Reasoning under Inconsistency: *Ex falso nihil non sequendum est*

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Layout

Part I - Introduction

- Conflicts are everywhere
- Why reasoning from inconsistent information proves difficult
- How to evaluate a paraconsistent inference relation $\mid\!\sim$

Handling Conflicting Pieces of Information

Designing principled approaches to deal with conflicting pieces of information is a **major task**

Because conflicts are **ubiquitous!**

- Conflicting preferences,
- Conflicting criteria,
- Conflicting norms,
- Conflicting beliefs,
- ...

Conflicting Preferences

A voting scenario (voters & alternatives)

- A : Ségolène $>$ François $>$ Nicolas
- B : François $>$ Nicolas $>$ Ségolène
- C : Nicolas $>$ François $>$ Ségolène

$\Rightarrow$ **Conflicts between voters preferences**

4 A, 2 B, 3 C: who should be elected?

$\Rightarrow$ No “good” solution for the vote issue in general (Arrow’s impossibility theorem)
$\Rightarrow$ But it ”works” nevertheless...

Conflicting Criteria

How (and why) to choose my next mobile phone?
Two options and three criteria

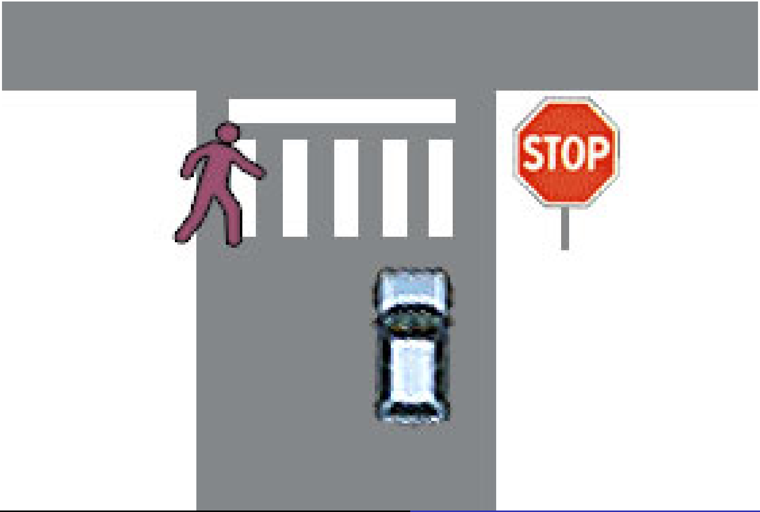
- price : StrawBerry $>$ iFaune
- size : iFaune $>$ StrawBerry
- design : iFaune $>$ Strawberry

$\Rightarrow$ **Conflicts between criteria**

Two undominated solutions: StrawBerry and iFaune

Conflicting Norms

The car dilemma (for young drivers)



Conflicting Beliefs

When did Aunt Agatha visit us last time?

- Mary : last week on Monday, together with Uncle Benny
- John : last week on Tuesday

⇒ **Conflicts between beliefs**

Assuming a unique actual “world”, at least one of the two sources has reported **some erroneous beliefs**

⇒ This departs from the other conflicts: there is no “true” preference or “true” norm!

Dealing with Inconsistent Beliefs

Handling inconsistent beliefs is necessary in many situations:

- consistency-based diagnosis,
- distributed databases,
- multi-agent systems,
- reasoning from conflicting arguments,
- reasoning using rules with exceptions,
- ...

Dealing with Inconsistent Beliefs

- **Active approaches:** use tests (or other forms of epistemic actions) to find the culprits
  - \+ The erroneous pieces of information can be removed
  - \- Requires tests to be available
- **Passive approaches:** do the best with the available information
  - \+ Does not need further information
  - \- Unexpected consequences can be derived (from incorrect pieces of belief)

Why is Reasoning under Inconsistency Difficult?

The problematic of inconsistency handling from the inference perspective comes from the **inadequacy of classical entailment** in this case:

- *Ex falso quodlibet sequitur:*

$$\alpha, \neg\alpha \models \beta$$

- $\models$ is **explosive** in presence of inconsistency!
- Model-theoretically: $\forall E, \emptyset \subseteq E$
- Proof-theoretically:
  - From $\neg\alpha$ derive $\neg\alpha \vee \beta$ ($\vee$-intro.)
  - From α and $\neg\alpha \vee \beta$, derive β (*disjunctive syllogism*)

The Unexpected Behaviour of $\models$

- Deduction trivializes from an inconsistent belief base
- A single, local inconsistency pollutes the whole base
- This calls for **other inference relations**

- This explains the strange title of this tutorial:

  Ex falso nihil non sequendum est
- **Deriving nothing is not mandatory!**

- **But determining what should be preserved is not obvious ...**

Aunt Agatha Revisited

When did Aunt Agatha visit us last time?

- Mary : last week on Monday, together with Uncle Benny
- John : last week on Tuesday

What can be concluded from this?

- No information
- Aunt Agatha visited us last week
- Aunt Agatha came together with Uncle Benny

What Should Be Concluded from an Inconsistent Belief Base?

No consensus!

Many approaches have been developed so far to address this issue:

- paraconsistent logics,
- belief revision,
- belief merging,
- knowledge integration,
- ...

What Should Be Concluded from an Inconsistent Belief Base?

Existing approaches are suited to various scenarios:

- Number of information sources: 1, 2, many,
- Preferences over beliefs (”preference to believe”),
- Reliability of the sources,
- ...

What Should Be Concluded from an Inconsistent Belief Base?

Formally: defining and evaluating an inference relation $\mid\!\sim$

- being infraclassical ($\mid\!\sim \subseteq \models$)
- taking account as much as possible of the available information (preferences, reliability, etc.)
- offering expected logical properties (forms of paraconsistency, preserving expected consequences, etc.)
- exhibiting a low computational complexity,
- ...

Preferences over Beliefs

- $\alpha \wedge \beta \models \bot$
- α is more certain than β
- Conclude α (but not β)!
- **Preferences (to believe) can be exploited to solve conflicts**
- Many formal settings (possibilistic logic, penalty logic and other weighted logics, ...)
- Inference is deduction from preferred consistent ”subbases”

Reliability of the Sources

- A piece of belief can be considered as certain as its incoming source is reliable
- Can be relativized by considering topics
  - Mary is more reliable than John according to the presence of Uncle Benny,
  - John is more reliable than Mary according to the day of the last visit of Aunt Agatha

Expected Logical Properties

System P Preferential logics: postulates for common-sense inference (supraclassical inference)

(Ref) $\Sigma \mid\!\sim \Sigma$

(LLE) If $\Sigma \mid\!\sim \Sigma'$ and $\Sigma' \mid\!\sim \Sigma$ and $\Sigma \mid\!\sim \phi$, then $\Sigma' \mid\!\sim \phi$

(RW) If $\Sigma \mid\!\sim \phi$ and ϕ implies ψ, then $\Sigma \mid\!\sim \psi$

(And) If $\Sigma \mid\!\sim \phi$ and $\Sigma \mid\!\sim \psi$, then $\Sigma \mid\!\sim (\phi \wedge \psi)$

(Or) If $\Sigma \mid\!\sim \phi$ and $\Sigma' \mid\!\sim \phi$, then $\Sigma \vee \Sigma' \mid\!\sim \phi$

(CM) If $\Sigma \mid\!\sim \phi$ and $\Sigma \mid\!\sim \psi$, then $\Sigma \wedge \phi \mid\!\sim \psi$

Expected Logical Properties

Paraconsistency issues

- Paraconsistency
  - Strong paraconsistency: $\forall \Sigma \exists \alpha \Sigma \not\mid\!\sim \alpha$
  - Weak paraconsistency: $\exists \Sigma \exists \alpha \Sigma \not\mid\!\sim \alpha$
- Preservation: if $\Sigma \not\models \bot$ then $\Sigma \mid\!\sim \alpha$ iff $\Sigma \models \alpha$
- Classical closure: $Cn_{\mid\sim}(\Sigma) = \{\alpha \mid \Sigma \mid\!\sim \alpha\}$ is classically consistent and classically closed
- ...

