Putting More Dynamics in Revision with Memory

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Abstract. We have proposed in previous works [14,15] a construction that allows to define operators for iterated revision from classical AGM revision operators. We called these operators *revision operators with memory* and show that the operators obtained have nice logical properties. But these operators can be considered as too conservative, since the revision policy of the agent, encoded as a faithful assignment, does not change during her life. In this paper we propose an extension of these operators, that aims to add more dynamics in the revision process.

1 Introduction

The predominant approaches for modelling belief change was proposed by Alchourrón, Gärdenfors and Makinson and is known as the AGM framework [1,10]. The core of this framework is a set of logical properties that a revision operator has to satisfy to guarantee a nice behaviour. A drawback of AGM definition of revision is that it is a static one, which means that, with this definition of revision operators, one can have a rational one step revision but the conditions for the iteration of the process are very weak. The problem is that AGM postulates state conditions only between the initial knowledge base, the new evidence and the resulting knowledge base. But the way to perform further revisions on the new knowledge base does not depend on the way the old knowledge base was revised.

Numerous proposals have tried to state a logical characterization that adequately models iterated belief change behaviour [6,8,9,14,17,19,20][1]. The core work on iterated revision is the proposal of Darwiche and Pearl [9] and its developments [4,11,13,16]. The main idea that is common to all of these works is that the belief base framework is not sufficient to encompass iterated revision, since one needs some additional information for coding the revision policy of the agent. So the need of *epistemic states* to encode the agent's "state of mind" is widely accepted. An epistemic state allows to code the agent's beliefs but also to code her relative confidence in alternative possible states of the world. Epistemic

[1] See also [22–24]. We do not adress this kind of operators in this paper since they require an additional numerical information with the new evidence.

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states can be represented by several means: pre-orders on interpretations [9,17], conditionals [6,9], epistemic entrenchments [19,23], prioritized belief bases [2,3], etc. In this work we will focus on the representation of epistemic states in terms of pre-orders on interpretations.

In [14,15], we define a family of revision operators that we have called *revision operators with memory*. These operators can be defined from any classical AGM revision operator [1,12] and they have good properties for iterated revision.

In fact revision operators with memory use the faithful assignment provided by the classical AGM revision operator as an *a priori* information. This *a priori* information is attached to the new evidence, and the completed information obtained is then incorporated to the old epistemic state with the usual *primacy of update* requirement. The ontology for this pre-processing step, associating an additional information to the incoming new evidence is the following. Suppose that the agent has no information (no belief) about the world and learns a (first) new evidence. Then, this new evidence alone can provide more change in the agent's mind than just the addition of a belief.

As an example, suppose that the agent learns $\varphi = a \wedge b \wedge c \wedge d$, where a, b, c, d are atomic formulae. Then her preferred worlds (the ones she finds the more plausible) will be the ones where the four atomic formulae are true. But it can be sensible for her to find the worlds where three of the atomic formulae are true more plausible than the ones where only two are, etc.

So the new evidence does not simply imply a partition between the believed worlds and the unbelieved ones, but defines several stratas, depending of the plausibility of each world, given the new evidence. We call this property *strong primacy of update*. This induced preferential information was given here by a "Dalal distance" policy[2], but more complex or realistic policies can be also used depending on the particular context.

The point in the definition of revision operators with memory, is that this *a priori* information, carried by a new evidence depends only on the new evidence by itself, and does not depend on the current agent's beliefs. Going back to the previous example, the fact that the worlds where three of the four atomic formulae are true are preferred to the ones where only two are, does not depend on any other information than the new evidence itself. So this *a priori* information has to be added to the new evidence before incorporating it in the agent's epistemic state.

More precisely, the revision policy of revision operators with memory is the following: the revision of the current epistemic state Φ – represented by a pre-order over possible worlds – by a new piece of information α – a formula – is the epistemic state (pre-order) obtained after the following two steps:

– First, take the pre-order $\leq_\alpha$ associated to α by the AGM revision operator (faithful assignment [12]) given at the beginning of the process.
– Second, take the lexicographical pre-order associated to $\leq_\alpha$ and Φ. The pre-order obtained in this way is the new epistemic state.

Note that there is a very static feature in this process: the way in which we associate a pre-order to the new piece of information is always the same; it is

[2] The Dalal distance [7] is a Hamming distance between interpretations.

given by the *fixed* AGM operator from which we start all the process. In some sense this is contrary to the principle of *priority of the new information*.

