Learning to Assign Degrees of Belief in Relational Domains

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Outline

1. Learning to Reason
   - The Knowledge Representation Approach
   - The Learning to Reason Approach
2. Exponentiated Gradient Learning to Reason
   - Two Key Ideas
   - The Algorithm
3. Tractable Query Languages
   - Decomposable Queries
   - Hitting Languages
   - Cluster Languages
4. Perspectives

Relational Vocabulary

A finite set of *relation symbols*, and a finite set of *constants*

- Background knowledge: a set $\mathcal{B}$ of ground atoms
- Relational interpretation: a subset I of $\mathcal{B}$

Example

Consider a simple logistic domain

- Constants: 20 objects 5 trucks and 4 cities.
- Relations: In(x, y), At(x, y).

The background knowledge contains 200 ground atoms

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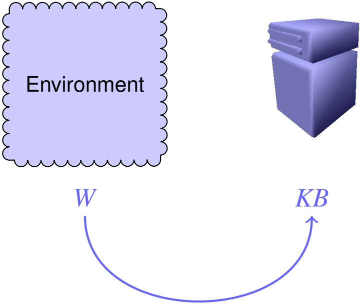


KR Approach

The reasoning agent is given a description of its environment

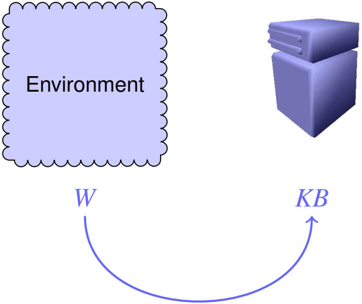
Environment

Distribution W on the space $2^{\mathcal{B}}$ of relational interpretations

Knowledge Base

A description KB of the environment W

- Logical theory
- Bayesian network



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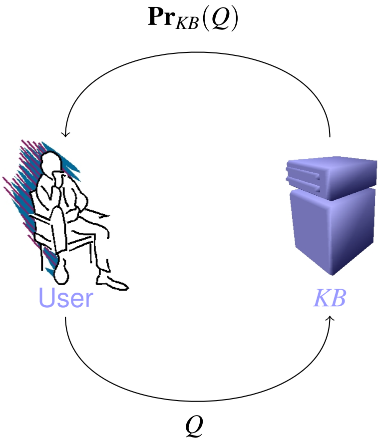


KR Approach

The agent is expected to evaluate any query with perfect precision

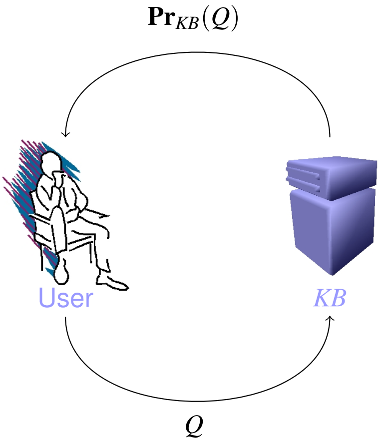
Degree of Belief

For any query Q, the probability of Q according to KB is

$$\mathbf{Pr}_{KB}(Q) = \sum_{I \models Q} \mathbf{Pr}_{KB}(I)$$

Example

$\mathbf{Pr}_{KB}(\mathsf{In}(\mathsf{o}_1, \mathsf{t}_1)) = \frac{3}{4}$ the agent believes that object o_1 is in the truck t_1 with probability $\frac{3}{4}$



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Complexity

The problem of evaluating the degree of belief of any query is #P-Hard

Simple Query Languages

The complexity is unchanged for very simple queries:

- Quantified literals: $\forall x\, \mathrm{In}(x, \mathrm{t}_1)$
- Ground atoms: $\mathrm{At}(\mathrm{c}_1, \mathrm{t}_1)$

Simple Representation Languages

The complexity is unchanged for simple representation languages:

- Horn Theories
- Monotone DNF Theories

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KR Approach

A sharp separation between *knowledge acquisition* and *query evaluation*. Knowledge is given a priori in order to correctly represent an environment

L2R Approach

The dependence between *knowledge acquisition* and *query evaluation* is made explicit. Knowledge is acquired a posteriori, by experience, in order to efficiently reason about queries

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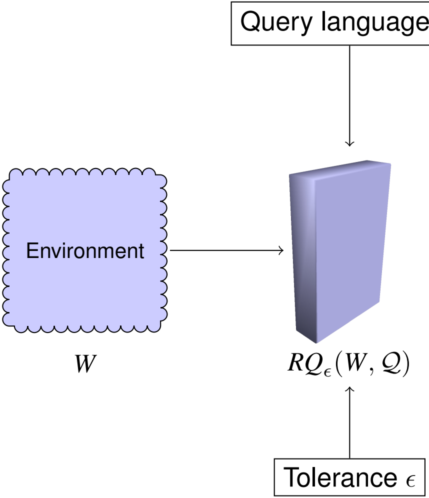
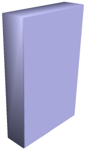


