Proceedings of the Sixteenth International Conference on
Principles of Knowledge Representation and Reasoning (KR 2018)

# On Belief Promotion

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### Abstract

We introduce a new class of belief change operators, named *promotion operators*. The aim of these operators is to enhance the acceptation of a formula representing a new piece of information. We give postulates for these operators and provide a representation theorem in terms of minimal change. We also show that this class of operators is a very general one, since it captures as particular cases belief revision, commutative revision, and (essentially) belief contraction.

## Introduction

Belief change theory aims at studying and characterizing operators that allow to modify the beliefs of an agent in light of new pieces of evidence. Such operators are mandatory for designing autonomous agents being able to face unexpected situations.

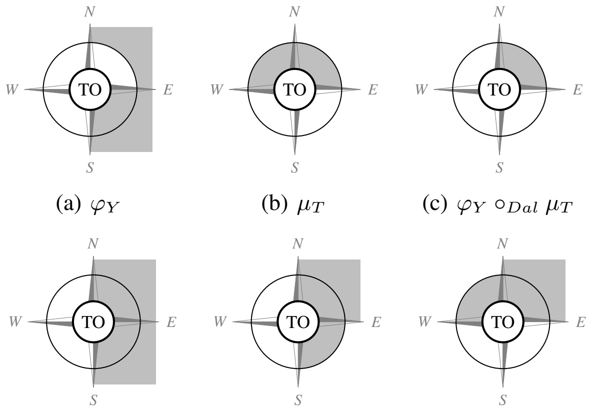
The most central belief change operators are revision operators and contraction operators. On the one hand, revision operators aim at adding a formula (representing new pieces of evidence) into the beliefs of an agent, while getting rid of beliefs that contradict the added formula. Thus a strong priority is given to the incoming pieces of evidence over the current beliefs. On the other hand, contraction operators aim at removing some pieces of belief (a formula) from the beliefs of an agent. Basically, one does not want any longer the formula to be entailed by the beliefs of the agent. As such, contraction can be interpreted as being a change operation that gives less priority to the change formula.

A number of additional change operators are related to some extent to these two main families of belief change operators. Those additional operators are typically obtained by considering other settings, based on distinct or supplementary assumptions. Among them are update operators (Katsuno and Mendelzon 1992), merging operators (Konieczny and Pino Pérez 2002) and iterated revision operators (Darwiche and Pearl 1997; Booth and Meyer 2006; Jin and Thielscher 2007). Contrastingly, if one sticks to the original belief change setting where the available information consists of the current beliefs of an agent and of a unique change formula, only few alternative views to revision and contraction have been pointed out so far. The main exception concerns some works (merely achieved in the nineties) around non-prioritized revision (Hansson 1997c). The pursued objective was to determine which (or which part of the) information given by the change formula should be accepted, but no new change operation was actually pointed out.

In this paper, a new class of change operators, called *promotion* operators, is defined. Promotion operators aim at "enhancing" the acceptation of the new piece of beliefs in the current beliefs of the agent, but do not require to accept it entirely in every case. While revision considers the new piece of beliefs as fully reliable, and contraction does not allow for acquiring new beliefs, promotion is a more general change operation that allows for a more moderate incorporation of a new piece of evidence into existing beliefs.

**Example 1.** Consider the following scenario. Yuko is currently at the town office (TO) and would like to reach the post office (PO). She does not precisely know where PO is, but she currently believes that (i) it is located east to TO, and (ii) there is a park located directly north-east to TO so that if PO happens to also be north-east to TO, then she will need time to reach her destination. On her way, she asks an inhabitant (named Takeshi) for more information, who tells her that PO is not far, north to TO. This contradicts Yuko's initial beliefs, and she seeks to promote the acceptation of Takeshi's claim into her initial beliefs. To represent Yuko's beliefs and Takeshi's claim, one can consider three propositional variables $n, w, f$, which respectively stand for "PO is located north to TO", "PO is located west to TO", and "PO is far from TO". Then Yuko's initial beliefs can be encoded by the propositional formula $\varphi_Y = \neg w \wedge (n \Rightarrow f)$ (Fig. 1(a)), and Takeshi's claim by $\mu_T = n \wedge \neg f$ (Fig. 1(b)).

Yuko has several options. First, she may choose to put trust into Takeshi's claim, and *revise* her initial beliefs $\varphi_Y$ by $\mu_T$. If she uses the Dalal revision operator $\circ_{Dal}$ (Dalal 1988), then her revised beliefs is a formula whose (unique) model is the closest one to $\varphi$ w.r.t. the Hamming distance, i.e., $\varphi_Y \circ_{Dal} \mu_T \equiv n \wedge \neg w \wedge \neg f$ (Fig. 1(c)). In this case, due to Yuko's trust into Takeshi's claim, her revised beliefs are inconsistent with her previous beliefs. Second, she may choose to weaken her beliefs so that to be consistent with Takeshi's claim by using a *contraction* operator, i.e., by contracting Yuko's beliefs by the negation of Takeshi's claim. Using the Dalal contraction operator $-_{Dal}$, one adds to the models of $\varphi_Y$ the model $n \wedge \neg w \wedge \neg f$, i.e., the closest one to $\varphi_Y$ w.r.t. the Hamming distance, and her beliefs become




(a) $\varphi_Y$ (b) $\mu_T$ (c) $\varphi_Y \circ_{Dal} \mu_T$

(d) $\varphi_Y -_{Dal} \neg\mu_T$ (e) $\varphi_Y \diamond_{Dal} \mu_T$ (f) $\varphi_Y \oplus \mu_T$

Figure 1: The graphical representation of the different pieces of beliefs from Example 1. In each case, the gray area represents the believed location of the post office (PO) w.r.t. the town office (TO).

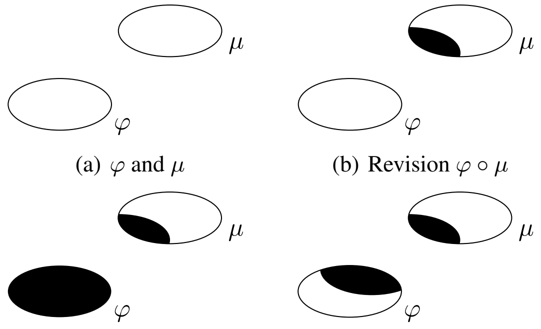
$\varphi_Y -_{Dal} \neg\mu_T \equiv \neg w$ (Fig. 1(d)). In this case, Yuko does not acquire new information since her contracted beliefs are entailed by her previous beliefs. Alternatively, she may want to promote Takeshi's claim into her initial beliefs, in such a way as to acquire new beliefs without putting an entire trust into Takeshi's claim. This process can be viewed as a midway between revision and contraction: one selects the closest models of $\mu_T$ to $\varphi_Y$, but allows also to select the closest models of $\varphi_Y$ to $\mu_T$. So for instance, using the Dalal commutative revision operator $\diamond_{Dal}$, her new beliefs would become $\varphi_Y \diamond_{Dal} \mu_T \equiv (\varphi_Y \circ_{Dal} \mu_T) \vee (\mu_T \circ_{Dal} \varphi_Y) \equiv \neg w \wedge (f \Rightarrow n)$ (Fig. 1(e)). This is the kind of result that is reasonable to expect when the relative certainty of $\mu_T$ and $\varphi_Y$ is not definite.

However, using a commutative revision operator may not be the only reasonable way to promote $\mu_T$ into $\varphi_Y$. Fig. 1(f) depicts an example of another reasonable choice for promoting $\mu_T$ into $\varphi_Y$ (denoted $\varphi_Y \oplus \mu_T$ in the figure), which is equivalent to the formula $n \wedge (w \Rightarrow \neg f)$. [1] Furthermore, there is no reason here to require promoting $\mu_T$ into $\varphi_Y$ to be equivalent to promoting $\varphi_Y$ into $\mu_T$. For instance, one may select some models of $\mu_T$ that are the closest ones to $\varphi_Y$ according to some revision operator, and select some models of $\varphi_Y$ that are the closest ones to $\mu_T$ according to a *different* revision operator. This is the kind of general change process that we intend to characterize in this paper.

The key ideas underlying (respectively) the revision, contraction and promotion operators are illustrated in Fig. 2. Since all those operators can be characterized semantically in terms of model selection, one can used Venn diagrams for representing sets of models.

Consider two (jointly inconsistent) formulas $\varphi$ and $\mu$ represented by their (disjoint) sets of models on Fig. 2(a).

[1]This example of promotion is formalized later in the paper.




(a) $\varphi$ and $\mu$ (b) Revision $\varphi \circ \mu$

(c) Contraction $\varphi - \neg\mu$ (d) Promotion of $\mu$ into $\varphi$

Figure 2: Revision, Contraction, Promotion.

Fig. 2(b) illustrates the revision of $\varphi$ by $\mu$, where one selects some models of $\mu$ as a result. Fig. 2(c) illustrates the contraction of $\varphi$ by $\neg\mu$, so that the result is the whole set of models of $\varphi$ union the selection of some models of $\mu$. Promotion operators (illustrated on Fig. 2(d)) can be seen as a midway between these two extreme scenarios: one does neither throw away all models of $\varphi$ (as in the revision case), nor retain all those models (as in the contraction case). Instead, one selects some models of $\varphi$ and some models of $\mu$.

Beyond the definition of the class of promotion operators and some interesting subclasses of it, the main contributions of the paper are as follows. We provide representation theorems in terms of minimal change for promotion operators. Then, we show that the class of promotion operators is very general, since it includes as particular cases many belief change operators. In particular revision operators (Alchourrón, Gärdenfors, and Makinson 1985; Gärdenfors 1988; Katsuno and Mendelzon 1991) are promotion operators and contraction operators are also essentially captured. We also discuss the close relationship with commutative revision operators (Liberatore and Schaerf 1998) (also called arbitration operators) and merging operators (Konieczny and Pino Pérez 2002).

The proofs of propositions are given in an appendix.

## Preliminaries

Our formal setting is finite propositional logic. Thus, we consider a language $\mathcal{L}$ defined from a finite set $\mathcal{V}$ of propositional variables and the usual connectives. The elements of $\mathcal{L}$ are called formulas. An interpretation $I$ is a mapping that assigns a truth value to every variable from $\mathcal{V}$. For any $\mu \in \mathcal{L}$, $I$ is a model of $\mu$, noted $I \models \mu$ iff it makes it true in the usual truth functional way; and $[\mu]$ denotes the set of models of $\mu$.

