A Global Constraint for Closed Frequent Pattern Mining

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Abstract. Discovering the set of closed frequent patterns is one of the fundamental problems in Data Mining. Recent Constraint Programming (CP) approaches for declarative itemset mining have proven their usefulness and flexibility. But the wide use of reified constraints in current CP approaches leads to difficulties in coping with high dimensional datasets. In this paper, we proposes the ClosedPattern global constraint to capture the closed frequent pattern mining problem without requiring reified constraints or extra variables. We present an algorithm to enforce *domain consistency* on ClosedPattern in polynomial time. The computational properties of this algorithm are analyzed and its practical effectiveness is experimentally evaluated.

1 Introduction

Frequent Pattern Mining is a well-known and perhaps the most popular research field of data mining. Originally introduced by Agrawal et al. [1], it plays a key role in many data mining applications. These applications include the discovery of frequent itemsets and association rules [1], correlations [2] and many other data mining tasks.

In practice, the number of frequent patterns produced is often huge and can easily exceed the size of the input dataset. Most frequent patterns are redundant and can be derived from other found patterns. Hence, *closed frequent patterns* have been introduced. They provide a concise and condensed representation that avoids redundancy. Discovering the set of closed frequent patterns is one of the fundamental problems in Data Mining. Several specialized approaches have been proposed to discover closed frequent patterns (e.g., A-Close algorithm [12], `CHARM` [17], `CLOSET` [13], `LCM` [14]).

Over the last decade, the use of the Constraint Programming paradigm (CP) to model and to solve Data Mining problems has received considerable attention [3,5,9]. The declarative aspect represents the key success of the proposed CP approaches. Doing so, one can add/remove any user-constraint without the need of developing specialized solving methods.

Related to the Closed Frequent Pattern Mining problem (CFPM), Guns et. al., propose to express the different constraints that we can have in Pattern Mining as a CP model [5]. The model is expressed on Boolean variables representing items and transactions, with a set of reified sum constraints. The reified model has become a de facto standard for many DM tasks. Indeed, the reified model had been adopted for the k-pattern sets [5]. The drawback is the wide use of reified constraints in the CP model, which makes the scalability of the approach questionable.

In the line of the work of Kemmar et al. [8], we propose in this paper the ClosedPattern global constraint. ClosedPattern does not require reified constraints and extra variables to encode and propagate the CFPM problem. ClosedPattern captures the particular semantics of the CFPM problem and domain consistency can be achieved on it using a polynomial algorithm. Experiments on several known large datasets show that our approach outperforms the reified model used in CP4IM [3] and is more scalable, which is a major issue for CP approaches. This result can be explained by the fact that ClosedPattern insures domain consistency.

The paper is organized as follows. Section 2 recalls preliminaries. Section 3 provides the context and the motivations for the ClosedPattern global constraint. Section 4 presents the global constraint ClosedPattern. Section 5 illustrates the power of the prunning algorithm compared with the reified model. Section 6 reports experiments. Finally, we conclude and draw some perspectives.

2 Background

In this section, we introduce some useful notions used in closed frequent pattern mining and constraint programming.

2.1 Closed frequent pattern mining

Let $\mathcal{I} = \{1, ..., n\}$ be a set of n *item* indices[1] and $\mathcal{T} = \{1, ..., m\}$ a set of m *transaction* indices. A pattern P (i.e., itemset) is a subset of $\mathcal{I}$. The language of patterns corresponds to $\mathcal{L}_{\mathcal{I}} = 2^{\mathcal{I}}$. A transaction database is a set $\mathcal{D} \subseteq \mathcal{I} \times \mathcal{T}$. The set of items corresponding to a transaction identified by t is denoted by $\mathcal{D}[t] = \{i \mid (i, t) \in \mathcal{D}\}$. A transaction t is an occurrence of some pattern P iff the set $\mathcal{D}[t]$ contains P (i.e., $P \subseteq \mathcal{D}[t]$).

The cover of P, denoted by $\mathcal{T}_{\mathcal{D}}(P)$, is the set of transactions containing P, that is, $\mathcal{T}_{\mathcal{D}}(P) = \{t \in \mathcal{T} \mid P \subseteq \mathcal{D}[t]\}$. Given $S \subseteq \mathcal{T}$ a subset of transactions, $\mathcal{I}_{\mathcal{D}}(S)$ is the set of common items of S, that is, $\mathcal{I}_{\mathcal{D}}(S) = \bigcap_{t \in S} \mathcal{D}[t]$. The (absolute) frequency of a pattern P is the size of its cover (i.e., $freq_{\mathcal{D}}(P) = |\mathcal{T}_{\mathcal{D}}(P)|$). Let $\theta \in \mathbb{N}^{+}$ be some given constant called a *minimum support*. A pattern P is frequent if $freq_{\mathcal{D}}(P) \geq \theta$.

Example 1. Consider the transaction database in Table 1. We have $\mathcal{T}_{\mathcal{D}}(C) = \{t_1, t_3\}$, $freq_{\mathcal{D}}(C) = 2$ and $\mathcal{I}_{\mathcal{D}}(\{t_1, t_3\}) = CH$.

[1] For the sake of readability, our examples refer to items by their names instead of their indices.

Table 1: A transaction database $\mathcal{D}$ (a) and its binary matrix (b).

(a)

Trans.	Items							
t_1		B	C				G	H
t_2	A			D				
t_3	A		C	D				H
t_4	A				E	F		
t_5		B			E	F	G	

(b)

Trans.	A	B	C	D	E	F	G	H
t_1	0	1	1	0	0	0	1	1
t_2	1	0	0	1	0	0	0	0
t_3	1	0	1	1	0	0	0	1
t_4	1	0	0	0	1	1	0	0
t_5	0	1	0	0	1	1	1	0

The closure of a pattern P in $\mathcal{D}$, denoted by $Clos(P)$, is the set of common items of its cover $\mathcal{T}_\mathcal{D}(P)$, that is, $Clos(P) = \mathcal{I}_\mathcal{D}(\mathcal{T}_\mathcal{D}(P))$. A pattern is closed if and only if $Clos(P) = P$.

Definition 1 (Closed Frequent Pattern Mining (CFPM)). *Given a transaction database $\mathcal{D}$ and a minimum support threshold θ, the closed frequent pattern mining problem is the problem of finding all patterns P such that $(freq_\mathcal{D}(P) \geq \theta)$ and $(Clos(P) = P)$.*

Example 2. For $\theta = 2$, the set of closed frequent patterns in Table 1 is $\emptyset\langle 5\rangle$,[2] $A\langle 3\rangle$, $AD\langle 2\rangle, BG\langle 2\rangle, CH\langle 2\rangle$ and $EF\langle 2\rangle$.

Closed frequent patterns provide a minimal representation of frequent patterns, i.e. we can derive all frequent patterns with their exact frequency value from the closed ones [12]. We now define the important notion of *full extension* that comes from pattern mining algorithms and that we will use later in this paper.

