Preemption Operators

Ph. Besnard[1] **É. Grégoire**[2] **S. Ramon**[2]

[1] IRIT CNRS (Toulouse, France)

[2] Université d'Artois, CRIL CNRS (Lens, France)

14th International Workshop on **N**on-**M**onotonic **R**easoning

8th June 2012 - Roma

Guideline

Guideline

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

➠ Preemption is quite **different from revision**

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **inconsistent with** g
- Aim : Giving up any belief in K **contradicting** g

➠ Preemption is quite **different from revision**

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 1 : Let K be a consistent belief base

1. $paul\_at\_office \vee paul\_at\_home$

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 1 : Let K be a consistent belief base

1 $paul\_at\_office \vee paul\_at\_home$

g $paul\_at\_office \vee paul\_at\_home \vee paul\_at\_club$

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 1 : Let K be a consistent belief base

1. $paul\_at\_office \vee paul\_at\_home$

g $paul\_at\_office \vee paul\_at\_home \vee paul\_at\_club$

➠ g conveys some **uncertainty** that *Paul's office or home* are where he is now

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 1 : Let K be a consistent belief base

1. ~~$paul\_at\_office \vee paul\_at\_home$~~

g $paul\_at\_office \vee paul\_at\_home \vee paul\_at\_club$

➠ The belief in K subsuming g should **no longer be deduced** from K

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 2 : Let K be a consistent belief base

1. $dana\_agrees \Rightarrow we\_begin\_tomorrow$

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 2 : Let K be a consistent belief base

1. $dana\_agrees \Rightarrow we\_begin\_tomorrow$

g $(dana\_agrees \wedge alexander\_agrees) \Rightarrow we\_begin\_tomorrow$

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 2 : Let K be a consistent belief base

1. $dana\_agrees \Rightarrow we\_begin\_tomorrow$

g $(dana\_agrees \wedge alexander\_agrees) \Rightarrow we\_begin\_tomorrow$

➠ g conveys some **uncertainty** that *Dana's agreement* is now sufficient

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Example 2 : Let K be a consistent belief base

1. ~~$dana\_agrees \Rightarrow we\_begin\_tomorrow$~~

g $(dana\_agrees \wedge alexander\_agrees) \Rightarrow we\_begin\_tomorrow$

➠ The belief in K subsuming g should **no longer be deduced** from K

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g
- Solution : Contract f, for all strict implicants f of g, then add g

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g
- Solution : **Contract f**, for all strict implicants f of g, then add g

➠ **Not enough** : introducing g might **enable strict implicants** f of g

Preemption [BGR11b, BGR11a]

- Context : Insertion of a new piece of information g into a belief base K **consistent with** g
- Aim : Giving up any belief in K **subsuming** g
- Solution : **Contract g** $\Rightarrow$ **f**, for all strict implicants f of g, then add g

Preemption vs. Revision [AGM85]

$K * g$: revision of K (a consistent belief base) by g (a formula)

$(\mathbf{K} * \mathbf{1})$	$K * g$ is a theory.	**(closure)**
$(\mathbf{K} * \mathbf{2})$	$g \in K * g$.	**(success)**
$(\mathbf{K} * \mathbf{3})$	$K * g \subseteq K + g$.	**(inclusion)**
$(\mathbf{K} * \mathbf{4})$	If $\neg g \notin K$ then $K + g \subseteq K * g$.	**(vacuity)**
$(\mathbf{K} * \mathbf{5})$	$K * g = K_\perp$ iff $\vdash \neg g$.	**(consistent)**
$(\mathbf{K} * \mathbf{6})$	If $g \equiv h$ then $K * g = K * h$.	**(extensionality)**

Existing operators for revision are inadequate for preemption

Preemption vs. Revision [AGM85]

$K * g$: revision of K (a consistent belief base) by g (a formula)

$(\mathbf{K} * \mathbf{1})$	$K * g$ is a theory.	**(closure)**
$(\mathbf{K} * \mathbf{2})$	$g \in K * g$.	**(success)**
$(\mathbf{K} * \mathbf{3})$	$K * g \subseteq K + g$.	**(inclusion)**
$(\mathbf{K} * \mathbf{4})$	If $\neg g \notin K$ then $K + g \subseteq K * g$.	**(vacuity)**
$(\mathbf{K} * \mathbf{5})$	$K * g = K_{\perp}$ iff $\vdash \neg g$.	**(consistent)**
$(\mathbf{K} * \mathbf{6})$	If $g \equiv h$ then $K * g = K * h$.	**(extensionality)**