We now recall the standard postulates for belief revision, belief contraction, and LS commutative revision, starting with the KM postulates for revision operators (Katsuno and Mendelzon 1991):

**Definition 1** (KM revision operator)**.** *A* KM revision operator $\circ$ *is a mapping associating every pair of formulas* $(\varphi, \mu)$ *with a formula* $\varphi \circ \mu$ *such that for all formulas* $\mu, \varphi, \mu', \varphi'$, *the following conditions are satisfied:*

**(R1)** $\varphi \circ \mu \models \mu$;
**(R2)** *If* $\varphi \wedge \mu \not\models \bot$, *then* $\varphi \circ \mu \equiv \varphi \wedge \mu$;
**(R3)** *If* $\mu \not\models \bot$, *then* $\varphi \circ \mu \not\models \bot$;
**(R4)** *If* $\varphi \equiv \varphi'$ *and* $\mu \equiv \mu'$, *then* $\varphi \circ \mu \equiv \varphi' \circ \mu'$;
**(R5)** $(\varphi \circ \mu) \wedge \mu' \models \varphi \circ (\mu \wedge \mu')$;
**(R6)** *If* $(\varphi \circ \mu) \wedge \mu' \not\models \bot$, *then* $\varphi \circ (\mu \wedge \mu') \models (\varphi \circ \mu) \wedge \mu'$.

The postulates for contraction operators (Alchourrón, Gärdenfors, and Makinson 1985; Gärdenfors 1988; Caridroit, Konieczny, and Marquis 2015) are:

**Definition 2** (CKM contraction operator)**.** *A* contraction operator $-$ *is a mapping associating every pair of formulas* $(\varphi, \mu)$ *with a formula* $\varphi - \mu$ *such that for all formulas* $\mu, \varphi, \mu', \varphi'$, *the following conditions are satisfied:*

**(C1)** $\varphi \models \varphi - \mu$;
**(C2)** *If* $\varphi \not\models \mu$, *then* $\varphi - \mu \models \varphi$;
**(C3)** *If* $\varphi - \mu \models \mu$, *then* $\models \mu$;
**(C4)** *If* $\varphi \models \mu$, *then* $(\varphi - \mu) \wedge \mu \models \varphi$;
**(C5)** *If* $\varphi \equiv \varphi'$ *and* $\mu \equiv \mu'$, *then* $\varphi - \mu \equiv \varphi' - \mu'$;
**(C6)** $\varphi - (\mu \wedge \mu') \models (\varphi - \mu) \vee (\varphi - \mu')$;
**(C7)** *If* $\varphi - (\mu \wedge \mu') \not\models \mu$, *then* $\varphi - \mu \models \varphi - (\mu \wedge \mu')$.

And the postulates for LS commutative revision (Liberatore and Schaerf 1998) are:

**Definition 3** (LS commutative revision operator)**.** *An* LS commutative revision operator $\diamond$ *is a mapping associating every pair of formulas* $(\varphi, \mu)$ *with a formula* $\varphi \diamond \mu$ *such that for all formulas* $\mu, \varphi, \mu', \varphi'$, *the following conditions are satisfied:*

**(A1)** $\varphi \diamond \mu \equiv \mu \diamond \varphi$;
**(A2)** $\varphi \wedge \mu \models \varphi \diamond \mu$;
**(A3)** *If* $\varphi \wedge \mu \not\models \bot$, *then* $\varphi \diamond \mu \models \varphi \wedge \mu$;
**(A4)** *If* $\varphi \vee \mu \not\models \bot$, *then* $\varphi \diamond \mu \not\models \bot$;[2]
**(A5)** *If* $\varphi \equiv \varphi'$ *and* $\mu \equiv \mu'$, *then* $\varphi \diamond \mu \equiv \varphi' \diamond \mu'$;
**(A6)** $\varphi \diamond (\mu \vee \mu') \equiv \begin{cases} \varphi \diamond \mu & \textit{or} \\ \varphi \diamond \mu' & \textit{or} \\ (\varphi \diamond \mu) \vee (\varphi \diamond \mu'); & \end{cases}$
**(A7)** $\varphi \diamond \mu \models \varphi \vee \mu$;
**(A8)** *If* $\varphi \not\models \bot$, *then* $(\varphi \diamond \mu) \wedge \varphi \not\models \bot$.

Let us finally recall the representation theorem for belief revision operators in terms of faithful assignments (Katsuno and Mendelzon 1991):

**Definition 4** (Faithful assignment)**.** *A* faithful assignment *is a mapping that associates with any formula* $\varphi$ *a pre-order* $\leq_\varphi$ *such that:*

1. *If* $I \models \varphi$ *and* $J \models \varphi$, *then* $I \simeq_\varphi J$
2. *If* $I \models \varphi$ *and* $J \not\models \varphi$, *then* $I <_\varphi J$
3. *If* $\varphi \equiv \varphi'$, *then* $\leq_\varphi = \leq_{\varphi'}$

*We say that a faithful assignment is* total *if it associates with any formula a total pre-order.*

[2] The original formulation of (A4) in (Liberatore and Schaerf 1998) also requires the reciprocal statement, but we omitted it here since it is implied by (A7).

**Proposition 1. (Katsuno and Mendelzon 1991)** *A revision operator* $\circ$ *satisfies postulates (R1)-(R6) if and only if there exists a total faithful assignment* $\varphi \mapsto \leq_\varphi$ *such that* $[\varphi \circ \mu] = \min([\mu], \leq_\varphi)$.

This theorem shows that belief revision operators select as result of the revision process the models of the change formula $\mu$ that are the most plausible ones w.r.t. $\varphi$ (i.e., the minimal ones w.r.t. $\leq_\varphi$).

## Promotion Operators

Our aim is to define promotion operators, i.e., operators that allow to "enhance" a formula $\mu$ in the current belief ($\varphi$) of an agent. In particular, the promoted beliefs of the agent (i.e., once $\mu$ has been taken into account) are expected to be consistent with $\mu$. Formally, a promotion operator is defined as follows:

**Definition 5** (Promotion operators)**.** *A* promotion operator $\ominus$ *is a mapping associating a pair of formulas* $(\varphi, \mu)$ *with a formula* $\varphi \ominus \mu$ *such that the following conditions are satisfied:*

**(P1)** $\varphi \ominus \mu \models \varphi \vee \mu$;
**(P2)** *If* $\varphi \wedge \mu \not\models \bot$, *then* $\varphi \ominus \mu \equiv \varphi \wedge \mu$;
**(P3)** *If* $\mu \not\models \bot$, *then* $(\varphi \ominus \mu) \wedge \mu \not\models \bot$;
**(P4)** *If* $\varphi \equiv \varphi'$ *and* $\mu \equiv \mu'$, *then* $\varphi \ominus \mu \equiv \varphi' \ominus \mu'$;
**(P5)** *If* $(\varphi \ominus \mu) \wedge \mu \wedge \gamma \not\models \bot$, *then* $[\varphi \ominus (\mu \wedge \gamma)] \wedge \mu \equiv (\varphi \ominus \mu) \wedge \mu \wedge \gamma$;
**(P6)** *If* $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \not\models \bot$, *then* $[(\varphi \wedge \gamma) \ominus \mu] \wedge \varphi \equiv (\varphi \ominus \mu) \wedge \varphi \wedge \gamma$;
**(P7)** *If* $[(\varphi \wedge \gamma) \ominus \mu] \wedge \varphi \not\models \bot$, *then* $(\varphi \ominus \mu) \wedge \varphi \not\models \bot$.

*A* consensual promotion operator $\oplus$ *is a promotion operator satisfying the following additional condition:*

**(P8)** *If* $\varphi \not\models \bot$, *then* $(\varphi \oplus \mu) \wedge \varphi \not\models \bot$.

Let us explain the meaning of these postulates. (P1) imposes that the promotion $\varphi \ominus \mu$ of $\mu$ in $\varphi$ implies the disjunction of the two formulas $\varphi$ and $\mu$. This can be seen as a minimal change requirement: no models outside those of the two formulas involved in the change operation can be considered in the result of the promotion process. (P2) states that if the conjunction of the two formulas $\varphi$ and $\mu$ is consistent, then the result of the promotion must be equivalent to this conjunction. This postulate can also be seen as a minimal change requirement: if $\varphi$ and $\mu$ are not conflicting, then one can keep them together. (P3) is a (weak) success postulate: one wants the result of the promotion process to be consistent with the new piece of information $\mu$. (P4) is a traditional irrelevance of syntax requirement: the result of the promotion process is the same if $\varphi$ or $\mu$ is replaced by a logically equivalent formula. (P5) is right conjunction. It describes the extent to which a promotion by $\mu$ can be related to a promotion by $\mu \wedge \gamma$. If $\mu \wedge \gamma$ is consistent with the result of the promotion by $\mu$, then the result of the promotion by $\mu \wedge \gamma$ of the same formula $\varphi$ will lead to an equivalent result (when the focus is laid on the models of $\mu \wedge \gamma$). This postulate is the counterpart of the (R5-R6) postulates for KM revision operators. (P6) is left conjunction. This is the dual of (P6) for $\varphi$: it describes the extent to which a promotion of $\varphi$ can be related to a promotion of $\varphi \wedge \gamma$. (P6) indicates that if $\varphi \wedge \gamma$ is consistent with the result of the promotion of $\varphi$, then the result of the promotion of $\varphi \wedge \gamma$ by the same formula $\mu$ will lead to an equivalent result (when the focus is laid on the models of $\varphi \wedge \gamma$). (P7) states that if the promotion of a formula is consistent with a logically weaker formula, then the promotion of this weaker formula by the same formula $\mu$ must also be consistent with this weaker formula.

These postulates define the class of promotion operators. If one adds (P8) to this set, then one obtains the subclass of *consensual promotion operators*, where less priority is given to $\mu$. Note that the promotion postulate (P3) is the only asymmetric postulate. Thus, when one adds its dual (P8) to the list of postulates, the two formulas $\varphi$ and $\mu$ get exactly the same status (however, this does not imply that the operator is commutative, i.e., satisfies (A1)).

