A Default Logic Patch for Default Logic

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A Default Logic Patch for Default Logic: Quid?

- *Context:* Merging of multiple information sources representing using default logic
- *Motivation:* When the standard-logic parts of the sources are contradictory, the resulting default theory trivializes
- *Proposal:* Handle the problem using the default logic framework itself

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Guidelines

Default Logic Framework (1)

- Reiter's Default logic and its major variants
- Default reasoning
  - To infer conclusion in the absence of the opposite
- Defeasible reasoning
  - Jump to default conclusions and be able to retract them
  - whenever additional information leads to inconsistency

Example (criminal investigation)

"Under a criminal investigation, any individual x on the crime scene is a suspect by default unless some evidence contradicts x's guilt. If such further evidence makes such a contradiction occur, x should not be suspected anymore".

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Default Logic Framework (2)

- Default-logic theory $\Gamma = (\Delta, \Sigma)$
  - A set of default rules (Δ), capturing pieces of defeasible reasoning
  - A set of standard-logic formulas (Σ), representing knowledge
- A default rule $d \in \Delta$ is a rule: $\dfrac{\alpha : \beta}{\gamma}$, where
  - α, β and γ are standard-logic formulas
  - α is the prerequisite, β the justification and γ the consequent
  - "Provided that the prerequisite can be established, and provided that the justification is consistently assumed w.r.t what is derived, infer the consequent"

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- $\Gamma = (\{\dfrac{\mathit{on\_crime\_scene} : \mathit{guilty}}{\mathit{suspect}}\}, \{\mathit{on\_crime\_scene}\})$

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Default Logic Framework (3)

- (possibly) several "extensions" can be obtained from $\Gamma = (\Delta, \Sigma)$
  - Contradictory consequents of defaults cannot belong to the same extension
  - Maximal consistent sets of infered formulas of Γ closed deductively
- Different forms of reasoning about a formula f
  - Credulously: f belongs to at least one extension of Γ
  - Skeptically: f belongs to all extensions of Γ

Example (criminal investigation)

- $\Gamma = (\{\frac{confession : guilty}{guilty}, \frac{alibi : \neg guilty}{\neg guilty}\}, \{confession, alibi\})$
  - $E_1 = Cn(\{confession, alibi, guilty\})$
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Guidelines

Merging of Default-Logic Theories

- How n ($n > 0$) default-logic theories should be merged?
  - n default theories $\Gamma_i = (\Delta_i, \Sigma_i)$ ($i \in [1..n]$) to be merged
  - Merging sets of defaults and sets of facts
- The resulting merged default theory is $\Gamma = (\cup_{i=1}^{n}\Delta_i, \cup_{i=1}^{n}\Sigma_i)$

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Default-Logic Trivialisation Issue

- In the merging process, local inconsistency can arise
- When $\cup_{i=1}^{n}\Sigma_i$ is inconsistent, the resulting default theory trivializes
- A possibly minor contradiction between sources should not cause the whole system to collapse

- Example (criminal investigation)
  - $\Gamma_1 = (\emptyset, \{confession, \neg right\_handed\})$
  - $\Gamma_2 = (\{\frac{confession : guilty}{guilty}\}, \{right\_handed\})$
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Removing Inconsistent Formulas

- Removing enough formulas from $\cup_{i=1}^{n}\Sigma_i$
- Capturing the Minimally Unsatisfiable Subformulas of $\cup_{i=1}^{n}\Sigma_i$
  - A *MUS* of $\cup_{i=1}^{n}\Sigma_i$ is one of its inconsistent subsets that cannot be made smaller
- Dropping formulas is unnecessarily destructive
  - Credulous reasoners might be interested in the Maximal Consistent Subsets of the various MUSes of $\cup_{i=1}^{n}\Sigma_i$
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Replacing Inconsistent Formulas

- To replace each formula f in the MUSes of $\cup_{i=1}^{n}\Sigma_i$ by a corresponding super-normal default $\frac{:f}{f}$
- Each formula f in the MUSes could be inferred if f could be consistently assumed

Definition (merged default theory)

Let us consider a non-empty set of n default theories of the form $\Gamma_i = (\Delta_i, \Sigma_i)$ to be merged. The resulting merged default theory is given by $\Gamma = (\Delta, \Sigma)$ where:

- $\Sigma = \cup_{i=1}^{n}\Sigma_i \setminus \cup MUS(\cup_{i=1}^{n}\Sigma_i)$,
- $\Delta = \cup_{i=1}^{n}\Delta_i \cup \{\frac{:f}{f} \mid f \in \cup MUS(\cup_{i=1}^{n}\Sigma_i)\}$.

Guidelines

Standard Boolean Case (1)

- Problem of merging sets of Boolean formulas
- Original approach to address this issue

Example (criminal investigation)

- $\Gamma_1 = (\emptyset, \{alibi\})$
- $\Gamma_2 = (\emptyset, \{\neg on\_crime\_scene \vee \neg alibi\})$
- $\Gamma_3 = (\emptyset, \{on\_crime\_scene\})$

- $\cup MUS(\cup_{i=1}^{n} \Sigma_i) = \{alibi, \neg on\_crime\_scene \vee \neg alibi, on\_crime\_scene\}$
- $\Gamma = (\{\frac{: on\_crime\_scene}{on\_crime\_scene}, \frac{: \neg on\_crime\_scene \vee \neg alibi}{\neg on\_crime\_scene \vee \neg alibi}, \frac{: alibi}{alibi}\}, \emptyset)$

- $E_1 = Cn(\{\neg on\_crime\_scene \vee \neg alibi, alibi\})$,
- $E_2 = Cn(\{\neg on\_crime\_scene \vee \neg alibi, on\_crime\_scene\})$,
- $E_3 = Cn(\{alibi, on\_crime\_scene\})$.