Definition 2 (Full extension). *The non-empty itemset Q is called a full extension of P iff $\mathcal{T}_\mathcal{D}(P) = \mathcal{T}_\mathcal{D}(P \cup Q)$.*

Definition 2 is at the key of the item merging property [15] stated as follows: If some pattern Q is a full extension of some pattern P, and none of the proper supersets of Q is a full extension of P, then $P \cup Q$ forms a closed pattern. In other words, a closed pattern can be defined as a pattern that does not possess a full extension.

Search Space Issues. In pattern mining, the search space contains $2^\mathcal{I}$ candidates. Given a large number of items $\mathcal{I}$, a naive search that consists of enumerating and testing the frequency of pattern candidates in a dataset is infeasible. The main property exploited by most algorithms to reduce the search space is that frequency is *monotone decreasing* with respect to extension of a set.

Property 1 (Anti-monotonicity of the frequency). Given a transaction database $\mathcal{D}$ over $\mathcal{I}$, and two patterns $X, Y \subseteq \mathcal{I}$. Then, $X \subseteq Y \Rightarrow freq_\mathcal{D}(Y) \leq freq_\mathcal{D}(X)$.

Hence, any subset (resp. superset) of a frequent (resp. infrequent) pattern is also a frequent (resp. infrequent) pattern.

[2] Value between $\langle . \rangle$ indicates the frequency of a pattern.

2.2 CFPM under constraints

Constraint-based pattern mining aims at extracting all patterns P of $\mathcal{L}_\mathcal{I}$ satisfying a query $q(P)$ (conjunction of constraints), which usually defines what we call a *theory* [10]: $Th(q) = \{P \in \mathcal{L}_\mathcal{I} \mid q(P)\ is\ true\}$. A common example is the frequency measure leading to the frequent pattern constraint. It is also possible to have other kind of (user-)constraints. For instance, constraints on the size of the returned patterns, $minSize(P, \ell_{min})$ constraint holds if and only if the number of items of P is greater than or equal to ℓ_{min}. Constraints on the presence of an item in a pattern $item(P, i)$ state that an item i must be in a pattern P.

2.3 CSP and global constraints

A constraint network is defined by a set of variables $X = \{x_1, \ldots, x_n\}$, each variable $x_i \in X$ having an associated finite domain $dom(x_i)$ of possible values, and a set of constraints $\mathcal{C}$ on X. A constraint $c \in \mathcal{C}$ is a relation that specifies the allowed combinations of values for its variables $X(c)$. An assignment σ is a mapping from variables in X to values in their domains. The *Constraint Satisfaction Problem* (CSP) consists in finding an assignment satisfying all constraints.

Domain Consistency (DC). Constraint solvers typically use backtracking search to explore the search space of partial assignments. At each assignment, filtering algorithms prune the search space by enforcing local consistency properties like domain consistency. A constraint c on $X(c)$ is domain consistent, if and only if, for every $x_i \in X(c)$ and every $d_i \in dom(x_i)$, there is an assignment σ satisfying c such that $x_i = d_i$.

Global constraints are constraints capturing a relation between a non-fixed number of variables. These constraints provide the solver with a better view of the structure of the problem. Examples of global constraints are AllDifferent, Regular and Among (see [7]). Except the case when for a given global constraint a Berge-acyclic decomposition exists, global constraints cannot be efficiently propagated by generic local consistency algorithms, which are exponential in the number of the variables of the constraint. Dedicated filtering algorithms are constructed to achieve polynomial time complexity in the size of the input, i.e., the domains and extra parameters. The aim of this paper is to propose a filtering algorithm for the frequent closed pattern constraint.

3 Context and Motivations

This section provides a critical review of ad-hoc specialized methods and CP approaches for CFPM, and motivates the proposition of a global constraint.

Specialized methods for CFPM. `CLOSE` [12] was the first algorithm proposed to extract closed frequent patterns (CFPs). It uses an apriori-like bottom-up method. Later, Zaki and Hsiao [17] proposed a depth-first algorithm based on a vertical database format e.g. `CHARM`. In [13], Pei et al. extended the FP-growth method to a method called `CLOSET` for mining CFPs. Lastly, Uno et al. [14] have proposed LCM, one of the fastest frequent itemset mining algorithm. It uses a hybrid representation based on vertical and horizontal representations. The milestone of LCM is a technique called *prefix preserving closure extension* (PPCE), which allows to generate a new frequent closed pattern from a previously obtained closed pattern. Let us explain the PPCE principle. Consider the closed pattern P. Let $P(i) = P \cap \{1, ..., i\}$ be the subset of P consisting of items no greater than i. The core index of P, denoted by $core(P)$, is the minimum index i such that $\mathcal{T}_{\mathcal{D}}(P(i)) = \mathcal{T}_{\mathcal{D}}(P)$. A pattern Q is PPCE of P if $Q = Clos(P \cup \{i\})$ and $P(i-1) = Q(i-1)$ for an item $i \notin P$ and $i > core(P)$. The completeness of PPCE is guaranteed by the following property: If Q is a nonempty closed itemset, then there is only one closed itemset P such that Q is a PPCE of P. The mining process using LCM is a depth first search where at each node we have a closed pattern P. LCM uses the PPCE technique as a branching strategy to jump from a closed pattern to other closed patterns by adding new items. With its specialized depth first search, LCM succeeds to enumerate very quickly the closed patterns. However, if the user considers other (user-)constraints on patterns, the search procedure should be revised. In fact, all these specialized proposals (e.g., Closet, Charm, LCM, etc.), though efficient, are ad-hoc methods suffering from the lack of genericity, since adding new constraints requires new implementations.

Reified constraint model for itemset mining. De Raedt et al. have proposed in [3] a CP model for itemset mining. They show how some constraints (e.g., frequency, maximality, closedness) can be modeled as CSP [11,6]. This modeling uses two sets of Boolean variables P and T: (1) Decision variables: item variables $P_1, P_2, ..., P_n$, where $P_i = 1$ if and only if item i is in the searched pattern; (2) Auxiliary variables: transaction variables $T_1, T_2, ..., T_m$, where $T_t = 1$ if and only if the searched pattern is in $\mathcal{D}[t]$.

The relationship between P and T, set of channeling constraints, is modeled by reified constraints stating that, for each transaction t, $(T_t = 1)$ iff P is a subset of $\mathcal{D}[t]$: $\forall t \in \mathcal{T} : (T_t = 1) \leftrightarrow \sum_{i \in \mathcal{I}} P_i(1 - \mathcal{D}[t, i]) = 0$ (arity $n+1$). The min frequency constraint is modeled us: $\forall i \in \mathcal{I} : (P_i = 1) \rightarrow \sum_{t \in T} T_t \mathcal{D}[t, i] \geq \theta$ (arity $m+1$). The closedness constraint is expressed with: $\forall i \in \mathcal{I} : (P_i = 1) \leftrightarrow \sum_{t \in T} T_t(1 - \mathcal{D}[t, i]) = 0$ (arity $m+1$). Such encoding has a major drawback since it requires $(m + n + n)$ reified constraints of arity $(n+1)$ and $(m+1)$ to encode the whole database. This constitutes a strong limitation especially when it comes to handle very large databases.