It is straightforward to show that:

**Proposition 2.** *If a promotion operator satisfies (P8), then it satisfies (P7).*

More properties can be established when focusing on commutative operators:

**Proposition 3.** *If a promotion operator satisfies (A1), then:*

- *it satisfies (P3) if and only if it satisfies (P8);*
- *it satisfies (P6) if and only if it satisfies (P5).*

One can also easily verify that the postulates characterizing promotion operators are not self-conflicting, i.e., the class of promotion operators is not empty. Indeed, a simple promotion operator can be defined as follows:

**Definition 6** (Drastic promotion operator)**.** *The* drastic promotion operator*, denoted $\ominus_D$, is defined as:*

$$\varphi \ominus_D \mu = \begin{cases} \varphi \wedge \mu & \textit{if consistent,} \\ \varphi \vee \mu & \textit{otherwise.} \end{cases}$$

**Proposition 4.** *The drastic promotion operator $\ominus_D$ is a consensual promotion operator in the sense of Definition 5, i.e., it satisfies postulates (P1-P8).*

This operator is clearly the weakest promotion operator, in the sense that for every $\varphi$ and $\mu$ any promotion operator $\ominus$ satisfies $\varphi \ominus \mu \models \varphi \ominus_D \mu$. In the following, we show how more interesting promotion operators can be designed.

## Representation Theorems

We first provide a representation theorem for promotion operators. This theorem shows that a promotion operator can be defined from two classical KM revision operators and a trigger function. It also provides a characterization of the promotion operators in terms of minimal change based on total faithful assignments.

Let us first introduce the notion of trigger function:

**Definition 7** (Trigger function)**.** *A* trigger function *is a mapping $\sigma$ associating any formula $\mu$ with a formula $\sigma(\mu)$ such that for all formulas $\mu$, $\mu'$:*

- $\mu \models \sigma(\mu)$,
- *if $\mu \equiv \mu'$ then $\sigma(\mu) \equiv \sigma(\mu')$.*

Applying the trigger function to the change formula $\mu$ generates a dilation of $\mu$ (i.e., a logically weaker formula) (Bloch and Lang 2000). The trigger function is used to decide whether or not $\varphi \ominus \mu$ is consistent with the old beliefs of the agent ($\varphi$).

**Proposition 5** (Representation theorem for $\ominus$)**.** *$\ominus$ is a promotion operator (i.e., it satisfies (P1-P7)) if and only if there exist two KM revision operators $\circ_1$, $\circ_2$, and a trigger function $\sigma$, such that for all formulas $\varphi$, $\mu$,*

$$\varphi \ominus \mu = (\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu))).$$

*$\ominus$ is said to be induced by $\circ_1$, $\circ_2$, and $\sigma$.*

Thanks to the representation theorem for KM revision operators (cf. Proposition 1), the previous result can be rephrased in terms of total faithful assignments:

**Proposition 6.** *$\ominus$ is a promotion operator (i.e., it satisfies (P1-P7)) if and only if there exist a trigger function $\sigma$, and two total faithful assignments $\varphi \mapsto \leq^1_\varphi$ and $\varphi \mapsto \leq^2_\varphi$, such that for all formulas $\varphi$, $\mu$,*

$$[\varphi \ominus \mu] = \begin{cases} \min([\mu], \leq^1_\varphi) \cup \min([\varphi], \leq^2_\mu) \\ \qquad \text{if } \varphi \wedge \sigma(\mu) \not\models \bot, \\ \min([\mu], \leq^1_\varphi) \quad \text{otherwise.} \end{cases}$$

Thus promoting $\mu$ into $\varphi$ consists in (i) including some worlds of $\mu$; and (ii) including some worlds of $\varphi$ provided that $\varphi$ is "close enough" to $\mu$ (where the admissible neighborhood of $\mu$ is made precise by $\sigma$). The representation theorem also shows that promoting $\mu$ in $\varphi$ using $\ominus$ carries out a $\mu$-promotion, in the sense of (Schwind et al. 2016), given that we always have $\varphi \wedge \mu \models \varphi \ominus \mu \models \varphi \vee \mu$.

Let us now give a representation theorem for consensual promotion operators. This theorem is similar to the previous one except that specifying a trigger function is not needed.

**Proposition 7** (Representation theorem for $\oplus$)**.** *$\oplus$ is a consensual promotion operator (i.e., it satisfies (P1-P8)) if and only if there exist two KM revision operators $\circ_1$, $\circ_2$, such that for all formulas $\varphi$, $\mu$, $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$.*

Rephrasing this result directly in terms of total faithful assignments we get:

**Proposition 8.** *$\oplus$ is a consensual promotion operator (i.e., it satisfies (P1-P8)) if and only if there exist two total faithful assignments $\varphi \mapsto \leq^1_\varphi$ and $\varphi \mapsto \leq^2_\varphi$, such that for all $\varphi$, $\mu$,*

$$[\varphi \oplus \mu] = \min([\mu], \leq^1_\varphi) \; \cup \; \min([\varphi], \leq^2_\mu).$$

Clearly enough, a consensual promotion operator $\oplus$ is a promotion operator for which the associated trigger function $\sigma$ satisfies $\sigma(\mu) \equiv \top$ for every formula $\mu$.

Given Proposition 7, it is natural to focus on the specific case when the two KM revision operators used to define $\oplus$ coincide:

**Definition 8** (Commutative promotion operator)**.** *A* commutative promotion operator *is a consensual promotion operator $\oplus$ induced by a single KM revision operator $\circ$ (i.e., $\circ_1 = \circ_2 = \circ$).*

A representation theorem for commutative promotion operators can be easily obtained by adding (A1) to (P1-P8).

Let us go back to our introductive example:

**Example 1 (continued).** We can easily see that the operator $\diamond_{Dal}$ introduced in Example 1 is a commutative promotion operator, since it is defined for every $\varphi, \mu$ as $\varphi \diamond_{Dal} \mu \equiv (\varphi \circ_{Dal} \mu) \vee (\mu \circ_{Dal} \varphi)$. Yet according to Proposition 7, any consensual promotion operator can be characterized by two KM revision operators $\circ_1, \circ_2$, so that one can easily define a consensual promotion operator $\oplus$ that is not commutative. For instance, let:

- $\circ_D$ be the drastic revision defined for every $\varphi, \mu$ as $\varphi \circ_D \mu = \varphi \wedge \mu$ if $\varphi \wedge \mu$ is consistent, $\varphi \circ_D \mu = \mu$ otherwise;
- $\circ_*$ be any revision operator associated with a total faithful assignment $\varphi \mapsto \leq_\varphi$ (cf. Definition 4) defined such that given the interpretation $\omega = n \wedge \neg w \wedge f$, we have $\omega <_{\mu_T} \omega'$ for any $\omega' \models \neg\mu_T, \omega' \neq \omega$.[3]

Then by defining the consensual promotion operator $\oplus$ as $\varphi \oplus \mu \equiv (\varphi \circ_D \mu) \vee (\mu \circ_* \varphi)$ for any $\varphi$, $\mu$, we get that:

$$\begin{aligned} \varphi_Y \oplus \mu_T &\equiv (\varphi_Y \circ_D \mu_T) \vee (\mu_T \circ_* \varphi_Y) \\ &\equiv ((\neg w \wedge (n \Rightarrow f)) \circ_D (n \wedge \neg f)) \\ &\quad \vee ((n \wedge \neg f) \circ_* (\neg w \wedge (n \Rightarrow f))) \\ &\equiv (n \wedge \neg f) \vee (n \wedge \neg w \wedge f) \\ &\equiv n \wedge (w \Rightarrow \neg f). \end{aligned}$$

In particular, this corresponds to the promotion of $\mu$ into $\varphi$ depicted in Fig. 1(f).

## Links with other Change Operators

In this section we show that belief revision is a special case of promotion, and we establish a correspondence between revision and a subclass of promotion operators. We also show that contraction operators merely correspond to special cases of promotion operators. Finally, we discuss some links with LS commutative revision operators and merging operators.

Let us start with belief revision:

**Proposition 9.** *Every KM revision operator is a promotion operator.*

Switching from revision operators to promotion operators leads to enlarge the family of belief change operators since promotion operators do not satisfy (R1) in the general case (for instance, the drastic promotion operator introduced in Definition 6 does not satisfy (R1)). Interestingly, KM revision operators are exactly the promotion operators satisfying (R1) :

**Proposition 10.** *Let $\ominus$ be a promotion operator. If $\ominus$ satisfies (R1), then $\ominus$ is a KM revision operator (it satisfies also (R2)-(R6)).*

The case of contraction of $\varphi$ by the negation of $\mu$ is a bit more complicated because of the trivial case when $\neg\mu$ is not entailed by $\varphi$, i.e., when $\varphi \wedge \mu$ is consistent. Indeed, if $\varphi \wedge \mu \not\models \bot$ then (C2) implies that $\varphi - \neg\mu \equiv \varphi$, but promotion operators demands (because of (P2)) that in this case the result has to be $\varphi \wedge \mu$. So stricto sensu, contraction operators are not promotion operators. However, this trivial case is not the significant one for "true" contraction, that makes sense merely when $\varphi \wedge \mu$ is not consistent. In such a non-trivial case, contraction operators behave as specific promotion operators. To make it more formal, one needs the notion of *conjunctive contraction operator* induced by a contraction operator:

**Definition 9** (Conjunctive contraction operator)**.** *Given a contraction operator $-$, the* conjunctive contraction operator $\div$ *induced by $-$ is defined as follows:*

$$\varphi \div \mu = \begin{cases} \varphi \wedge \mu & \text{if } \varphi \wedge \mu \not\models \bot, \\ \varphi - \neg\mu & \text{otherwise.} \end{cases}$$

The change achieved by applying $\div$ on $\varphi$ when $\mu$ is the change formula is essentially a contraction (by the negation of $\mu$), since in the non-trivial case, i.e., when $\varphi \wedge \mu \models \bot$, the two operators $\div$ and $-$ give the same results, i.e., $\varphi \div \mu \equiv \varphi - \neg\mu$. The only difference between $\div$ and $-$ thus concerns the trivial case when contraction requires strictly no change (the result must be equivalent to $\varphi$) whereas for promotion operators more information is expected to be preserved (the result must be $\varphi \wedge \mu$). Accordingly, one can safely use the $\div$ operator in the Levi identity[4] instead of the corresponding contraction operator $-$ for defining the corresponding revision operator $\circ_-$. To be more precise, if $-$ is a contraction operator, then $\circ_-$ defined as $\varphi \circ_- \mu = (\varphi - \neg\mu) \wedge \mu$ can also be defined using $\div$: $\varphi \circ_- \mu \equiv (\varphi \div \mu) \wedge \mu$.

