SOLVING WCSP BY EXTRACTION OF MINIMAL UNSATISFIABLE CORES

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Introduction

Context:

- CSP and Weighted CSP frameworks
- WCSP algorithms are often more complex than their CSP counterparts (due to management of costs)

Goal:

- Benefit from efficient CSP algorithms developped for more than two decades

Principle:

- Solve WCSP by iteratively generating and solving classical CSPs (greedy approach)
- The sequence of CSPs is enumerated according to an increasing cost order related to the WCSP
- Minimal Unsatisfiable Cores (MUC) identify the soft constraints whose costs must be increased

Background

CSP framework:

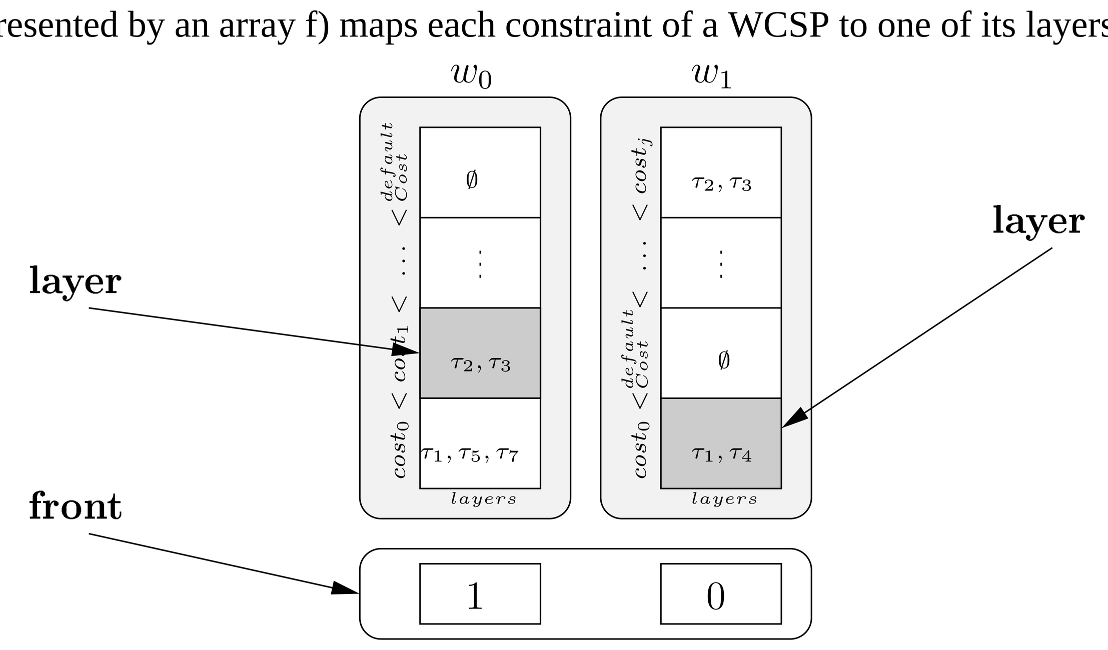
- A CSP is satisfiable iff it admits at least one solution
- An Unsatisfiable Core is an unsatisfiable subset of constraints
- A core is a **Minimal Unsatisfiable Core (MUC)** iff each strict subset is satisfiable

WCSP framework:

- Extension of CSP
- (X, C, k): C is a set of soft constraints (cost functions), $k > 0$ is either a natural integer or $+\infty$
- $\forall a, b \in \{0, \dots, k\}, a \oplus b = min(k, a + b)$
- Goal: find a complete instantiation with minimal cost (optimisation problem)
- The current methods to solve WCSPs: branch and bound tree search combined with the use of soft local consistencies (EDAC, etc. by cost tranfer) for estimating minimal costs of sub-problems during search