We propose in the next section the ClosedPattern global constraint to encode both the minimum frequency constraint and the closedness constraint. This global constraint requires neither reified constraints nor auxiliary variables.

4 ClosedPattern Constraint

This section presents the ClosedPattern global constraint for the CFPM problem.

4.1 Definition and filtering

Let P be the unknown pattern we are looking for. The unknown pattern P is encoded with Boolean item variables $P_1, ..., P_n$. In the rest of the paper we will denote by σ the partial assignment obtained from the variables $P_1, ..., P_n$ that have a singleton domain. We will also use the following subsets of items:

- present items: $\sigma^+ = \{j \in 1..n \mid P_j = 1\}$,
- absent items: $\sigma^- = \{j \in 1..n \mid P_j = 0\}$,
- other items: $\sigma^* = \{1..n\} \setminus (\sigma^+ \cup \sigma^-)$.

σ^* is the set of free items (non instantiated variables). If $\sigma^* = \emptyset$ then σ is a complete assignment.

The global constraint ClosedPattern ensures both the minimum frequency property and the closedness property.

Definition 3 (ClosedPattern global constraint). *Let $P_1, \ldots, P_n$ be binary item variables. Let $\mathcal{D}$ be a transaction database and θ a minimum support. Given a complete assignment σ on $P_1, \ldots, P_n$,* ClosedPattern$_{\mathcal{D},\theta}(\sigma)$ *holds if and only if $freq_{\mathcal{D}}(\sigma^+) \geq \theta$ and σ^+ is closed.*

Example 3. Consider the transaction database of Table 1a with $\theta = 2$. Let $P = \langle P_1, \ldots, P_8 \rangle$ with $dom(P_i) = \{0, 1\}$ for $i \in 1..8$. Consider the closed pattern AD encoded by $P = \langle 10010000 \rangle$, where $\sigma^+ = \{A, D\}$ and $\sigma^- = \{B, C, E, F, G, H\}$. ClosedPattern$_{\mathcal{D},2}(P)$ holds because $freq_{\mathcal{D}}(\{A, D\}) \geq 2$ and $\{A, D\}$ is closed.

Let σ be a partial assignment of variables P and i a free item. We use the vertical representation of the dataset, denoted $\mathcal{V}_{\mathcal{D}}$ where for each item, the transactions containing it are stored: $\forall i \in \mathcal{I}, \mathcal{V}_{\mathcal{D}}(i) = \mathcal{T}_{\mathcal{D}}(\{i\})$. We denote by $\mathcal{V}_{\mathcal{D}}^{\sigma^+}(i)$ the cover of item i within the current cover of a pattern σ^+:

$$\mathcal{V}_{\mathcal{D}}^{\sigma^+}(i) = \mathcal{T}_{\mathcal{D}}(\sigma^+ \cup \{i\}) = \mathcal{T}_{\mathcal{D}}(\sigma^+) \cap \mathcal{T}_{\mathcal{D}}(\{i\}).$$

We need to define extensible assignments.

Definition 4 (Extensible assignment). *Given a constraint* ClosedPattern$_{\mathcal{D},\theta}$ *on $P_1, \ldots, P_n$, a partial assignment is said to be extensible if and only if it can be extended to a complete assignment of $P_1, \ldots, P_n$ that satisfies* ClosedPattern$_{\mathcal{D},\theta}$.

We show when a partial assignment is extensible with respect to ClosedPattern constraint.

Proposition 1. *Let σ be a partial assignment of variables in $P_1, \ldots, P_n$. σ is an extensible partial assignment if and only if $freq_{\mathcal{D}}(\sigma^+) \geq \theta$ and $\nexists j \in \sigma^-$ such that $\{j\}$ is a full extension of σ.*

Proof. According to the anti-monotonicity property of the frequency (cf. Property 1), if the partial assignment σ is infrequent (i.e., $freq_{\mathcal{D}}(\sigma^+) < \theta$), it cannot, under any circumstances, be extended to a closed pattern.

Given now a frequent partial assignment σ (i.e., $freq_{\mathcal{D}}(\sigma^+) \geq \theta$), let us take $j \in \sigma^-$ such that $\{j\}$ is a full extension of σ. It follows that $\mathcal{T}_{\mathcal{D}}(\sigma^+) = \mathcal{T}_{\mathcal{D}}(\sigma^+ \cup \{j\}) = \mathcal{V}_{\mathcal{D}}^{\sigma^+}(j)$. Therefore, $Clos(\sigma^+) = Clos(\sigma^+ \cup \{j\})$. Since σ^+ without j (j being in σ^-) cannot be extended to a closed pattern, the result follows. If there is no item $j \in \sigma^-$ such that $\{j\}$ is a full extension of σ, then the current assignment σ can be definitely extended to a closed itemset by adopting a full extension to form a closed pattern. □

We now give the ClosedPattern filtering rules by showing when a value of a given variable is inconsistent.

Proposition 2 (ClosedPattern filtering rules). *Let σ be an extensible partial assignment of variables in $P_1, \ldots, P_n$, and P_j ($j \in \sigma^*$) be a free variable. The following two cases characterize the inconsistency of the values 0 and 1 of P_j:*

- $0 \notin dom(P_j)$ *iff:* $\{j\}$ *is a full extension of* σ. *(rule 1)*
- $1 \notin dom(P_j)$ *iff:* $\begin{cases} |\mathcal{V}_{\mathcal{D}}^{\sigma^+}(j)| < \theta \quad \vee & \textit{(rule 2)} \\ \exists k \in \sigma^-, \mathcal{V}_{\mathcal{D}}^{\sigma^+}(j) \subseteq \mathcal{V}_{\mathcal{D}}^{\sigma^+}(k). & \textit{(rule 3)} \end{cases}$

Proof. Let σ be an extensible partial assignment and P_j be a free variable.

$0 \notin dom(P_j)$: ($\Rightarrow$) Let 0 be an inconsistent value. In this case, P_j can only take value 1. It means that $Clos(\sigma^+) = Clos(\sigma^+ \cup \{j\})$. Thus, $\mathcal{T}_{\mathcal{D}}(\sigma^+) = \mathcal{T}_{\mathcal{D}}(\sigma^+ \cup \{j\})$. By definition 2, $\{j\}$ is a full extension of σ.
($\Leftarrow$) Let $\{j\}$ be a full extension of σ, which means that $Clos(\sigma^+) = Clos(\sigma^+ \cup \{j\})$ (def. 2). The value 0 is inconsistent where j cannot be in σ^- (prop.1).

$1 \notin dom(P_j)$: ($\Rightarrow$) Let 1 be an inconsistent value. This can be the case if the frequency of the current pattern σ^+ is set up below the threshold θ by adding the item j (i.e., $|\mathcal{V}_{\mathcal{D}}^{\sigma^+}(j)| < \theta$). Or, $\sigma^+ \cup \{j\}$ cannot be extended to a closed itemset: this is the case when there exists an item $k \in \sigma^-$ such that at each time the item j belongs to a transaction in the database, k belongs as well ($\mathcal{V}_{\mathcal{D}}^{\sigma^+}(j) \subseteq \mathcal{V}_{\mathcal{D}}^{\sigma^+}(k)$). Conversly, the lack of k (i.e., $k \in \sigma^-$) implies the lack of j as well. This means that: ($P_k = 0 \Rightarrow P_j = 0$).
($\Leftarrow$) This is a direct consequence of Proposition 1. □

The first rule takes its origin from item merging [15]. The second rule is a basic rule derived from the property of anti-monotonicity of the frequency (Property 1). To the best of our knowledge, the third rule is a new rule taking its originality from the reasoning made on absent items.