Expected Logical Properties

- No consensus on the expected properties ($\neq$ voting)
- **Some of them are conflicting!**
  (ako Impossibility Theorem)
  - (Ref):
    $$\Sigma = \mathit{Monday}(\mathit{Agatha}) \wedge \neg \mathit{Monday}(\mathit{Agatha}) \mid\!\sim \mathit{Monday}(\mathit{Agatha}) \wedge \neg \mathit{Monday}(\mathit{Agatha})$$
  - (RW): incompatible with (Ref) when "implies" is w.r.t. $\models$
    $$\Sigma \mid\!\sim \Sigma \text{ and } \Sigma \models \alpha \Rightarrow \Sigma \mid\!\sim \alpha$$
  - Classical closure: the $\mid\!\sim$ closure of Σ is not classically consistent if (Ref) is satisfied

Complexity

In the propositional case, the decision problem for $\mid\!\sim$ is typically **complete** for one of these classes:

- P (tractability)
- coNP (the baseline, corresponding to $\models$)
- Θ_2^p
- Δ_2^p
- Π_2^p (the second level of PH)

How to Avoid Explosion

$$\Sigma \models \bot \Rightarrow \forall \alpha, \Sigma \models \alpha$$

Two main approaches:

- Weakening $\models$: consider $\mid\!\sim \subseteq \models$
- Weakening Σ: "turn to" a consistent belief base Σ' such that $\Sigma \models \Sigma'$ and take advantage of classical deduction
  (remark: no need to compute Σ' explicitly)

Part II - Weakening Deduction

- A case study: the paraconsistent logic *THREE*
- Several inference relations
  - $\models_3$
  - $\mid\!\sim_3$
  - $\models_3^S$
  - $\mid\!\sim_3^c$

Paraconsistent Logics

- **Weakening $\models$**
- Suited to ”canonical” inconsistencies
- **A single source of information represented as a single formula, and nothing else!**
- Several approaches
  - Weakening the proof theory of classical logic
  - Enlarging the set of classical interpretations

Tweety the Bird

- $\Sigma = \ bird(Tweety) \wedge \neg bird(Tweety) \wedge hasWings(Tweety)$
  $\wedge (bird(Tweety) \Rightarrow flies(Tweety))$
- $\Sigma \mid\!\sim ?$
  - Surely $hasWings(Tweety)$
  - Maybe $flies(Tweety)$
  - Eventually $bird(Tweety)$ (and $\neg bird(Tweety)$) or none of them
  - Surely not $red(Tweety)$

Multivalued Paraconsistent Logics

- **Regaining consistency through a relaxation of the notion of model**
- 3 or 4 "truth values", including $\frac{1}{2}$ (both 0 and 1)
- A glimpse at *THREE*
  - d'Ottaviano and da Costa, CRAS, 1970,
  - Belnap, Modern Uses of Multiple-Valued Logic, 1977,
  - Priest, AIJ, 1989,
  - ...

Syntax of *THREE*

THREE (as in Arieli and Avron, AIJ, 1998)

- Connectives $\neg$, $\Box$, $\wedge$, $\vee$ and constant $\top$, $\bot$
- $PROP^3_{PS}$ defined inductively from *PS* and the connectives in the usual way
- Other connectives can be incorporated as syntactic sugars

$$\phi \Rightarrow \psi =_{def} \Box(\neg\phi) \vee \psi$$

- The set of connectives under consideration is **functionally complete** for $THREE = \{0, \frac{1}{2}, 1\}$

Semantics of *THREE* (1/3)

- $I : PS \rightarrow THREE$ 3-interpretation
- $I(\Sigma)$ defined in a truth-functional way (even for $\Sigma = \Box\psi$)
- I is a **3-model** of Σ iff $I(\Sigma) \neq 0$
- Two designated "truth values": 1 and $\frac{1}{2}$
- $\Sigma \models_3 \gamma$ iff every 3-model of Σ is a 3-model of γ

Semantics of *THREE* (2/3)

- $I(\top) = 1$, $I(\bot) = 0$
- Negation

A	$\neg A$
1	0
$\frac{1}{2}$	$\frac{1}{2}$
0	1

$\neg$

- Disjunction

A \\ B	1	$\frac{1}{2}$	0
1	1	1	1
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
0	1	$\frac{1}{2}$	0

$\vee$

Semantics of *THREE* (3/3)

- Conjunction

$A \backslash B$	1	$\frac{1}{2}$	0
1	1	$\frac{1}{2}$	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
0	0	0	0

$\wedge$

- Necessity

A	$\Box A$
1	1
$\frac{1}{2}$	0
0	0

$\Box$

$\models_3$-Inference: An Example

- $\Sigma = bird(T(weety)) \wedge \neg bird(T) \wedge hasWings(T) \wedge (bird(T) \Rightarrow flies(T))$
- $\Sigma \models_3$?
  - $hasWings(Tweety)$
  - $bird(Tweety)$
  - $\neg bird(Tweety)$
  - $flies(Tweety)$
- $\Sigma \not\models_3 red(Tweety)$

$\models_3$-Inference: Some Properties

- $\models_3 \subseteq \models$
- $\models_3$ satisfies P ("implies" is w.r.t. $\models_3$), is monotonic and (weakly) paraconsistent
- $\models_3$ satisfies the *modus ponens* schema but not the *disjunctive syllogism* schema

$$\Sigma = bird(T) \wedge (\neg bird(T) \vee flies(T)) \not\models_3 flies(T)$$

$\models_3$-Inference: An Example

- $\Sigma \not\models_3 flies(Tweety)$
- $\Sigma = bird(Tweety) \wedge (\neg bird(Tweety) \vee flies(Tweety))$ has five 3-models:
  - I s.t. $I(bird(Tweety)) = I(flies(Tweety)) = 1$
  - J s.t. $K(bird(Tweety)) = 1$ and $J(flies(Tweety)) = \frac{1}{2}$
  - K s.t. $K(bird(Tweety)) = \frac{1}{2}$ and $K(flies(Tweety)) = 0$
  - L s.t. $L(bird(Tweety)) = \frac{1}{2}$ and $L(flies(Tweety)) = \frac{1}{2}$
  - M s.t. $M(bird(Tweety)) = \frac{1}{2}$ and $M(flies(Tweety)) = 1$
- $K(flies(Tweety)) = 0$

$\models_3$-Inference: Some Properties

- $\models_3$ does not necessarily coincide with $\models$ in the restricted case when Σ is classically consistent (the preservation property does not hold)
- $\models_3$ does not satisfy the classical closure property

$\mid\!\sim_3$-Inference

Priest, Studia Logica, 1991

- 3-models which are "**as classical as possible**" are preferred
- $I \leq J$ iff $\{x \in PS \mid I(x) = \frac{1}{2}\} \subseteq \{x \in PS \mid J(x) = \frac{1}{2}\}$
- $\Sigma \mid\!\sim_3 \gamma$ iff every 3-model of Σ that is **minimal w.r.t.** $\leq$ is a 3-model of γ

$\mid\!\sim_3$-Inference: An Example

- $\Sigma = bird(Tweety) \wedge (\neg bird(Tweety) \vee flies(Tweety))$
- Σ has five 3-models:
  - I s.t. $I(bird(Tweety)) = I(flies(Tweety)) = 1$
  - J s.t. $K(bird(Tweety)) = 1$ and $J(flies(Tweety)) = \frac{1}{2}$
  - K s.t. $K(bird(Tweety)) = \frac{1}{2}$ and $K(flies(Tweety)) = 0$
  - L s.t. $L(bird(Tweety)) = \frac{1}{2}$ and $L(flies(Tweety)) = \frac{1}{2}$
  - M s.t. $M(bird(Tweety)) = \frac{1}{2}$ and $M(flies(Tweety)) = 1$
- $min(3 - mod(\Sigma), \leq) = \{I\}$
- $\Sigma \mid\!\sim_3 flies(Tweety)$

$\mid\!\sim_3$-Inference: Some Properties

- $\mid\!\sim_3$ satisfies P ("implies" is w.r.t. $\models_3$), is nonmonotonic and (weakly) paraconsistent
- $\mid\!\sim_3$ satisfies the preservation property
- $\mid\!\sim_3$ does not satisfy the classical closure property
- Other preference criteria can be considered (see e.g. Arieli and Avron, AIJ, 1998)

