A Branching Heuristics for Quantified Renamable Horn Formulas

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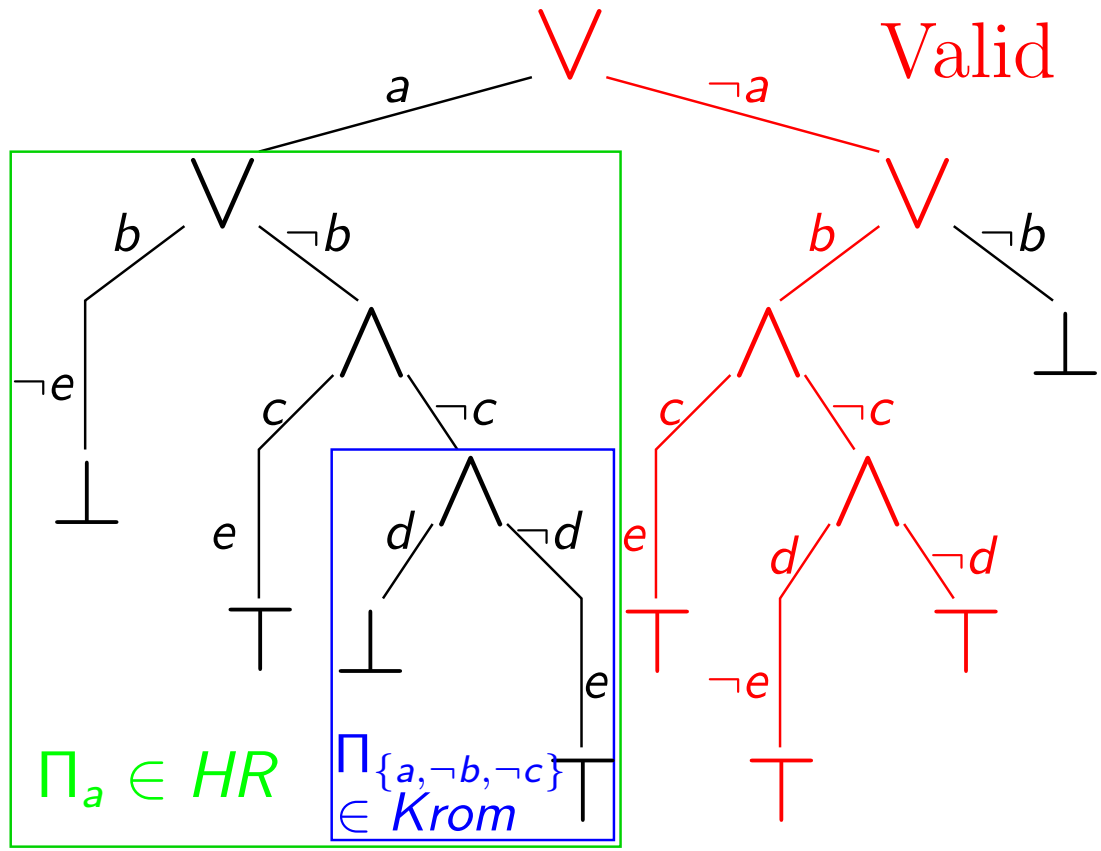
$$\Pi = \exists\ a, b\ \forall\ c, d\ \exists\ e$$

$$\left[\begin{array}{ll} (b \vee c \vee \neg d \vee e) & \wedge \\ (\neg a \vee \neg b \vee \neg e) & \wedge \\ (\neg a \vee d \vee e) & \wedge \\ (\neg c \vee e) & \wedge \\ (a \vee b \vee \neg c) & \wedge \\ (c \vee \neg d \vee \neg e) & \end{array}\right]$$



Recognition of renamable Horn formulas

Definition *($\Rightarrow$ et $\textsc{Clos}(I)$)*

- $I \Rightarrow t$ iff $\exists C \in \Phi$ s.t. $I \in C$, $\neg t \in C$ and $I \neq \neg t$
- $\textsc{Clos}(I)$ denotes the set $\{t \ / \ I \Rightarrow^* t\}$

Proposition *(Proposition 1.1 from Hébrard 1994)*

A renaming R is Horn iff it's closed, i.e. $\forall I \in R$, $\textsc{Clos}(I) \subseteq R$

If $\textsc{Clos}(I)$ is contradictory (contains a literal and its opposite) and $I \in R$, then R isn't closed

Heuristics Δ

Definition *(Contradiction's distance of Horn renamability)*

$$\delta_l = \begin{cases} 0 \text{ if } \nexists v | l \Rightarrow v \\ 1 \text{ if } l \Rightarrow t \text{ and } l \Rightarrow \neg t \\ 1 + min(\{\delta_v | l \Rightarrow v\}) \text{ otherwise} \end{cases}$$