➠ $(\mathbf{K} * \mathbf{4})$ expresses that no information must be expelled when K and g are not contradictory

Guideline

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **(K $\circledast$ 1)** $K \circledast g$ is a theory. **(closure)**
- **(K $\circledast$ 2)** $g \in K \circledast g$. **(success of insertion)**
- **(K $\circledast$ 3)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **(K $\circledast$ 4)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **(K $\circledast$ 5)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **(K $\circledast$ 6)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

$(\mathbf{K} \circledast \mathbf{1})$	$K \circledast g$ is a theory.	**(closure)**
$(\mathbf{K} \circledast \mathbf{2})$	$g \in K \circledast g$.	**(success of insertion)**
$(\mathbf{K} \circledast \mathbf{3})$	$f \notin K \circledast g$ for all clausal strict implicants f of g.	**(success of preemption)**
$(\mathbf{K} \circledast \mathbf{4})$	$K \circledast g \subseteq K + g$.	**(inclusion)**
$(\mathbf{K} \circledast \mathbf{5})$	If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$.	**(vacuity)**
$(\mathbf{K} \circledast \mathbf{6})$	If $g \equiv h$ then $K \circledast g = K \circledast h$.	**(extensionality)**

➠ $(\mathbf{K} \circledast \mathbf{1})$, $(\mathbf{K} \circledast \mathbf{2})$, $(\mathbf{K} \circledast \mathbf{4})$ and $(\mathbf{K} \circledast \mathbf{6})$ are shared with revision

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **($\mathbf{K} \circledast \mathbf{1}$)** $K \circledast g$ is a theory. **(closure)**
- **($\mathbf{K} \circledast \mathbf{2}$)** $g \in K \circledast g$. **(success of insertion)**
- **($\mathbf{K} \circledast \mathbf{3}$)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **($\mathbf{K} \circledast \mathbf{4}$)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **($\mathbf{K} \circledast \mathbf{5}$)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **($\mathbf{K} \circledast \mathbf{6}$)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

➡ Only ($\mathbf{K} \circledast \mathbf{3}$) and ($\mathbf{K} \circledast \mathbf{5}$) are in contrast with revision

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **($\mathbf{K} \circledast \mathbf{1}$)** $K \circledast g$ is a theory. **(closure)**
- **($\mathbf{K} \circledast \mathbf{2}$)** $g \in K \circledast g$. **(success of insertion)**
- **($\mathbf{K} \circledast \mathbf{3}$)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **($\mathbf{K} \circledast \mathbf{4}$)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **($\mathbf{K} \circledast \mathbf{5}$)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **($\mathbf{K} \circledast \mathbf{6}$)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

($\mathbf{K} \circledast \mathbf{1}$)	$K \circledast g$ is a theory.	**(closure)**
($\mathbf{K} \circledast \mathbf{2}$)	$g \in K \circledast g$.	**(success of insertion)**
($\mathbf{K} \circledast \mathbf{3}$)	$f \notin K \circledast g$ for all clausal strict implicants f of g.	**(success of preemption)**
($\mathbf{K} \circledast \mathbf{4}$)	$K \circledast g \subseteq K + g$.	**(inclusion)**
($\mathbf{K} \circledast \mathbf{5}$)	If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$.	**(vacuity)**
($\mathbf{K} \circledast \mathbf{6}$)	If $g \equiv h$ then $K \circledast g = K \circledast h$.	**(extensionality)**

➡ ($\mathbf{K} \circledast \mathbf{3}$) expresses that no **clausal strict implicant** f of g is allowed

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **(K $\circledast$ 1)** $K \circledast g$ is a theory. **(closure)**
- **(K $\circledast$ 2)** $g \in K \circledast g$. **(success of insertion)**
- **(K $\circledast$ 3)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **(K $\circledast$ 4)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **(K $\circledast$ 5)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **(K $\circledast$ 6)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

➡ (**K** $\circledast$ **3**) cannot be extended to **all strict implicants** of g

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

(K ⊛ 1)	$K \circledast g$ is a theory.	**(closure)**
(K ⊛ 2)	$g \in K \circledast g$.	**(success of insertion)**
(K ⊛ 3)	$f \notin K \circledast g$ for all clausal strict implicants f of g.	**(success of preemption)**
(K ⊛ 4)	$K \circledast g \subseteq K + g$.	**(inclusion)**
(K ⊛ 5)	If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$.	**(vacuity)**
(K ⊛ 6)	If $g \equiv h$ then $K \circledast g = K \circledast h$.	**(extensionality)**

Property 3 : Let $\circledast$ satisfy (**K ⊛ 1**) and (**K ⊛ 2**).