Complexity Results

Cadoli and Schaerf, AMAI, 1996
Coste-Marquis and Marquis, KR'02

$\Sigma \overset{?}{\vdash} \gamma$	**general case**	$\{\neg, \wedge, \vee\}$ **fragment**	**CNF formulae**
$\models_3$	coNP-complete	coNP-complete	$\in$ P
$\mid\!\sim_3$	Π^p_2-complete	Π^p_2-complete	Π^p_2-complete

$\models_3^S$-Inference

Schaerf and Cadoli, AIJ, 1995
Marquis and Porquet, AMAI, 2003

- Key idea: designing a relation between $\models_3$ and $\mid\sim_3$ by fixing **a priori** the set S of propositional symbols which must be classically interpreted
- Syntax of *THREE*
- $I : PS \rightarrow \mathit{THREE}$ S-3-interpretation if $\forall x \in S$, $I(x) \neq \frac{1}{2}$
- $I(\Sigma)$ defined in a truth-functional way
- I is an ***S*-3-model** of Σ iff $I(\Sigma) \neq 0$
- $\Sigma \models_3^S \gamma$ iff every S-3-model of Σ is an S-3-model of γ

$\models_3^S$-Inference: An Example

- $\Sigma = bird(Omer) \land (\neg bird(Omer) \lor flies(Omer)) \land(\neg flies(Omer) \lor hasWings(Omer))$
- $S = \{bird(Omer)\}$
- $\Sigma \models_3^S bird(Omer) \land flies(Omer)$
- $\Sigma \not\models_3^S hasWings(Omer)$

$\models_3^S$-Inference: Some Properties

- $\models_3^S$ satisfies P ("implies" is w.r.t. $\models_3$), is monotonic and (weakly) paraconsistent
- $\models_3^S$ satisfies neither the preservation property nor the classical closure one
- $\models_3^S$ satisfies some instances of the *disjunctive syllogism* schema
- For every S, S' s.t. $S \subseteq S'$, we have $\models_3 \subseteq \models_3^S \subseteq \models_3^{S'} \subseteq \models$
- For CNF formulae and S of bounded size, deciding whether $\Sigma \models_3^S \gamma$ is tractable (FPT)

Beyond $\models_3^S$-Inference

- In some situations, $\models_3^S$ is **still explosive**

$$bird(Tweety) \land \neg bird(Tweety) \models_3^{\{bird(Tweety)\}} bird(Omer)$$

- S' is a **consistent subset** of S w.r.t. Σ iff $\Sigma \models_3^S \not\models_3^S \bot$
- Policies for **selecting preferred consistent subsets** of S can be defined
- This leads to nonmonotonic inference relations that are (weakly) paraconsistent, tractable when $|S|$ is of bounded size, and non explosive in many cases for which $\models_3^S$ is

$\mid\!\sim^c_3$-Inference

Coste-Marquis and Marquis, JELIA'08

- *IncForg*(Σ) is the set of 2-interpretations I which are as close as possible to a 3-interpretation $J \in min(3 - mod(\Sigma), \leq)$:

$$\forall x \in PS, \text{ if } J(x) \neq \frac{1}{2}, \text{ then } J(x) = I(x)$$

- Computing *IngForg*(Σ) amounts to projecting each preferred 3-model of Σ on the variables classically interpreted in it (hence, **forgetting inconsistency**) and interpreting the resulting partial interpretations in a classical way
- $\Sigma \mid\!\sim^c_3 \alpha$ iff $\forall I \in IncForg(\Sigma)$, $I(\alpha) = 1$

$\mid\!\sim^c_3$-Inference: An Example

- $\Sigma = bird(W(oodstock)) \wedge (\neg bird(W) \vee hasWings(W))$
  $\wedge flies(W) \wedge \neg flies(W)$
- $\min(3-mod(\Sigma), \leq) = \{I\}$ s.t.
  $I(bird(W)) = I(hasWings(W)) = 1$, $I(flies(W)) = \frac{1}{2}$
- $IncForg(\Sigma) = \{J, K\}$ s.t.
  $J(bird(W)) = J(hasWings(W)) = 1$, $J(flies(W)) = 0$ and
  $K(bird(W)) = K(hasWings(W)) = 1$, $K(flies(W)) = 1$
- $\Sigma \mid\!\sim^c_3 bird(W) \wedge hasWings(W)$
- $\Sigma \not\mid\!\sim^c_3 flies(W)$
- $\Sigma \not\mid\!\sim^c_3 \neg flies(W)$

$\mid\!\sim_3^c$-Inference: Some Properties

- $\mid\!\sim_3^c$ satisfies all the properties of system P ("implies" is w.r.t. $\models$), except reflexivity
- $\mid\!\sim_3^c$ is strongly paraconsistent and satisfies the preservation property and the classical closure property
- $\mid\!\sim_3^c$ is relevant both to the paraconsistent approach to reasoning under inconsistency (where $\models$ is weakened) and to the approaches where Σ is weakened and $\models$ is kept (see part III of the presentation)

Complexity of $\mid\!\sim^c_3$-Inference

- $\mid\!\sim^c_3$-INFERENCE is Π^p_2-complete
- Under the restriction when Σ is a DNF formula, $\mid\!\sim^c_3$-INFERENCE is coNP-complete
- Under the restriction when Σ is a DNF formula and γ is a CNF formula, $\mid\!\sim^c_3$-INFERENCE is in P

"Compiling" Σ

Compute $cl(\Sigma)$

First Turn Σ into a strongly equivalent DNF

Step 1 Remove from the DNF every term α s.t. $inc(\alpha)$ is not minimal w.r.t. set inclusion

Step 2 Remove in every term of the resulting DNF every l s.t. its complementary literal also occurs in the term then remove every empty term

$$Cn_{\mid\sim_3^c}(\Sigma) = Cn_{\models}(cl(\Sigma))$$

Complexity of $\mid\!\sim^c_3$-Inference: An Example

- $\Sigma = bird(W) \wedge (\neg bird(W) \vee hasWings(W)) \wedge flies(W) \wedge \neg flies(W)$
- Σ is strongly 3-equivalent to the DNF $(bird(W) \wedge \neg bird(W) \wedge flies(W) \wedge \neg flies(W)) \vee$ $(bird(W) \wedge hasWings(W) \wedge flies(W) \wedge \neg flies(W))$
- Step 1 removes the first term
- Step 2 removes $flies(W) \wedge \neg flies(W)$ from the second one
- $cl(\Sigma) = bird(W) \wedge hasWings(W)$
- $Cn_{\models_c}(\Sigma) \;=\; Cn_{\models}(cl(\Sigma))$

Part III - Weakening Beliefs

- Mechanisms for ”recovering consistency”
- Belief revision
  - Two sources of information
  - The incoming information is considered prioritary
- Belief merging
  - *n* sources of information
  - Integrity constraints

Consistency "Recovering"

Weakening belief bases and taking advantage of classical deduction: many approaches

- A set $\Sigma = \{\varphi_1, \ldots, \varphi_n\}$ of formulae (beliefs)
- Each φ_i is classically interpreted
- Cannot be interpreted conjunctively when inconsistent!