Learning Interface

Help the agent in finding a representation KB of W that is computationally efficient for some target query language $\mathcal{Q}$



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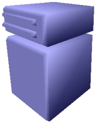


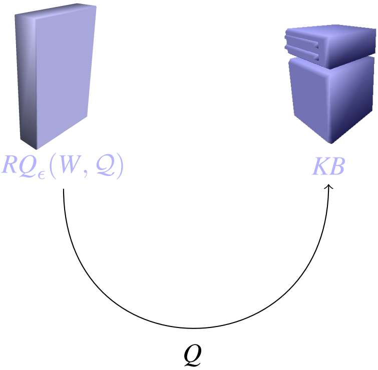
KB

Grace Period

Repeated game between the agent and the interface

1. Receive a query $Q \in \mathcal{Q}$
2. Predict $\hat{y} = \mathbf{Pr}_{KB}(Q)$
3. Receive $y = \mathbf{Pr}_W(Q)$

   If $L(y, \hat{y}) > \epsilon$ update KB



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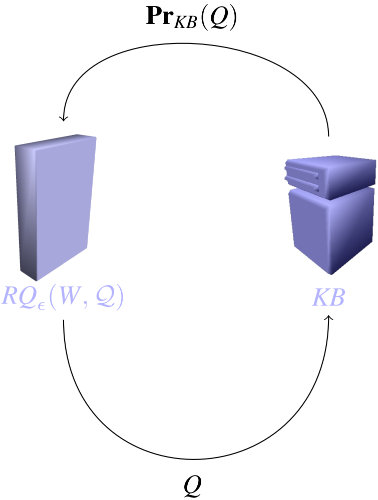

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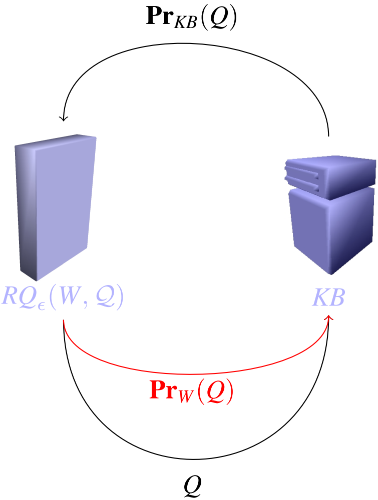

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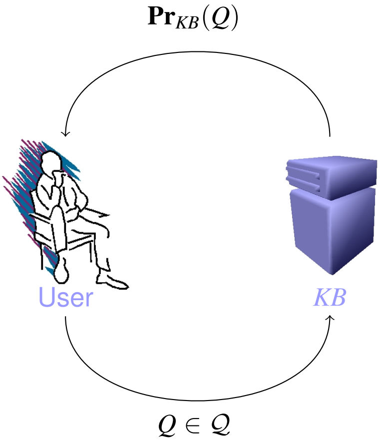

Operational Period

The reasoning performance of the agent is measured according to

- the same target query language $\mathcal{Q}$
- the same tolerance parameter ϵ

Polynomial Mistake Bound

For any possible sequence of queries in $\mathcal{Q}$, the total number of mistakes made by the L2R algorithm must be $\mathrm{poly}(|\mathcal{B}|, \frac{1}{\epsilon})$

Polynomial Complexity

For any possible query Q in $\mathcal{Q}$, the L2R algorithm must evaluate $\mathbf{Pr}_{KB}(Q)$ in $\mathrm{poly}(|\mathcal{B}|, |Q|, \frac{1}{\epsilon})$ time

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Outline

1st Idea

Use an exponentiated gradient strategy to update knowledge

2nd Idea

Use a weighted model counting approach to evaluate queries

Weighted Atoms

The vocabulary is extended with a set $\{\mathrm{q}_1, \mathrm{q}_2, \ldots\}$ of weighted atoms

- Standard Atom: $weight(\mathrm{a}) = 1$
- Weighted Atom: $weight(\mathrm{q}) \geq 0$

Weighted Interpretation

An interpretation that possibly contains weighted atoms

$$weight(I) = \prod_{A \in I} weight(A)$$

Weighted Formula

A relational expression F over the extended vocabulary

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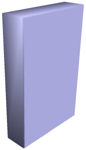

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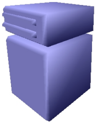

