Merging Qualitative Constraints Networks Using Propositional Logic

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Abstract. In this paper we address the problem of merging qualitative constraints networks ($QCNs$). We propose a rational merging procedure for $QCNs$. It is based on translations of $QCNs$ into propositional formulas, and take advantage of propositional merging operators.

1 Introduction

Representing and reasoning about time and space is an important task in many domains such as natural language processing, geographic information systems, computer vision, robot navigation. Several qualitative approaches have been proposed so far to represent spatial or temporal entities and their relations [1,24,21,18,19]. The majority of these formalisms use qualitative constraints networks ($QCNs$ for short) as a representation language.

In some applications, especially multi-agent ones, spatial or temporal information comes from different sources, i.e. each source provides a spatial or temporal QCN representing relative positions of objects. The multiplicity of sources providing spatial or temporal information makes that the underlying $QCNs$ are generally conflicting. A way to address the conflict issue consists in defining a merging operator which takes as input a set of $QCNs$ $\mathcal{N} = \{N_1, \cdots, N_m\}$ modeling the information provided by the different sources and returns a consistent set of spatial or temporal information corresponding to the global information deduced from the information of the different sources.

Merging multiple sources information has attracted much attention in the framework of (weighted) propositional logic [22,23,12,13,14,11,3,2]. Inspired from these works, Condotta et al. [5] have proposed a first merging approach to $QCNs$. In this paper, we propose a new merging procedure for $QCNs$ by first translating each QCN into a propositional formula and then merging these formulas using propositional merging operators [12,13]. The new approach can benefit from recent advances on merging propositional formulas [9,10]. Different translations have been proposed in literature. Initially such translations have been defined to tackle the consistency problem for $QCNs$ in propositional logic. Note that such translations do not exist for all qualitative formalisms. For example, Nebel and Bürckert [19] represent constraints of interval algebra by a set of propositional

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clauses. Other more generic translations [20,4] allow to represent $QCNs$ defined on a qualitative formalism for which the closure by weak composition is complete for the consistency problem.

The aim of this paper is to characterize such translations and study the behavior of propositional merging operators on a set of propositional formulas resulting from the translation of a set of $QCNs$. The rest of this paper is organized as follows. We present in Section 2 some necessary background on qualitative formalisms for representing space or time. In Section 3, we describe the problem and briefly recall the merging procedure given in [5]. Then we present in Section 4 a merging procedure based on the translation of $QCNs$ into propositional formulas. We show that this procedure is equivalent to the one proposed in [5]. In Section 5 we propose rationality postulates for merging $QCNs$ and show how one can define a rational $QCNs$ merging operator from propositional merging operators thanks to more generic translations. Lastly we conclude.

2 Background on Qualitative Formalisms

Let $\mathcal{B}$ be a finite set of binary relations (called basic relations) over a domain $\mathcal{D}$. Each of these basic relations represents a particular qualitative position between two elements of $\mathcal{D}$. We suppose that these basic relations are complete and mutually exclusive, namely two elements of $\mathcal{D}$ satisfy one and only one basic relation of $\mathcal{B}$. The weak composition $r_1 \diamond r_2$ between two basic relations $r_1, r_2 \in \mathcal{B}$ is defined by the set $\{r : \exists x, y, z \in \mathcal{D}, x\ r_1\ y, y\ r_2\ z, x\ r\ z\}$. $\mathcal{A}$ denotes the set $2^{\mathcal{B}}$, i. e. , the set of all subsets of $\mathcal{B}$. An element $R \in \mathcal{A}$ is a set of basic relations between two elements of $\mathcal{D}$. Thus we have $X\ R\ Y \Leftrightarrow \exists r \in R : X\ r\ Y$. For illustration, we consider the Point Algebra [24] which considers relations between two points of the rational line. Figure 1 details the three basic relations of the Point Algebra, forming the set $\mathcal{B}_{pt}$.

Relation	Symbol	Illustration
precedes	<	
follows	>	
same	=	

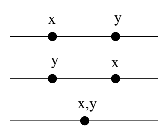

Fig. 1. The 3 basic relations of the Point Algebra

Pieces of knowledge about a set of spatial or temporal entities can be represented by means of qualitative constraints networks ($QCNs$ for short). A QCN N is a pair (V, C), where $V = \{v_0, \cdots, v_{n-1}\}$ is a finite set of variables representing the spatial or temporal entities and C is a mapping which associates to each pair of variables (v_i, v_j), with $i < j$, an element R of $\mathcal{A}$. R represents the set of all possible basic relations between v_i and v_j. We write C_{ij} instead of

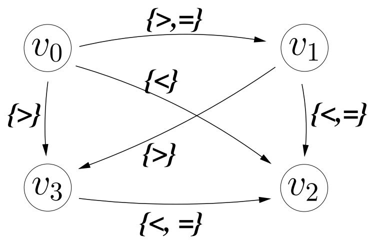

Fig. 2. N_1, a QCN of the Point Algebra

$C(v_i, v_j)$ for short. A QCN N_1 defined over 4 variables within the Point Algebra is depicted on Fig. 2.