$f \notin K \circledast g$ for all strict implicants f of g iff $K \circledast g$ is logically eq. with g.

➡ (**K ⊛ 3**) cannot be extended to **all strict implicants** of g

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

(**K** $\circledast$ **1**) $K \circledast g$ is a theory. **(closure)**

(**K** $\circledast$ **2**) $g \in K \circledast g$. **(success of insertion)**

(**K** $\circledast$ **3**) $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**

(**K** $\circledast$ **4**) $K \circledast g \subseteq K + g$. **(inclusion)**

(**K** $\circledast$ **5**) If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**

(**K** $\circledast$ **6**) If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

(**K** $\circledast$ **1**) $K \circledast g$ is a theory. **(closure)**

(**K** $\circledast$ **2**) $g \in K \circledast g$. **(success of insertion)**

(**K** $\circledast$ **3**) $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**

(**K** $\circledast$ **4**) $K \circledast g \subseteq K + g$. **(inclusion)**

(**K** $\circledast$ **5**) If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**

(**K** $\circledast$ **6**) If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

(**K** $\circledast$ **5**) expresses that if no $g \Rightarrow f$ is in K, preemption amounts to expansion

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

(K ⊛ 1)	$K \circledast g$ is a theory.	**(closure)**
(K ⊛ 2)	$g \in K \circledast g$.	**(success of insertion)**
(K ⊛ 3)	$f \notin K \circledast g$ for all clausal strict implicants f of g.	**(success of preemption)**
(K ⊛ 4)	$K \circledast g \subseteq K + g$.	**(inclusion)**
(K ⊛ 5)	If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$.	**(vacuity)**
(K ⊛ 6)	If $g \equiv h$ then $K \circledast g = K \circledast h$.	**(extensionality)**

(K ⊛ 5) does not require a proviso about the negation of g not to be in K

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

(**K** $\circledast$ **1**) $K \circledast g$ is a theory. **(closure)**

(**K** $\circledast$ **2**) $g \in K \circledast g$. **(success of insertion)**

(**K** $\circledast$ **3**) $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**

(**K** $\circledast$ **4**) $K \circledast g \subseteq K + g$. **(inclusion)**

(**K** $\circledast$ **5**) If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**

(**K** $\circledast$ **6**) If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Property 4

If $g \Rightarrow f \notin K$ for all clausal strict implicants f of g then $\neg g \notin K$.

➡ (**K** $\circledast$ **5**) does not require a proviso about the negation of g not to be in K

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

(K $\circledast$ 1) $K \circledast g$ is a theory. **(closure)**

(K $\circledast$ 2) $g \in K \circledast g$. **(success of insertion)**

(K $\circledast$ 3) $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**

(K $\circledast$ 4) $K \circledast g \subseteq K + g$. **(inclusion)**

(K $\circledast$ 5) If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**

(K $\circledast$ 6) If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **(K ⊛ 1)** $K \circledast g$ is a theory. **(closure)**
- **(K ⊛ 2)** $g \in K \circledast g$. **(success of insertion)**
- **(K ⊛ 3)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **(K ⊛ 4)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **(K ⊛ 5)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **(K ⊛ 6)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Property 1 : Let ⊛ satisfy (K ⊛ 1), (K ⊛ 2) and (K ⊛ 3).

$K \circledast g = K_\perp$ iff $\vdash \neg g$.

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

($\mathbf{K \circledast 1}$)	$K \circledast g$ is a theory.	**(closure)**
($\mathbf{K \circledast 2}$)	$g \in K \circledast g$.	**(success of insertion)**
($\mathbf{K \circledast 3}$)	$f \notin K \circledast g$ for all clausal strict implicants f of g.	**(success of preemption)**
($\mathbf{K \circledast 4}$)	$K \circledast g \subseteq K + g$.	**(inclusion)**
($\mathbf{K \circledast 5}$)	If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$.	**(vacuity)**
($\mathbf{K \circledast 6}$)	If $g \equiv h$ then $K \circledast g = K \circledast h$.	**(extensionality)**

Property 1 : Let $\circledast$ satisfy ($\mathbf{K \circledast 1}$), ($\mathbf{K \circledast 2}$) and ($\mathbf{K \circledast 3}$).