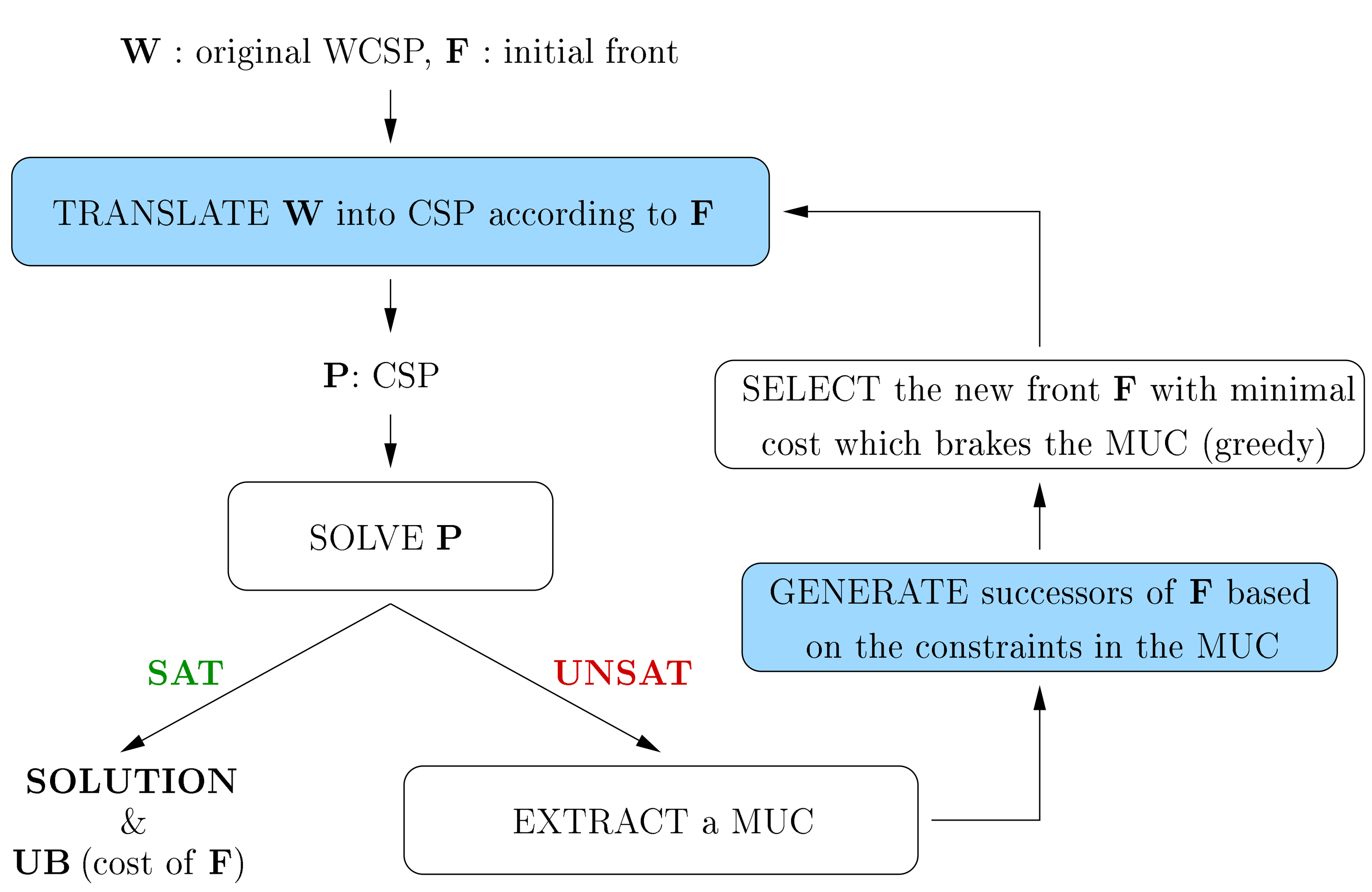
Layers and fronts

- Focus on soft table constraints (explicit and implicit tuples), but the method can be easily extended to other kinds of constraints
- A **layer** contains all tuples having the same cost
- A **front** (represented by an array f) maps each constraint of a WCSP to one of its layers



