Preemption Operators

Philippe Besnard
IRIT, CNRS UMR 5505
Toulouse, France
besnard@irit.fr

Éric Grégoire **Sébastien Ramon**
Université Lille-Nord de France, Artois
CRIL, CNRS UMR 8188
Lens, France
{gregoire,ramon}@cril.fr

ABSTRACT

We introduce a family of ***operators for belief change that aim at making a new piece of information to be preemptive so that any former belief subsuming it is given up****. That is, the current belief base is to be altered even in the case that it is logically consistent with the new piece of information.* ***Existing operators for belief revision are inadequate for this purpose*** *because they amount to set-theoretic union in a contradiction-free case.* ***We propose a series of postulates for such preemption operators. We show that a preemption operator can be defined as a multiple contraction followed by an expansion****, drawing on operators from belief revision.*

1. INTRODUCTION

Formalizing **belief change** is a major topic in Artificial Intelligence. **Belief revision is dedicated to the special case that a new piece of information must be taken into account**, as a statement to be inserted in the belief base.

Should the current belief base be contradicted by the new piece of information, then the current belief base must undergo some modifications before it can simply be unioned with the new piece of information, resulting in a new current belief base.

However, **should the new piece of information be preemptive in a belief base that it can be deduced from**, then some information must be taken out –this may happen to be necessary even though the current belief base is logically consistent with the new piece of information.

2. PRELIMINARIES

We consider **classical logic** throughout. We assume a **propositional language** $\mathcal{L}$ built via the **connectives** $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), and $\Rightarrow$ (material implication). A **literal** is a propositional variable or its negation. A **clausal formula** (called a *clause*) is a finite disjunction of literals. Lowercase letters denote formulas of $\mathcal{L}$ whereas uppercase letters denote sets of formulas, these being called **belief bases**.

$\vdash$ denotes **deduction**, i.e., $A \vdash p$ denotes that p is a **logical consequence** of A. A **theory** A is a deductively closed set of formulas, $A = \{p \mid A \vdash p\}$. It is assumed throughout that belief bases are deductively closed. Two formulas p and q are **logically equivalent**, written $p \equiv q$, iff $p \vdash q$ and $q \vdash p$. $\vdash p$ means that p is **tautologous** and $\vdash \neg p$ that p is a **contradiction**. $\top$ stands for a **tautology** and $\bot$ stands for a **contradiction**. As usual, a set of formulas A is **consistent** iff $A \nvdash \bot$. $K_\bot$ is the **trivial belief base**, i.e., it consists of all formulas of $\mathcal{L}$. $K_\top$ is the **tautologous belief base**, i.e., it consists of the tautologous formulas of $\mathcal{L}$.

The concept of **prime implicant** is central in this paper : f is a **strict implicant** of g iff $f \vdash g$ and $g \nvdash f$.

3. PREEMPTION (BGR11b, BGR11a)

- **Context** : Insertion of a new piece of information g into a belief base K **consistent with** g.
- **Aim** : Giving up any belief in K **subsuming** g.
- **Solution** : **Contract** $\mathbf{g \Rightarrow f}$, for all strict implicants f of g, then add g.

Example 1 *Let K be a consistent belief base s.t. :*

1. $paul\_at\_office \vee paul\_at\_home$,

g $paul\_at\_office \vee paul\_at\_home \vee paul\_at\_club$.

Example 2 *Let K be a consistent belief base s.t. :*

1. $dana\_agrees \Rightarrow we\_begin\_tomorrow$,

g $(dana\_agrees \wedge alexander\_agrees) \Rightarrow we\_begin\_tomorrow$.

➠ g conveys some **uncertainty** that *Paul's office or home* are where he is now (Example 1), and that *Dana's agreement* is now sufficient (Example 2).

➠ The belief in K subsuming g should **no longer be deduced** from K.

➠ Note that **only contract** f**, for all strict implicants** f **of** g **is not enough** : introducing g might enable strict implicants f of g!