Consistency "Recovering"

- "," is a **specific connective**
  Konieczny, Lang, Marquis, IJCAI'05
- **Two main issues**
  - **How to associate Σ with one/many consistent sets of formulae $\Sigma_1, \ldots, \Sigma_k$ representing the meaning of Σ?**
  - **How to reason from the consistent sets of formulae $\Sigma_1, \ldots, \Sigma_k$ representing the meaning of Σ?**

Preserving Information

- $\Sigma = \{\varphi_1, \ldots, \varphi_n\} \rightarrow \Sigma_1 = \emptyset$
- Σ_1 is a consistent set
- **Too many information are lost!** Drastic weakening mechanisms should be avoided
- Preserving non conflicting pieces of belief
- $\Sigma = \{bird(Tweety), \neg bird(Tweety), flies(Tweety)\}$: $flies(Tweety)$ is an expected consequence
- $\Sigma = \{bird(Tweety) \wedge red(Tweety), \neg bird(Tweety)\}$: $red(Tweety)$ is an expected consequence

Inference Principles

- **Skeptical inference :** $\Sigma \mid\!\sim \alpha$ iff $\forall \Sigma_i \in \{\Sigma_1, \ldots, \Sigma_k\}$, we have $\Sigma_i \models \alpha$
- **Credulous inference :** $\Sigma \mid\!\sim \alpha$ iff $\exists \Sigma_i \in \{\Sigma_1, \ldots, \Sigma_k\}$, we have $\Sigma_i \models \alpha$
- **Argumentative inference :** $\Sigma \mid\!\sim \alpha$ iff $\exists \Sigma_i \in \{\Sigma_1, \ldots, \Sigma_k\}$, we have $\Sigma_i \models \alpha$ and $\forall \Sigma_i \in \{\Sigma_1, \ldots, \Sigma_k\}$, we have $\Sigma_i \not\models \neg\alpha$
- **Majoritarian inference :** $\Sigma \mid\!\sim \alpha$ iff for a strict majority of $\Sigma_i \in \{\Sigma_1, \ldots, \Sigma_k\}$, we have $\Sigma_i \models \alpha$
- ...

An Example

$\Sigma = \{b(T) \wedge f(T), \neg f(T) \wedge r(T), \neg hW(T), f(T) \vee hW(T)\}$

- Two "minimal inconsistent subsets" (w.r.t. $\subseteq$)
  - $\{b(T) \wedge f(T), \neg f(T) \wedge r(T)\}$
  - $\{\neg f(T) \wedge r(T), \neg hW(T), f(T) \vee hW(T)\}$
- Assume Σ associated with its "maxcons = maximal consistent subsets" (w.r.t. $\subseteq$)
  - $\{b(T) \wedge f(T), \neg hW(T), f(T) \vee hW(T)\}$
  - $\{\neg f(T) \wedge r(T), \neg hW(T)\}$
  - $\{\neg f(T) \wedge r(T), f(T) \vee hW(T)\}$

An Example

$\Sigma = \{b(T) \wedge f(T), \neg f(T) \wedge r(T), \neg hW(T), f(T) \vee hW(T)\}$

- **Skeptical inference :** has valuable logical properties (especially (And))
- This is not the case for the other inference principles
  - **Credulous inference :** $\Sigma \mid\!\sim f(T)$, $\Sigma \mid\!\sim hW(T)$ but $\Sigma \not\mid\!\sim f(T) \wedge hW(T)$
  - **Argumentative inference :** $\Sigma \mid\!\sim f(T)$, $\Sigma \mid\!\sim r(T)$, but $\Sigma \not\mid\!\sim f(T) \wedge r(T)$
  - **Majoritarian inference :** $\Sigma \mid\!\sim \neg f(T)$, $\Sigma \mid\!\sim \neg hW(T)$ but $\Sigma \not\mid\!\sim \neg f(T) \wedge \neg hW(T)$
- This explains why **skeptical inference is the main choice**

Old Good Classical Deduction can be Safely Used

- **The preservation property is expected:** if $\Sigma = \{\varphi_1, \ldots, \varphi_n\}$ is classically consistent, then the meaning of Σ should be given by a unique Σ_1 equivalent to Σ!
- **The classical closure property is ensured** (when skeptical inference is used Σ is interpreted as the disjunction $\Sigma_1 \vee \ldots \vee \Sigma_k$ which is classically consistent

Mechanisms for Consistency "Recovering"

- **Belief inhibition:** $\{\alpha, \beta\} \rightarrow \{\alpha\}$
- **Dilation:** $\{\alpha, \beta\} \rightarrow \{\alpha, [\beta]^{+1}\}$
- **Forgetting:** $\{\alpha, \beta\} \rightarrow \{\alpha, \exists X.\beta\}$
- Disjunction: $\{\alpha, \beta\} \rightarrow \{\alpha \vee \beta\}$
- Introducing abnormalities (to be minimized): $\{\alpha, \beta\} \rightarrow \{\alpha, \neg ab \vee \beta\}$
- Renaming techniques (with a principled reintroduction of $x \Leftrightarrow x'$): $\{\alpha, \beta\} \rightarrow \{\alpha, \beta'\}$

Belief Inhibition

Rescher and Manor, Theory and Decision, 1970
Nebel, KR'91 (and many others)

- **Removing some beliefs:** $\Sigma = \{\alpha, \beta\} \rightarrow \{\alpha\}$
- **Preserving information:** removing as few pieces of beliefs as possible
- Which ones? Exploit preferences/priorities (when available!)
- Whatever the case, may lead to several (exponentially many) consistent subsets of Σ
- **Many selection policies** for characterizing them

Belief Inhibition: Some Policies

1. Remove every α involved in a conflict (an inconsistent subset of Σ minimal w.r.t. $\subseteq$)
   - Leads to a single Σ_i
   - Is very cautious
   - Skeptical inference is computationally expensive (Π_2^p-complete) (but compilable to coNP)
2. Focus on the "maxcons": the consistent subsets of Σ which are **maximal w.r.t.** $\subseteq$
   - Leads to exponentially many Σ_i in the worst case
   - Is not as cautious as the previous policy
   - Skeptical inference is computationally expensive (Π_2^p-complete)

Belief Inhibition: Some Policies

3. Focus on the consistent subsets of Σ which are **maximal w.r.t. cardinality**
   - Leads to exponentially many Σ_i in the worst case
   - Is not as cautious as the previous policy (preserves more information)
   - Skeptical inference is "mildly" expensive (Θ_2^p-complete)

An Example

$\Sigma = \{b(T) \wedge f(T), \neg f(T) \wedge r(T), \neg hW(T), f(T) \vee hW(T)\}$

1. "free consequences": every formula of Σ is involved in a conflict!
   $\Sigma \mid\!\sim \alpha$ iff α is valid
   Σ is "interpreted" as $\top$
2. "subset maximality":
   $\Sigma \mid\!\sim \alpha$ iff $(b(T) \wedge f(T) \wedge \neg hW(T)) \vee (\neg f(T) \wedge r(T)) \models \alpha$
3. "cardinality maximality":
   $\Sigma \mid\!\sim \alpha$ iff $(b(T) \wedge f(T) \wedge \neg hW(T)) \models \alpha$

A Glimpse at Dilation

Bloch and Lang, IPMU 1998

- **Weakening some beliefs:** $\Sigma = \{\alpha, \beta\} \rightarrow \{\alpha, [\beta]^{+1}\}$
- $[\beta]^{+1}$: any formula at Hamming distance $\leq$ 1 of β
  - $\alpha = \neg b(T)$
  - $\beta = b(T) \wedge f(T)$
  - $[\beta]^{+1} \equiv (b(T) \wedge f(T)) \vee (\neg b(T) \wedge f(T)) \vee (b(T) \wedge \neg f(T))$
  - $[\beta]^{+1} \equiv b(T) \vee f(T)$

A Glimpse at Dilation

- **Preserve more information than belief inhibition (in general)**
- $\Sigma = \{\alpha, \beta\}$ has two "maximal consistent subsets" w.r.t. $\subseteq$
  - $\{\alpha(= \neg b(T))\}$
  - $\{\beta(= b(T) \wedge f(T))\}$
- Using such a policy, Σ is "interpreted" as $(\alpha \vee \beta) \equiv \neg b(T) \vee f(T)$: $\Sigma \not\models f(T)$
- Solving the conflict by dilating α or dilating β:
- Σ is "interpreted" as $\beta \vee (\alpha \wedge (b(T) \vee f(T)) \equiv f(T)$

A Glimpse at Forgetting

Lang and Marquis, AIJ, 2010

- **Weakening some beliefs:** $\Sigma = \{\alpha, \beta\} \rightarrow \{\alpha, \exists X.\beta\}$
- **Preserve more information than belief inhibition (in general)**
- $\exists X.\beta$ is the logically strongest consequence of β which is independent of X
- β is independent of X iff there exists γ such that $\gamma \equiv \beta$ and $Var(\gamma) \cap X = \emptyset$
- $\exists X.\beta$ is equivalent to the formula given by
  - $\exists \emptyset.\beta \equiv \beta$
  - $\exists \{x\}.\beta \equiv (\beta \mid x) \vee (\beta \mid \neg x)$
  - $\exists X \cup \{x\}.\beta \equiv \exists X.(\exists \{x\}.\beta)$