In this work we solve this problem. In order to do that we take the revision policy as an epistemic state and naturally this revision policy will change progressively with the successive revisions. The new process can be described in the following manner: first of all, an epistemic state is composed by a faithful assignment, say f, and a distinguished formula ϕ. When α, the new evidence, arrives, we revise as follows:

(i) the new distinguished formula ϕ' will be a formula having as models the minimal models of α with respect to the $f(\phi)$ pre-order.
(ii) The new assignment f' will coincide with f on the formulas not equivalent to ϕ'. On formulas equivalent to ϕ' it will be the lexicographical pre-order associated to $f(\alpha)$ and $f(\phi)$.

Thus, this method allows to incorporate the changes step by step in a very natural way. This process agrees with the postulate of primacy of the new information. Unlike our original revision operators with memory that mix the new piece of information with the oldest information (which is static), our present operators mix the new piece of information with the current epistemic state.

The rest of the paper is organized as follows: in Sect. 2, we recall the logical characterization of iterated revision operators of Darwiche and Pearl. In Sect. 3, we recall the definition of revision operators with memory and state the general logical results. Then, in Sect. 4, we show how to add more dynamics to revision operators with memory. We conclude in Sect. 5 with some general remarks.

2 Iterated Revision Postulates

We give here a formulation of AGM postulates for belief revision *à la* Katsuno and Mendelzon [12]. More exactly, we give a formulation of these postulates in terms of epistemic states [9]. The epistemic states framework is an extension of the belief bases one. Intuitively an epistemic state can be seen as a composed information: the beliefs of the agent, plus all the information that the agent needs about how to perform revision (preference ordering, conditionals, etc.). Then we give the additional iteration postulates proposed by Darwiche and Pearl [9].

2.1 Formal Preliminaries

We will work in the finite propositional case. A belief base φ is a finite set of formulae, which can be considered as the formula that is the conjunction of its formulae. The set of all interpretations is denoted $\mathcal{W}$. Let φ be a formula, $Mod(\varphi)$ denotes the set of models of φ, *i.e.* $Mod(\varphi) = \{I \in \mathcal{W} : \ I \models \varphi\}$.

A pre-order $\leq$ is a reflexive and transitive relation, and $<$ is its strict counterpart, i.e. $I < J$ if and only if $I \leq J$ and $J \not\leq I$. As usual, $\simeq$ is defined by $I \simeq J$ iff $I \leq J$ and $J \leq I$. A pre-order is total if and only if $\forall I, J$, $I \leq J$ or $J \leq I$.

To each epistemic state Ψ is associated a belief base $Bel(\Psi)$ which is a propositional formula representing the objective (logical) part of Ψ. The models of Ψ are the models of its associated belief base, thus $Mod(\Psi) = Mod(Bel(\Psi))$. Let Ψ be an epistemic state and μ be a sentence denoting the new information. $\Psi \circ \mu$ denotes the epistemic state resulting of the revision of Ψ by μ. For reading convenience we will write respectively $\Psi \vdash \mu$, $\Psi \wedge \mu$ and $I \models \Psi$ instead of $Bel(\Psi) \vdash \mu$, $Bel(\Psi) \wedge \mu$ and $I \models Bel(\Psi)$.

Two epistemic states are equivalent, noted $\Psi \equiv \Psi'$, if and only if their objective parts are equivalent formulae, *i.e.* $Bel(\Psi) \leftrightarrow Bel(\Psi')$. Two epistemic states are equal, noted $\Psi = \Psi'$, if and only if they are identical. Thus equality is stronger than equivalence.