Definition 1. *Let $N = (V, C)$ be a QCN.*

- *A* consistent instantiation *of N over $V' \subseteq V$ is a mapping α from V' to $\mathcal{D}$ such that $\alpha(v_i)\ C_{ij}\ \alpha(v_j)$, $\forall v_i, v_j \in V'$.*
- *N is* consistent *iff there exists a consistent instantiation of N over V.*
- *N is* $\diamond$-closed *iff $\forall v_i, v_j, v_k \in V$, $C_{ij} \subseteq C_{ik} \diamond C_{kj}$.*
- *A* sub-network *of N is a QCN $N' = (V, C')$, where $C'_{ij} \subseteq C_{ij}$, $\forall i, j \in \{0, \cdots, n-1\}$.*
- *A* consistent scenario *of N is a consistent sub-network of N, in which each constraint is composed of one and only one basic relation of $\mathcal{B}$.*

Figure 3.a depicts a consistent scenario σ of N_1 given in Fig. 2. Figure 3.b depicts a consistent instantiation of σ.

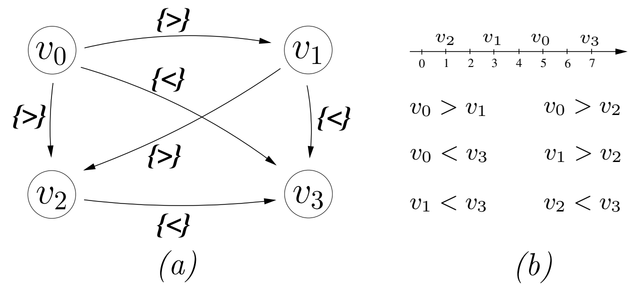

Fig. 3. A consistent scenario σ (a) and a consistent instantiation of σ (b)

$[N]$ denotes the set of consistent scenarios of a QCN N. N^V_{ALL} denotes the QCN defined on the set of variables V in which each constraint corresponds to the set $\mathcal{B}$. Thus the set of consistent scenarios defined on a set V corresponds to the set $[N^V_{ALL}]$.

3 Merging $QCNs$

Let $\mathcal{N} = \{N_1, \ldots, N_m\}$ be a set of $QCNs$ defined on the same set of variables $V = \{v_0, \ldots, v_{n-1}\}$ and on the same qualitative algebra having $\mathcal{B}$ as the set of basic relations. The problem we consider consists in merging the different information modeled by the QCN of $\mathcal{N}$. A natural way to solve this problem is to consider the set $\bigcap_{N_i \in \mathcal{N}} [N_i]$ as the result of merging, i.e., the set of consistent scenarios belonging to each QCN of $\mathcal{N}$. However this set may be empty due to the multiplicity of sources providing information. It is necessary to define a more parsimonious merging method in order to get a consistent result. The problem of merging $QCNs$ has been addressed in [5] where the authors propose a merging procedure inspired from propostional merging [22,12]. We first recall the merging process in propositional setting before we recall the merging procedure of $QCNs$ developped in [5].

3.1 Merging Propositional Bases

We consider a propositional language $PROP$ defined on a finite alphabet of variables V. An *interpretation* is a mapping from V to $\{0, 1\}$. We denote by $\mathcal{W}$ the finite set of all possible interpretations. An interpretation ω is a *model* of a formula ϕ (denoted $\omega \models \phi$) if and only if it makes the formula true. A *knowledge base* K is a finite set of propositional formulas $\{\phi_1, \cdots, \phi_m\}$. We consider K as logically equivalent to the conjunction of its formulas: $K = \phi_1 \wedge \cdots \wedge \phi_m$. K is *consistent* iff $\exists \omega \in \mathcal{W}$ such that $\omega \models K$. If K_1 and K_2 are two knowledge bases, we denote $K_1 \equiv K_2$ when two knowledge bases K_1, K_2 are logically equivalent. A multiset of knowledge bases $\{K_1, \cdots, K_m\}$ is called a *profile*. Two profiles $\mathcal{K}_1$ and $\mathcal{K}_2$ are *equivalent*, denoted $\mathcal{K}_1 \equiv \mathcal{K}_2$, if there exists a bijection f between $\mathcal{K}_1$ and $\mathcal{K}_2$ such that $\forall K \in \mathcal{K}_1$, $K \equiv f(K)$. $\sqcup$ is the union operator for multisets.