A Glimpse at Forgetting

- $\Sigma = \{\alpha, \beta\}$
  - $\alpha = \neg b(T)$
  - $\beta = b(T) \wedge f(T)$
  - $\exists\{b(T)\}.\beta \equiv ((b(T) \wedge f(T)) \mid \neg b(T)) \vee ((b(T) \wedge f(T)) \mid b(T))$
  - $\exists\{b(T)\}.\beta \equiv \bot \vee f(T) \equiv f(T)$

A Glimpse at Forgetting

- **Preserve more information than belief inhibition (in general)**
- $\Sigma = \{\alpha, \beta\}$ has two "maximal consistent subsets" w.r.t. $\subseteq$
  - $\{\alpha (= \neg b(T))\}$
  - $\{\beta (= b(T) \wedge f(T))\}$
- Using such a policy, Σ is "interpreted" as $(\alpha \vee \beta) \equiv \neg b(T) \vee f(T)$: $\Sigma \not\models f(T)$
- Solving the conflict by forgetting $b(T)$ in α or in β:
- Σ is "interpreted" as $\beta \vee (\alpha \wedge f(T)) \equiv f(T)$

Belief Revision with Tweety

A belief revision scenario: two sources, the incoming information being prioritary

- $\Sigma = bird(Tweety) \wedge (bird(Tweety) \Rightarrow flies(Tweety))$
  $\wedge (bird(Tweety) \Rightarrow hasWings(Tweety))$
- The new piece of evidence $\phi = \neg flies(Tweety)$ (assumed more reliable than Σ) is available
- What should be concluded from the revised base $\Sigma \circ \phi$?
  - Nothing except ϕ and all valid formulae
  - $bird(Tweety)$
  - $(bird(Tweety) \Rightarrow flies(Tweety))$
  - $hasWings(Tweety)$
- **Extra-logical information are required!**

Postulates for Belief Revision

Alchourrón, Gärdenfors and Makinson, JSL, 1985

K belief set (deductively closed set of formulae) – ϕ, ψ formulae

(K1) $K \circ \phi$ is a belief set

(K2) $\phi \in K \circ \phi$

(K3) $K \circ \phi \subseteq K + \phi$

(K4) If $\neg\phi \notin K$, then $K + \phi \subseteq K \circ \phi$

(K5) $K \circ \phi = PROP_{PS}$ iff ϕ is contradictory

(K6) If $\phi \equiv \psi$, then $K \circ \phi = K \circ \psi$

(K7) $K \circ (\phi \wedge \psi) \subseteq (K \circ \phi) + \psi$

(K8) If $\neg\psi \notin K \circ \phi$, then $(K \circ \phi) + \psi \subseteq K \circ (\phi \wedge \psi)$

Postulates for Belief Revision

Katsuno and Mendelzon, AIJ, 1991
(equivalent to AGM in the finite case)

(R1) $\Sigma \circ \phi \models \phi$

(R2) If $\Sigma \wedge \phi$ is consistent then $\Sigma \circ \phi \equiv \Sigma \wedge \phi$

(R3) If ϕ is consistent then $\Sigma \circ \phi$ is consistent

(R4) If $\Sigma_1 \equiv \Sigma_2$ and $\phi_1 \equiv \phi_2$ then $\Sigma_1 \circ \phi_1 \equiv \Sigma_2 \circ \phi_2$

(R5) $(\Sigma \circ \phi) \wedge \psi \models \Sigma \circ (\phi \wedge \psi)$

(R6) If $(\Sigma \circ \phi) \wedge \psi$ is consistent then $\Sigma \circ (\phi \wedge \psi) \models (\Sigma \circ \phi) \wedge \psi$

AGM Belief Revision Operators

- Many characterizations (representation theorems) based on subbases selection, epistemic entrenchment or **faithful assignments**
- $\circ$ satisfies (R1) ... (R6) iff there exists a faithful assignment f s.t. $f(\Sigma)$ is complete and $mod(\Sigma \circ \phi) = min(mod(\phi), f(\Sigma))$
  - $f : \Sigma \mapsto \leq_\Sigma$ a preorder over $BOOL^{PS}$
  - If $I \models \Sigma$ and $J \models \Sigma$, then $I =_\Sigma J$
  - If $I \models \Sigma$ and $J \not\models \Sigma$, then $I <_\Sigma J$
  - If $\Sigma \equiv \Sigma'$, then $f(\Sigma) = f(\Sigma')$
- **Many AGM operators exist**

AGM Belief Revision Operators: Some Examples

- Drastic revision $\circ$
  - $I =_\Sigma J$ iff ($I \models \Sigma$ and $J \models \Sigma$) or ($I \not\models \Sigma$ and $J \not\models \Sigma$)
  - $I <_\Sigma J$ iff $I \models \Sigma$ and $J \not\models \Sigma$
  - Belief inhibition at work!
- $\Sigma = \mathit{bird}(T) \wedge (\mathit{bird}(T) \Rightarrow \mathit{flies}(T)) \wedge (\mathit{bird}(T) \Rightarrow \mathit{hasWings}(T))$
- $\Sigma \circ \neg \mathit{flies}(T) \equiv \neg \mathit{flies}(T)$
- $\Sigma \circ \phi \equiv \Sigma \wedge \phi$ if consistent, and $\equiv \phi$ otherwise

AGM Belief Revision Operators: Some Examples

- Dalal's operator $\circ_D$ AAAI'88
  - $I \leq_\Sigma J$ iff $d_H(I, \Sigma) \leq d_H(J, \Sigma)$
  - $d_H(I, \Sigma) = min(d_H(I, K) \mid K \models \Sigma)$
  - $d_H(I, K)$ is the Hamming distance between I and K
  - Belief dilation at work!
- $\Sigma = bird(T) \wedge (bird(T) \Rightarrow flies(T)) \wedge (bird(T) \Rightarrow hasWings(T))$
- $\Sigma \circ_D \neg flies(T) \equiv bird(T) \wedge hasWings(T) \wedge \neg flies(T)$

System P

"Belief revision and nonmonotonic logic are two sides of the same coin" P. Gärdenfors, ECAI'90
Rationality postulates (system P**)** and its representation theorems: Kraus, Lehmann and Magidor, AIJ, 1990

(Ref) $\Sigma \mid\!\sim \Sigma$
(LLE) If $\Sigma \equiv \Sigma'$ and $\Sigma \mid\!\sim \phi$, then $\Sigma' \mid\!\sim \phi$
(RW) If $\Sigma \mid\!\sim \phi$ and $\phi \models \psi$, then $\Sigma \mid\!\sim \psi$
(And) If $\Sigma \mid\!\sim \phi$ and $\Sigma \mid\!\sim \psi$, then $\Sigma \mid\!\sim (\phi \wedge \psi)$
(Or) If $\Sigma \mid\!\sim \phi$ and $\Sigma' \mid\!\sim \phi$, then $\Sigma \vee \Sigma' \mid\!\sim \phi$
(CM) If $\Sigma \mid\!\sim \phi$ and $\Sigma \mid\!\sim \psi$, then $\Sigma \wedge \phi \mid\!\sim \psi$

AGM Belief Revision and Reasoning with Exceptions

Connections with belief revision:
Makinson and Gärdenfors, Logic of Theory Change, 1989

$$\phi \mid\!\sim^{\circ}_{K} \psi \text{ iff } \psi \in K \circ \phi$$

- If $\circ$ satisfies the AGM postulates, then $\mid\!\sim^{\circ}_{K}$ is rational and satisfies the consistency preservation rule:
  If $\phi \mid\!\sim^{\circ}_{K} \bot$, then ϕ is contradictory
- If $\mid\!\sim$ is rational and satisfies the consistency preservation rule, then there exists a belief revision operator $\circ$ and a belief set K s.t. $\mid\!\sim = \mid\!\sim^{\circ}_{K}$

Nevertheless, nonmonotonicity and paraconsistency are two **logically independent notions**