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The EG-L2R Algorithm

Start with $KB = \varnothing$. In each trial,

1. Receive a query $Q \in \mathcal{Q}$
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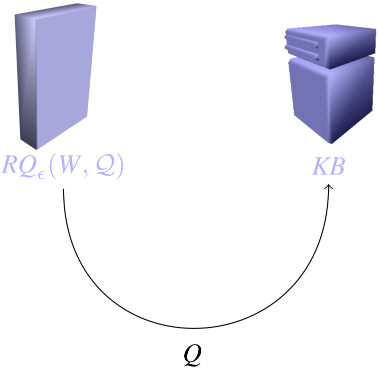

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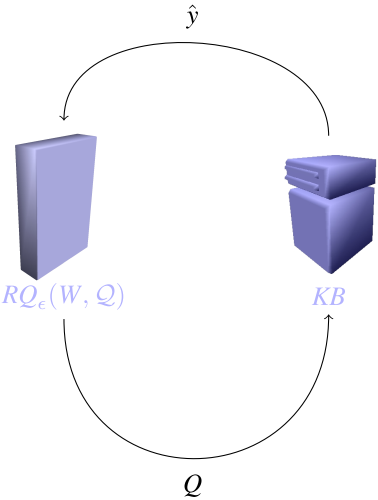

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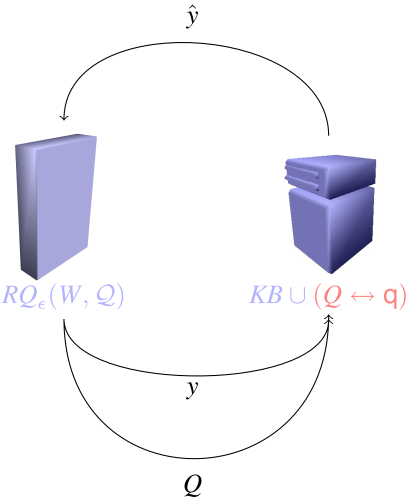

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Polynomial Mistake Bound

The total number of mistakes made by EG-L2R is bounded by

$$\frac{|\mathcal{B}|}{2\epsilon}$$

Polynomial Size Representation

Let l be the largest size of any query in $\mathcal{Q}$. Then the size of KB is bounded by

$$\frac{l|\mathcal{B}|}{2\epsilon}$$

Outline

Quantified Atom

Atomic formula where each variable occurs in the scope of a quantifier $\forall$ or $\exists$

$\forall x \, \mathsf{In}(x, \mathsf{t}_1)$ $\qquad$ $\exists y \, \mathsf{At}(y, \mathsf{t}_1))$

Decomposable Query

Conjunction (or disjunction) of pairwise independent quantified literals

$$\forall x \, \mathsf{In}(x, \mathsf{t}_1) \wedge \exists y \, \mathsf{At}(y, \mathsf{t}_1)$$

Complexity

The number of models of any decomposable query Q can be evaluated in $O(|\mathcal{B}||Q|)$ time

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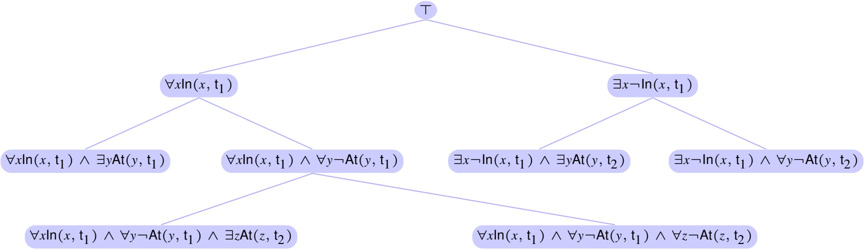
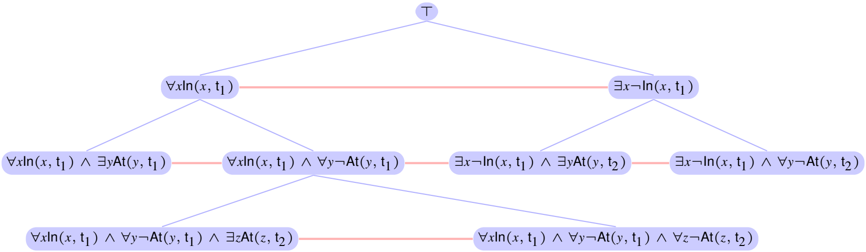


Hitting Language

Set of decomposable queries that are pairwise comparable under entailment or insatisfiable