A merging operator Δ is a mapping which associates a propositional formula to a profile $\mathcal{K}$ and a propositional formula IC representing integrity constraints. A logical characterization of merging operators under integrity constraints has been proposed in [13], by means of a set of rationality postulates. The result of merging is denoted $\Delta_{IC}(\mathcal{K})$. For example, the first postulate (see Section 5) expresses that the propositional formula representing the result of merging should pick its models in the set of models of IC, namely $\Delta_{IC}(\mathcal{K}) \models IC$.

Merging operators in the propositional logic framework [17,12,13,14] are often based on a pseudo-distance d which is a mapping from $\mathcal{W} \times \mathcal{W}$ to $\mathbb{N}$ such that $\forall \omega, \omega'$ we have $d(\omega, \omega') = d(\omega', \omega)$ and $d(\omega, \omega) = 0$. Merging propositional knowledge bases is then a three step process. First, the "distance" between an interpretation ω and a knowledge base K is defined as follows: $d(\omega, K) = \min_{\omega' \models K} d(\omega, \omega')$. Then an aggregation operator denoted $\otimes$ [17,12,22,23] is used to compute the distance between an interpretation ω and a profile $\mathcal{K}$. This distance is defined by $d_{\otimes}(\omega, \mathcal{K}) = \otimes\{d(\omega, K) \mid K \in \mathcal{K}\}$. Lastly, the result of merging, denoted $\Delta_{IC}^{\otimes;d}(\mathcal{K})$, is the set of models of IC which are the closest to $\mathcal{K}$ w.r.t. d. Formally, we have $\Delta_{IC}^{\otimes;d}(\mathcal{K}) = \{\omega \models IC \mid \nexists \omega' \models IC, d(\omega', \mathcal{K}) < d(\omega, \mathcal{K})\}$.

3.2 Merging $QCNs$

Inspired from propositional operators described in the previous subsection, Condotta et al. [5] have defined an operator for merging a set of $QCNs$ $\mathcal{N}$ in a similar way. The merging process also follows three steps.

The first step consists in computing a local distance between each consistent scenario of $[N^V_{ALL}]$ and each QCN of $\mathcal{N}$. The distance between a scenario σ and a QCN N is the minimum distance between σ and all consistent scenarios of N.

$$d(\sigma, N) = \begin{cases} \min\{d^{QCN}(\sigma, \sigma') \mid \sigma' \in [N]\} & \text{if } N \text{ is consistent,} \\ 0 & \text{otherwise.} \end{cases}$$

Thus we need to define a distance between scenarios. Such a distance is a mapping from $[N^V_{ALL}] \times [N^V_{ALL}]$ to $\mathbb{N}$ such that $\forall \sigma, \sigma' \in [N^V_{ALL}]$,

$$\begin{cases} d^{QCN}(\sigma, \sigma') = d^{QCN}(\sigma', \sigma) \\ d^{QCN}(\sigma, \sigma) = 0. \end{cases}$$

Different distances between scenarios have been defined in [5]. Some of them are inspired from distances between interpretations as defined in the propositional logic framework (e.g. drastic distance, Hamming distance. See Section 4). Other more specific distances have also been defined in the context of $QCNs$ (e.g. the conceptual neighborhood distance [5]).

The second step consists in aggregating local distances computed in the previous step in order to compute a global distance between each consistent scenario of $[N^V_{ALL}]$ and $\mathcal{N}$. Different aggregation operators have been defined in literature. For example, the majority operator $\sum$ [17], which computes the sum of local distances, favors the point of view of the majority of sources. Arbitration operator $\mathcal{MAX}$ [23], which returns the greatest distance, has a more consensual behavior. The global distance between a scenario σ and a set $\mathcal{N}$ of $QCNs$ is defined by $d_\otimes(\sigma, \mathcal{N}) = \otimes\{d(\sigma, N) \mid N \in \mathcal{N}\}$, where $\otimes$ is an aggregation operator.

The result of merging, denoted $\Theta^{\otimes, d^{QCN}}(\mathcal{N})$, is the set of consistent scenarios of $[N^V_{ALL}]$ which are the "closest" to $\mathcal{N}$. These are consistent scenarios which have a minimal global distance to $\mathcal{N}$. Formally,

$$\Theta^{\otimes, d^{QCN}}(\mathcal{N}) = \{\sigma \in [N^V_{ALL}] \mid \nexists \sigma' \in [N^V_{ALL}], d(\sigma', \mathcal{N}) < d(\sigma, \mathcal{N})\}.$$

4 A Merging Procedure of *QCNs* Based on a Propositional Translation

4.1 Characterization of a Translation

We consider fixed a set of variables V and a qualitative formalism defined on a set of basic relations $\mathcal{B}$. We denote $QCN^V_{\mathcal{B}}$ the set of $QCNs$ defined on the qualitative formalism given by $\mathcal{B}$ and V. We call translation a mapping from $QCN^V_{\mathcal{B}}$ to the set of propositional formulas $PROP$. The main advantage of existing translations proposed in literature [19,20,4] is to benefit from works made around the SAT problem in order to solve the consistency problem for $QCNs$. A translation τ has to satisfy at least the following property:

Property 1. $\forall N \in QCN^V_{\mathcal{B}}$, *$\tau(N)$ is satisfiable if and only if N is consistent.*

We mean that a QCN has to admit a consistent scenario if and only if its associated propositional formula admits a model. However this property is insufficient in our context when considered alone. Indeed the merging process of $QCNs$ described in the previous section is based on distances between consistent scenarios while merging propositional bases is based on distances between interpretations. Therefore we suppose that τ also satisfies the following property:

Property 2

(a.) $\forall N \in QCN_{\mathcal{B}}^{V}$, $\tau(N) \models \tau(N_{ALL}^{V})$,
(b.) *There is a bijection* μ_τ *from the set of models of* $\tau(N_{ALL}^{V})$ *to* $[N_{ALL}^{V}]$ *such that* $\forall N \in QCN_{\mathcal{B}}^{V}$, $\{\mu_\tau(\omega) \mid \omega \models \tau(N)\} = [N]$.

The last property makes it possible to identify (through a bijection μ_τ) a consistent scenario with an interpretation ω of $\mathcal{W}$ if $\omega \models \tau(N_{ALL}^{V})$, and that $\forall N \in QCN_{\mathcal{B}}^{V}$, $\tau(N)$ represents through its models the set of consistent scenarios of N.

We now give an example of such a translation. Let $\tau_{Sup}(N)$ be the translation of a QCN $N = (V, C) \in QCN_{\mathcal{B}}^{V}$ using the support encoding [8,6,20]. The propositional formula $\tau_{Sup}(N)$ is built on the set of propositional variables $V_T = \{r_{ij} \mid r \in \mathcal{B}, 0 \leq i < j \leq n-1\}$. The propositional variable r_{ij} is valuated to *true* if and only if the basic relation r is satisfied for the constraint between the two variables v_i and v_j of V. We say that l is a literal of V_T if and only if l is a variable of V_T or its negation. $\forall N = (V, C) \in QCN_{\mathcal{B}}^{V}$, $\tau_{Sup}(N)$ is the conjunction of the following clauses:

- $\bigvee_{r \in C_{ij}} r_{ij}$, $\forall 0 \leq i < j \leq n-1$ (*at least one*),
- $\neg r_{ij} \vee \neg s_{ij}$, $\forall 0 \leq i < j \leq n-1$, $\forall r, s \in \mathcal{B}, r \neq s$ (*at most one*),
- $\neg r_{ik} \vee \neg s_{kj} \vee \bigvee_{t \in (r \diamond s) \cap C_{ij}} t$, $\forall 0 \leq i < k < j \leq n-1$, $\forall r \in C_{ik}$, $\forall s \in C_{kj}$ (*supports*).

Since $\mathcal{B}$ is a given set of a constant number of basic relations, the size of the translation $\tau_{Sup}(N)$ only depends on the number of variables n of the QCN N. Indeed the number of propositional variables of V_T is in $O(n^2)$, the number of clauses generated by $\tau_{Sup}(N)$ is in $O(n^3)$ and the number of literals in any clause of $\tau_{Sup}(N)$ is in $O(1)$.

If $\omega \models \tau_{Sup}(N)$, then ω represents the consistent scenario $\sigma = (V, C')$ of N such that $\forall 0 \leq i < j \leq n-1$, C'_{ij} is defined by the basic relation $r \in C_{ij}$ such that the value of the propositional variable r_{ij} in ω is *true*. *At least one* and *at most one* clauses certify that each constraint of σ is composed of one and only one basic relation of the associated constraint in N, namely σ is a scenario of N. The consistency of σ is given by the presence of *supports* clauses which guarantee the $\diamond$-closure of the scenario (we consider qualitative algebras in which $\diamond$-closed scenarios are consistent). Thus τ_{Sup} satisfies Properties 1 and 2. Since the formula $\tau_{Sup}(\sigma)$ admits exactly one model, we can represent it by a conjunction of literals of V_T.

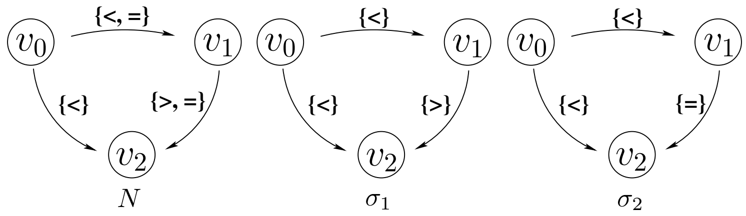

Fig. 4. A QCN N and its two consistent scenarios σ_1 and σ_2

Example. Figure 4 depicts a QCN N defined on Point Algebra where $\mathcal{B}_{pt} = \{<, =, >\}$ and $V = \{v_0, v_1, v_2\}$. N admits two consistent scenarios σ_1 and σ_2 depicted in the same figure.