Belief Merging

- A profile $\Sigma = \langle \phi_1, \ldots, \phi_n \rangle$ of *n* belief bases
- Belief bases are typically conflicting one another
- Some integrity constraints μ (encoding physical laws or norms) which must be satisfied
- $\Delta_\mu(\Sigma)$ is a belief base representing **a global, consistent view of the available information**

Rationality Postulates for Belief Merging

Konieczny and Pino Pérez, JLC, 2002

Δ is an **Integrity Constraint merging operator** (IC merging operator) if and only if it satisfies the following properties :

(IC0) $\Delta_\mu(\Sigma) \models \mu$

(IC1) If μ is consistent, then $\Delta_\mu(\Sigma)$ is consistent

(IC2) If $\bigwedge \Sigma$ is consistent with μ, then $\Delta_\mu(\Sigma) \equiv \bigwedge \Sigma \wedge \mu$

(IC3) If $\Sigma_1 \equiv \Sigma_2$ and $\mu_1 \equiv \mu_2$, then $\Delta_{\mu_1}(\Sigma_1) \equiv \Delta_{\mu_2}(\Sigma_2)$

(IC4) If $\phi_1 \models \mu$ and $\phi_2 \models \mu$, then $\Delta_\mu(\langle \phi_1, \phi_2 \rangle) \wedge \phi_1 \not\models \bot$ then $\Delta_\mu(\langle \phi_1, \phi_2 \rangle) \wedge \phi_2 \not\models \bot$

(IC5) $\Delta_\mu(\Sigma_1) \wedge \Delta_\mu(\Sigma_2) \models \Delta_\mu(\Sigma_1 \sqcup \Sigma_2)$

(IC6) If $\Delta_\mu(\Sigma_1) \wedge \Delta_\mu(\Sigma_2)$ is consistent, then $\Delta_\mu(\Sigma_1 \sqcup \Sigma_2) \models \Delta_\mu(\Sigma_1) \wedge \Delta_\mu(\Sigma_2)$

(IC7) $\Delta_{\mu_1}(\Sigma) \wedge \mu_2 \models \Delta_{\mu_1 \wedge \mu_2}(\Sigma)$

(IC8) If $\Delta_{\mu_1}(\Sigma) \wedge \mu_2$ is consistent, then $\Delta_{\mu_1 \wedge \mu_2}(\Sigma) \models \Delta_{\mu_1}(\Sigma)$

Majority vs. Arbitration

Ally, Brian and Charles want to spend the evening together. Brian and Ally want to go out for diner and to the movie. Charles wants to stay at home.

Majority restaurant and movie

Ally	+ +
Brian	+ +
Charles	– –

Arbitration restaurant xor movie

Ally	+
Brian	+
Charles	–

Majority - Arbitration

An IC operator is a **majority operator** if it satisfies (*Maj*):

(Maj) $\exists n\ \Delta_{\mu}(\Sigma_1 \sqcup \Sigma_2^n) \models \Delta_{\mu}(\Sigma_2)$

An IC operator is an **arbitration operator** if it satisfies (*Arb*)

(Arb) $\Delta_{\mu_1}(\langle\phi_1\rangle) \equiv \Delta_{\mu_2}(\langle\phi_2\rangle)$
$\Delta_{\mu_1 \Leftrightarrow \neg\mu_2}(\langle\phi_1, \phi_2\rangle) \equiv \mu_1 \Leftrightarrow \neg\mu_2$
$\mu_1 \not\models \mu_2$
$\mu_2 \not\models \mu_1$
$\Rightarrow \Delta_{\mu_1 \vee \mu_2}(\langle\phi_1, \phi_2\rangle) \equiv \Delta_{\mu_1}(\langle\phi_1\rangle)$

Syncretic Assignments

A **syncretic assignment** is a total function mapping each profile Σ to a total pre-order $\leq_\Sigma$ over interpretations such that:

1) If $I \models \Sigma$ and $J \models \Sigma$, then $I \simeq_\Sigma J$

2) If $I \models \Sigma$ and $J \not\models \Sigma$, then $I <_\Sigma J$

3) If $\Sigma_1 \equiv \Sigma_2$, then $\leq_{\Sigma_1} = \leq_{\Sigma_2}$

4) $\forall I \models \phi_1 \; \exists J \models \phi_2 \; J \leq_{\langle \phi_1, \phi_2 \rangle} I$

5) If $I \leq_{\Sigma_1} J$ and $I \leq_{\Sigma_2} J$, then $I \leq_{\Sigma_1 \sqcup \Sigma_2} J$

6) If $I <_{\Sigma_1} J$ and $I \leq_{\Sigma_2} J$, then $I <_{\Sigma_1 \sqcup \Sigma_2} J$

Majority and Fair Assignments

A **majority syncretic assignment** is a syncretic assignment which satisfies:

7) If $I <_{\Sigma_2} J$, then $\exists n\ I <_{\Sigma_1 \sqcup \Sigma_2^n} J$

A **fair syncretic assignment** is a syncretic assignment which satisfies:

8) $I <_{\langle \phi_1 \rangle} J$, $I <_{\langle \phi_2 \rangle} L$, $J \simeq_{\langle \phi_1, \phi_2 \rangle} L \Rightarrow I <_{\langle \phi_1, \phi_2 \rangle} J$

Arbitration

Preferring ”median” worlds

Arbitration

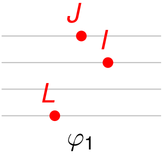
Preferring "median" worlds

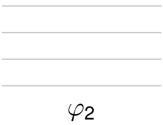
Arbitration

Preferring "median" worlds

Arbitration

Preferring ”median” worlds



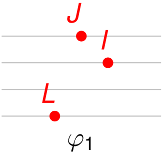


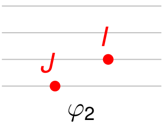
Arbitration

Preferring "median" worlds

Arbitration

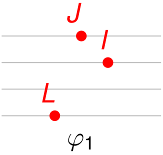
Preferring ”median” worlds

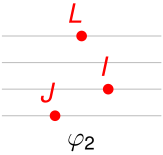




Arbitration

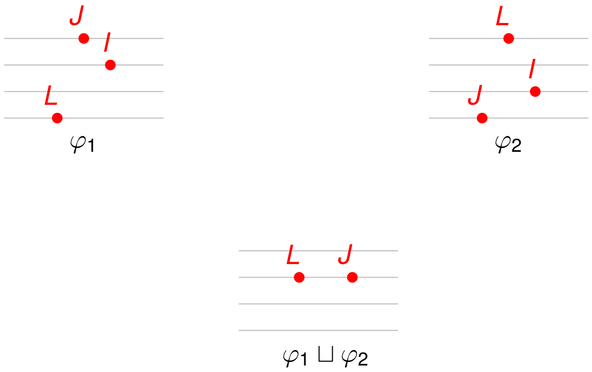
Preferring "median" worlds





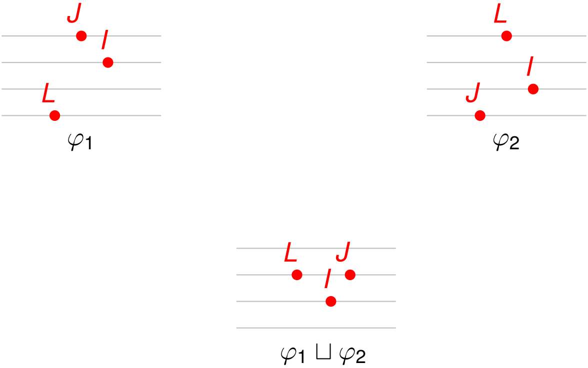
Arbitration

Preferring ”median” worlds



Arbitration

Preferring "median" worlds



Belief Merging and Belief Revision

- **IC merging generalizes belief revision**
- one source φ + some constraints μ
  $\rightarrow$ a profile Σ of *n* sources + some constraints μ
- **Theorem** If $\triangle$ is an IC merging operator (it satisfies **(IC0-IC8)**), then the operator $\circ_\triangle$, defined as $\varphi \circ_\triangle \mu = \triangle_\mu(\varphi)$, is an AGM revision operator (it satisfies **(R1-R6)**)