4. PREEMPTION VS. REVISION (AGM85)

Revision (AGM85)
The postulates for the revision of K by g, denoted $K * g$, are:

$(\mathbf{K*1})$	$K * g$ is a theory.	**(closure)**
$(\mathbf{K*2})$	$g \in K * g$.	**(success)**
$(\mathbf{K*3})$	$K * g \subseteq K + g$.	**(inclusion)**
$(\mathbf{K*4})$	If $\neg g \notin K$ then $K + g \subseteq K * g$.	**(vacuity)**
$(\mathbf{K*5})$	$K * g = K_\bot$ iff $\vdash \neg g$.	**(consistent)**
$(\mathbf{K*6})$	If $g \equiv h$ then $K * g = K * h$.	**(extensionality)**
$(\mathbf{K*7})$	$K * (g \wedge h) \subseteq (K * g) + h$.	**(conjunctive inclusion)**
$(\mathbf{K*8})$	If $\neg h \notin K * g$ then $(K * g) + h \subseteq K * (g \wedge h)$.	**(conjunctive vacuity)**

$(K*7)$-$(K*8)$ are additional postulates devoted to minimal change.

➠ $(\mathbf{K*4})$ expresses that no information must be expelled when K and g are **not contradictory**.

➠ **Existing operators for revision are inadequate for preemption.**

5. POSTULATES

Preemption
The postulates for the preemption of g in K, denoted $K \circledast g$, are:

$(\mathbf{K\circledast 1})$	$K \circledast g$ is a theory.	**(closure)**
$(\mathbf{K\circledast 2})$	$g \in K \circledast g$.	**(success of insertion)**
$(\mathbf{K\circledast 3})$	$f \notin K \circledast g$ for all clausal strict implicants f of g.	**(success of preemption)**
$(\mathbf{K\circledast 4})$	$K \circledast g \subseteq K + g$.	**(inclusion)**
$(\mathbf{K\circledast 5})$	If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$.	**(vacuity)**
$(\mathbf{K\circledast 6})$	If $g \equiv h$ then $K \circledast g = K \circledast h$.	**(extensionality)**

➠ $(\mathbf{K\circledast 1})$, $(\mathbf{K\circledast 2})$, $(\mathbf{K\circledast 4})$ and $(\mathbf{K\circledast 6})$ are **shared with revision**.
$(\mathbf{K\circledast 1})$ shows that the output of preemption is required to be deductived closed.
$(\mathbf{K\circledast 2})$ shows that the new piece of information is meant to be part of the resulting belief base.
$(\mathbf{K\circledast 4})$ shows that preempting never introduces beliefs beyond those in (the deductive closure of) the expansion of K by g.
$(\mathbf{K\circledast 6})$ shows that the outcome of preempting does not depend on the "syntax" of g.

➠ $(\mathbf{K\circledast 3})$ is **in contrast with revision**.
$(\mathbf{K\circledast 3})$ expresses that no **clausal strict implicant** f of g is allowed in the output of preemption.
$(\mathbf{K\circledast 3})$ cannot be extended to **all strict implicants** of g.

Property 3 *Let $\circledast$ satisfy $(\mathbf{K\circledast 1})$-$(\mathbf{K\circledast 2})$. Then, $f \notin K \circledast g$ for all strict implicants f of g iff $K \circledast g$ is logically equivalent with g.*

➠ $(\mathbf{K\circledast 5})$ is **in contrast with revision**.
$(\mathbf{K\circledast 5})$ expresses that if no $g \Rightarrow f$ is in K, preemption amounts to expansion.
$(\mathbf{K\circledast 5})$ does not require a proviso about the negation of g not to be in K (idem about the disjunctions of $g \Rightarrow f_i$ not to be in K, for distinct clausal strict implicants f_i's of g).

Property 4 *If $g \Rightarrow f \notin K$ for all clausal strict implicants f of g then $\neg g \notin K$.*

Property 5 *Let K and g be such that $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g. Then, there exist no clausal strict implicants i and j of g such that $(i \vee j) \not\equiv g$ and $(g \Rightarrow i) \vee (g \Rightarrow j) \in K$.*

➠ Two others interesting properties!