Example 4. Following Example 3, consider a partial assignment σ such that the variable P_1 is set to 0 (item A). That is, $\sigma^- = \{A\}$ and $\sigma^+ = \emptyset$. Value 1 from $dom(P_4)$ (item D) is inconsistent because the lack of A implies the lack of D in $\mathcal{D}$ (i.e., $\mathcal{V}_{\mathcal{D}}^{\sigma^+}(D) \subseteq \mathcal{V}_{\mathcal{D}}^{\sigma^+}(A)$). Let now $P_1 = 0, P_4 = 0$, that is, $\sigma^- = \{A, D\}$ and $\sigma^+ = \emptyset$. If the variable P_3 is set to 1 (item C), value 1 from $P_i, i = \{2, 5, 6, 7\}$ (items B, E, F, G) is inconsistent because $|\mathcal{V}_{\mathcal{D}}^{\sigma^+}(P_i)| < 2$, and value 0 from P_8 (item H) is also inconsistent because $\{H\}$ is a full extension of σ.

Algorithm 1: FILTER-CLOSEDPATTERN($\mathcal{V}_\mathcal{D}, \theta, \sigma, P$)

1 **Input**: $\mathcal{V}_\mathcal{D}$: vertical database; θ : minimum support
2 **InOut**: $P = \{P_1 \dots P_n\}$: Boolean item variables; σ : current assignment.
3 **begin**
4 **if** $(\ |\mathcal{T}_\mathcal{D}(\sigma^+)| < \theta)$ **then return** $false$;
5 **if** $\exists\ i \in \sigma^- : |\mathcal{V}_\mathcal{D}^{\sigma^+}(i)| = |\mathcal{T}_\mathcal{D}(\sigma^+)|$ **then return** $false$;
6 **foreach** $i \in \sigma^*$ **do**
7   **if** $(|\mathcal{V}_\mathcal{D}^{\sigma^+}(i)| = |\mathcal{T}_\mathcal{D}(\sigma^+)|)$ **then**
8         $dom(P_i) \leftarrow dom(P_i) - \{0\}$;
9         $\sigma^+ \leftarrow \sigma^+ \cup \{i\}$; $\quad \sigma^* \leftarrow \sigma^* \setminus \{i\}$;
10   **else if** $(|\mathcal{V}_\mathcal{D}^{\sigma^+}(i)| < \theta)$ **then**
11         $dom(P_i) \leftarrow dom(P_i) - \{1\}$;
12         $\sigma^- \leftarrow \sigma^- \cup \{i\}$; $\quad \sigma^* \leftarrow \sigma^* \setminus \{i\}$;
13 **foreach** $i \in \sigma^-$ **do**
14   **foreach** $j \in \sigma^* : \mathcal{V}_\mathcal{D}^{\sigma^+}(j) \subseteq \mathcal{V}_\mathcal{D}^{\sigma^+}(i)$ **do**
15         $dom(P_j) \leftarrow dom(P_j) - \{1\}$;
16         $\sigma^- \leftarrow \sigma^- \cup \{j\}$; $\quad \sigma^* \leftarrow \sigma^* \setminus \{j\}$;
17 **return** $true$;

4.2 CLOSEDPATTERN Filtering Algorithm

In this section, we present the algorithm FILTER-CLOSEDPATTERN (Algorithm 1) for enforcing domain consistency on the CLOSEDPATTERN constraint. FILTER-CLOSEDPATTERN incrementally maintains the internal data structures $\sigma =< \sigma^+, \sigma^-, \sigma^* >$ and the corresponding cover $\mathcal{T}_\mathcal{D}(\sigma^+)$. Using these two structures, one can check if an item is present or not in the vertical dataset $\mathcal{V}_\mathcal{D}$.

Algorithm 1 takes as input the vertical dataset $\mathcal{V}_\mathcal{D}$, a minimum support threshold θ, the current partial assignment σ on P where $\sigma^* \neq \emptyset$, and the variables P. As output, Algorithm 1 reduces the domains of P_i's and therefore, increases σ^+ and/or σ^-, and decreases σ^*.

The algorithm starts by checking if the current partial assignment is extensible or not (Proposition 1). This is performed by checking 1) if the size of the current cover is greater than the minimum support (line 4) and 2) if no item of a variable already instantiated to zero is a full extension of σ^+ (line 5).

Lines 6-12 are a straightforward application of *rules 1* and *2* of Proposition 2. For each non-instantiated variable, 1) we check if value 0 is consistent: that item is not a full extension (lines 6-9), and 2) we check if value 1 is consistent: the new cover size by adding that item remains greater than θ (lines 10-12).

Finally, lines 13-16 implement *rule 3* of Proposition 2. We prune value 1 from each free item variable $i \in \sigma^*$ such that its cover is a superset of the cover of an absent item $j \in \sigma^-$ $(\mathcal{V}_\mathcal{D}^{\sigma^+}(i) \subseteq \mathcal{V}_\mathcal{D}^{\sigma^+}(j))$.

Theorem 1. *Given a transaction database $\mathcal{D}$ of n items and m transactions, and a threshold minsup θ. Algorithm* FILTER-CLOSEDPATTERN *enforces domain* *consistency on the* ClosedPattern *constraint, or proves that it is inconsitent in time* $O(n^2 \times m)$ *with a space complexity of* $O(n \times m)$.

Proof. **DC:** Filter-ClosedPattern implements exactly Proposition 1 and the three rules given in Proposition 2. Thus Filter-ClosedPattern ensures domain consistency (see the description of Algorithm 1).
Time: Let $n = |\mathcal{I}|$ and $m = |\mathcal{T}|$. First, we need to compute $\mathcal{T}_{\mathcal{D}}(\sigma^+)$ which requires at most $O(n \times m)$. This is done only once. The cover $\mathcal{V}_{\mathcal{D}}^{\sigma^+}(i)$ can be computed by intersecting $\mathcal{T}_{\mathcal{D}}(\sigma^+)$ (already computed) and $\mathcal{T}_{\mathcal{D}}(\{i\})$ (given by the vertical representation) within at most $O(m)$. Checking *rules 1* and *2* on all free variables can be done in $O(n \times m)$ (lines 6-12). However, checking *rule 3* is cubic at lines 13-16 (i.e., $O(n \times (n \times m))$), where checking if a cover $\mathcal{V}_{\mathcal{D}}^{\sigma^+}(i)$ is a subset of another cover can be done in $O(m)$. Finally, the worst case complexity is $O(n \times (n \times m))$.
Space: The space complexity of Filter-ClosedPattern lies in the storage of $\mathcal{V}_{\mathcal{D}}$, σ and the cover $\mathcal{T}$ data structures. The vertical representation $\mathcal{V}_{\mathcal{D}}$ requires at most $n \times m$ space. In the worst case, we have to store n items within σ and m transactions within $\mathcal{T}$. That is, the worst case space complexity is $O(n \times m + n + m) = O(n \times m)$. □

During the solving process in depth first search, the whole space complexity is $O(n \times (m + n))$ because (1) the depth is at most n; (2) σ and $\mathcal{T}$ require $O(n \times (m + n))$; (3) the vertical representation is the same data used all along the solving process $O(n \times m)$; (4) $O(n \times (m+n)) + O(n \times m) = O(n \times (m+n))$.