Hébrard's original algorithm : Depth-First Search
Our algorithm : Breadth-First Search

Definition *(Δ)*

- $\Delta_x = 1024 \times \delta_x \times \delta_{\neg x} + \delta_x + \delta_{\neg x}$
- choose a variable x minimizing Δ_x
- choose the literal x if $\delta_x < \delta_{\neg x}$, $\neg x$ otherwise

Rationale : by assigning the worst variable, we remove it from the formula

Experimental results : methodology

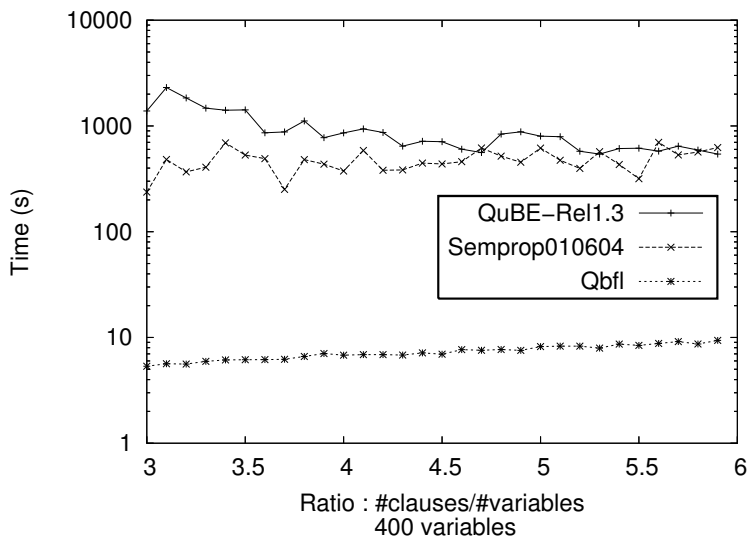
- PIV 3GHz, 512 MB de RAM, Linux Fedora
- Qbfl : our solver, version 1.7
  (`http://www.cril.univ-artois.fr/~letombe/qbfl`)
  - Extension of DPLL
  - Use Limmat (1.3) as a SAT oracle (winner in SAT 2002 competition)
  - Recognize renamable Horn formulas
  - Several heuristics (including Δ)
- QuBE-Rel : Giunchiglia *et al.* 2001, version 1.3
  (`http://www.star.dist.unige.it/~qube`)
- Semprop : Letz 2002, version 010604
  (`http://www4.in.tum.de/~letz/semprop`)

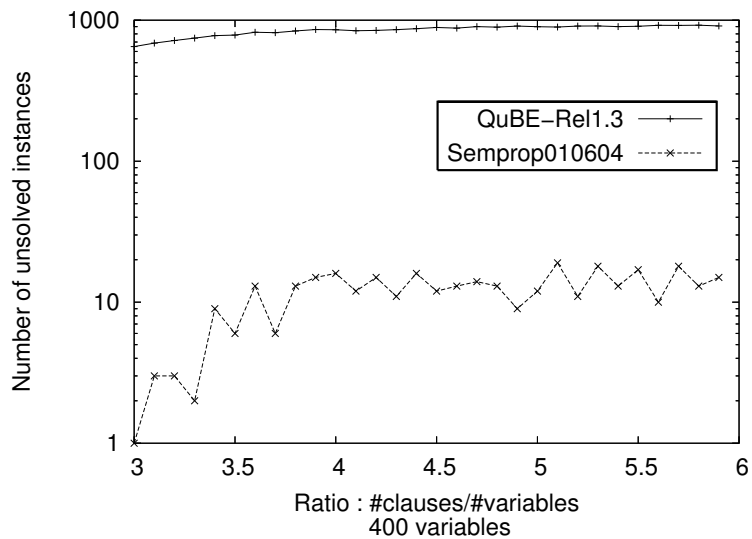
Experimental results : random generation of polynomial classes benchmarks

- Horn ($\mathcal{QHF}$s) and renamable Horn (ren$\mathcal{QHF}$s) formulas randomly generated
- 2 generators (`http://www.cril.univ-artois.fr/~letombe/qbfg`)
- $\mathcal{QHF}$: for each clause
  - random size
  - random positive literal
  - complete with random negative literals
- ren$\mathcal{QHF}$:
  - random number of renamed variables
  - random renamed variables
  - same parameters than $\mathcal{QHF}$s
- Prefix $\forall X \exists Y$, $2 < |X| < 2/3\ |V|$