$\tau_{Sup}(N)$ is built on $V_T = \{r_{ij} \mid r \in \{<, =, >\}, 0 \leq i < j \leq 2\}$. It is composed of the following clauses:

$$\begin{cases} (<_{01} \vee =_{01}), (<_{02}), (>_{12} \vee =_{12}) & \text{(at least one)} \\ (\neg <_{01} \vee \neg =_{01}), (\neg =_{01} \vee \neg >_{01}), (\neg <_{01} \vee \neg >_{01}), & \\ (\neg <_{12} \vee \neg =_{12}), (\neg =_{12} \vee \neg >_{12}), (\neg <_{12} \vee \neg >_{12}), & \\ (\neg <_{02} \vee \neg =_{02}), (\neg =_{02} \vee \neg >_{02}), (\neg <_{02} \vee \neg >_{02}) & \text{(at most one)} \\ (\neg <_{01} \vee \neg >_{12} \vee <_{02} \vee =_{02} \vee >_{02}), (\neg <_{01} \vee \neg =_{12} \vee <_{02}), & \\ (\neg =_{01} \vee \neg >_{12} \vee >_{02}), (\neg =_{01} \vee \neg =_{12} \vee =_{02}) & \text{(supports)} \end{cases}$$

$\tau_{Sup}(N)$ admits exactly two models, which represents (by $\mu_{\tau_{Sup}}$) the consistent scenarios σ_1 and σ_2 of N, i.e., $\tau_{Sup}(\sigma_1)$ and $\tau_{Sup}(\sigma_2)$.

$\tau_{Sup}(\sigma_1)$ is equivalent to the following conjunction of literals of V_T:

$$(\mathbf{<_{01}} \wedge \neg =_{01} \wedge \neg >_{01} \wedge \neg <_{12} \wedge \neg =_{12} \wedge \mathbf{>_{12}} \wedge \mathbf{<_{02}} \wedge \neg =_{02} \wedge \neg >_{02}).$$

$\tau_{Sup}(\sigma_2)$ is equivalent to the following conjunction of literals of V_T:

$$(\mathbf{<_{01}} \wedge \neg =_{01} \wedge \neg >_{01} \wedge \neg <_{12} \wedge \mathbf{=_{12}} \wedge \neg >_{12} \wedge \mathbf{<_{02}} \wedge \neg =_{02} \wedge \neg >_{02}).$$

4.2 The Merging Process

We now consider a set $\mathcal{N} = \{N_1, \ldots, N_m\}$ of $QCNs \in QCN^V_{\mathcal{B}}$. Our merging procedure is a three step process. We first encode each QCN N_i ($i \in \{1, \ldots, m\}$) into a propositional formula $\tau(N_i)$. Then, we apply an IC merging operator $\Delta^{\otimes,d}_{IC}$ on the resulting set $\mathcal{K}$ of propositional formulas, with $IC = \tau(N^V_{ALL})$. Lastly the set of interpretations resulting from this merging will represent the subset of consistent scenarios of $[N^V_{ALL}]$ resulting from the merging of $\mathcal{N}$.

Recall that the first requirement for defining a propositional merging operator is to define a local distance between interpretations. Given a distance d^{QCN} between scenarios, we define a distance d^{PROP} between interpretations as follows:

Definition 2. *Let τ be a translation satisfying Properties 1 and 2 and a d^{QCN} be a distance between scenarios of $[N^V_{ALL}]$. We define the distance d^{PROP} between models ω and ω' of $\tau(N^V_{ALL})$ by $d^{PROP}(\omega, \omega') = d^{QCN}(\mu_\tau(\omega), \mu_\tau(\omega'))$.*

This definition is intuitively derived from Property 2.b.

Differents proposals can be made to define distance d^{QCN} between scenarios [5]. We recall here the Hamming distance between scenarios and the Hamming distance between interpretations.

Definition 3 (Hamming distance). *The Hamming distance between scenarios σ and σ', denoted $d_H^{QCN}(\sigma, \sigma')$, is the number of constraints that are different in the two scenarios. Formally,*

$$d_H^{QCN}(\sigma, \sigma') = |\{(v_i, v_j) \in V : \sigma(i,j) \neq \sigma'(i,j), i < j\}|,$$

where $|E|$ is the number of elements of the set E. The Hamming distance between two interpretations ω and ω', denoted $d_H(\omega, \omega')$, is the number of propositional variables of V_T which differ between the two interpretations. Formally,

$$d_H(\omega, \omega') = |\{x \in V_T : \omega(x) \neq \omega'(x)\}|,$$

where $\omega(x)$ is the truth value of the literal x in ω.