$K \circledast g = K_{\perp}$ iff $\vdash \neg g$.

➠ Property shared with revision (postulate ($\mathbf{K * 5}$))

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **($\mathbf{K} \circledast \mathbf{1}$)** $K \circledast g$ is a theory. **(closure)**
- **($\mathbf{K} \circledast \mathbf{2}$)** $g \in K \circledast g$. **(success of insertion)**
- **($\mathbf{K} \circledast \mathbf{3}$)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **($\mathbf{K} \circledast \mathbf{4}$)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **($\mathbf{K} \circledast \mathbf{5}$)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **($\mathbf{K} \circledast \mathbf{6}$)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Postulates

$K \circledast g$: preemption of g (a clause) over K (a consistent belief base)

- **(K ⊛ 1)** $K \circledast g$ is a theory. **(closure)**
- **(K ⊛ 2)** $g \in K \circledast g$. **(success of insertion)**
- **(K ⊛ 3)** $f \notin K \circledast g$ for all clausal strict implicants f of g. **(success of preemption)**
- **(K ⊛ 4)** $K \circledast g \subseteq K + g$. **(inclusion)**
- **(K ⊛ 5)** If $(g \Rightarrow f) \notin K$ for all clausal strict implicants f of g then $K + g \subseteq K \circledast g$. **(vacuity)**
- **(K ⊛ 6)** If $g \equiv h$ then $K \circledast g = K \circledast h$. **(extensionality)**

Property 2 : Let ⊛ satisfy (K ⊛ 1) and (K ⊛ 2).

Then, if $\vdash g$ then $K \circledast g = K_\top$.

Guideline

Characterization

- Preemption could be approximated as **multiple contraction followed by expansion** (similarly to Levi's identity [G$\ddot{8}$8])

Characterization

- Preemption could be approximated as **multiple contraction followed by expansion** (similarly to Levi's identity [G8̈8])

➠ Multiple contraction [FH94] permits **to contract by a set of information** ($K \ominus \Lambda$)

Characterization

- Preemption could be approximated as **multiple contraction followed by expansion** (similarly to Levi's identity [G̈88])

Definition 1 ($|||$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$K \; ||| \; g = (K \ominus \{g \Rightarrow f_i\}_{i=1,2,..}) + g.$

Characterization

- Preemption could be approximated as **multiple contraction followed by expansion** (similarly to Levi's identity [G88])

Definition 1 ($|||$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$K \mid\mid\mid g = (K \ominus \{g \Rightarrow f_i\}_{i=1,2,..}) + g$.

Theorem 1

If $\ominus$ satisfies $(K\ominus 1)-(K\ominus 4)$ and $(K\ominus 6)$, and if $+$ satisfies $(K+1)-(K+6)$, then $|||$ satisfies $(K \circledast 1) - (K \circledast 6)$.

Characterization

- Preemption could be approximated as **multiple contraction followed by expansion** (similarly to Levi's identity [G̈88])

Definition 1 ($|||$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$K \; ||| \; g = (K \ominus \{g \Rightarrow f_i\}_{i=1,2,..}) + g.$

Theorem 2

Every operator satisfying $(K \circledast 1) - (K \circledast 6)$ can be written as an $|||$ operator s.t. $\ominus$ satisfies $(K \ominus 1) - (K \ominus 4)$ and $(K \ominus 6)$, and $+$ satisfies $(K + 1) - (K + 6)$.

Guideline

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|^{p}_{q}$)

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|_q^p$)

Could be captured as a **contraction** by $q \Rightarrow p$ followed by an **expansion** by q

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|^p_q$)
- Preemption could be captured as the **set-theoretic intersection of replacement operations**

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|^p_q$)
- Preemption could be captured as the **set-theoretic intersection of replacement operations**

Definition 2 ($|||_H$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$K \; |||_H \; g = \bigcap_{f \in \{f_1, f_2, \ldots, f_n, \ldots\}} K|^f_g$.