Experimental results : QBFs

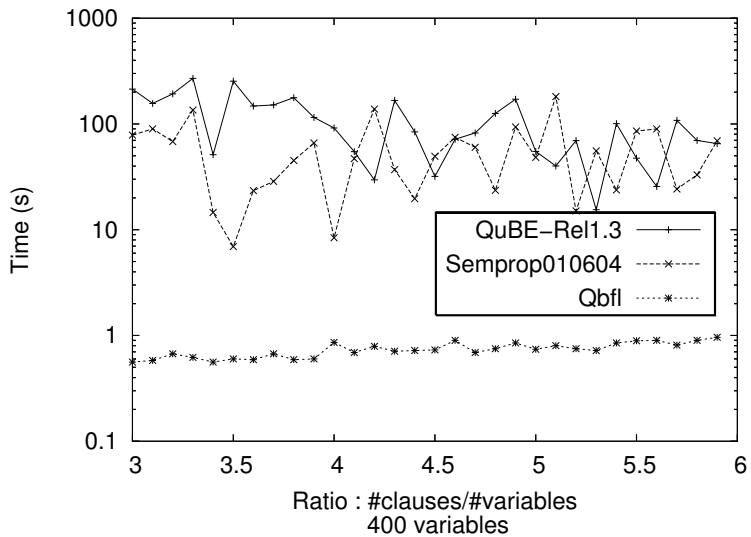
- 1000 instances generated per point
- Accumulated time for **solved instances**
- TimeOut : 60 s
- 400 variables
- 1200 à 2360 clauses

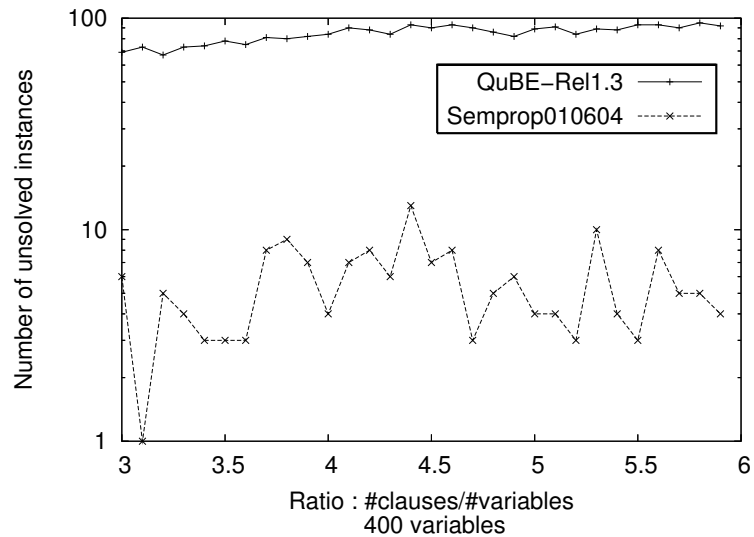




Experimental results : renQBFs

- 100 instances generated per point
- Accumulated time for **solved instances**
- TimeOut : 60 s
- 400 variables
- 1200 à 2360 clauses





Results on benchmarks from the QBF *2004 evaluation*

- Benchmarks available on `http://www.qbflib.org`
- Ayari (72), Castellini (169), Mneimneh and Sakallah (202), **Pan** (378), Rintanen (67), Scholl and B. Becker (64)
- TimeOut : 900 s

Instance type	Jeroslow-Wang				Renamable Horn Δ			
	%solved		%RH		%solved		%RH	
	k_*_n	k_*_p	k_*_n	k_*_p	k_*_n	k_*_p	k_*_n	k_*_p
k_branch_n/p	4.76	4.76	25.26	24.56	**9.52**	4.76	9.29	9.33
k_d4_n/p	4.76	9.52	13.25	26.33	4.76	**14.28**	5.63	25.34
k_dum_n/p	4.76	4.76	11.69	11.71	**23.80**	**14.28**	11.51	11.40
k_grz_n/p	0	0	-	-	**61.90**	0	11.94	-
k_lin_n/p	9.52	9.52	20.00	11.88	9.52	**19.04**	24.26	8.65
k_path_n/p	9.52	14.28	5.47	7.43	**14.28**	**19.04**	12.12	7.64
k_ph_n/p	23.80	19.04	2.66	10.06	23.80	19.04	11.73	6.89
k_poly_n/p	9.52	4.76	0	12.91	**14.28**	**9.52**	6.66	11.04
k_t4p_n/p	4.76	0	11.37	-	4.76	**4.76**	26.62	26.02
Total v/f	7.93	7.40	11.21	14.98	**18.51**	**11.64**	13.30	13.28
Total	7.67		13.09		**15.07**		13.29	

Results on benchmarks from the QBF *2005 evaluation*

Solver	Horn	Renamable Horn
GRL	62	31
qbfbdd	55	0
qbflHR	156	84
QChaffLearn	52	24
QMRes	0	0
quantor	8	7
run-openqbf	20	5
run-semprop	103	23
run-ssolve	148	83
sKizzo_v0.4	30	18
sKizzo_v0.5	29	18
WalkQSAT	37	13
yquaffle	73	57