That stated, we can show that $\div$ operators are promotion operators:

**Proposition 11.** *Every conjunctive contraction operator $\div$ is a consensual promotion operator.*

The other way around, consensual promotion operators can also be defined in terms of contraction (conjunctive or not):

**Proposition 12.** *Let $\oplus$ be the consensual promotion operator induced by two KM revision operators $\circ_1$ and $\circ_2$. Let $-_1$ and $-_2$ be the contraction operators associated with $\circ_1$ and $\circ_2$ using the Harper identity[5]. Let $\div_1$ and $\div_2$ be the conjunctive contraction operators induced by $-_1$ and $-_2$ (respectively). For every $\varphi$ and $\mu$, we have*

$$\varphi \oplus \mu \equiv (\varphi -_1 \neg\mu) \wedge (\mu -_2 \neg\varphi) \equiv (\varphi \div_1 \mu) \wedge (\mu \div_2 \varphi).$$

This proposition sheds some light on the inferential power of consensual promotion, by providing a framing of the resulting formula. Indeed, since $-_2$ satisfies (C1), $\mu -_2 \neg\varphi$ is entailed by $\mu$. Because $\varphi \circ_1 \mu \equiv (\varphi -_1 \neg\mu) \wedge \mu$ thanks to Levi identity, we get that

$$\varphi \circ_1 \mu \models \varphi \oplus \mu \models \varphi -_1 \neg\mu.$$

[3] Note that $\circ_* \neq \circ_{Dal}$.

[4] Let $\div$ be a contraction operator, the Levy identity is $\varphi \circ \mu = (\varphi \div \neg\mu) \wedge \mu$ (Gärdenfors 1988)

[5] Let $\circ$ be a revision operator, the Harper identity is $\varphi \div \mu = (\varphi \circ \mu) \vee \varphi$ (Caridroit, Konieczny, and Marquis 2015; Gärdenfors 1988)

Furthermore, since $\varphi \circ_1 \mu \equiv \varphi \oplus \mu \equiv \varphi \div_1 \mu \equiv \varphi \wedge \mu$ when $\varphi \wedge \mu$ is consistent, and since $\varphi \div_1 \mu \equiv \varphi -_1 \neg\mu$ in the remaining case, we also have

$$\varphi \circ_1 \mu \models \varphi \oplus \mu \models \varphi \div_1 \mu.$$

Concerning commutative revision (aka arbitration) operators (Liberatore and Schaerf 1998) a direct relationship can be established. Indeed, every LS commutative revision operator $\diamond$ can be defined as $\varphi \diamond \mu \equiv (\varphi \circ \mu) \vee (\mu \circ \varphi)$ for some KM revision operator $\circ$ (Liberatore and Schaerf 1998; Konieczny and Pino Pérez 2002), and Proposition 7 shows that consensual promotion operators can be defined as $(\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$. As a direct consequence:

**Proposition 13.** *Every LS commutative revision operator is a commutative promotion operator.*

In addition, commutative promotion operators are "almost" LS commutative revision operators.

**Proposition 14.** *Every commutative promotion operator satisfies (A1-A5) and (A7-A8).*

In fact, as illustrated by Proposition 7, commutative promotion operators are "true" commutative revision operators. LS commutative revision operators ask for the additional property (A6) that prevents some revision operators from being used.

Noteworthy, a class of consensual promotion operators can also be obtained from IC merging operators $\Delta$ (Konieczny and Pino Pérez 2002). In a nutshell, an IC merging operator considers a vector of formulas $E = \langle \varphi_1, \ldots, \varphi_m \rangle$ and an integrity constraint $IC$, and associates with them a consistent new formula $\Delta_{IC}(E)$ which is consistent with $IC$ and aims to represent the point of view of the group $E$ (see (Konieczny and Pino Pérez 2002) for more details on the underlying IC postulates):

**Proposition 15.** *Let $\Delta$ be an IC merging operator (Konieczny and Pino Pérez 2002), and let $\ominus$ be the operator defined by $\varphi \ominus \mu = \Delta_{\varphi \vee \mu}(\langle \varphi, \mu \rangle)$ for every $\varphi$ and $\mu$. Then $\ominus$ is a commutative promotion operator.*

Finally, we would like to discuss the relationships between promotions operators and two other kinds of operators that allow softer change than revision operators: non-prioritized revision operators and improvement operators.

Several studies about non-prioritized revision operators exist (see (Hansson 1997b) for an overview). The basic idea on which such operators are based is to get rid of postulate (R1), i.e., the success of revision is no longer ensured, so the new formula is not always accepted after revision. For instance, for screened revision (Makinson 1997), a classical revision is performed only if the change formula does not threat a specific part of the old beliefs (the core). (R1) is also questioned by credibility-limited revision operators (Hansson et al. 2001; Booth et al. 2012) which are operators satisfying the following (Relative success) postulate: $\varphi \circ \mu \models \mu$ or $\varphi \circ \mu \equiv \varphi$. While this postulate requires the revised beliefs to entail the new piece of evidence $\mu$ or not to change anything, our postulate (P3) requires promoted beliefs to be consistent with $\mu$ when $\mu$ is consistent. Obviously enough, an operator satisfying (P3) and (Relative success) would satisfy the success postulate (R1), showing that the two approaches weaken (R1) in quite different ways. In Hansson's semi-revision (Hansson 1997a), the new formula $\mu$ is first added to the initial beliefs, and in case of inconsistency $\mu$ or some of the initial beliefs are deleted. Semi-revision departs significantly from promotion since it is based on a different setting: initial beliefs are represented through a belief base, i.e., a set of formulas that is not logically closed. As a consequence, semi-revision is not required to satisfy the syntax independence postulate (P4), unlike promotion operators.

Another framework where the success postulate is not expected is the one of improvement operators (Konieczny and Pino Pérez 2008; Konieczny, Medina Grespan, and Pino Pérez 2010). The underlying purpose is similar to the one considered in this paper, that is, enhancing the change formula into the beliefs of the agent without doing it too drastically (i.e., without being forced to accept it entirely). However, improvement differs from promotion on several aspects. On the one hand, when using a promotion operator, it is ensured that at least one model of the change formula $\mu$ is a model of the result of the promotion process, whereas this cannot be guaranteed when improvement operators are used instead. On the other hand, promoting $\mu$ into $\varphi$ can lead to pieces of belief that would not be allowed by an improvement operator: namely, using promotion, one can select some models of $\mu$ and some models of $\varphi$, but not necessarily all of them (in the general case, i.e., when $\varphi \wedge \mu$ is not consistent). Contrastingly, with improvement, only two cases are possible: keeping all models of $\varphi$ or removing them all, but no selection of the models of $\varphi$ is possible (again, in the general case, i.e., when $\varphi \wedge \mu$ is not consistent). Last but not least, improvement operators consider epistemic states, i.e., they are defined in the Darwiche and Pearl's framework (Darwiche and Pearl 1997), while promotion operators are defined in the classical AGM framework, where the pieces of belief are propositional formulas (Katsuno and Mendelzon 1991), and not more complex objects.

## Acknowledgments

This work was supported by JSPS KAKENHI Grant Number JP17K12746.

## Conclusion

We have presented in this paper a new class of belief change operators, promotion operators, and some representation theorems for them. This is, as far as we know, the most general family of belief change operators ever introduced, since it captures as special cases classical KM revision operators, Liberatore and Shaerf commutative revision operators, and (essentially) belief contraction operators. More precisely, our approach offers a unifying setting for these three families, that allows one to better understand the principle of minimal change in belief change, and encompasses new change operators, which can be seen as in-between revision and (conjunctive) contraction. From the new perspective on belief change given in the paper, revision and contraction can be viewed as two "extreme" cases of change operators;

choosing an operator that is strictly in-between is appropriate when a more cautious operator is expected.

In perspective, we plan to find a direct characterization of the class of conjunctive contraction operators introduced in the paper, which looks interesting on its own. Another research perspective is to look at the iterated change problem. For belief revision and contraction this requires to shift to epistemic states for the beliefs of the agent, while keeping a formula for the new evidence (Darwiche and Pearl 1997). Since promotion operators provide a more symmetrical treatment to the two pieces of information, this may require to use epistemic states on both sides.

## Appendix: proofs of propositions

**Proofs of Proposition 2 and Proposition 3**. Direct by definition of the postulates. □

**Proof of Proposition 4**. (P1), (P2) and (P4) are directly satisfied by definition of $\ominus_D$. Then, one can remark that $\ominus_D$ satisfies (A1), that allows us to take advantage of Proposition 3 in the rest of the proof. Let us now show that the remaining postulates are satisfied.

(P3) One can verify by definition of $\ominus_D$ that $(\varphi \ominus_D \mu) \wedge \mu \equiv \mu$, whether $\varphi \wedge \mu$ is consistent or not. So if $\mu$ is consistent, then $(\varphi \ominus_D \mu) \wedge \mu$ is also consistent. Hence, $\ominus_D$ satisfies (P3).

(P8) By Proposition 3, since $\ominus_D$ satisfies (A1) and (P3).