Proposition 3 (Backtrack-free). *Extracting the total number of closed frequent patterns, noted C, is backtrack-free with a complexity in* $O(C \times n^2 \times m)$ *using* Filter-ClosedPattern *to propagate the* ClosedPattern *constraint.*

Proof. Filter-ClosedPattern ensures DC at each node of the search tree. Hence, the closed frequent patterns are guaranteed to be produced in a backtrack-free manner. The explored search tree is a binary full tree where each node is either a leaf (a solution) or possesses exactly two child nodes. The number of nodes is thus in $O(2 \times C)$. Knowing that ensuring DC is in $O(n^2 \times m)$, extracting the total number of closed frequent patterns is in $O(C \times n^2 \times m)$. □

4.3 Data structures

To represent the transactional dataset and the cover of items, we adopted the vertical representation format [16]. Our implementation is in `or-tools` solver [3].
Static structures: for each item $i \in \mathcal{I}$, $\mathcal{T}_{\mathcal{D}}(i) = \{t \in \mathcal{T} \mid i \in t\}$ is stored as a bitset of size m. If $t \in \mathcal{T}_{\mathcal{D}}(i)$, then the associated bit is set to 1 (0 otherwise).
Dynamic structure: a vector $Memo$ of bitsets is used to store the cover of each partial solution. $Memo$ is a vector of size $n + 1$, since at the beginning of the search, the partial assignment is empty. Let σ be a partial assignment. Each time a (new) variable P_i is instantiated, the cover of the new partial assignment

[3] `https://developers.google.com/optimization/`

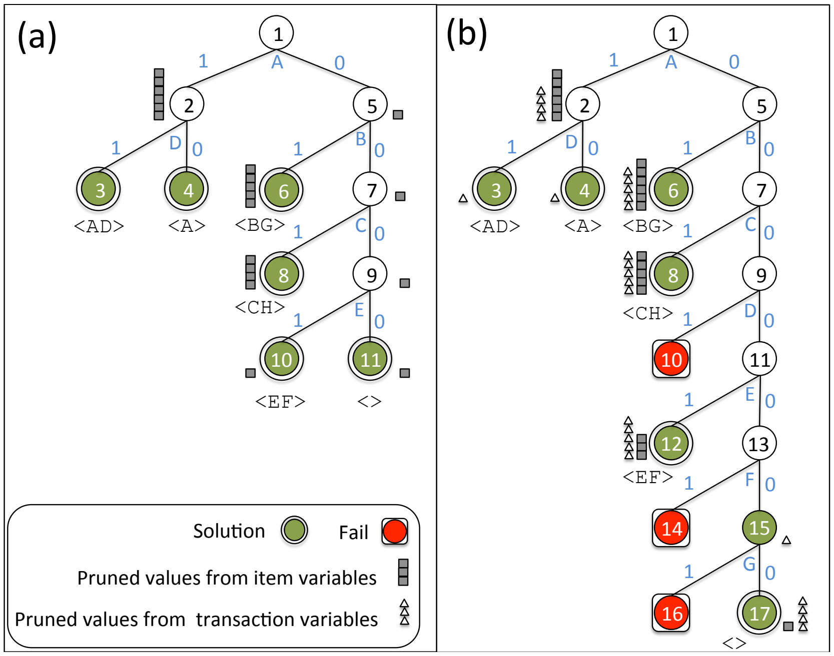


Fig. 1: (a) ClosedPattern and (b) Reified Constraint Model (RCM)

$\sigma \cup \{i\}$ is stored in $Memo$. If $P_i = 0$, the cover remains the same: $\mathcal{T}_\mathcal{D}(\sigma^+ \cup \{i\}) = \mathcal{T}_\mathcal{D}(\sigma^+)$. If $P_i = 1$, the cover of the new partial solution $\sigma \cup \{i\}$ is computed by a bitwise-AND between $\mathcal{T}_\mathcal{D}(\sigma^+)$ and $\mathcal{T}_\mathcal{D}(\{i\})$, and stored in $Memo$.

Backtracking. First, all P_i, as well as their domains $dom(P_i)$, are fully maintained by the `or-tools` backtracking. Then, a single value (the current index of the vector $Memo$) is asked to be managed by the `or-tools` backtracking. Each time a partial solution is extended (from σ to $\sigma \cup \{i\}$), the current index of $Memo$ is memorised. When a backtrack occurs (from $\sigma \cup \{i\}$ to σ), this value is restored by `or-tools` giving access to the cover of the (restored) partial solution.

Rule 3. The inclusion between two covers $\mathcal{V}_\mathcal{D}^{\sigma^+}(i) \subseteq \mathcal{V}_\mathcal{D}^{\sigma^+}(j)$ is rewritten as $\mathcal{V}_\mathcal{D}^{\sigma^+}(i) \cap \mathcal{V}_\mathcal{D}^{\sigma^+}(j) = \mathcal{V}_\mathcal{D}^{\sigma^+}(i)$, and the intersection is performed by a bitwise-AND.

5 Running example

In this section, we illustrate the propagation of our ClosedPattern constraint and the difference that exists comparing to the use of a simple Reified Constraint Model (denoted by RCM, and detailed in Section 3). For that, let us take the transactional dataset given in Table 1: Five transactions t_1 to t_5 and eight items from A to H.

Figure 1 shows the tree search explored using ClosedPattern (**part(a)**) and the tree search explored using a reified model (**part(b)**) to extract closed frequent patterns at minimum support $\theta = 2$. Here, both approaches use the same branching heuristics, namely `Lex` on variables and `Max_val` on values. First of all, it is worth noticing that the search space that can be explored using ClosedPattern is defined only on decision variables (item variables), whereas the reified model adds a further dimension with auxiliary variables (transaction variables).

At the root node (node 1), no pruning is done since all items are frequent and no item is a full extension of the empty pattern (see Table 1). Thereafter, ClosedPattern and RCM are acting in the same manner on the branch $A = 1$. With $A = 1$, B, C, E, F, G, H become infrequent. That is, the 1 values are pruned (*rule 2*). With RCM on node 2, the pruning on the five decision variables (the item variables) induce a pruning on four auxiliary variables (transaction variables). On the branch $A = 1$, two solutions are found: $\langle AD \rangle$ and $\langle A \rangle$.