Some Representation Theorems

Konieczny and Pino Pérez, JLC, 2002

Theorem A merging operator Δ is

- an IC merging operator

if and only if there exists

- a syncretic assignment

that maps each profile Σ to a complete preorder $\leq_\Sigma$ such that $mod(\Delta_\mu(\Sigma)) = min(mod(\mu), \leq_\Sigma)$

Some Representation Theorems

Konieczny and Pino Pérez, JLC, 2002

Theorem A merging operator Δ is

- an IC majority merging operator

if and only if there exists

- a majority syncretic assignment

that maps each profile Σ to a complete preorder $\leq_\Sigma$ such that
$mod(\Delta_\mu(\Sigma)) = min(mod(\mu), \leq_\Sigma)$

Some Representation Theorems

Konieczny and Pino Pérez, JLC, 2002

Theorem A merging operator Δ is

- an IC arbitration operator

if and only if there exists

- a fair syncretic assignment

that maps each profile Σ to a complete preorder $\leq_\Sigma$ such that $mod(\Delta_\mu(\Sigma)) = min(mod(\mu), \leq_\Sigma)$

Belief Merging: the SBB View

- The available data can be viewed as a two-strata **stratified belief base**
$$\langle \{\mu\}, \bigcup \{\varphi_i \mid \varphi_i \in \Sigma\} \rangle$$
- This expresses that μ is prioritary to each belief base φ_i
- The mechanisms for "recovering consistency", especially belief inhibition, can be easily generalized to stratified belief bases
- μ has to be taken for sure and only belief bases can be weakened
- However this does not always give the expected results...

Formula-Based Merging

Baral, Kraus, Minker, IEEE TKDE, 1991
Baral, Kraus, Minker, Subrahmanian, Comp. Intell., 1992

$\text{MAXCONS}(\Sigma, \mu)$ = the set of "maxcons" M from $\Sigma \wedge \mu$ s.t. $\mu \in M$

- $\triangle_{\mu}^{C1}(\Sigma) = \text{MAXCONS}(\Sigma, \mu)$
- $\triangle_{\mu}^{C3}(\Sigma) = \{M \mid M \in \text{MAXCONS}(\Sigma, \top) \text{ and } M \wedge \mu \text{ consistent}\}$
- $\triangle_{\mu}^{C4}(\Sigma) = \text{MAXCONS}_{card}(\Sigma, \mu)$
- $\triangle_{\mu}^{C5}(\Sigma) = \{M \wedge \mu \mid M \in \text{MAXCONS}(\Sigma, \top) \text{ and } M \wedge \mu \text{ consistent}\}$ if this set is nonempty and μ otherwise

Formula-Based Merging: Some Postulates

	IC0	IC1	IC2	IC3	IC4	IC5	IC6	IC7	IC8
$\triangle^{C1}$	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$	
$\triangle^{C3}$					$\surd$	$\surd$		$\surd$	$\surd$
$\triangle^{C4}$	$\surd$	$\surd$	$\surd$					$\surd$	$\surd$
$\triangle^{C5}$	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$	$\surd$

Taking Account for the Information Distribution

- n sources ... but the origin of each piece of belief is lost!
- No integrity constraints ($\mu \equiv \top$)
- Each source but one states $b(T)$
- The remaining one states $\neg b(T)$
- There are two "maxcons" w.r.t. $\subseteq$:
  - one equivalent to $b(T)$
  - one equivalent to $\neg b(T)$
- Σ is "interpreted" as $\top$
- **If $n > 2$, the majority of sources states $b(T)$!**

Formula-Based Merging: Selection Functions

Konieczny, KR'00

Use a selection function to choose only the "best maxcons" M

- Take into account the distribution of the information among the sources

Example:

- $dist_{\cap}(M, \varphi) = card(\varphi \cap M)$
- $dist_{\cap,\Sigma}(M, \Sigma) = \sum_{\varphi \in \Sigma} dist_{\cap}(M, \varphi)$

Formula-Based Merging: Selection Functions

	IC0	IC1	IC2	IC3	IC4	IC5	IC6	IC7	IC8
$\triangle^{C1}$	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$	
$\triangle^{\cap,\Sigma}$	$\surd$	$\surd$	$\surd$			$\surd$	$\surd$	$\surd$	$\surd$

Model-Based Merging

Konieczny and Pino Pérez, JLC, 2002

Select the models of μ that are the closest ones to the given profile Σ

$$I \leq_{\Sigma}^{d_x} J \text{ iff } d_x(I, \Sigma) \leq d_x(J, \Sigma)$$

d_x can be computed using:

- a ”distance” between worlds d
- an aggregation function f

"Distances"

- Distance between worlds
  - $d(I, J) = d(J, I)$
  - $d(I, J) = 0$ iff $I = J$
- Distance between a world and a base
  - $d(I, \varphi) = \min_{J \models \varphi} d(I, J)$
- Distance between a world and a profile
  - $d_{d,f}(I, \Sigma) = f(d(I, \varphi_1), \ldots d(I, \varphi_n))$

"Distances": Some Examples

- Drastic distance d_D:

  $d_D(I, J) = 0$ if $I = J$, $= 1$ otherwise

- Hamming distance d_H:

  $d_H(I, J)$ is the number of atoms on which I and J differ

Aggregation Functions

- An **aggregation function** f is a total function that associates a positive number to any tuple of positive numbers such that:
  - If $x \leq y$, then $f(x_1, \ldots, x, \ldots, x_n) \leq f(x_1, \ldots, y, \ldots, x_n)$ *(monotony)*
  - $f(x_1, \ldots, x_n) = 0$ if and only if $x_1 = \ldots = x_n = 0$ *(minimality)*
  - $f(x) = x$ *(identity)*
- Examples of aggregation function:
  - max, *leximax*, Σ, Σ^n, *leximin*, ...

Distance-Based Merging

Konieczny, Lang and Marquis, AIJ, 2004

Theorem Let d be a "distance" between interpretation and f be an aggregation function. $\triangle^{d,f}$ satisfies (IC0), (IC1), (IC2), (IC7) and (IC8).

Theorem A merging operator $\triangle^{d,f}$ satisfies (IC0-IC8) if and only if f satisfies:

- For any permutation σ, $f(x_1, \ldots, x_n) = f(\sigma(x_1, \ldots, x_n))$ *(symmetry)*
- If $f(x_1, \ldots, x_n) \leq f(y_1, \ldots, y_n)$, then $f(x_1, \ldots, x_n, z) \leq f(y_1, \ldots, y_n, z)$ *(composition)*
- If $f(x_1, \ldots, x_n, z) \leq f(y_1, \ldots, y_n, z)$, then $f(x_1, \ldots, x_n) \leq f(y_1, \ldots, y_n)$ *(decomposition)*

Distance-Based Merging: An Example

$\mu = ((S \wedge T) \vee (S \wedge P) \vee (T \wedge P)) \Rightarrow I$

- $\varphi_1 = \varphi_2 = S \wedge T \wedge P$
- $\varphi_3 = \neg S \wedge \neg T \wedge \neg P \wedge \neg I$
- $\varphi_4 = T \wedge P \wedge \neg I$

- $mod(\varphi_1) = \{(1,1,1,1),(1,1,1,0)\}$
- $mod(\varphi_3) = \{(0,0,0,0)\}$
- $mod(\varphi_4) = \{(1,1,1,0),(0,1,1,0)\}$

Distance-Based Merging: An Example

	φ_1	φ_2	φ_3	φ_4	$d_{d_H,Max}$	$d_{d_H,\Sigma}$	d_{d_H,Σ^2}	$d_{d_H,GMax}$
$(0,0,0,0)$	3	3	0	2	3	8	22	(3,3,2,0)
$(0,0,0,1)$	3	3	1	3	3	10	28	(3,3,3,1)
$(0,0,1,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,0,1,1)$	2	2	2	2	2	8	16	(2,2,2,2)
$(0,1,0,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,1,0,1)$	2	2	2	2	2	8	16	(2,2,2,2)
$(0,1,1,1)$	1	1	3	1	3	6	12	(3,1,1,1)
$(1,0,0,0)$	2	2	1	2	2	7	13	(2,2,2,1)
$(1,0,0,1)$	2	2	2	3	3	9	21	(3,2,2,2)
$(1,0,1,1)$	1	1	3	2	2	7	15	(3,2,1,1)
$(1,1,0,1)$	1	1	3	2	3	7	15	(3,2,1,1)
$(1,1,1,1)$	0	0	4	1	4	5	17	(4,1,0,0)