2.2 AGM Postulates for Epistemic States

Let Ψ be an epistemic state and μ and φ be formulae. An operator $\circ$ that maps an epistemic state Ψ and a formula μ to an epistemic state $\Psi \circ \mu$ is said to be a revision operator on epistemic states if it satisfies the following postulates [9]:

(R*1) $\Psi \circ \mu \vdash \mu$
(R*2) If $\Psi \wedge \mu \nvdash \bot$, then $\Psi \circ \mu \leftrightarrow \Psi \wedge \mu$
(R*3) If $\mu \nvdash \bot$, then $\Psi \circ \mu \nvdash \bot$
(R*4) If $\Psi_1 = \Psi_2$ and $\mu_1 \leftrightarrow \mu_2$, then $\Psi_1 \circ \mu_1 \equiv \Psi_2 \circ \mu_2$
(R*5) $(\Psi \circ \mu) \wedge \varphi \vdash \Psi \circ (\mu \wedge \varphi)$
(R*6) If $(\Psi \circ \mu) \wedge \varphi \nvdash \bot$, then $\Psi \circ (\mu \wedge \varphi) \vdash (\Psi \circ \mu) \wedge \varphi$

This is nearly the Katsuno and Mendelzon formulation of AGM postulates [12]; the only differences are that we work with epistemic states instead of belief bases and that postulate (R*4) is weaker than its AGM counterpart. See [9] for a full motivation of this definition.

A representation theorem states how revisions can be characterized in terms of pre-orders on interpretations. In order to give such a semantical representation, the concept of faithful assignment on epistemic states is defined.

Definition 1. *A function that maps each* epistemic state Ψ *to a pre-order* $\leq_\Psi$ *on interpretations is called a* faithful assignment over epistemic states *if and only if: 1. If* $I \models \Psi$ *and* $J \models \Psi$*, then* $I \simeq_\Psi J$
2. If $I \models \Psi$ *and* $J \not\models \Psi$*, then* $I <_\Psi J$
3. If $\Psi_1 = \Psi_2$*, then* $\leq_{\Psi_1} = \leq_{\Psi_2}$

Now the reformulation of the Katsuno and Mendelzon [12] representation theorem in terms of epistemic states is:

Proposition 1 ([9]). *A revision operator* $\circ$ *satisfies postulates (R*1-R*6) if and only if there exists a faithful assignment (over epistemic states) that maps each epistemic state* Ψ *to a total pre-order* $\leq_\Psi$ *such that*

$$Mod(\Psi \circ \mu) = \min(Mod(\mu), \leq_\Psi)$$

Notice that this theorem gives information only on the objective part of the resulting epistemic state, but does not allow to know what is the pre-order associated with $\Psi \circ \mu$, *i.e.* we can not identify the new epistemic state, but only its associated belief base $Mod(\Psi \circ \mu)$. Making the parallel with the classical Katsuno and Mendelzon representation theorem (*cf* Definition 2 and [12]), that allows to define exactly what is the belief base $Mod(\Psi \circ \mu)$[3], the last theorem is only a weak representation theorem.

2.3 Darwiche and Pearl Postulates

A strong limitation of AGM revision postulates is that they impose very weak constraints on the iteration of the revision process. Darwiche and Pearl [8,9] proposed postulates for iterated revision. The aim of these postulates is to keep as much as possible of conditional beliefs (a conditional belief can be expressed as "if μ would be the case, then φ must be true") of the old belief base. These conditional beliefs are encoded in the total pre-orders on interpretations. So, besides postulates (R*1-R*6), a revision operator has to satisfy:

(C1) If $\varphi \vdash \mu$, then $(\Psi \circ \mu) \circ \varphi \equiv \Psi \circ \varphi$
(C2) If $\varphi \vdash \neg\mu$, then $(\Psi \circ \mu) \circ \varphi \equiv \Psi \circ \varphi$
(C3) If $\Psi \circ \varphi \vdash \mu$, then $(\Psi \circ \mu) \circ \varphi \vdash \mu$
(C4) If $\Psi \circ \varphi \nvdash \neg\mu$, then $(\Psi \circ \mu) \circ \varphi \nvdash \neg\mu$

These postulates can be explained as follows: (C1) states that if two pieces of information arrive and if the second implies the first, the second alone would give the same belief base. (C2) says that when two contradictory pieces of information arrive, the second alone would give the same belief base. (C3) states that an information should be retained after revising by a second information such that, when revising the current belief base by it, the first one holds. (C4) says that no piece of information can contribute to its own denial.

3 Revision Operators with Memory

A "classical" AGM revision operator is equivalent to a faithful assignment over belief bases as stated in the following theorem [12].