(P6) Assume that $(\varphi \ominus_D \mu) \wedge \varphi \wedge \gamma$ is consistent. Let us consider two cases: first, assume that $\varphi \wedge \mu$ is consistent. Then $(\varphi \ominus_D \mu) \wedge \varphi \wedge \gamma \equiv \varphi \wedge \mu \wedge \gamma$. This means that $\varphi \wedge \mu \wedge \gamma$ is consistent, so we also get that $[(\varphi \wedge \gamma) \ominus_D \mu] \wedge \varphi \equiv \varphi \wedge \mu \wedge \gamma$. Hence, $[(\varphi \wedge \gamma) \ominus_D \mu] \wedge \varphi \equiv (\varphi \ominus_D \mu) \wedge \varphi \wedge \gamma$. now, assume that $\varphi \wedge \mu$ is inconsistent. Then on the one hand, $(\varphi \ominus_D \mu) \wedge \varphi \wedge \gamma \equiv (\varphi \vee \mu) \wedge \varphi \wedge \gamma \equiv \varphi \wedge \gamma$. And on the other hand, since $\varphi \wedge \mu$ is inconsistent, we have that $\varphi \wedge \gamma \wedge \mu$ is also inconsistent, and so $[(\varphi \wedge \gamma) \ominus_D \mu] \wedge \varphi \equiv [(\varphi \wedge \gamma) \vee \mu] \wedge \varphi \equiv (\varphi \wedge \gamma) \vee (\mu \wedge \varphi) \equiv \varphi \wedge \gamma$. Hence, $[(\varphi \wedge \gamma) \ominus_D \mu] \wedge \varphi \equiv (\varphi \ominus_D \mu) \wedge \varphi \wedge \gamma$. This shows that $\ominus_D$ satisfies (P6).

(P5) By Proposition 3, since $\ominus_D$ satisfies (A1) and (P6).

(P7) Assume that $((\varphi \wedge \gamma) \ominus_D \mu) \wedge \varphi$ is consistent. We fall into two cases. Assume first that $\varphi \wedge \mu$ is consistent. Then $(\varphi \ominus_D \mu) \wedge \varphi \equiv \varphi \wedge \mu$, which is consistent. Assume now that $\varphi \wedge \mu$ is inconsistent. Then $(\varphi \ominus_D \mu) \wedge \varphi \equiv (\varphi \vee \mu) \wedge \varphi \equiv \varphi$. Yet $\varphi$ is consistent since we assumed that $((\varphi \wedge \gamma) \ominus_D \mu) \wedge \varphi$ is consistent. Hence, $(\varphi \ominus_D \mu) \wedge \varphi$, so $\ominus_D$ satisfies (P7).

This concludes the proof. □

**Proof of Proposition 5**. **(If)** Let $\circ_1$ and $\circ_2$ be KM revision operators, $\sigma$ be a trigger function, and $\ominus$ be an operator defined as $\varphi \ominus \mu = (\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))$. We show that $\ominus$ satisfies (P1-P7):

(P1) By (R1), $\varphi \circ_1 \mu \models \mu$, and $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \models \varphi \wedge \sigma(\mu)$, so $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \models \varphi$. Thus $(\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu))) \models \varphi \vee \mu$. Hence, $\varphi \ominus \mu \models \varphi \vee \mu$. So $\ominus$ satisfies (P1).

(P2) Assume that $\varphi \wedge \mu \not\models \bot$. First, by (R2), (i) $\varphi \circ_1 \mu \equiv \varphi \wedge \mu$. Second, by (R2), $\mu \circ_2 \varphi \equiv \varphi \wedge \mu$. So $(\mu \circ_2 \varphi) \wedge \sigma(\mu) \equiv \varphi \wedge \mu \wedge \sigma(\mu)$, which is equivalent to $\varphi \wedge \mu$ since $\mu \models \sigma(\mu)$. Then since $(\mu \circ_2 \varphi) \wedge \sigma(\mu) \not\models \bot$, by (R5) and (R6) it is also equivalent to $\mu \circ_2 (\varphi \wedge \sigma(\mu))$. Hence, (ii) $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \equiv \varphi \wedge \mu$. Therefore, by (i) and (ii) above, $(\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu))) \equiv \varphi \wedge \mu$. Hence, $\varphi \ominus \mu \equiv \varphi \wedge \mu$. So $\ominus$ satisfies (P2).

(P3) Assume that $\mu \not\models \bot$. By (R1), $\varphi \circ_1 \mu \models \mu$. Thus $((\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))) \wedge \mu \not\models \bot$. Hence, $(\varphi \ominus \mu) \wedge \mu \not\models \bot$. So $\ominus$ satisfies (P3).

(P4) Assume that $\varphi \equiv \varphi'$ and $\mu \equiv \mu'$. By (R4), $\varphi \circ_1 \mu \equiv \varphi' \circ_1 \mu'$, and since $\sigma(\mu) \equiv \sigma(\mu')$, $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \equiv \mu' \circ_2 (\varphi' \wedge \sigma(\mu'))$. Thus $(\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu))) \equiv (\varphi' \circ_1 \mu') \vee (\mu' \circ_2 (\varphi' \wedge \sigma(\mu')))$. Hence, $\varphi \ominus \mu \equiv \varphi' \ominus \mu'$. So $\ominus$ satisfies (P4).

(P5) is proved similarly as (P6) below.

(P6) Assume first that $\varphi \wedge \mu \not\models \bot$. By (P2), $\varphi \ominus \mu \equiv \varphi \wedge \mu$. If $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \not\models \bot$, then $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \equiv \varphi \wedge \mu \wedge \gamma$, and then $\varphi \wedge \gamma \wedge \mu \not\models \bot$, so $[(\varphi \wedge \gamma) \ominus \mu] \wedge \varphi \equiv \varphi \wedge \gamma \wedge \mu$, thus $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \equiv [(\varphi \wedge \gamma) \ominus \mu] \wedge \varphi$.

Now, assume that $\varphi \wedge \mu \models \bot$ and $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \not\models \bot$. We have $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \equiv (\varphi \circ_1 \mu \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))) \wedge \varphi \wedge \gamma$. By (R1), $\varphi \circ_1 \mu \models \mu$, and since $\varphi \wedge \mu \models \bot$, $(\varphi \circ_1 \mu) \wedge \varphi \models \bot$. So $(\varphi \circ_1 \mu \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))) \wedge \varphi \wedge \gamma \equiv (\mu \circ_2 (\varphi \wedge \sigma(\mu))) \wedge \varphi \wedge \gamma$. By (R1), $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \models \varphi \wedge \sigma(\mu)$, so $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \models \varphi$, and then $(\mu \circ_2 (\varphi \wedge \sigma(\mu))) \wedge \varphi \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu))$. Thus $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \wedge \varphi \wedge \gamma \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu)) \wedge \gamma$. We got that $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu)) \wedge \gamma$. Yet by hypothesis, $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \not\models \bot$, so $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \wedge \gamma \not\models \bot$. Then by (R5) and (R6), we get that $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \wedge \gamma \equiv \mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu))$. Yet by (R1), $\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu)) \models \varphi \wedge \gamma \wedge \sigma(\mu)$, thus $\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu)) \models \varphi$. So $\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu)) \equiv (\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu))) \wedge \varphi$. Then, by (R1) $(\varphi \wedge \gamma) \circ_1 \mu \models \mu$, and since $\varphi \wedge \mu \models \bot$, we have $((\varphi \wedge \gamma) \circ_1 \mu) \wedge \varphi \models \bot$. So we can write $(\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu))) \wedge \varphi \equiv [((\varphi \wedge \gamma) \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu)))] \wedge \varphi$. By definition of $\ominus$, this is equivalent to $((\varphi \wedge \gamma) \ominus \mu) \wedge \varphi$. Overall, we got that $(\varphi \ominus \mu) \wedge \varphi \wedge \gamma \equiv ((\varphi \wedge \gamma) \ominus \mu) \wedge \varphi$. So $\ominus$ satisfies (P6).

(P7) Assume first that $\varphi \wedge \mu \not\models \bot$. By (P2), $\varphi \ominus \mu \equiv \varphi \wedge \mu$, so $(\varphi \ominus \mu) \wedge \varphi \equiv \varphi \wedge \mu$, and thus $(\varphi \ominus \mu) \wedge \varphi \not\models \bot$. So assume now that $\varphi \wedge \mu \models \bot$ and $((\varphi \wedge \gamma) \ominus \mu) \wedge \varphi \not\models \bot$ (the precondition of (P7)). We must prove that $(\varphi \ominus \mu) \wedge \varphi \not\models \bot$. We know that $((\varphi \wedge \gamma) \ominus \mu) \wedge \varphi \equiv (((\varphi \wedge \gamma) \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu)))) \wedge \varphi \not\models \bot$. By (R1), $(\varphi \wedge \gamma) \circ_1 \mu \models \mu$, and since $\varphi \wedge \mu \models \bot$, $((\varphi \wedge \gamma) \circ_1 \mu) \wedge \varphi \models \bot$. So $(((\varphi \wedge \gamma) \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu)))) \wedge \varphi \equiv (\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu))) \wedge \varphi \not\models \bot$. So $\mu \circ_2 (\varphi \wedge \gamma \wedge \sigma(\mu))$ is consistent, and by (R1), $\varphi \wedge \gamma \wedge \sigma(\mu) \not\models \bot$. Thus $\varphi \wedge \sigma(\mu) \not\models \bot$. So by (R1) and (R3), $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \not\models \bot$ and $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \models \varphi$. So $(\mu \circ_2 (\varphi \wedge \sigma(\mu))) \wedge \varphi \not\models \bot$, thus $((\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))) \wedge \varphi \not\models \bot$, which means that $(\varphi \ominus \mu) \wedge \varphi \not\models \bot$. So $\ominus$ satisfies (P7).

**(Only If)** Let $\ominus$ be an operator satisfying (P1-P7). Define a trigger function $\sigma$ such that for every formula $\mu$ and every interpretation $I$, $I \models \sigma(\mu)$ iff $I \models \alpha_I \ominus \mu$, where $\alpha_I$ denotes any formula such that $[\alpha_I] = \{I\}$. Define the revision operator $\circ_1$ as $\varphi \circ_1 \mu \equiv (\varphi \ominus \mu) \wedge \mu$. and define $\circ_2$ as $\varphi \circ_2 \mu \equiv (\mu \ominus \varphi) \wedge \mu$ if $(\mu \ominus \varphi) \wedge \mu \not\models \bot$, otherwise $\varphi \circ_2 \mu \equiv \mu$. Let us verify first that $\circ_1$ and $\circ_2$ satisfy (R1-R6).

(R1) By definition of $\circ_1$, $\varphi \circ_1 \mu \equiv (\varphi \ominus \mu) \wedge \mu$, thus $\varphi \circ_1 \mu \models \mu$. So $\circ_1$ satisfies (R1). By definition of $\circ_2$, we have $\varphi \circ_2 \mu \equiv (\mu \ominus \varphi) \wedge \mu$ or $\varphi \circ_2 \mu \equiv \mu$. In both cases, $\varphi \circ_2 \mu \models \mu$. So $\circ_2$ satisfies (R1).