Given a translation τ, thanks to Definition 2 we can define a distance between the interpretations $\tau(\sigma)$ and $\tau(\sigma')$ equivalent to $d_H^{QCN}(\sigma, \sigma')$ for all scenarios σ, σ' of $[N_{ALL}^V]$. For the translation τ_{Sup} we have the following result.

Proposition 1. $\forall \sigma, \sigma' \in [N_{ALL}^V]$, $2 \cdot d_H^{QCN}(\sigma, \sigma') = d_H(\tau_{Sup}(\sigma), \tau_{Sup}(\sigma'))$.

Thus we can associate to the distance d_H^{QCN} between scenarios the distance d_H^{PROP} such that $\forall \omega, \omega'$ models of $\tau_{Sup}(N_{ALL}^V)$, $d_H^{PROP}(\omega, \omega') = (1/2) \cdot d_H(\omega, \omega')$.

A specific distance in the context of $QCNs$ has been defined in [5]. This distance, called the neighborhood distance, considers the notion of proximity between basic relations of a qualitative algebra [7]. A neighborhood between basic relations is often represented by a conceptual neighborhood graph. Some lattice structures allowing to determine these graphs have been defined in the literature [15,16]. The conceptual neighborhood is more precise and suitable than Hamming distance in the context of $QCNs$. Using the translation τ_{Sup}, we cannot directly define a corresponding distance between interpretations. However this will be possible if we add to τ_{Sup} some additional clauses encoding the conceptual neighborhood graph. We do not give further details on this issue due to the lack of space.

The next steps in the merging process of propositional bases consist in aggregating local distances computed in the previous step, using the aggregation operator used for merging the $QCNs$ under consideration. Therefore, we have the following result:

$$\forall \sigma \in [N_{ALL}^V]\ \sigma \in \Theta^{\otimes, d^{QCN}}(\mathcal{N}) \text{ iff } \tau(\sigma) \in \Delta_{\tau(N_{ALL}^V)}^{\otimes, d^{PROP}}(\mathcal{K}).$$

Figure 5 summarizes the merging procedure.

In the next section we describe a set of rationality postulates for $QCNs$ merging and properties of the merging operator using a more generic class of translations.

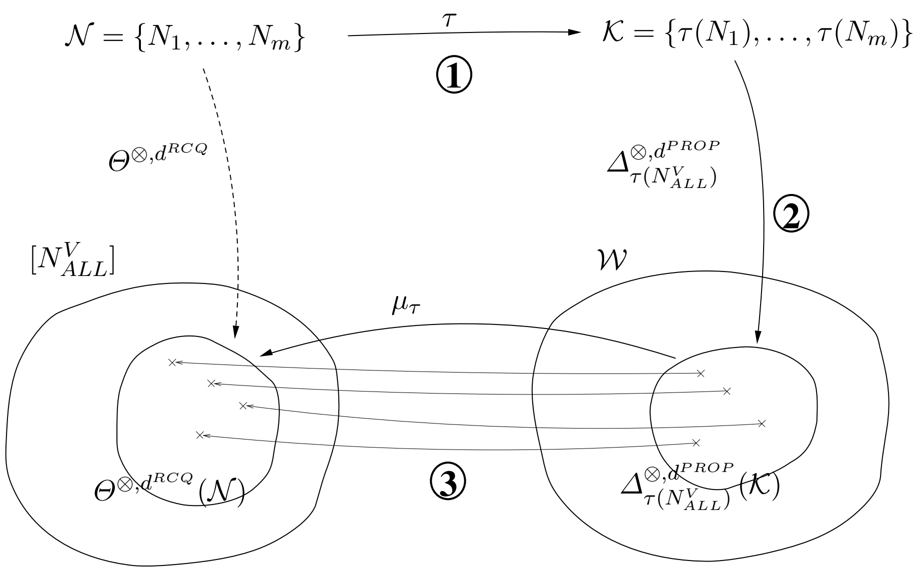

Fig. 5. $QCNs$ merging procedure

5 Merging $QCNs$: Rationality Postulates and Rational Operators

In the propositional setting, a number of postulates characterizing the rational belief merging operators have been given [13]. The purpose of this section is to present similar postulates for merging $QCNs$ and to show how one can define rational operators for merging $QCNs$ from propositional merging operators thanks to translations. We first recall rationality postulates given in the propositional setting. We denote by $\bigwedge \mathcal{K}$ the conjunction of the knowledge bases of the profile $\mathcal{K}$.