Property 1 *Let $\circledast$ satisfy $(\mathbf{K\circledast 1})$-$(\mathbf{K\circledast 3})$. Then, $K \circledast g = K_\bot$ iff $\vdash \neg g$.*

Property 2 *Let $\circledast$ satisfy $(\mathbf{K\circledast 2})$-$(\mathbf{K\circledast 3})$. Then, if $\vdash g$ then $K \circledast g = K_\top$.*

6. CHARACTERIZATION

- Preemption could be approximated as **multiple contraction followed by expansion** (similarly to Levi's identity (Gä88)).
- Multiple contraction (FH94) permits **to contract by a set of information** $(K \ominus \Lambda)$.

Definition 1 ($|||$ operator) *Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.*

$$K \mathbin{|||} g = (K \ominus \{g \Rightarrow f_i\}_{i=1,2,..}) + g.$$

Theorem 1 *If $\ominus$ satisfies $(K\ominus 1)-(K\ominus 4)$ and $(K\ominus 6)$, and if $+$ satisfies $(K+1)-(K+6)$, then $|||$ satisfies $(K\circledast 1)-(K\circledast 6)$.*

Theorem 2 *Every $\circledast$ operator satisfying $(K\circledast 1)-(K\circledast 6)$ can be written as an $|||$ operator s.t. $\ominus$ satisfies $(K\ominus 1)-(K\ominus 4)$ and $(K\ominus 6)$, and $+$ satisfies $(K+1)-(K+6)$.*

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APPENDIX

Let K be a consistent base, Λ be a set of formulas and g be a formula.

Expansion (AGM85)
The postulates for the expansion of K by g, denoted $K + g$, are:

$(\mathbf{K+1})$	$K + g$ is a theory.	**(closure)**
$(\mathbf{K+2})$	$g \in K + g$.	**(success)**
$(\mathbf{K+3})$	$K \subseteq K + g$.	**(inclusion)**
$(\mathbf{K+4})$	If $g \in K$ then $K + g = K$.	**(vacuity)**
$(\mathbf{K+5})$	If $K \subseteq H$ then $K + g \subseteq H + g$.	**(monotony)**
$(\mathbf{K+6})$	$K + g$ is the smallest set satisfying $(K+1)$ to $(K+5)$.	**(minimality)**

Multiple contraction (FH94)
The postulates for the multiple contraction of K by Λ, $K \ominus \Lambda$, are:

$(\mathbf{K\ominus 1})$	$K \ominus \Lambda$ is a theory.	**(closure)**
$(\mathbf{K\ominus 2})$	$K \ominus \Lambda \subseteq K$.	**(inclusion)**
$(\mathbf{K\ominus 3})$	If $\Lambda \cap K = \emptyset$ then $K \ominus \Lambda = K$.	**(vacuity)**
$(\mathbf{K\ominus 4})$	If $\Lambda \cap Cn(\emptyset) = \emptyset$ then $\Lambda \cap (K \ominus \Lambda) = \emptyset$.	**(success)**
$(\mathbf{K\ominus 5})$	$K \subseteq Cn((K \ominus \Lambda) \cup \Lambda)$.	**(recovery)**
$(\mathbf{K\ominus 6})$	If $\Lambda \cong \Theta$ then $K \ominus \Lambda = K \ominus \Theta$.	**(extensionality)**
$(\mathbf{K\ominus 7})$	$(K \ominus \Lambda) \cap (K \ominus \Theta) \subseteq K \ominus (\Lambda \cap \Theta)$.	**(intersection)**
$(\mathbf{K\ominus 8})$	If $\varphi \notin K \ominus \Theta$ then $K \ominus \Theta \subseteq K \ominus (\Theta \cup \{\varphi\})$.	**(non-deterioration)**
$(\mathbf{K\ominus 9})$	If $\Lambda \cap (K \ominus \Theta) = \emptyset$ then $K \ominus \Theta \subseteq K \ominus (\Lambda \cup \Theta)$.	**(conjonction)**

$\Lambda \cong \Theta$ means that for every element of Λ there exists a logically equivalent element of Θ, and *vice versa*. Also, $(K\ominus 7)$, $(K\ominus 8)$ and $(K\ominus 9)$ are additional postulates devoted to minimal change.