Definition 2. *A function that maps each* belief base φ *to a pre-order* $\leq_\varphi$ *on interpretations is called a* faithful assignment <u>over belief bases</u> *if and only if:*

1. *If* $I \models \varphi$ *and* $J \models \varphi$*, then* $I \simeq_\varphi J$
2. *If* $I \models \varphi$ *and* $J \not\models \varphi$*, then* $I <_\varphi J$
3. *If* $\varphi_1 \leftrightarrow \varphi_2$*, then* $\leq_{\varphi_1} = \leq_{\varphi_2}$

[3] Recall that classical AGM operators are functions that map a belief base and a formula to a belief base, which is (completely) defined by the theorem, whereas Proposition 1 concerns operators that are functions which map an epistemic state and a formula to an epistemic state, that is not completely defined by the theorem.

It is important to note that, in what follows, we have two distinct kinds of faithful assignments: one over belief bases and one over epistemic states.

Proposition 2 ([12]). *A revision operator* $\circ$ *satisfies classical AGM postulates (R1-R6)*[4] *if and only if there exists a faithful assignment (over belief bases) that maps each belief base* φ *to a total pre-order* $\leq_\varphi$ *such that:* $Mod(\varphi \circ \mu) = \min(Mod(\mu), \leq_\varphi)$.

So one can define a revision operator directly by defining the corresponding faithful assignment over belief bases. It is the case for most distance-based revision operators such as Dalal operator [7,12].

More precisely we say that a revision operator $\circ$ is defined from a distance d iff the following conditions hold:

- d is a (pseudo-)distance, that is d is a function $d : \mathcal{W} \times \mathcal{W} \mapsto I\!N$ which satisfies: $d(I, J) = d(J, I)$ and $d(I, J) = 0$ iff $I = J$.
- The distance between an interpretation I and a belief base φ is defined as:

$$d(I, \varphi) = \min \{d(I, J) : J \models \varphi\}$$

- This distance induces a faithful assignment: $I \leq_\varphi J$ iff $d(I, \varphi) \leq d(J, \varphi)$
- And the revision operator is defined by $Mod(\varphi \circ \mu) = \min(Mod(\mu), \leq_\varphi)$

One can check that the assignment obtained like this is a faithful assignment and thus that all operators defined in this way satisfy AGM postulates. It can also be easily checked that operators defined in this way do not satisfy many of the iterated revision postulates.

Now we will give a construction that allows, from a given faithful assignment (*i.e.* from a given classical AGM revision operator), to define another revision operator that satisfies AGM postulates but also most of the iterated revision postulates.

First, let us notice that an epistemic state can be represented by a total pre-order on interpretations as suggested by Proposition 1 and by several related works (*cf e.g.* [3,9]). So, with this particular representation (identifying the epistemic state Ψ with a pre-order $\leq_\Psi$), the belief base $Bel(\Psi)$ is simply the formula whose models are minimal for the pre-order, that is $Bel(\Psi) = \min(\mathcal{W}, \leq_\Psi)$. And the other interpretations are ordered according to their relative plausibility for the agent. For example, $I \leq_\Psi J$ means that the agent that is in the epistemic state Ψ considers I as at least as plausible as J. It is this preferential information that can be used to encompass the iterated revision behaviour, by considering revision operators as functions that map a pre-order (epistemic state) and a formula (new information) into a new pre-order (epistemic state). This idea is the mainstay in most of iterated revision works [4,9,11,13,16,19,23].

So, using this representation by means of pre-orders on interpretations and Proposition 1 we will define a familly of revision operators as follows:

[4] It is the same set of postulates than (R*1-R*6) but expressed for belief bases instead of epistemic states (*cf* [12]).

Definition 3. *Suppose that we have a function that maps each belief base φ to a pre-order $\leq_\varphi$. Then, we define the epistemic state (the pre-order) $\Psi \circ \varphi$ resulting of the revision of Ψ by the new information φ as:*

$$I \leq_{\Psi \circ \varphi} J \text{ iff } I <_\varphi J \text{ or } (I \simeq_\varphi J \text{ and } I \leq_\Psi J)$$

Then one can check that:

Proposition 3 ([15]). *If the function that maps each belief base φ to a total pre-order $\leq_\varphi$ is a faithful assignment over belief bases, then the revision operator on epistemic states defined in Definition 3 satisfies postulates (R*1-R*6). We will call such operators* revision operators with memory*.*