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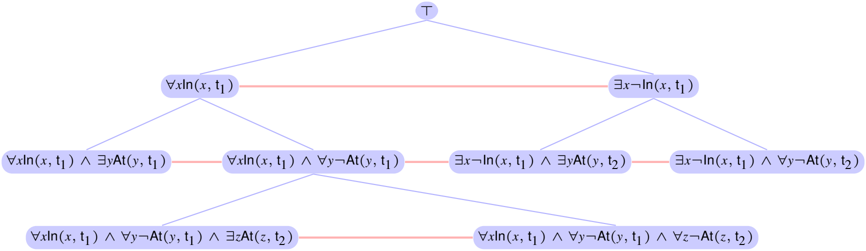

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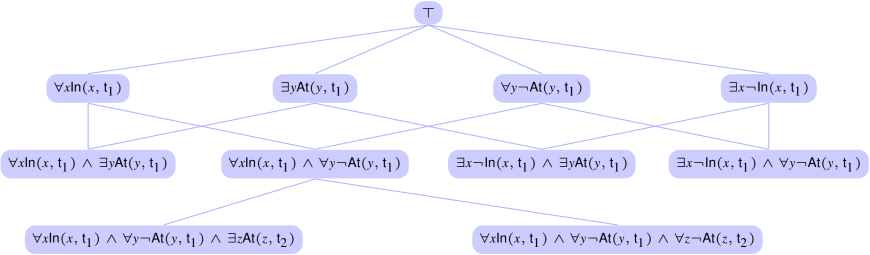
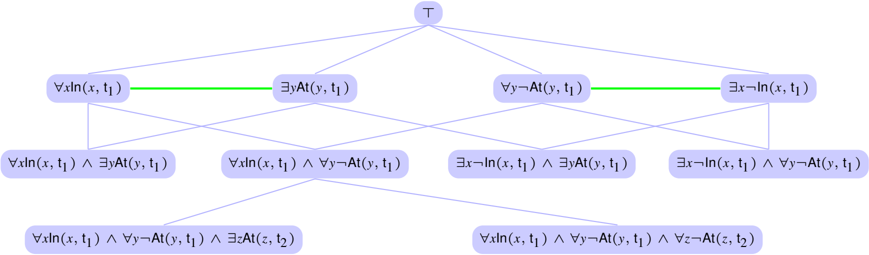
Learnability

There exists an efficient L2R algorithm for any probabilistic reasoning problem $(W, \mathcal{Q})$ where $\mathcal{Q}$ is an hitting query language



Cluster Language

Set $\mathcal{Q}$ of decomposable queries that are pairwise comparable or independent or insatisfiable



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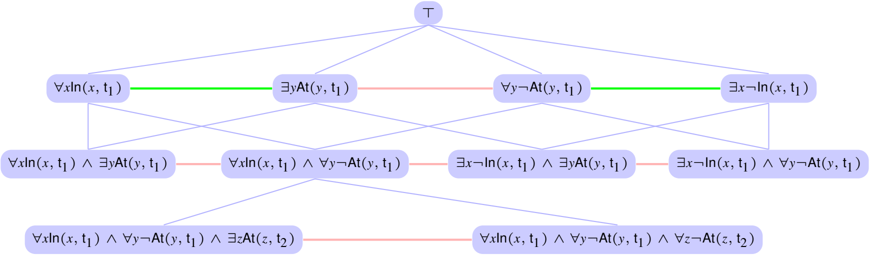

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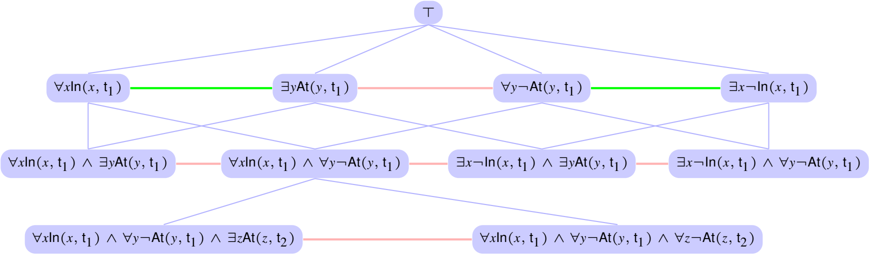

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Outline

Application

Inductive Knowledge Compilation: Learning a computationally efficient representation of a logical theory (or Bayesian network) for some frequent queries supplied by users

Extensions

- Extending the scope of quantifiers

$$\forall x, y \, (\mathsf{In}(x, y) \rightarrow \mathsf{Truck}(x))$$

- Parameterized Cluster-Width

$$\forall x \, \mathsf{In}(x, \mathsf{t}_1) \wedge \exists y \mathsf{At}(y, \mathsf{t}_1) \qquad \exists x \, \mathsf{In}(x, \mathsf{t}_2) \wedge \mathsf{At}(\mathsf{c}_1, \mathsf{t}_1)$$

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