Definition 4. *Let $\mathcal{K}, \mathcal{K}_1$ and $\mathcal{K}_2$ be three profiles, K_1, K_2 be consistent knowledge bases and IC, IC_1, IC_2 be propositional formulas. Δ is an IC merging operator iff it satisfies the following postulates.*

- ***(IC0)*** $\Delta_{IC}(\mathcal{K}) \models IC$.
- ***(IC1)*** *If IC is consistent, then $\Delta_{IC}(\mathcal{K})$ is consistent.*
- ***(IC2)*** *If $\bigwedge \mathcal{K} \wedge IC$ is consistent, then $\Delta_{IC}(\mathcal{K}) \equiv \bigwedge \mathcal{K} \wedge IC$.*
- ***(IC3)*** *If $\mathcal{K}_1 \equiv \mathcal{K}_2$ and $IC_1 \equiv IC_2$, then $\Delta_{IC_1}(\mathcal{K}_1) \equiv \Delta_{IC_2}(\mathcal{K}_2)$.*
- ***(IC4)*** *If $K_1 \models IC$ and $K_2 \models IC$, then if $\Delta_{IC}(\{K_1, K_2\}) \wedge K_1$ is consistent then $\Delta_{IC}(\{K_1, K_2\}) \wedge K_2$ is consistent.*
- ***(IC5)*** $\Delta_{IC}(\mathcal{K}_1) \wedge \Delta_{IC}(\mathcal{K}_2) \models \Delta_{IC}(\mathcal{K}_1 \sqcup \mathcal{K}_2)$.
- ***(IC6)*** *If $\Delta_{IC}(\mathcal{K}_1) \wedge \Delta_{IC}(\mathcal{K}_2)$ is consistent, then $\Delta_{IC}(\mathcal{K}_1 \sqcup \mathcal{K}_2) \models \Delta_{IC}(\mathcal{K}_1) \wedge \Delta_{IC}(\mathcal{K}_2)$.*
- ***(IC7)*** $\Delta_{IC_1}(\mathcal{K}) \wedge IC_2 \models \Delta_{IC_1 \wedge IC_2}(\mathcal{K})$.
- ***(IC8)*** *If $\Delta_{IC_1}(\mathcal{K}) \wedge IC_2$ is consistent, then $\Delta_{IC_1 \wedge IC_2}(\mathcal{K}) \models \Delta_{IC_1}(\mathcal{K}) \wedge IC_2$.*

For the sake of generality, we consider that information conveyed by a QCN N is not reduced to the set $[N]$ of its consistent scenarios (as it is the case in [5]) but more generally to a subset $\langle N\rangle$ of $QCN_{\mathcal{B}}^{V}$ (this enables us for taking some context-dependent information into account). In the same vein, the result of the merging is defined as a subset of $QCN_{\mathcal{B}}^{V}$ (instead of a subset of $[N_{ALL}^{V}]$). Thus we now define a $QCNs$ merging operator Θ as a mapping which associates to a finite subset $\mathcal{N}$ of $QCN_{\mathcal{B}}^{V}$, a subset of $QCN_{\mathcal{B}}^{V}$.

The following rationality postulates are the direct counterparts in the $QCNs$ setting of the postulates ***(IC0) - (IC8)*** from [13] and the postulates ***(A1) - (A6)*** from [12], for propositional merging. Before presenting them, we first need to define a notion of equivalence between $QCNs$ and between sets of $QCNs$: Two $QCNs$ N and N' are said to be *equivalent*, denoted $N \equiv N'$, iff $\langle N\rangle = \langle N'\rangle$. Two subsets $\mathcal{N}_1$ and $\mathcal{N}_2$ of $QCNs$ are said to be *equivalent*, denoted $\mathcal{N}_1 \equiv \mathcal{N}_2$, iff there exists a bijection f from $\mathcal{N}_1$ to $\mathcal{N}_2$ such that $\forall N_1 \in \mathcal{N}_1$, $N_1 \equiv f(N_1)$.

Definition 5. *Let $\mathcal{N}, \mathcal{N}_1$ and $\mathcal{N}_2$ be finite sets of $QCNs$, and let N_1, N_2 be two consistent $QCNs$. Θ is a $QCNs$ merging operator iff it satisfies the following postulates:*

(N1) $\Theta(\mathcal{N}) \neq \emptyset$.
(N2) *If* $\bigcap_{N_i\in\mathcal{N}} \langle N_i\rangle \neq \emptyset$, *then* $\Theta(\mathcal{N}) = \bigcap_{N_i\in\mathcal{N}} \langle N_i\rangle$.
(N3) *If* $\mathcal{N}_1 \equiv \mathcal{N}_2$, *then* $\Theta(\mathcal{N}_1) = \Theta(\mathcal{N}_2)$.
(N4) *If* $\Theta(\{N_1, N_2\}) \cap \langle N_1\rangle \neq \emptyset$, *then* $\Theta(\{N_1, N_2\}) \cap \langle N_2\rangle \neq \emptyset$.
(N5) $\Theta(\mathcal{N}_1) \cap \Theta(\mathcal{N}_2) \subseteq \Theta(\mathcal{N}_1 \sqcup \mathcal{N}_2)$.
(N6) *If* $\Theta(\mathcal{N}_1) \cap \Theta(\mathcal{N}_2) \neq \emptyset$, *then* $\Theta(\mathcal{N}_1 \sqcup \mathcal{N}_2) \subseteq \Theta(\mathcal{N}_1) \cap \Theta(\mathcal{N}_2)$.