So, with Definition 3, one can start from any epistemic state (total pre-order over interpretations) and carry on iterated revisions. A particular epistemic state we can mention is the "empty" epistemic state, where the agent has no belief and no preferential information, that is, such that $\forall I, J \in \mathcal{W}\ I \simeq J$. We will denote by Ξ this epistemic state. So, the objective part of this epistemic state is $Bel(\Xi) = \top$. It can be considered as the epistemic state generalisation of $\top$ for the belief base framework, since they are both neutral elements for the corresponding operators: $\Xi \circ \varphi \equiv \varphi$ (as $\top \circ \varphi \equiv \varphi$ in the belief base framework). One can consider that all agents start with this epistemic state (we will consider this in the examples). Concerning iteration postulates:

Proposition 4 ([15]). *Revision operators with memory satisfy postulates (C1), (C3) and (C4).*

It can be also easily checked that (C2) is satisfied by a unique revision operator with memory, since it demands (in the presence of the other revision postulates), that the pre-order associated to a belief base by the faithful assignment on belief base used in Definition 3 is a two-level pre-order with the models of the belief base at the lowest level and the counter-models at the higher one. This operator will be presented in the next section.

So most of our revision operators with memory do not satisfy (C2). But we do not consider this as a drawback. We rather think that it is (C2) that is not fully satisfactory. In fact C2 demands that a piece of information that is accepted but later contradicted is completely discarded. One could argue for a more subtle behavior where only the contradicted part (so not all the formula) is discarded. See [15,17] for more explanations on this point. For instance suppose that you learn a big conjunction $a \wedge b \wedge \ldots \wedge z$ and that later you learn $\neg a$. Couldn't be natural to try to keep $b \wedge \ldots \wedge z$? Or should we discard it completely as required by (C2) ? According to us (C2) should not be regarded as a first class requirement (conversely to other postulates), but as an optional property that makes a distinction between two kinds of revision operators: the ones that consider that contradicting a piece of information amounts to discrediting its source, and then to discard it completely, and the ones that have a more subtle behavior and that only remove the contradicting parts of the pieces of information.

For a more complete logical characterization of this family of operators see [14].

3.1 Basic Memory Operator

Let us define the assignment that maps each belief base to a pre-order in the following way:

Definition 4. *Let φ be a belief base, the basic pre-order $\leq^b_\varphi$ associated to φ is defined as: $I \leq^b_\varphi J$ if and only if $I \models \varphi$ or ($I \not\models \varphi$ and $J \not\models \varphi$)*

So we have what we call a basic order, which is a two-level order (at most), with the models of φ at the lowest level and the other worlds at the highest level.

Definition 5. *The* basic memory operator *is the memory operator obtained from this assignment (i.e. the operator obtained by Definitions 4 and 3).*

It is worthy to note that if one uses this faithful assigment (Definition 4) to define a classical AGM operator (Proposition 2), one obtains the *full meet revision operator* which is not a good operator. But, even with this basic order on belief bases in the revision with memory framework, one can build very complex epistemic states. This is due to revision memory. The assignment of Definition 4 is a faithful assignment on belief bases; with Propositions 3 and 4, it is easy to show that:

Proposition 5 ([15]). *The only revision operator with memory that satisfies (R*1-R*6) and (C1–C4) is the basic memory revision operator.*

This operator has been already studied in the literature under different particular representations: in [19] with epistemic entrenchments, in [2,21] with polynomials and syntactic belief bases. Finally, we can note that Liberatore has shown [18] that several problems are computationally simpler for the basic memory operator than for the other iterated belief revision proposals (including Boutilier's natural revision [5], Lehmann's ranking revision [17] and Williams' transmutations [23]).

3.2 Dalal Memory Operator

We use in this section the Hamming distance d_H between interpretations[5]. The Dalal distance between an interpretation I and a belief base φ is defined as $d_D(I, \varphi) = \min_{J \models \varphi}(d_H(I, J))$.

Let's define the assignment that maps each belief base to a pre-order in the following way:

Definition 6. *Let φ be a belief base, the pre-order $\leq^d_\varphi$ associated to φ is defined as: $I \leq^d_\varphi J$ if and only if $d_D(I, \varphi) \leq d_D(J, \varphi)$*

So we have a pre-order with the models of φ at the lowest level and the other worlds in the higher levels, according to their Dalal distance.

[5] The Hamming distance between two interpretations is the number of propositional letters on which the two interpretations differ.

