$.*

Theorem 2. *Every $\circledast$ operator satisfying $(K \circledast 1) - (K \circledast 6)$ can be written as an $|||$ operator s.t. $\ominus$ satisfies $(K \ominus 1) - (K \ominus 4)$ and $(K \ominus 6)$, and $+$ satisfies $(K + 1) - (K + 6)$.*

ACKNOWLEDGMENTS

Part of this work has been supported by the *Région Nord/Pas-de-Calais* and the EC under a FEDER project.

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A AGM OPERATORS

Let K be a consistent base, Λ be a set of formulas and g be a formula.

Expansion *[1]*
The postulates for the expansion of K by g, denoted $K + g$, are:

$(\mathbf{K+1})$	*$K + g$ is a theory.*	***(closure)***
$(\mathbf{K+2})$	*$g \in K + g$.*	***(success)***
$(\mathbf{K+3})$	*$K \subseteq K + g$.*	***(inclusion)***
$(\mathbf{K+4})$	*If $g \in K$ then $K + g = K$.*	***(vacuity)***
$(\mathbf{K+5})$	*If $K \subseteq H$ then $K + g \subseteq H + g$.*	***(monotony)***
$(\mathbf{K+6})$	*$K + g$ is the smallest set satisfying $(K + 1)$ to $(K + 5)$.*	***(minimality)***

Revision *[1]*
The postulates for the revision of K by g, denoted $K * g$, are:

$(\mathbf{K*1})$	*$K * g$ is a theory.*	***(closure)***
$(\mathbf{K*2})$	*$g \in K * g$.*	***(success)***
$(\mathbf{K*3})$	*$K * g \subseteq K + g$.*	***(inclusion)***
$(\mathbf{K*4})$	*If $\neg g \notin K$ then $K + g \subseteq K * g$.*	***(vacuity)***
$(\mathbf{K*5})$	*$K * g = K_\perp$ iff $\vdash \neg g$.*	***(consistent)***
$(\mathbf{K*6})$	*If $g \equiv h$ then $K * g = K * h$.*	***(extensionality)***
$(\mathbf{K*7})$	*$K * (g \wedge h) \subseteq (K * g) + h$.*	***(conjunctive inclusion)***
$(\mathbf{K*8})$	*If $\neg h \notin K * g$ then $(K * g) + h \subseteq K * (g \wedge h)$.*	***(conjunctive vacuity)***

$(K*7)$-$(K*8)$ are additional postulates devoted to minimal change.

Multiple contraction *[4]*
The postulates for the multiple contraction of K by Λ, $K \ominus \Lambda$, are:

$(\mathbf{K\ominus 1})$	*$K \ominus \Lambda$ is a theory.*	***(closure)***
$(\mathbf{K\ominus 2})$	*$K \ominus \Lambda \subseteq K$.*	***(inclusion)***
$(\mathbf{K\ominus 3})$	*If $\Lambda \cap K = \emptyset$ then $K \ominus \Lambda = K$.*	***(vacuity)***
$(\mathbf{K\ominus 4})$	*If $\Lambda \cap Cn(\emptyset) = \emptyset$ then $\Lambda \cap (K \ominus \Lambda) = \emptyset$.*	***(success)***
$(\mathbf{K\ominus 5})$	*$K \subseteq Cn((K \ominus \Lambda) \cup \Lambda)$.*	***(recovery)***
$(\mathbf{K\ominus 6})$	*If $\Lambda \cong \Theta$ then $K \ominus \Lambda = K \ominus \Theta$.*	***(extensionality)***
$(\mathbf{K\ominus 7})$	*$(K \ominus \Lambda) \cap (K \ominus \Theta) \subseteq K \ominus (\Lambda \cap \Theta)$.*	***(intersection)***
$(\mathbf{K\ominus 8})$	*If $\varphi \notin K \ominus \Theta$ then $K \ominus \Theta \subseteq K \ominus (\Theta \cup \{\varphi\})$.*	***(non-deterioration)***
$(\mathbf{K\ominus 9})$	*If $\Lambda \cap (K \ominus \Theta) = \emptyset$ then $K \ominus \Theta \subseteq K \ominus (\Lambda \cup \Theta)$.*	***(conjonction)***

$\Lambda \cong \Theta$ means that for every element of Λ there exists a logically equivalent element of Θ, and vice versa*. Also, $(K \ominus 7)$, $(K \ominus 8)$ and $(K \ominus 9)$ are additional postulates devoted to minimal change.*