Distance-Based Merging: An Example

	φ_1	φ_2	φ_3	φ_4	$d_{d_H,Max}$	$d_{d_H,\Sigma}$	d_{d_H,Σ^2}	$d_{d_H,GMax}$
$(0,0,0,0)$	3	3	0	2	3	8	22	(3,3,2,0)
$(0,0,0,1)$	3	3	1	3	3	10	28	(3,3,3,1)
$(0,0,1,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,0,1,1)$	2	2	2	2	2	8	16	(2,2,2,2)
$(0,1,0,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,1,0,1)$	2	2	2	2	2	8	16	(2,2,2,2)
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Distance-Based Merging: some Operators

Let d be any "distance" between worlds

- $\triangle^{d,\max}$ operators satisfy (IC0-IC5), (IC7), (IC8) and (Arb)
- $\triangle^{d,\text{GMIN}}$ operators are IC merging operators
- $\triangle^{d,\text{GMAX}}$ operators are arbitration operators
- $\triangle^{d,\Sigma}$ and $\triangle^{d,\Sigma^n}$ operators are majority operators

Beyond Distance-Based Merging

Distance-based merging operators

- take account for the information distribution
- offer expected logical properties
  (many of them are IC merging operators)
- are "mildly" complex
  (inference is typically Θ_2^p-complete or Δ_2^p-complete)
- ... but cannot take inconsistent bases into account!

$\Rightarrow$ **Add a second aggregation step (intra-sources):**
DA^2 **merging operators**

Konieczny, Lang and Marquis, AIJ, 2004

Belief Merging and Judgment Aggregation

- A set $N = \{1, \ldots n\}$ of individuals
- A set $X = \{\alpha_1, \ldots, \alpha_m\}$ of logical formulae, called the **agenda**
- Each individual i gives her (consistent) **judgment set** about the agenda: $J_i : X \to \{0, 1\}$
- **Question**: how to define a consistent **judgment of the group** $J = f(J_1, \ldots, J_n)$ from the judgment sets of the individuals?

Judgment Aggregation: An Example

Doctrinal Paradox / Discursive Paradox

- α: good researcher
- β: good teacher
- $\gamma \Leftrightarrow \alpha \wedge \beta$: hire the candidate

	α	β	γ
1	1	0	0
2	0	1	0
3	1	1	1
majority	**1**	**1**	**0**

Majority does not lead to a consistent judgment!

An Impossibility Theorem

- **Universal Domain**. The judgment aggregation function should accept any profile of individual judgment sets (complete, consistent, deductively closed)
- **Collective Rationality**. The judgment aggregation function produces consistent and complete collective judgment sets
- **Anonymity**. The result should be invariant under any permutation of individuals in N
- **Systematicity**. For any formulae $\alpha, \beta \in X$, and any profiles $(J_1, \ldots J_n)$, $(J'_1, \ldots J'_n)$, if for all individuals i, $\alpha \in J_i$ iff $\beta \in J'_i$, then $\alpha \in f(J_1, \ldots J_n)$ iff $\beta \in f(J'_1, \ldots J'_n)$

Theorem [List-Pettit, Economics and Philosophy, 2002] There is no judgment aggregation function satisfying universal domain, collective rationality, anonymity and systematicity

Comparing Belief Merging with Judgment Aggregation

- **Input:** Profile of bases vs. profile of individual judgments
- **Computational Complexity:** NP-hard vs. in P
- **Logical Properties:** Good vs. bad (no consistency guarantee)

Judgment aggregation: a quick-and-dirty approach to belief merging?

Part IV - Conclusion

- Some concluding remarks
- Some perspectives for further research

Conclusion

- **Inconsistency handling is a fundamental issue for AI**
- Many different approaches have been defined so far
  - Different pieces of data (one/several source(s), preferences, etc.)
  - Different mechanisms

Challenging Issues

- Inferring from an inconsistent belief base is typically hard from a computational point of view
- Approaches must be designed for **circumventing the intractability** of paraconsistent inference (restriction, approximation, compilation, ...)

Challenging Issues

- Defining **a set of rationality postulates** suited to the different scenarios where handling inconsistency is needed
- Turn out to be a hard task, even for a single scenario (e.g. belief merging)
- Not independent of the computational dimension
- Intuitions stemming from social choice theory

Other Postulates for Belief Merging: Unanimity

Everaere, Konieczny and Marquis, AIJ, 2010

Unanimity: If everyone agrees on a merits of a candidate, so does the aggregation result

- Unanimity on models

(UnaM) If $I \models \mu$ and if $\forall \varphi \in \Sigma$, $I \models \varphi$, then $I \models \triangle_\mu(\Sigma)$

- This is a consequence of (IC2)

- Unanimity on consequences

(UnaF) If $\exists \varphi \in \Sigma$ s.t. $\mu \wedge \varphi$ is consistent, then if $\forall \varphi \in \Sigma$, $\varphi \models \alpha$, then $\triangle_\mu(\Sigma) \models \alpha$

(Disj) If $\bigvee \Sigma$ is consistent with μ, then $\triangle_\mu(\Sigma) \models \bigvee \Sigma$

Other Postulates for Belief Merging: Truth Tracking

Everaere, Konieczny and Marquis, ECAI'08

- **Belief merging operators prove suited to goal merging as well**
- How belief merging differs from goal merging?

- **Truth tracking:**
  ability of a merging operator to identify the "true world" /* "in the limit"

Condorcet Jury Theorem

- **Condorcet Jury Theorem:** n agents (supposed to be independent and reliable) voting on a single yes/no alternative
- **Simple majority** is the truth tracking method in this case

- In belief merging, more than two worlds are considered in general
- ... and agents beliefs may have more than a single model!

Extending Condorcet Jury Theorem

- **Reliability has to be generalized:**
  agent i is reliable when for all $I \neq I^*$, $P(I^* \models \varphi_i) > P(I \models \varphi_i)$
- Let $\{I^*, I_1, \ldots, I_{k-1}\}$ be a set of possible worlds and let Σ be a profile from a set of n independent, homogenous, reliable individuals. Then the probability that the true world is identified by the majority tends to 1 as the group size increases, i.e., $\forall i \in \{1, \ldots, k-1\}$,

$$P(s_a(I^*) > s_a(I_i)) \xrightarrow[n \to \infty]{} 1$$

- $s_a(I)$ is the number of bases φ_i satisfied by I
- **Approval voting** is the appropriate truth tracking method for voting on k ($k > 2$) alternatives

Other Postulates for Belief Merging: Truth Tracking

(TT) Let Σ be a profile from n independent, homogeneous, reliable agents.

$$P(mod(\triangle(\Sigma)) = \{I^*\}) \xrightarrow[n \to \infty]{} 1$$

- $\triangle^{d_H,Gmax}$ does not satisfy **(TT)**
- $\triangle^{d_H,\Sigma}$ does not satisfy **(TT)**
- $\triangle^{d_D,\Sigma}$ satisfies **(TT)**
- For any "distance" d, $\triangle^{d,Gmin}$ satisfies **(TT)**

Other Postulates for Belief Merging: Language Independence

Marquis and Schwind, IJCAI'11

- **Language independence is robustness to (some) substitutions**
- σ from $X = \{x_1, \ldots, x_n\}$ to $PROP_Y$ satisfies *SIN* (symbol insensitivity) iff σ is a bijection from X to Y

- There exist belief revision (resp. belief merging) operators which satisfy all AGM/KM postulates (resp. all KP postulates) **but are not *SIN*-language independent**

Other Postulates for Belief Merging: Language Independence

- This calls for additional postulates:
  - **(SIN-R)** $\sigma(\alpha \circ \beta) \equiv \sigma(\alpha) \circ \sigma(\beta)$
  - **(SIN-M)** $\sigma(\Delta_{\mu}(\Sigma)) \equiv \Delta_{\sigma(\mu)}(\sigma(\Sigma))$

- Every belief revision/merging operator based on a decomposable distance is *SIN*-language independent
- Other results are available for other forms of language independence

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