(N1) ensures that the result of the merging is non-trivial. ***(N2)*** requires $\Theta(\mathcal{N})$ to be the set of $QCNs$ shared by $\langle N_i\rangle$ $\forall N_i \in \mathcal{N}$, when this set is non-empty. ***(N3)*** is a syntax-irrelevance principle. ***(N4)*** is an equity postulate: it asks that the merging operator does not exploit any hidden preferences between two $QCNs$ to be merged. ***(N5)*** and ***(N6)*** state that, if there exists a non-empty set E of $QCNs$ shared by the mergings of two groups $\mathcal{N}_1$ and $\mathcal{N}_2$, then the merging of the joint groups must be this set E.

We now show how one can define $QCNs$ merging operators Θ satisfying all those postulates ***(N1) - (N6)*** from propositional IC merging operators Δ thanks to translations τ. We first need to slightly modify the notion of translation presented in the previous section so that to ensure the existence of a bijection from the set of models of the propositional formula $\tau(N)$ to $\langle N\rangle$:

Definition 6. *A translation τ is a mapping from $QCN_{\mathcal{B}}^{V}$ to $PROP$ satisfying Property 1 and such that:*

(a.) $\exists\varphi_\tau \in PROP : \forall N \in QCN_{\mathcal{B}}^{V}, \tau(N) \models \varphi_\tau$.
(b.) There exists a bijection μ_τ from the set of models of φ_τ to $QCN_{\mathcal{B}}^{V}$ such that $\forall N \in QCN_{\mathcal{B}}^{V}$, $\{\mu_\tau(\omega) \mid \omega \models \tau(N)\} = \langle N\rangle$.

The conditions on τ ensure the existence of a propositional formula φ_τ associated to τ, such that every model of φ_τ corresponds (in a bijective way via μ_τ) to a QCN of $QCN_{\mathcal{B}}^{V}$. For instance, one can have $\varphi_\tau = \tau(N_{ALL}^{V})$, provided that

$\forall N \in QCN^V_{\mathcal{B}}$, $\tau(N) \models \tau(N^V_{ALL})$. In addition, $\forall N \in QCN^V_{\mathcal{B}}$, the set of models of $\tau(N)$ must be in bijection via μ_τ with $\langle N \rangle$. Observe that the notion of translation defined in the previous section satisfies the requirements of Definition 6 assuming that $\forall N$, $\langle N \rangle = [N]$.

Definition 7. *Let τ be a translation in the sense of Definition 6 and Δ_{IC} an IC merging operator (i.e., a propositional merging operator satisfying **(IC0) - (IC8)**) with $IC = \varphi_\tau$. The $QCNs$ merging operator Θ induced by τ and Δ_{IC} is defined by: let $\mathcal{N} = \{N_1, \ldots, N_m\}$ be a set of $QCNs$, we have*

$$\Theta(\mathcal{N}) = \{\mu_\tau(\omega) \mid \omega \models \Delta_{\varphi_\tau}(\tau(N_1), \ldots, \tau(N_m))\}.$$

(IC0) ensures that every model of $\Delta_{\varphi_\tau}(\tau(N_1), \ldots, \tau(N_m))$ is associated to a QCN from $QCN^V_{\mathcal{B}}$ via μ_τ. We have:

Proposition 2. *Every $QCNs$ merging operator induced by a translation (in the sense of Definition 6) and an IC merging operator satisfies **(N1) - (N6)**.*

6 Conclusion

Using a particular class of propositional distance-based merging operators, we have shown that the $QCNs$ merging operator presented in [5] can be reduced to propositional merging. Thus we can retrieve some interesting results from the widely studied topic of merging propositional bases to our work. For example, we directly get a characterization of the complexity of the process [11]. Moreover an efficient implementation of propositional merging operators has recently been proposed [9] while the implementation of the merging method developed in [5] is hard in practice.

Our method is valid for qualitative formalisms in which the closure by weak composition is complete for its consistency problem, however it is not appropriate if no translation allows a propositional formula to capture the set of consistent scenarios of the translated QCN. In addition, we have proposed a set of rationality postulates for $QCNs$ merging operators. These postulates are satisfied if we use an appropriate translation from $QCNs$ to propositional formulas and a particular class of propositional merging operators.

This work can be extended in several directions. Given a context and a particular definition of the set $\langle N \rangle$ for all N, one can define and study some appropriate translations. Another perspective is to study properties about the $QCNs$ merging operator using other classes of propositional merging operators.

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