Definition 7. *The* Dalal memory operator *is the memory operator obtained from this assignment (i.e. the operator obtained by Definitions 6 and 3).*

We can show through a simple example that this operator differs from the classical Dalal revision operator [7,12]. Let a and b be two propositional letters and consider for example the sequence $\Psi = \Xi \circ a \circ b \circ \neg(a \wedge b)$. The classical Dalal operator gives $Bel(\Psi) = (a \wedge \neg b) \vee (\neg a \wedge b)$, whereas Dalal memory operator gives $Bel(\Psi) = (\neg a \wedge b)$. This behaviour seems more natural since at the next to last step we learned that b was true, and it is normal to keep some credit for this evidence in the following step. It is in this way, that our operators use revision with "memory".

4 Dynamical Revision Operators with Memory

For revision operators with memory, the revision policy is fixed once the operator is chosen. For example for the Dalal memory operator, the way to associate a pre-order to each new evidence is completely determined at the beginning of the process by the Dalal distance.

So, whereas the aim of revision operators with memory is to give a strong preference to the new evidence, one can object that the faithful assignment used to associate a pre-order to the new evidence does not change and so, that an old information is used in each revision step.

The solution to cope with this objection is to find a way to change the faithful assignment during the course of revisions. Such a solution will be given in this section. So, first, let's sum up the way revision operators with memory work:

- The definition of a particular operator lies in the chosen faithful assignment over belief bases. Let's call f such an assignment. So, for each formula φ, f associates a total pre-order $f(\varphi)$ (also noted $\leq_{f(\varphi)}$) satisfying the conditions of Definition 2.
- Each time a new evidence φ comes, the operator associates to it its corresponding pre-order $f(\varphi)$.
- The new epistemic state is the result of incorporating the pre-order in the old epistemic state, giving preference to the new evidence (*i.e.* to the pre-order) by using a lexicographical order: $\leq_{\Phi\circ\varphi} = \leq_{lex(f(\varphi),\Phi)}$[6].

So, what we want now is to be able to change f during the agent's life. That is, to dynamically change the revision policy of the agent, so that when a new evidence comes, it is not always associated to the same pre-order.

The idea is to start from an *a priori* faithful assignment over belief bases such as for revision by memory operators, but then to modify it at each revision step. To be able to do that, we have to use a more general definition of epistemic states. The (representation of) epistemic states we use for revision operators with memory are pre-orders on interpretations $\leq_\Phi$, from which we can extract the corresponding belief base $Bel(\Phi) = min(\mathcal{W}, \leq_\Phi)$.

[6] Where $I \leq_{lex(\leq_1,\leq_2)} J$ means $I <_1 J$ or ($I \simeq_1 J$ and $I \leq_2 J$).

For dynamical revision operators with memory, the representation of an epistemic state we use is a couple $\Phi = (\varphi, f)$, where φ is the current belief base and f is the current faithful assignment (So, with this representation, we can extract the pre-order corresponding to the belief base: $f(\varphi)$, and straightforwardly $Bel(\Phi) = \varphi$).

As for classical revision operators with memory, to define a particular dynamical revision with memory operator, one needs an initial, *a priori* faithful assignment over belief bases (*i.e.* a classical AGM revision operator), that will encode the initial revision policy of the agent.

So let's define dynamical revision operators with memory:

Definition 8. *Let $\Phi = (\varphi, f)$ be an epistemic state and let μ be a formula denoting a new evidence. We define the new epistemic state $\Phi \circ \mu$, resulting of the* dynamical revision with memory *of Φ by μ, as $\Phi \circ \mu = (\varphi', f')$, where φ' is a formula whose models are* $\min(Mod(\mu), f(\varphi))$*, and f' is a function (faithful assignment over belief bases) that is identical to f for each belief base ψ except when $\psi \leftrightarrow \varphi'$. In this case $f'(\psi)$ is defined as:*

$$I \leq_{f'(\psi)} J \textit{ iff } I <_{f(\mu)} J \textit{ or } (I \simeq_{f(\mu)} J \textit{ and } I \leq_{f(\varphi)} J)$$

So, pointwise, the dynamical operators work exactly the same way as memory operators. The difference is that they also change the given faithful assignment over belief bases at each step. One could believe that the difference between the two families of operators is not huge, since the corresponding pre-orders (faithful assignment) used change only for one value at each step. But as we will see next the dynamical revision operators with memory satisfy the following postulate that the revision operators with memory do not (always) satisfy (*cf* Example 1):

(C5) If $Bel(\Phi) \leftrightarrow \mu$ then $\Phi \circ \mu = \Phi$.