Branching on $A = 0$ (node 5), the value 1 of D is pruned with *rule 3* of ClosedPattern. From Table 1, we have $D \Rightarrow A$, and a branching on $A = 0$ reduces D to 0. Here, we can say that the DC is maintained on node 5 using ClosedPattern, which is not the case using RCM. The same observation can be made on nodes 7, 9 and 11.

Let us take the node 6, here the branching on $A = 0$ and thereafter on $B = 1$ will make C, D, E, F, H infrequent (*rule 2*). Moreover, (*rule 1*) can be applied since G is a full extension of B (i.e., we cannot have a frequent closed pattern including B without G). That is, the value 0 is pruned from the domain of G, which allows us to reach the solution $\langle BG \rangle$. The same observation can be made on node 8.

To sum up, Filter-ClosedPattern maintains DC at each node and thus, enumerates the solutions backtrack-free (no fails). The same cannot be said with RCM because the *rule 3* is never covered in this example and there are 3 fails.

6 Experiments

We made several experiments to compare and evaluate our global constraint with the state of the art methods (CP and specialized methods).

Benchmark datasets. We selected several real and synthetic datasets [17,4] from FIMI repository[4] with large size. These datasets have varied characteristics representing different application domains. Table 2 reports for each dataset, the number of transactions $|\mathcal{T}|$, the number of items $|\mathcal{I}|$, the average size of transactions $|\widehat{\mathcal{T}}|$, its density ρ (i.e., $|\widehat{\mathcal{T}}|/|\mathcal{I}|$) and its *size* (i.e., $|\mathcal{T}| \times |\mathcal{I}|$). We note that the datasets are presented according to their size. They represent various numbers of transactions, numbers of items, densities. We have datasets that are very dense like Chess and Connect (resp. 49% and 33%), and others that are very sparse like Retail and BMS-Web-View1 (resp. 0.06% and 0.5%). Note that we have datasets of sizes going from $\approx 10^5$ to more than 10^9.

[4] `http://fimi.ua.ac.be/data/`

Table 2: Dataset Characteristics.

Dataset	$\|\mathcal{T}\|$	$\|\mathcal{I}\|$	$\widehat{\|\mathcal{T}\|}$	ρ	type of data	*size*
Chess	3 196	75	37	49%	game steps	**239 700**
Splice1	3 190	287	60	21%	genetic sequences	**915 530**
Mushroom	8 124	119	23	19%	species of mushrooms	**966 756**
Connect	67 557	129	43	33%	game steps	**8 714 853**
BMS-Web-View1	59 602	497	2.5	0.5%	web click stream	**29 622 194**
T10I4D100K	100 000	1 000	10	1%	synthetic dataset	**100 000 000**
T40I10D100K	100 000	1 000	40	4%	synthetic dataset	**100 000 000**
Pumsb	49 046	7 117	74	1%	census data	**349 060 382**
Retail	88 162	16 470	10	0.06%	retail market basket data	**1 452 028 140**

Experimental protocol. The implementation of our approach was carried out in the `or-tools` solver. All experiments were conducted on an Intel Xeon E5-2680 @ 2.5 GHz with 128 Gb of RAM with a timeout of 3600s. For each dataset, we decreased the (relative) θ threshold until it is impossible to extract all closed patterns within the allocated time/memory. We have implemented two variants of ClosedPattern constraint: **(i)** ClosedPattern-dc ensuring DC with *rules 1,2 and 3* (cubic prunning). **(ii)** ClosedPattern-wc ensuring a weaker consistency with only *rules 1 and 2* (quadratic prunning). Comparisons are made with: **(i)** `CP4IM`, the state-of-the-art on CP approaches, that uses an RCM model. **(ii)** `LCM`, the state-of-the-art on specialized methods.

For ClosedPattern and `CP4IM`, we use the same branching heuristics, namely `Lex` on variables and `Max_val` on values. We experimented using the available distributions of `LCM-v5.3`,[5] and `CP4IM`,[6] with `Gecode` as the underlying solver of `CP4IM`. Table 3 gives a comparison between ClosedPattern (wc and dc versions), `CP4IM` and `LCM`. We report the number of closed patterns $\#C$ of each instance, the number of propagations, the number of nodes, the CPU times in seconds and the number of failures.

ClosedPattern (dc vs wc). Despite the pruning complexity, dc clearly dominates wc in terms of CPU times (except for *BMS1* dataset where both are more or less equivalent). For instance, on the *pumsb* dataset with $\theta = 70\%$, dc is about 4 times faster than wc. As a second observation, the use of *rule 3* can reduce drastically the number of explored nodes and thus, number of propagations. For instance, we note a reduction of 38% on explored nodes on *connect* and 98% on *splice1* compared to wc.

ClosedPattern vs `CP4IM`. If we compare ClosedPattern with `CP4IM`, the main observation is that dc outperforms significantly `CP4IM` at all levels. In terms of CPU times and without counting the Out-of-memory instances, we can observe 23 instances (out of 30) with a speed-up factor between 2 and 15. Factors of 27 to 45 are noted for six instances and for one instance, we have a factor of 182. The weaker version (wc) is also better than `CP4IM` except two instances in *connect* and two instances in *chess*. In terms of number of propagations, we observe a gain factor within a range from 13 to 300. This is because of

[5] `http://research.nii.ac.jp/~uno/codes.htm`
[6] `https://dtai.cs.kuleuven.be/CP4IM/`

Table 3: ClosedPattern vs CP4IM vs LCM. (OOM: Out Of Memory; TO: TimeOut; (1): ClosedPattern-wc; (2): ClosedPattern-dc; (3): CP4IM)