(R2) Assume that $\varphi \wedge \mu \not\models \bot$. By (P2), $\varphi \ominus \mu \equiv \varphi \wedge \mu$. Thus $\varphi \circ_1 \mu \equiv (\varphi \ominus \mu) \wedge \mu \equiv \varphi \wedge \mu$. So $\circ_1$ satisfies (R2). And $(\mu \ominus \varphi) \wedge \mu \equiv \varphi \wedge \mu \not\models \bot$, thus $\varphi \circ_2 \mu \equiv (\mu \ominus \varphi) \wedge \mu \equiv \varphi \wedge \mu$. So $\circ_2$ satisfies (R2).

(R3) Assume that $\mu \not\models \bot$. By (P3), $(\varphi \ominus \mu) \wedge \mu \not\models \bot$, thus $\varphi \circ_1 \mu \not\models \bot$. So $\circ_1$ satisfies (R3). Now, if $(\mu \ominus \varphi) \wedge \mu \not\models \bot$, then $\varphi \circ_2 \mu \equiv (\mu \ominus \varphi) \wedge \mu$, and thus $\varphi \circ_2 \mu \not\models \bot$. So assume that $(\mu \ominus \varphi) \wedge \mu \models \bot$. In this case, $\varphi \circ_2 \mu \equiv \mu$, yet we assumed $\mu \not\models \bot$, so $\varphi \circ_2 \mu \not\models \bot$. Hence, $\circ_2$ satisfies (R3).

(R4) Assume that $\varphi \equiv \varphi'$ and $\mu \equiv \mu'$. By (P4), $\varphi \ominus \mu \equiv \varphi' \ominus \mu'$. Thus $\varphi \circ_1 \mu \equiv (\varphi \ominus \mu) \wedge \mu \equiv (\varphi' \ominus \mu') \wedge \mu' \equiv \varphi' \circ_1 \mu'$. So $\circ_1$ satisfies (R4). Then, if $(\mu \ominus \varphi) \wedge \mu \not\models \bot$, $\varphi \circ_2 \mu \equiv (\mu \ominus \varphi) \wedge \mu \equiv (\mu' \ominus \varphi') \wedge \mu'$ by (P4), which is consistent and thus equivalent to $\varphi' \circ_2 \mu'$. Otherwise, $(\mu \ominus \varphi) \wedge \mu \models \bot$, and then $\varphi \circ_2 \mu \equiv \mu \equiv \mu'$. This is equivalent to $\varphi' \circ_2 \mu'$, since $(\mu \ominus \varphi) \wedge \mu \models \bot$ and by (P4) $(\mu' \ominus \varphi') \wedge \mu' \models \bot$. In both cases, $\varphi \circ_2 \mu \equiv \varphi' \circ_2 \mu'$. So $\circ_2$ satisfies (R4).

(R5-R6) We first need to show that if $(\varphi \circ_1 \mu_1) \wedge \mu_2 \not\models \bot$, $\varphi \circ_1 (\mu_1 \wedge \mu_2) \equiv (\varphi \circ_1 \mu_1) \wedge \mu_2$. So assume $(\varphi \circ_1 \mu_1) \wedge \mu_2 \not\models \bot$. By definition of $\circ_1$, $(\varphi \circ_1 \mu_1) \wedge \mu_2 \equiv (\varphi \ominus \mu_1) \wedge \mu_1 \wedge \mu_2 \equiv ((\varphi \ominus \mu_1) \wedge \mu_2) \wedge (\mu_1 \wedge \mu_2)$, which is consistent, thus equivalent to $(\varphi \ominus (\mu_1 \wedge \mu_2)) \wedge (\mu_1 \wedge \mu_2)$ by (P5), i.e., equivalent to $\varphi \circ_1 (\mu_1 \wedge \mu_2)$ by definition of $\circ_1$. We got that $(\varphi \circ_1 \mu_1) \wedge \mu_2 \equiv \varphi \circ_1 (\mu_1 \wedge \mu_2)$. So $\circ_1$ satisfies (R5-R6).

Now we need to show that if $(\varphi \circ_2 \mu_1) \wedge \mu_2 \not\models \bot$, $\varphi \circ_2 (\mu_1 \wedge \mu_2) \equiv (\varphi \circ_2 \mu_1) \wedge \mu_2$. So assume $(\varphi \circ_2 \mu_1) \wedge \mu_2 \not\models \bot$. We fall into two cases:

• case (i): $(\mu_1 \ominus \varphi) \wedge \mu_1 \not\models \bot$. Then $(\varphi \circ_2 \mu_1) \wedge \mu_2 \equiv (\mu_1 \ominus \varphi) \wedge \mu_1 \wedge \mu_2 \equiv ((\mu_1 \ominus \varphi) \wedge \mu_1 \wedge \mu_2) \wedge \mu_2$, which is consistent, so by (P6) it is equivalent to $((\mu_1 \wedge \mu_2) \ominus \varphi) \wedge \mu_1 \wedge \mu_2$, which is equivalent to $\varphi \circ_2 (\mu_1 \wedge \mu_2)$, by definition of $\circ_2$ and since it is consistent.

• case (ii): $(\mu_1 \ominus \varphi) \wedge \mu_1 \models \bot$. Then $(\varphi \circ_2 \mu_1) \wedge \mu_2 \equiv \mu_1 \wedge \mu_2$. We need to show that it is also equivalent to $\varphi \circ_2 (\mu_1 \wedge \mu_2)$. Yet since $(\mu_1 \ominus \varphi) \wedge \mu_1 \models \bot$, by (P7) we get that $((\mu_1 \wedge \mu_2) \ominus \varphi) \wedge \mu_1 \models \bot$, thus $((\mu_1 \wedge \mu_2) \ominus \varphi) \wedge \mu_1 \wedge \mu_2 \models \bot$. By definition of $\circ_2$, we get that $\varphi \circ_2 (\mu_1 \wedge \mu_2) \equiv \mu_1 \wedge \mu_2$, that was left to be shown.

If $(\varphi \circ_2 \mu_1) \wedge \mu_2 \not\models \bot$, we have shown in both cases (i) and (ii) above that $\varphi \circ_2 (\mu_1 \wedge \mu_2) \equiv (\varphi \circ_2 \mu_1) \wedge \mu_2$. So $\circ_2$ satisfies (R5) and (R6).

Let us now verify that $\sigma$ is a trigger function according to Definition 7. We need to show that for all formulas $\mu, \mu'$ such that $\mu \equiv \mu'$, (i) $\mu \models \sigma(\mu)$ and (ii) $\sigma(\mu) \equiv \sigma(\mu')$. Let $\mu$ and $\mu'$ be two equivalent formulas:

(i) Let $I$ be an interpretation such that $I \models \mu$, and let us show that $I \models \sigma(\mu)$. Since $I \models \mu$, $\alpha_I \wedge \mu \not\models \bot$. By (P2), $\alpha_I \ominus \mu \equiv \alpha_I \wedge \mu$. And by definition of $\sigma$, $I \models \sigma(\mu)$.

(ii) Let $I$ be an interpretation. We have $I \models \sigma(\mu)$ iff $I \models \alpha_I \ominus \mu$ (by definition of $\sigma$) iff $I \models \alpha_I \ominus \mu'$ (by (P4)) iff $I \models \sigma(\mu')$ (by definition of $\sigma$). Thus $\sigma(\mu) \equiv \sigma(\mu')$.

Let us now show that for all formulas $\varphi, \mu$, $\varphi \ominus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))$. By (P1), $\varphi \ominus \mu \equiv (\varphi \ominus \mu) \wedge (\varphi \vee \mu) \equiv ((\varphi \ominus \mu) \wedge \mu) \vee ((\varphi \ominus \mu) \wedge \varphi) \equiv (\varphi \circ_1 \mu) \vee ((\varphi \ominus \mu) \wedge \varphi)$ by definition of $\circ_1$. What remains to be shown is that $(\varphi \ominus \mu) \wedge \varphi \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu))$. We fall into two cases:

• case (i): $(\varphi \ominus \mu) \wedge \varphi \not\models \bot$. Let us first prove that $(\varphi \ominus \mu) \wedge \varphi \models \sigma(\mu)$. Let $I \models (\varphi \ominus \mu) \wedge \varphi$. We have $I \models (\varphi \ominus \mu) \wedge \varphi \wedge \alpha_I$. By (P6), $I \models ((\varphi \wedge \alpha_I) \ominus \mu) \wedge \varphi$. Thus $I \models (\alpha_I \ominus \mu) \wedge \varphi$, so $I \models \alpha_I \ominus \mu$, i.e., $I \models \sigma(\mu)$. We have just proved that $(\varphi \ominus \mu) \wedge \varphi \models \sigma(\mu)$. So $(\varphi \ominus \mu) \wedge \varphi \equiv (\varphi \ominus \mu) \wedge \varphi \wedge \sigma(\mu) \equiv ((\varphi \wedge \sigma(\mu)) \ominus \mu) \wedge \varphi \wedge \sigma(\mu)$ (by (P6), since it is consistent), which by definition of $\circ_2$ is equivalent to $\mu \circ_2 (\varphi \wedge \sigma(\mu))$, since it is consistent. Hence, $(\varphi \ominus \mu) \wedge \varphi \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu))$.

• case (ii): $(\varphi \ominus \mu) \wedge \varphi \models \bot$. Since we want to prove that $(\varphi \ominus \mu) \wedge \varphi \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu))$, it is enough to show that $\mu \circ_2 (\varphi \wedge \sigma(\mu)) \models \bot$. By (R3), this boils down to show that $\varphi \wedge \sigma(\mu) \models \bot$. Toward a contradiction, assume that $\varphi \wedge \sigma(\mu) \not\models \bot$, and let $I \models \varphi \wedge \sigma(\mu)$. By definition of $\sigma$, $I \models \alpha_I \ominus \mu$. Yet $I \models \varphi$, so $I \models ((\varphi \wedge \alpha_I)) \ominus \mu) \wedge \varphi$. So $((\varphi \wedge \alpha_I)) \ominus \mu) \wedge \varphi \not\models \bot$, and by (P7), $(\varphi \ominus \mu) \wedge \varphi \not\models \bot$. This contradicts $(\varphi \ominus \mu) \wedge \varphi \models \bot$. Therefore, $(\varphi \ominus \mu) \wedge \varphi \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu))$.