This axiom says that the current epistemic state does not change in all cases where the new piece of information coincides with the observable part of this epistemic state. Note that this axiom is almost trivial in the classical AGM framework[7]. But in the framework of complex epistemic states it is not the case. In fact, as we already mentioned, the revision operators with memory do not (always) satisfy (C5) as can be seen in the following example.

Example 1. *We are reasoning about an electronic circuit with two components, the left one and the right one. The propositional variable l means that the left component is working, and r encodes the fact the right component is working. Suppose we start from the Dalal classical AGM revision operator. Let Φ be the epistemic state with observable part being the following formula:* "only one of the two components is working" *($Bel(\Phi) = (l \wedge \neg r) \vee (\neg l \wedge r)$). Let μ be the formula expressing that* "the component on the left is not working" *($\mu = \neg l$). The beliefs of the epistemic state after the revision $\Phi \circ \mu$ using Dalal memory operator is* "only the component on the right is working"*. The other (conditional)*

[7] In that framework, it is a consequence of the other axioms.

information of this epistemic state can be described by the conditionals "if the component on the right is not working then the two components are bad" *and* "if the components on the left is working then only the component on the left is working". *Now if we revise this current epistemic state by the fact that* "only the component on the right is working" *($\varphi = \neg l \wedge r$), which is indeed the beliefs of the current epistemic state, we obtain a different epistemic state in which for instance we have the conditional* "if the component on the left is working then the two components are working". *On the contrary, the current epistemic state does not change after revision by this new information when the operator is the dynamical Dalal revision with memory operator.*

We illustrate this example below. In order to do that consider a language $\mathcal{L}$ with only two propositional letters l and r. We will denote interpretations simply by the truth assignment, i.e. 10 *denotes the interpretation mapping l to true and r to false. Two interpretations are equivalent, with respect to the pre-order, if they appear at the same level. An interpretation I is better than another interpretation J ($I < J$) if it appears at a lower level. $\circ_{MD}$ denotes the Dalal revision with memory operator and $\circ_{DMD}$ the dynamical Dalal with memory operator.*

Let's see the pre-order associated to some belief bases by the faithful assignment over belief bases given by the Dalal distance:

$$\leq^D_\Phi = \begin{array}{c} 11\ 00 \\ 01\ 10 \end{array} \qquad \leq^D_\mu = \begin{array}{c} 10\ 11 \\ 00\ 01 \end{array} \qquad \leq^D_\varphi = \begin{array}{c} 10 \\ 11\ 00 \\ 01 \end{array}$$

And the epistemic states reached by the operators are:

$$\leq_\Phi = \begin{array}{c} 11\ 00 \\ 01\ 10 \end{array} \qquad \leq_{\Phi \circ_{MD} \mu} = \begin{array}{c} 11 \\ 10 \\ 00 \\ 01 \end{array} \qquad \leq_{\Phi \circ_{MD} \mu \circ_{MD} \varphi} = \begin{array}{c} 10 \\ 11 \\ 00 \\ 01 \end{array}$$

$$\leq_\Phi = \begin{array}{c} 11\ 00 \\ 01\ 10 \end{array} \qquad \leq_{\Phi \circ_{DMD} \mu} = \begin{array}{c} 11 \\ 10 \\ 00 \\ 01 \end{array} \qquad \leq_{\Phi \circ_{DMD} \mu \circ_{DMD} \varphi} = \begin{array}{c} 11 \\ 10 \\ 00 \\ 01 \end{array}$$

Note that the idea here is that, when the agent receives a new evidence that she has met before, the repetition of this evidence suggests that the old beliefs of the agent were correct, and so she holds on to the last pre-order that corresponds to this evidence.

In fact, if one considers the definition of iterated revision operators according to Darwiche and Pearl (cf Sect. 2.2), it amounts to say that we change the revision operators at each step, since the corresponding faithful assignment over epistemic states changes at each step. So, in a sense, dynamical revision operators with memory are definable by a family of revision operators with memory (that corresponds to the set of all faithful assignments over belief bases reached by the course of revisions).