$\mathcal{D}$	θ	#C	#Propagations			#Nodes			Time (s)				Failures		
	(%)	(≈)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	LCM	(1)	(2)	(3)
chess	50	10^5	4 037 618	**920 377**	9 201 740	4 037 211	**738 901**	738 907	7.26	**3.21**	10.97	0.32	1 649 155	0	3
	40	10^6	15 931 714	**3 411 690**	30 541 475	15 929 751	**2 733 667**	2 733 735	26.37	**12.27**	40.85	1.44	6 598 042	0	34
	30	10^6	71 744 915	**13 298 679**	105 040 377	71 734 057	**10 679 631**	10 681 739	109.36	**45.92**	136.31	6.07	30 527 213	0	1 054
	20	10^7	305 446 222	**56 550 872**	385 399 747	305 374 413	**45 837 171**	45 901 933	480.52	**187.89**	467.52	27.55	1.29×10^8	0	32 381
	10	10^8	1.51×10^9	**304 335 522**	1.66×10^9	1.51×10^9	**247 960 091**	249 411 325	2 287.98	**969.40**	1 950.51	141.55	6.34×10^8	0	725 617
splice1	20	10^2	54 055	**25 078**	913 779	53 991	**487**	**487**	**0.10**	0.59	22.59	0.04	26 752	0	0
	10	10^3	303 405	**57 623**	1 810 466	303 359	**3 211**	**3 211**	0.18	**0.14**	25.54	0.07	150 074	0	0
	5	10^4	5 013 077	**1 706 269**	36 211 067	5 013 031	**63 935**	**63 935**	3.39	**3.23**	138.54	1.46	2 474 548	0	0
	1	10^7	754 074 683	**193 156 761**	2.43×10^9	754 067 617	**13 454 755**	13 467 247	502.77	**400.10**	1 652.41	53.59	3.70×10^8	0	6 246
mushroom	30	10^2	6 323	**1 530**	430 016	6 179	**853**	1 039	**0.46**	0.64	0.77	0.00	2 663	0	93
	20	10^3	34 375	**4 414**	1 031 778	34 113	**2 405**	3 083	0.92	**0.56**	2.36	0.01	15 854	0	339
	10	10^4	148 429	**17 456**	2 771 719	147 623	**9 793**	13 281	0.38	**0.15**	5.32	0.04	68 915	0	1 744
	5	10^4	496 643	**46 165**	5 574 143	494 559	**25 707**	36 495	0.84	**0.41**	10.98	0.09	234 426	0	5 394
	1	10^5	2 347 734	**178 834**	13 813 312	2 339 109	**103 343**	168 999	3.42	**1.74**	24.06	0.22	1 117 883	0	32 828
	0.5	10^5	3 896 158	**264 660**	18 230 838	3 883 697	**156 613**	265 781	5.00	**3.62**	29.84	0.34	18 635 42	0	54 584
	0.1	10^5	7 148 533	**516 041**	31 222 435	7 130 827	**328 233**	529 289	9.96	**5.40**	41.38	0.47	3 401 297	0	100 528
	0.05	10^5	8 001 925	**613 712**	36 520 438	7 983 119	**407 763**	622 145	9.94	**6.37**	43.69	0.51	3 787 678	0	107 191
connect	90	10^3	12 377	**10 272**	1 865 236	9 725	**6 973**	**6 973**	2.45	**0.92**	7.10	0.22	1 376	0	0
	80	10^4	60 312	**48 156**	9 455 886	44 771	**30 223**	**30 223**	9.35	**1.65**	16.57	0.31	7 274	0	0
	70	10^4	158 817	**120 184**	24 969 205	115 335	**71 761**	**71 761**	24.26	**4.09**	33.72	0.40	21 787	0	0
	60	10^5	297 175	**240 634**	51 648 114	202 637	**136 699**	**136 699**	45.23	**7.30**	45.73	0.39	32 969	0	0
	50	10^5	570 870	**469 442**	98 602 318	378 603	**260 223**	**260 223**	85.99	**14.53**	110.19	0.52	59 190	0	0
	40	10^5	1 139 466	**886 722**	177 653 293	762 579	**478 781**	**478 781**	159.04	**27.32**	153.39	0.83	141 899	0	0
	30	10^5	2 173 593	**1 752 199**	313 288 691	1 400 609	**920 823**	**920 823**	263.64	**49.97**	304.52	1.37	239 893	0	0
	20	10^6	8 440 998	**5 637 016**	633 429 969	6 092 697	**2 966 399**	**2 966 399**	678.95	**157.40**	712.68	4.37	1 563 149	0	0
	10	10^7	55 152 559	**31 370 763**	1.76×10^9	41 641 659	**16 075 555**	**16 075 555**	3 110.90	**760.71**	2 597.89	17.70	12 783 052	0	0
BMS1	0.20	10^3	311 638	**105 403**	OOM	250 959	**1 677**	OOM	**111.04**	145.72	OOM	0.04	124 671	0	OOM
	0.16	10^3	493 069	**116 902**	OOM	432 419	**2 617**	OOM	**161.60**	218.88	OOM	0.06	214 939	0	OOM
	0.12	10^3	921 446	**134 436**	OOM	860 811	**4 971**	OOM	239.08	**202.46**	OOM	0.05	427 940	0	OOM
	0.08	10^4	3 638 431	**200 049**	OOM	3 577 735	**20 787**	OOM	2 441.84	**1 899.70**	OOM	0.11	1 778 490	0	OOM
T10*	10	1	**1 000**	**1 000**	OOM	**1**	**1**	OOM	0.46	**0.24**	OOM	0.03	0	0	OOM
	1	10^2	155 166	**76 955**	OOM	154 559	**805**	OOM	3.75	**1.14**	OOM	0.14	76 894	0	OOM
	0.5	10^3	509 852	**174 015**	OOM	509 441	**2 185**	OOM	10.29	**3.11**	OOM	0.42	253 647	0	OOM
	0.1	10^3	11 228 933	**520 218**	OOM	11 228 667	**53 625**	OOM	258.44	**11.30**	OOM	0.94	5 587 527	0	OOM
	0.05	10^4	21 004 412	**1 257 876**	OOM	21 003 915	**93 987**	OOM	477.22	**27.96**	OOM	1.30	10 454 964	0	OOM
	0.01	10^5	130 686 806	**17 313 642**	OOM	130 673 093	**566 795**	OOM	1 471.32	**905.17**	OOM	3.49	65 053 149	0	OOM
T40*	10	10^2	8 712	**4 900**	OOM	7 801	**177**	OOM	**0.89**	1.13	OOM	0.43	3 818	0	OOM
	5	10^2	97 982	**47 983**	OOM	97 289	**643**	OOM	2.34	**1.78**	OOM	1.31	48 328	0	OOM
	1	10^5	19 932 041	**937 766**	OOM	19 931 799	**130 477**	OOM	599.49	**25.78**	OOM	21.32	9 900 663	0	OOM
	0.5	10^6	TO	**6 394 931**	OOM	TO	**2 551 883**	OOM	TO	**953.58**	OOM	13.31	TO	0	OOM
pumsb	95	10^2	7 981	**7 438**	OOM	877	**223**	OOM	2.85	**1.45**	OOM	0.20	328	0	OOM
	90	10^3	17 429	**11 074**	OOM	10 323	**2 933**	OOM	44.92	**13.37**	OOM	0.29	3 695	0	OOM
	85	10^4	61 459	**29 149**	OOM	54 331	**17 027**	OOM	155.79	**58.84**	OOM	0.27	18 652	0	OOM
	80	10^4	176 955	**89 805**	OOM	169 743	**66 615**	OOM	640.59	**133.97**	OOM	0.33	51 564	0	OOM
	75	10^5	472 624	**249 382**	OOM	465 267	**202 165**	OOM	1 010.43	**271**	OOM	0.48	131 551	0	OOM
	70	10^5	969 374	**567 929**	OOM	961 763	**482 517**	OOM	2 150.80	**509.79**	OOM	0.69	239 623	0	OOM
retail	10	10	16 501	**16 490**	OOM	37	**19**	OOM	**1**	2.55	OOM	0.06	9	0	OOM
	1	10^2	31 089	**19 852**	OOM	14 695	**329**	OOM	11.03	**4.02**	OOM	0.10	7 188	0	OOM
	0.5	10^3	205 688	**56 868**	OOM	189 475	**1 233**	OOM	61.41	**12.73**	OOM	0.32	94 156	0	OOM
	0.1	10^4	23 520 823	**4 506 219**	OOM	23 506 749	**15 901**	OOM	**495.52**	796.82	OOM	0.80	11 745 679	0	OOM
	0.05	10^4	114 357 067	**18 342 468**	OOM	114 345 005	**40 229**	OOM	**2 352.14**	2 645.06	OOM	1.07	57 152 804	0	OOM