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|^p_q$)
- Preemption could be captured as the **set-theoretic intersection of replacement operations**

Definition 2 ($|||_H$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$$K \; |||_H \; g = \bigcap_{f \in \{f_1, f_2, \ldots, f_n, \ldots\}} K|^f_g.$$

➠ Properties directly follow from the properties of replacement operators

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|_q^p$)
- Preemption could be captured as the **set-theoretic intersection of replacement operations**

Definition 2 ($|||_H$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$K \; |||_H \; g = \bigcap_{f \in \{f_1, f_2, \ldots, f_n, \ldots\}} K|_g^f$.

Properties of $|||_H$ operator

$g \in K \; |||_H \; g$.
$K \; |||_H \; g \cap \{f_1, f_2, \ldots, f_n, \ldots\} = \emptyset$.

Hansson's replacement operators [Han09]

- Permits **to replace** in K **a proposition** p **by a proposition** q ($K|^p_q$)
- Preemption could be captured as the **set-theoretic intersection of replacement operations**

Definition 2 ($|||_H$ operator) : Let $\{f_1, f_2, \ldots, f_n, \ldots\}$ be the set of all clausal strict implicants of g.

$$K \; |||_H \; g = \bigcap_{f \in \{f_1, f_2, \ldots, f_n, \ldots\}} K|^f_g.$$

Properties of $|||_H$ operator

$$g \in K \; |||_H \; g.$$
$$K \; |||_H \; g \cap \{f_1, f_2, \ldots, f_n, \ldots\} = \emptyset.$$

➡ **Warning** : It might be the case that some **preemption operators cannot be written as a $|||_H$ operator**

Guideline

Conclusions & Perspectives

Conclusions :

- Introduction of operators $\circledast$ for evolution of belief bases that are **not guided by inconsistency**

Conclusions & Perspectives

Conclusions :

- Introduction of operators $\circledast$ for evolution of belief bases that are **not guided by inconsistency**
- Proposition of postulates for $\circledast$ **when the new information is a clause**

Conclusions & Perspectives

Conclusions :

- Introduction of operators $\circledast$ for evolution of belief bases that are **not guided by inconsistency**
- Proposition of postulates for $\circledast$ **when the new information is a clause**
- Characterization of $\circledast$ in term of **multiple contraction followed by expansion**

Conclusions & Perspectives

Conclusions :

- Introduction of operators $\circledast$ for evolution of belief bases that are **not guided by inconsistency**
- Proposition of postulates for $\circledast$ **when the new information is a clause**
- Characterization of $\circledast$ in term of **multiple contraction followed by expansion**
- Study of a related work : **Hansson's replacement operators**

Conclusions & Perspectives

Conclusions :

- Introduction of operators $\circledast$ for evolution of belief bases that are **not guided by inconsistency**
- Proposition of postulates for $\circledast$ **when the new information is a clause**
- Characterization of $\circledast$ in term of **multiple contraction followed by expansion**
- Study of a related work : **Hansson's replacement operators**

Perspectives :

- What about **non-clausal g** ?

Conclusions & Perspectives

Conclusions :

- Introduction of operators $\circledast$ for evolution of belief bases that are **not guided by inconsistency**
- Proposition of postulates for $\circledast$ **when the new information is a clause**
- Characterization of $\circledast$ in term of **multiple contraction followed by expansion**
- Study of a related work : **Hansson's replacement operators**

Perspectives :

- What about **non-clausal g** ?
- What about **representation theorem** ?

References

Carlos Alchourrón, Peter Gärdenfors, and David Makinson.
On the logic of theory change : partial meet contraction and revision functions.
Journal of Symbolic Logic, 50(2) :510–530, 1985.

Philippe Besnard, Éric Grégoire, and Sébastien Ramon.
Enforcing logically weaker knowledge in classical logic.
In 5th International Conference on Knowledge Science Engineering and Management (KSEM'11), pages 44–55. LNAI 7091, Springer, 2011.

Philippe Besnard, Éric Grégoire, and Sébastien Ramon.
Overriding subsuming rules.
In 11^{th} European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'11), pages 532–544. LNAI 6717, Springer, 2011.

André Fuhrmann and Sven Ove Hansson.
A survey of multiple contractions.
Journal of Logic, Language and Information, 3(1) :39–76, 1994.

Peter Gärdenfors.
Knowledge in flux : modeling the dynamics of epistemic states, volume 103.
MIT Press, 1988.

Sven Ove Hansson.
Replacement—a sheffer stroke for belief change.
Journal of Philosophical Logic, 38(2) :127–149, 2009.