Concerning the logical properties of this family of operators, it is easy to check the following:

Theorem 6. *A dynamical revision operator with memory satisfies (R*1)–(R*6). It satisfies (C1), (C3), (C4) and (C5) but it never satisfies (C2).*

Finally, as another example, let's see the behaviour of the full meet revision operator $\circ_B$, the basic memory operator $\circ_{MB}$ and the dynamical basic memory operator $\circ_{DMB}$ (they are all built from the same faithful assignment over belief bases) on the same situations.

Example 2. *Consider a language $\mathcal{L}$ with only two propositional letters a and b (considered in that order for the valuations). Let's see the pre-order associated to some belief bases by the faithful assignment over belief bases given by the Basic distance:*

$$\leq^B_a = \begin{matrix} 00\ 01 \\ 10\ 11 \end{matrix} \qquad \leq^B_b = \begin{matrix} 00\ 10 \\ 01\ 11 \end{matrix}$$

$$\leq^B_{a \wedge b} = \begin{matrix} 00\ 01\ 10 \\ 11 \end{matrix} \qquad \leq^B_{\neg a} = \begin{matrix} 10\ 11 \\ 00\ 01 \end{matrix}$$

And the epistemic states reached by the operators are:

$$\leq_{a \circ_{MB} b} = \leq_{a \circ_{DMB} b} = \begin{matrix} 00 \\ 10 \\ 01 \\ 11 \end{matrix} \qquad \leq_{a \circ_{MB} b \circ_{MB} \neg a} = \leq_{a \circ_{DMB} b \circ_{DMB} \neg a} = \begin{matrix} 10 \\ 11 \\ 00 \\ 01 \end{matrix}$$

$$\leq_{a \circ_{MB} b \circ_{MB} \neg a \circ_{MB} a \wedge b} = \begin{matrix} 10 \\ 00 \\ 01 \\ 11 \end{matrix} \qquad \leq_{a \circ_{DMB} b \circ_{DMB} \neg a \circ_{DMB} a \wedge b} = \begin{matrix} 00 \\ 10 \\ 01 \\ 11 \end{matrix}$$

So we have that:

$$a \circ_B b \circ_B \neg a \circ_B a \wedge b \circ_B \neg b \equiv \neg b$$
$$a \circ_{MB} b \circ_{MB} \neg a \circ_{MB} a \wedge b \circ_{MB} \neg b \equiv \neg a \wedge \neg b$$
$$a \circ_{DMB} b \circ_{DMB} \neg a \circ_{DMB} a \wedge b \circ_{DMB} \neg b \equiv a \wedge \neg b$$

As noted previously, the full meet revision operator $\circ_{\mathrm{B}}$ does not have a very good behaviour: each time the new evidence contradicts the current beliefs, the new beliefs are only the logical consequences of the new evidence. So, it absolutely does not consider the previous revisions. With the revision operator with memory $\circ_{\mathrm{MB}}$, the agent is able to build complex epistemic states (pre-orders), that lead to a satisfactory behaviour for iterated revision. With this operator, the two evidences $\neg a$ and $\neg b$ recently learned lead to this belief base. With the dynamical revision operator with memory $\circ_{\mathrm{DMB}}$, the evidence learned at the next to last step ($a \wedge b$) recalls the agent the last time she had this belief (after $a \circ_{\mathrm{DMB}} b$), and this modifies her epistemic state.

5 Conclusion

It is worthy to note that the two families of operators defined, revision with memory and dynamical revision with memory, are revision operators in the sense of Darwiche and Pearl, that is, they map an epistemic state and a formula (new evidence) to an epistemic state. We have shown that one can use any standard AGM revision operator and turns it to a DP iterated revision operator using revision with memory and dynamical revision with memory (C2 is not satisfied, but this postulate is criticizable).

Note that [3] considers revision of epistemic states by epistemic states. In this work, even if at the end of the process we work with two pre-orders, the second one is obtained from the input, that is a single formula (as usual in AGM/DP framework), by a pre-processing step.

It is interesting also to note that our definition of epistemic states for dynamical revision with memory is more complicated than usual DP ones: so this work illustrates that one can encode sublter behaviours with more complicated epistemic states. Studying this kind of generalized epistemic states and its application seems to be an interesting research issue.

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