Standard Boolean Case (2)

- No formula is lost in the merging process as in the standard merging approaches
  - Any formula in $\cup MUS(\cup_{i=1}^{n}\Sigma_i)$ belongs to at least one extension
  - No extension contains $\cup MUS(\cup_{i=1}^{n}\Sigma_i)$

Proposition (1)

Let $n > 1$. Consider n finite default theories $\Gamma_i = (\Delta_i, \Sigma_i)$ s.t. $\cup_{i=1}^{n}\Delta_i$ is empty and $\cup_{i=1}^{n}\Sigma_i$ is inconsistent. Let Γ denote the resulting merged default theory.

- *There exists no extension of Γ that contains $\cup MUS(\cup_{i=1}^{n}\Sigma_i)$, but*
- *for any satisfiable formula f in $\cup MUS(\cup_{i=1}^{n}\Sigma_i)$, there exists an extension of Γ containing f.*

Standard Boolean Case (3)

- The intersection of all extensions do not coincide with the unique extension of the theory where the MUSes are removed
- Mimics a case analysis process that allows inferences to be entailed that would be dropped in standard merging approaches
- A skeptical reasoner will be able to infer at least all formulas infered in standard merging approaches

Proposition (2)

Let $n > 1$. Consider n finite default theories $\Gamma_i = (\Delta_i, \Sigma_i)$ to be merged. Let $\cap_j E_j$ denote the set-theoretic intersection of all extensions of the resulting fused default theory $\Gamma = (\Delta, \Sigma)$. Let E denote the unique extension of $\Gamma' = (\emptyset, \Sigma)$. If Δ_i is empty for $i = 1..n$, then $E \subseteq \cap_j E_j$.

Guidelines

Normal Default Theories

- Extension of Proposition 1 to normal default theories holds
  - Normal default theories enjoy semi-monotonicity property
  - We only add supernormal defaults to $\cup_{i=1}^{n}\Delta_i$
- Extension of Proposition 2 to normal theories does not hold
  - The unique extension of standard merging approaches is not necessarily contained in the intersection of all the extensions
  - Removing MUSes prevents the application of initial (normal) defaults whose prerequisite belongs to MUSes

Proposition (3)

Let $n > 1$. Consider n finite normal default theories $\Gamma_i = (\Delta_i, \Sigma_i)$ to be fused and $\Gamma' = (\cup_{i=1}^{n}\Delta_i, \cup_{i=1}^{n}\Sigma_i \setminus \cup MUS(\cup_{i=1}^{n}\Sigma_i))$. For any extension E of Γ', there exists an extension of the resulting merged default theory that contains E.

General Default Theories (1)

- Extension to Proposition 3 does not hold in general case
  - Not ensure that we shall obtain supersets of the extensions of the standard merging approaches
  - Semi-monotonicity does not hold: some variants of Reiter's default logic ensure this property

Example

- $\Gamma = (\Delta, \{on\_crime\_scene, \neg on\_crime\_scene\})$
- $\Delta = \{\frac{: \neg suspect}{\neg suspect}, \frac{on\_crime\_scene : \neg alibi}{suspect}, \frac{\neg on\_crime\_scene : \neg alibi}{suspect}\}$
- $\Gamma' = (\Delta, \emptyset)$ exhibits one extension $E = Cn\ (\{\neg suspect\})$
- $\Gamma = (\Delta \cup \{\frac{: on\_crime\_scene}{on\_crime\_scene}, \frac{: \neg on\_crime\_scene}{\neg on\_crime\_scene}\}, \emptyset)$ does not contain any extension containing $\neg suspect$

General Default Theories (2)

- Extension to Proposition 2 does not hold in general case
  - It may happen that consistent formulas of MUSes are in no extension
  - Semi-monotonicity does not hold: some variants of Reiters default logic ensure this property

Example

- $\Gamma_1 = (\emptyset, \{suspect, \neg alibi\})$
- $\Gamma_2 = (\{\frac{\neg alibi : on\_crime\_scene}{suspect}\}, \{\neg suspect\})$
- $\cup MUS(\cup_{i=1}^{n} \Sigma_i) = \{suspect, \neg suspect\}$
- $\Gamma = (\{\frac{: suspect}{suspect}, \frac{: \neg suspect}{\neg suspect}, \frac{\neg alibi : on\_crime\_scene}{suspect}\}, \{\neg alibi\})$
- The unique extension of Γ is $E = Cn(\{suspect, \neg alibi\})$, which does not contain $\neg suspect$

Guidelines

Complexity Issues

- Computing MUSes is computationally heavy (Σ_2^p-*complete*)
- The whole process (finding, replacing and reasoning)
  - Boolean credulous default reasoning: Σ_2^p-*complete*
  - Boolean skeptical default reasoning: Π_2^p-*complete*
- Efficient techniques to compute all MUSes
- Cannot afford to compute the set-theoritical union of all MUSes
  - Don't replace all formulas in all MUSes (MUS per MUS iteration)
  - Strict Inconsistent Cover (approximation technique)
  - Super-set of all MUSes Ω

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Conclusion and Future Works

- Handle the trivialisation issue of default logic
- No distinction between initial defaults and introduced defaults
- Defaults were of the same epistemological nature
  - Defaults are introduce to weaken deficient pieces of knowledge
  - Introduced defaults are formulas accepted by default
- New defaults should be given a higher (resp. lower) priority
  - Resort to a form of prioritized default logic
  - Extend default theories with a partial order

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