We have shown that in both cases (i) and (ii) above, $(\varphi \ominus \mu) \wedge \varphi \equiv \mu \circ_2 (\varphi \wedge \sigma(\mu))$, from which we conclude that $\varphi \ominus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu)))$. □

**Proof of Proposition 6**. This proof follows from Proposition 5 and from the usual representation theorem for belief revision (Proposition 1). It is easy to check that an operator defined from a trigger function and two total faithful assignements as

$$[\varphi \ominus \mu] = \begin{cases} \min([\mu], \leq^1_\varphi) \cup \min([\varphi], \leq^2_\mu) \\ \qquad \text{if } \varphi \wedge \sigma(\mu) \not\models \bot, \\ \min([\mu], \leq^1_\varphi) \quad \text{otherwise,} \end{cases}$$

satisfies (P1-P7).

For the converse implication, suppose that $\ominus$ is a promotion operator. From Proposition 7 we have that $\varphi \ominus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma(\mu))$. There are two cases.

First if $\varphi \wedge \sigma(\mu)$ is inconsistent, then by (R1) we know that $\mu \circ_2 (\varphi \wedge \sigma(\mu))$ is inconsistent. So $\varphi \ominus \mu \equiv \varphi \circ_1 \mu$. Now from Proposition 1, there exists a total faithful assignment $\varphi \mapsto \leq^1_\varphi$ s.t. $[\varphi \circ_1 \mu] = \min([\mu], \leq^1_\varphi)$. So $[\varphi \ominus \mu] = \min([\mu], \leq^1_\varphi)$. The second case is when $\varphi \wedge \sigma(\mu)$ is consistent, then from Proposition 1 there exists $\varphi \mapsto \leq^1_\varphi$ and $\varphi \mapsto \leq'_\varphi$ and $[\varphi \ominus \mu] = \min([\mu], \leq^1_\varphi) \cup \min([\varphi \wedge \sigma(\mu)], \leq'_\mu)$. We will look at two subcases. First suppose that $\varphi \wedge \mu$ is consistent, then $\min([\varphi \wedge \sigma(\mu)], \leq'_\mu) = \min([\varphi], \leq'_\mu)$, so $[\varphi \ominus \mu] = \min([\mu], \leq^1_\varphi) \cup \min([\varphi], \leq'_\mu)$. Now, if $\varphi \wedge \mu$ is inconsistent, define $\leq^2$ from $\leq'$. For any formula $\alpha \not\equiv \mu$, $\leq^2_\alpha = \leq'_\alpha$. For $\mu$ define $I \leq^2_\mu J$ as:

- If $I \models \varphi \wedge \sigma(\mu)$ and $J \models \varphi \wedge \neg\sigma(\mu)$ and $I \simeq'_\mu J$, then $I <^2_\mu J$,
- If $I \models \varphi \wedge \sigma(\mu)$, $J \models \varphi \wedge \neg\sigma(\mu)$, $K \models \neg\varphi$ and $I \simeq'_\mu J \simeq'_\mu K$, then $I <^2_\mu K$,
- Otherwise $I \leq^2_\mu J$ iff $I \leq'_\mu J$.

We have that $\mu \mapsto \leq^2_\mu$ is a total faithful assignement, and we have $\min([\varphi \wedge \sigma(\mu)], \leq'_\mu) = \min([\varphi], \leq^2_\mu)$. So there exists $\varphi \mapsto \leq^1_\varphi$ and $\varphi \mapsto \leq'_\varphi$ s.t.

$$[\varphi \ominus \mu] = \min([\mu], \leq^1_\varphi) \cup \min([\varphi], \leq^2_\mu).$$

We have shown that there exists $\varphi \mapsto \leq^1_\varphi$ and $\varphi \mapsto \leq^2_\varphi$ s.t.

$$[\varphi \ominus \mu] = \begin{cases} \min([\mu], \leq^1_\varphi) \cup \min([\varphi], \leq^2_\mu) \\ \qquad \text{if } \varphi \wedge \sigma(\mu) \not\models \bot, \\ \min([\mu], \leq^1_\varphi) \quad \text{otherwise.} \end{cases}$$

□

**Proof of Proposition 7**. **(If)** Let $\circ_1$ and $\circ_2$ be KM revision operators, and $\oplus$ be an operator defined as $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$. We show that $\oplus$ satisfies (P1-P8). Since $\circ_2$ satisfies (R4), $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 (\varphi \wedge \sigma_\top(\mu))$, where $\sigma_\top(\mu)$ is defined as $\sigma_\top(\mu) = \top$ for every formula $\mu$. By Proposition 5, $\oplus$ satisfies (P1-P7). What remains to be shown is that $\oplus$ satisfies (P8). Assume $\varphi \not\models \bot$. By (R1), $\mu \circ_2 \varphi \models \varphi$ and by (R3), $\mu \circ_2 \varphi \not\models \bot$. So $(\mu \circ_2 \varphi) \wedge \varphi \not\models \bot$. Thus $((\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)) \wedge \varphi \not\models \bot$. Hence, $(\varphi \oplus \mu) \wedge \varphi \not\models \bot$. So $\oplus$ satisfies (P8).

**(Only If)** Let $\oplus$ be an operator satisfying (P1-P8). Define an operator $\circ_1$ as $\varphi \circ_1 \mu \equiv (\varphi \oplus \mu) \wedge \mu$ and an operator $\circ_2$ as $\varphi \circ_2 \mu \equiv (\mu \oplus \varphi) \wedge \mu$.

Let us define an additional operator $\circ'_2$ as $\varphi \circ'_2 \mu \equiv (\mu \oplus \varphi) \wedge \mu$ if $(\mu \oplus \varphi) \wedge \mu \not\models \bot$, otherwise $\varphi \circ'_2 \mu \equiv \mu$. Lastly, let us define a trigger function $\sigma$ as $\sigma(\mu) \equiv \top$ for every $\mu$.

Since $\oplus$ satisfies (P1-P8), from Proposition 5 we get that $\circ_1$ and $\circ'_2$ are KM revision operators, and $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ'_2 (\varphi \wedge \sigma(\mu)))$, i.e., $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ'_2 \varphi)$, since for every formula $\mu$, $\sigma(\mu) \equiv \top$.

What we need to show is that $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$, but since $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ'_2 \varphi)$, what remains to be shown is that $\varphi \circ_2 \mu \equiv \varphi \circ'_2 \mu$. We fall into two cases:

• case (i): $(\mu \oplus \varphi) \wedge \mu \not\models \bot$. Then by definition, $\varphi \circ'_2 \mu \equiv (\mu \oplus \varphi) \wedge \mu \equiv \varphi \circ_2 \mu$.

• case (ii): $(\mu \oplus \varphi) \wedge \mu \models \bot$. First, since $\varphi \circ_2 \mu \equiv (\mu \oplus \varphi) \wedge \mu$ by definition, we have that $\varphi \circ_2 \mu \models \bot$. Second, $\varphi \circ'_2 \mu \equiv \mu$ by definition. Yet since $(\mu \oplus \varphi) \wedge \mu \models \bot$ and since $\oplus$ satisfies (P8), we get $\mu \models \bot$. Thus $\varphi \circ'_2 \mu \models \bot$. We got that $\varphi \circ_2 \mu \equiv \varphi \circ'_2 \mu$. We have showed that for all formulas $\varphi, \mu$, $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$. □

**Proof of Proposition 8**. This proof follows directly from Proposition 7 and from the usual representation theorem for belief revision (Proposition 1). From Proposition 7 we have that $\varphi \oplus \mu \equiv (\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$. Now from Proposition 1 we know that there exists a total faithful assignment $\varphi \mapsto \leq^1_\varphi$ s.t. $\varphi \circ_1 \mu = \min([\mu], \leq^1_\varphi)$, and similarly for $\circ_2$ we know that there exists a total faithful assignment $\varphi \mapsto \leq^2_\varphi$ s.t. $\varphi \circ_2 \mu = \min([\mu], \leq^2_\varphi)$. So putting this together we obtain that there exists $\varphi \mapsto \leq^1_\varphi$ and $\varphi \mapsto \leq^2_\varphi$ s.t. $\varphi \oplus \mu = \min([\mu], \leq^1_\varphi) \cup \min([\varphi], \leq^2_\mu)$. □

**Proof of Proposition 9**. Let $\circ$ be a KM revision operator. We need to show that $\circ$ is a promotion operator. Let us define the trigger function $\sigma$ as $\sigma(\mu) = \mu$ for every formula $\mu$. Let us define the operator $\ominus$ as $\varphi \ominus \mu = (\varphi \circ \mu) \vee (\mu \circ (\varphi \wedge \sigma(\mu)))$. According to Proposition 5, by construction, $\ominus$ is a promotion operator. Yet for all formulas $\varphi, \mu$, we fall into one of the following two cases:

• case (i): $\varphi \wedge \mu$ is consistent. Then:

$$\begin{aligned} \varphi \ominus \mu &\equiv (\varphi \circ \mu) \vee (\mu \circ (\varphi \wedge \sigma(\mu))) && \text{(by def. of } \ominus) \\ &\equiv (\varphi \circ \mu) \vee (\mu \circ (\varphi \wedge \mu)) && \text{(by def. of } \sigma) \\ &\equiv (\varphi \wedge \mu) \vee (\mu \wedge (\varphi \wedge \mu)) && \text{(by (R2))} \\ &\equiv \varphi \wedge \mu \\ &\equiv \varphi \circ \mu && \text{(by (R2)).} \end{aligned}$$

• case (ii): $\varphi \wedge \mu$ is inconsistent. Then:

$$\begin{aligned} \varphi \ominus \mu &\equiv (\varphi \circ \mu) \vee (\mu \circ (\varphi \wedge \sigma(\mu))) && \text{(by def. of } \ominus) \\ &\equiv (\varphi \circ \mu) \vee (\mu \circ (\varphi \wedge \mu)) && \text{(by def. of } \sigma) \\ &\equiv (\varphi \circ \mu) \vee (\mu \circ \bot) \\ &\equiv \varphi \circ \mu && \text{(by (R1)).} \end{aligned}$$