- **Cost of a front**: sum of costs associated with the selected layers

General principle of the greedy approach

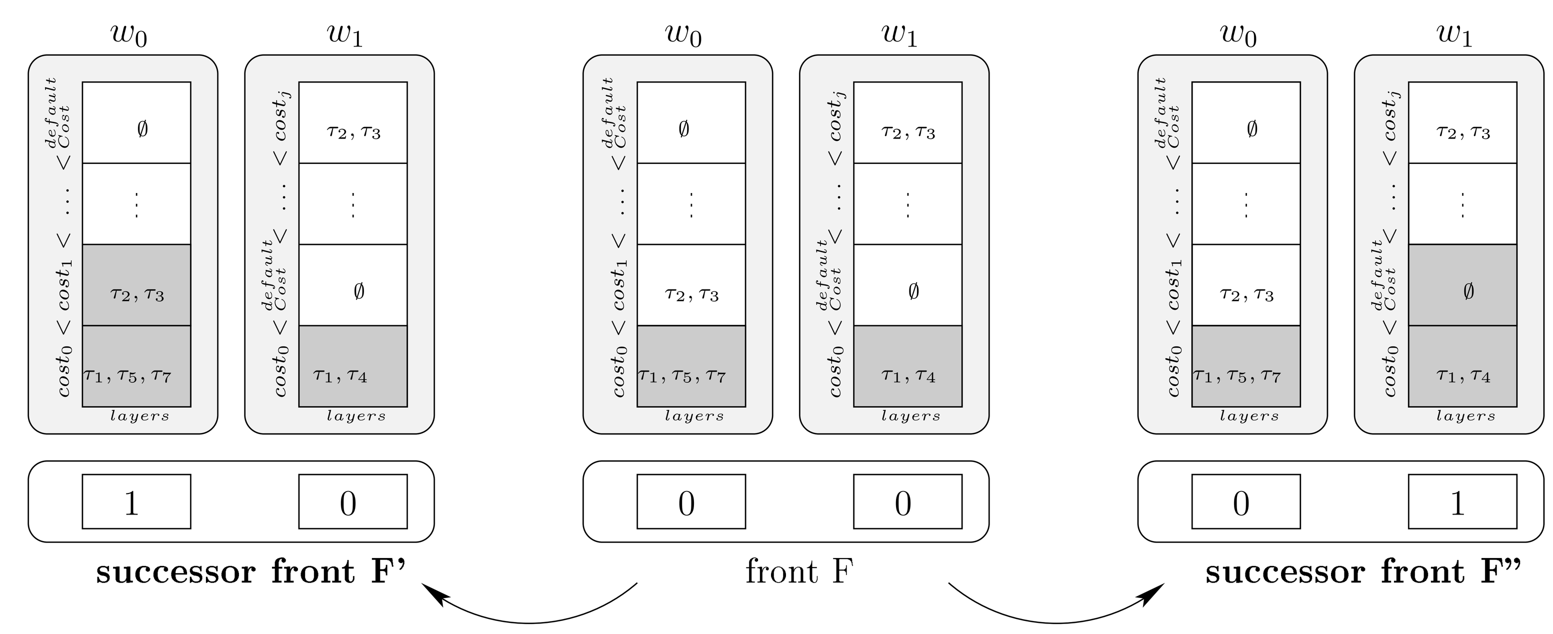


Algorithm

GMR (W: WCSP)

foreach $w \in constraints(W)$ **do**
  $f[w] \leftarrow 0$;
repeat
  $P \leftarrow toCSP_{\leq}(W, f)$;
  $sol \leftarrow solveCSP(P)$;
  if $sol \neq \bot$ **then**
    return sol ;
  else
    $M \leftarrow extractMUC(P)$;
    $W' \leftarrow restrict(W, M)$;
    $f \leftarrow relax(W', f)$;
until $sol \neq \bot$;

- $toCSP_{\leq}(W, f)$
  - Translates WCSP into CSP: converting soft constraints into hard constraints according to a front f
  - Considering a soft constraint w and the front f: a hard constraint is obtained by selecting as allowed tuples in w the tuples of the layers whose index is **less than or equal** to $f[w]$
  - Representation: extension, positive table (supports) / negative table (conflicts)
  - Default cost Layer forbidden (resp. allowed) ⇒ hard positive (resp. negative) constraint
- $extractMUC(P)$
  - A dichotomic approach is used to extract MUCs of unsatisfiable CSPs
- $relax(W', f)$
  - MUCs are broken by generating **successors** of the front considering only the constraints of MUCs



- A **successor** of a front differs only by the incrementation of the allowed layers for one constraint

Experiments

Instances		AbsCon GMR	AbsCon EDAC	ToulBar2 INCOP	ToulBar2 EDAC
spot5/spot5-404	CPU	4.99	$\bot$	$\bot$	217
	UB	118	**114**	**114**	**114**
spot5/spot5-412	CPU	18.8	$\bot$	$\bot$	$\bot$
	UB	33,403	43,390	**32398**	37,399
spot5/spot5-505	CPU	12	$\bot$	$\bot$	$\bot$
	UB	22,266	28,258	**21266**	25,268
spot5/spot5-509	CPU	32.2	$\bot$	$\bot$	$\bot$
	UB	37469	48,475	**37462**	46,477
spot5/spot5-1403	CPU	142.5	$\bot$	$\bot$	$\bot$
	UB	**481,266**	517,260	482267	507,265
celar/graph-05	CPU	16.6	$\bot$	$\bot$	0.62
	UB	**221**	4,645	243	**221**
celar/scen-06-20	CPU	68.5	$\bot$	$\bot$	67.9
	UB	3,402	8,594	3166	**3,163**
celar/scen-07	CPU	209.9	$\bot$	$\bot$	$\bot$
	UB	426,423	31,230K	**394006**	505,731

CPU time (in seconds) to prove optimality on various selected instances
(time-out of 600 seconds set per instance, $\bot$: time-out reached)

Conclusion and future work

- Conclusion
  - Original greedy approach: solve WCSP through successive resolutions of CSPs
  - Focus the cost increase on the sole constraints in the Minimal Unsatisfiable Cores extracted
  - Promising results when compared to other state-of-the-art approaches
- Future work
  - Complete approach based on the same principles (work in progress)