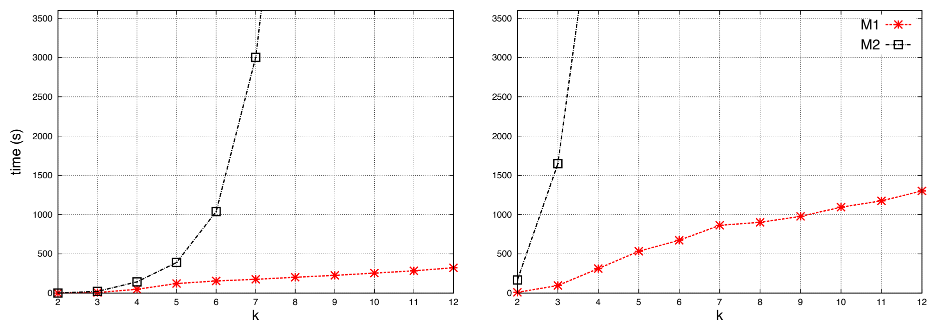
the huge number of propagator calls for reified constraints comparing to one propagator call using ClosedPattern. The number of explored nodes is also reduced, sometimes by half (e.g., *mushroom*), using ClosedPattern-dc. This is not a surprise as its number of explored nodes is optimal. Another observation is the experimental validation of Proposition 3: ClosedPattern-dc extracts the closed patterns in a *backtrack-free* manner. All solutions are enumerated without any fail (see failures column in Table 3). CP4IM requires an important number of backtracks on most datasets. On *connect* and three instances of *splice1* we can observe that ClosedPattern-dc and CP4IM explore the same number of nodes. The reified model on such dense datasets and using (Lex\Max_val) heuristics is able to prune all inconsistent values. This result is confirmed by the number of failures always equal to zero (backtrack-free). Even if the WC version is faster, CP4IM remains better in terms of pruning (#nodes). Finally, the major drawback using the reified model for frequent pattern extraction is the memory consumption. We denote 25 Out-of-memory (out of 51 instances). The Out-of-memory state is due to the huge number of reified constraints. For instance, if we take the *T40I10D100K* dataset, the CP model produced by CP4IM contains $|\mathcal{T}| + |\mathcal{I}| = 101\ 000$ variables, $|\mathcal{T}| = 100\ 000$ reified constraints to express the channeling constraints, $2\times|\mathcal{I}| = 2\times1\ 000$ reified constraints to express the closure and frequency constraints. This means that the CP solver has to load in memory a CP model of 102 000 reified constraints expressed on 101 000 variables, which represents the *size* given in Table 2. From Table 2, we observe that Gecode is not able to handle CP4IM models on datasets of size greater than $\approx 10^7$ (greater than *connect* dataset).

ClosedPattern vs LCM. In terms of CPU times, LCM remains the leader on basic queries. However, ClosedPattern is quite competitive as a declarative approach. For instance, if we take *chess* dataset, LCM is 15 times faster than ClosedPattern-DC on average, where it is more than 120 times faster on average comparing to CP4IM. ClosedPattern pruning acts only within the current node of the search tree, without imposing any condition on the main search algorithm such as variable or value orderings. This allows to consider new constraints and let the main search algorithm adopt the best heuristics favouring the whole solving. To illustrate our point, we propose to model a particular problem (k-pattern sets) in a declarative manner where LCM could not meet this need.

k-patterns instance. A promising road to discover useful patterns is to impose constraints on a set of k related patterns (k-pattern sets) [5,9]. In this setting, the interest of a pattern is evaluated w.r.t. a set of patterns. We propose to model and solve a particular instance, coined `dist_kpatterns_lb_ub`. Here, we aim at finding k closed patterns $\{P^1, \ldots, P^k\}$ s.t:
(i) $\forall i \in [1, k] : \text{ClosedPattern}(P^i)$ (Closed Frequent Patterns),
(ii)$\forall i, j \in [1, k] : P^i \cap P^j = \emptyset$ (all distinct patterns constraints),
(iii) $\forall i \in [1, k] : \texttt{lb} < |P^i| < \texttt{ub}$ (min and max size constraints).

Figure 2 shows a comparison between two models, M1 using ClosedPattern for the CFPM part of the problem, M2 using a CP4IM implementation. We selected two dataset instances where CP4IM does not reach the Out-of-memory state and where we have a reasonable number of closed patterns, *chess* with $\theta = 80\%$ (5084 closed patterns) and *connect* with $\theta = 90\%$ (3487 closed patterns), and we have varied k with a timeout of $3600s$. After a few preliminary tests, the bounds lb and ub on the size of the patterns were set to 2 and 10 respectively. On *chess*, model M1 is robust and scales well: it is linear on k and never exceeds 6 *min* even with $k = 12$ ($323.53s$). M2 follows an exponential scale and goes beyond the timeout with only $k = 8$ ($7222.41s$). The same observation can be made on the **connect** instance, but in a more pronounced way on the exponential scale followed by M2. With $k = 4$, M2 goes beyond the timeout with $5428.05s$ whereas M1 confirms its linear behavior when varying k from 2 to 12.

For such problems, a baseline can be the use of specialized methods with postprocessing. One can imagine **(i)** the use of `LCM` to extract the total number of closed patterns and **(ii)** a generate-and-test search trying to find distinct patterns of a given size. Such approach can be very expensive. Here, the postprocessing will generate all the possible k combinations of closed patterns. For instance, we recall that for *chess* with $\theta = 80\%$ we have 5084 closed patterns. With $k = 12$ and using `M1` , we need less than 6 *min*, where using the baseline we have to cope with a massive number of combinations. Thus, this last experiment confirms that if `LCM` is faster on basic queries (e.g., asking for closed frequent with given size), it cannot cope with complex queries. It would need to think and to propose an adhoc solution whereas CP enables a novice DM-user to express his query as constraints.



chess ($\theta = 80\%, \texttt{lb} = 2, \texttt{ub} = 10$) *connect* ($\theta = 90\%, \texttt{lb} = 2, \texttt{ub} = 10$)

Fig. 2: `dist_kpatterns_lb_ub` instance using ClosedPattern and `CP4IM`.

7 Conclusion

In this paper we have introduced a new global constraint for Closed Frequent Pattern Mining. The ClosedPattern constraint captures the particular semantic of the CFPM problem, namely the minimum frequency and closedness of patterns. To propagate efficiently this global constraint, we have first defined three filtering rules that ensure domain consistency. Second, we have defined a filtering algorithm that establishes domain consistency in a *cubic* time complexity and *quadratic* space complexity. We have implemented this filtering algorithm into the `or-tools` solver using a vertical representation of datasets and smart data structures. We have conducted an experimental study on several real and synthetic datasets, showing the efficiency and the scalability of the global constraint compared to a reified constraints approach such as `CP4IM`. Finally, to show the applicability and the flexibility of ClosedPattern compared to specialized methods we performed experiments on an instance of k-pattern set problem where ClosedPattern is integrated with a set of constraints.

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