In both cases, we get that $\varphi \circ \mu \equiv \varphi \ominus \mu$. Then since $\ominus$ is a promotion operator, $\circ$ is a promotion operator as well. This concludes the proof. □

**Proof of Proposition 10**. Let $\ominus$ be a promotion operator, i.e., it satisfies postulates (P1-P7), and assume that $\ominus$ satisfies (R1). We must prove that $\ominus$ is a KM revision operator, i.e., that it satisfies postulates (R1-R6). (R1) is satisfied by assumption, and (R2) and (R4) are identical to (P2) and (P4), respectively, so $\ominus$ trivially satisfies (R2) and (R4). Moreover, (R3) is a direct consequence of (P3). What remains to be shown is that postulates (R5) and (R6) are satisfied. Both (R5) and (R6) are trivially satisfied if $\varphi \ominus (\mu_1 \wedge \mu_2)$ is inconsistent, so assume that $\varphi \ominus (\mu_1 \wedge \mu_2)$ is consistent. We must prove that $(\varphi \ominus \mu_1) \wedge \mu_2 \equiv \varphi \ominus (\mu_1 \wedge \mu_2)$. Yet from (R1), $\varphi \ominus \mu_1 \models \mu_1$, so $(\varphi \ominus \mu_1) \wedge \mu_2 \equiv (\varphi \ominus \mu_1) \wedge \mu_1 \wedge \mu_2$. Then $(\varphi \ominus \mu_1) \wedge \mu_1 \wedge \mu_2$ is consistent, and by (P5) we get that $(\varphi \ominus \mu_1) \wedge \mu_1 \wedge \mu_2 \equiv [\varphi \ominus (\mu_1 \wedge \mu_2)] \wedge \mu_1$. Yet by (R1), $\varphi \ominus (\mu_1 \wedge \mu_2) \models \mu_1 \wedge \mu_2$, so $\varphi \ominus (\mu_1 \wedge \mu_2) \models \mu_1$, and then $[\varphi \ominus (\mu_1 \wedge \mu_2)] \wedge \mu_1 \equiv \varphi \ominus (\mu_1 \wedge \mu_2)$. we have just proved that $(\varphi \ominus \mu_1) \wedge \mu_2 \equiv \varphi \ominus (\mu_1 \wedge \mu_2)$, which shows that $\ominus$ satisfies (R5) and (R6). This concludes the proof. □

**Proof of Proposition 11**. Let $\div$ be a conjunctive contraction operator. We need to show that $\div$ is a consensual promotion operator. Let us define the operator $\oplus$ as $\varphi \oplus \mu = (\varphi \circ_\div \mu) \vee (\mu \circ_D \varphi)$, where (i) $\circ_\div$ is the KM revision operator associated with $\div$ defined as $\varphi \circ_\div \mu = (\varphi \div \mu) \wedge \mu$, and (ii) $\circ_D$ is the KM drastic revision operator defined as $\varphi \circ_D \mu = \varphi \wedge \mu$ if $\varphi \wedge \mu$ is consistent, otherwise $\varphi \circ_D \mu = \mu$. According to Proposition 7, by construction, $\oplus$ is a consensual promotion operator. Yet for all formulas $\varphi, \mu$, we fall into one of the following two cases:

• case (i): $\varphi \wedge \mu$ is consistent. Then:

$$\begin{aligned} \varphi \oplus \mu &\equiv (\varphi \circ_\div \mu) \vee (\mu \circ_D \varphi) && \text{(by def. of } \oplus) \\ &\equiv (\varphi \circ_\div \mu) \vee (\mu \wedge \varphi) && \text{(by def. of } \circ_D) \\ &\equiv (\varphi \wedge \mu) \vee (\mu \wedge \varphi) && \text{(since } \circ_\div \text{ sat. (R2))} \\ &\equiv \varphi \wedge \mu \\ &\equiv \varphi \div \mu && \text{(by def. of } \div). \end{aligned}$$

• case (ii): $\varphi \wedge \mu$ is inconsistent. Then:

$$\begin{aligned} \varphi \oplus \mu &\equiv (\varphi \circ_\div \mu) \vee (\mu \circ_D \varphi) && \text{(by def. of } \oplus) \\ &\equiv (\varphi \circ_\div \mu) \vee \varphi && \text{(by def. of } \circ_D) \\ &\equiv ((\varphi \div \mu) \wedge \mu) \vee \varphi \\ &\equiv ((\varphi -_\div \neg\mu) \wedge \mu) \vee \varphi \\ &\equiv (\varphi -_\div \neg\mu) \wedge \varphi) \vee (\varphi \wedge \mu) \\ &\equiv (\varphi -_\div \neg\mu) \wedge \varphi \\ &\equiv \varphi -_\div \neg\mu && \text{(by (C1))} \\ &\equiv \varphi \div \mu && \text{(by def. of } \div). \end{aligned}$$

In both cases, we get that $\varphi \div \mu \equiv \varphi \oplus \mu$. Then since $\oplus$ is a consensual promotion operator, $\div$ is a consensual promotion operator as well. This concludes the proof. $\square$

**Proof of Proposition 12**. Using Harper identity, we have $(\varphi -_1 \neg\mu) \wedge (\mu -_2 \neg\varphi) \equiv ((\varphi \circ_1 \mu) \vee \varphi) \wedge ((\mu \circ_2 \varphi) \vee \mu)$. Distributing $\wedge$ over $\vee$, we get as an equivalent formula $((\varphi\circ_1\mu)\wedge(\mu\circ_2\varphi))\vee((\varphi\circ_1\mu)\wedge\mu)\vee((\mu\circ_2\varphi)\wedge\varphi)\vee(\varphi\wedge\mu)$. There are two cases:

- $\varphi \wedge \mu$ is consistent: in this case, we have $\varphi \circ_1 \mu \equiv \mu \circ_2 \varphi \equiv \varphi \wedge \mu$ since $\circ_1$ and $\circ_2$ satisfies (R2), hence the above formula is also equivalent to $\varphi \wedge \mu$. We also have $\varphi \oplus \mu \equiv \varphi \wedge \mu$, $\varphi \div_1 \mu \equiv \varphi \wedge \mu$, and $\varphi \div_2 \mu \equiv \varphi \wedge \mu$. Hence $\varphi \oplus \mu \equiv (\varphi -_1 \neg\mu) \wedge (\mu -_2 \neg\varphi) \equiv (\varphi \div_1 \mu) \wedge (\mu \div_2 \varphi)$, as expected.
- $\varphi \wedge \mu$ is inconsistent: in this case, the formula can be simplified into $((\varphi\circ_1\mu)\wedge(\mu\circ_2\varphi))\vee((\varphi\circ_1\mu)\wedge\mu)\vee((\mu\circ_2\varphi)\wedge\varphi)$. Since $\circ_1$ and $\circ_2$ satisfies (R1), we have $(\varphi \circ_1 \mu) \wedge \mu \equiv \varphi \circ_1 \mu$ and $(\mu \circ_2 \varphi) \wedge \varphi \equiv \mu \circ_2 \varphi$. Hence the formula can be further simplified into $(\varphi \circ_1 \mu) \vee (\mu \circ_2 \varphi)$, which is equivalent to $\varphi \oplus \mu$. Finally, when $\varphi \wedge \mu$ is inconsistent, we also have that $\varphi -_1 \neg\mu \equiv \varphi \div_1 \mu$ and $\varphi -_2 \neg\mu \equiv \varphi \div_2 \mu$, and this completes the proof. $\square$

**Proof of Proposition 13**. From (Liberatore and Schaerf 1998; Konieczny and Pino Pérez 2002), we already know that every LS commutative revision operator $\diamond$ can be defined as $\varphi \diamond \mu \equiv (\varphi \circ \mu) \vee (\mu \circ \varphi)$ for some KM revision operator $\circ$. The result is a direct consequence of Definition 8 and Proposition 7. $\square$

**Proof of Proposition 14**. Let $\oplus$ be a commutative promotion operator. So $\oplus$ is defined as $\varphi \oplus \mu \equiv (\varphi \circ \mu) \vee (\mu \circ \varphi)$. Then the fact that $\oplus$ satisfies (A1-A5) and (A7-A8) is a direct consequence of (Konieczny and Pino Pérez 2002) (Definition 45, Theorem 46 and Theorem 47). $\square$

**Proof of Proposition 15**. Let $\Delta$ be an IC merging operator (Konieczny and Pino Pérez 2002), and let $\ominus$ be an operator defined as $\varphi \ominus \mu = \Delta_{\varphi\vee\mu}(\langle\varphi, \mu\rangle)$. From (Konieczny and Pino Pérez 2002) (Theorem 46), we get that $\varphi\ominus\mu = (\varphi\circ\mu)\vee(\mu\circ\varphi)$, for some KM revision operator $\circ$. So by Definition 8, $\ominus$ is a commutative consensual promotion operator. $\square$

## References


Alchourrón, C. E.; Gärdenfors, P.; and Makinson, D. 1985. On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic* 50:510–530.

Bloch, I., and Lang, J. 2000. Towards mathematical morpho-logics. In *Proceedings of the eigth International Conference on Information Processing and Management of Uncertainty in Knowledge based Systems (IPMU'00)*, 1405–1412.

Booth, R., and Meyer, T. 2006. Admissible and restrained revision. *Journal of Artificial Intelligence Research* 26:127–151.

Booth, R.; Fermé, E.; Konieczny, S.; and Pérez, R. P. 2012. Credibility-limited revision operators in propositional logic. In *Proceedings of the Thirteenth International Conference on Principles of Knowledge Representation and Reasoning (KR'12)*.

Caridroit, T.; Konieczny, S.; and Marquis, P. 2015. Contraction in propositional logic. In *Proceedings of the Thirteenth European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'15)*, 186–196.

Dalal, M. 1988. Investigations into a theory of knowledge base revision: Preliminary report. In *Proceedings of the Seventh National Conference on Artificial Intelligence (AAAI'88)*, 475–479.

Darwiche, A., and Pearl, J. 1997. On the logic of iterated belief revision. *Artificial Intelligence* 89:1–29.

Gärdenfors, P. 1988. *Knowledge in flux*. MIT Press.

Hansson, S. O.; Fermé, E.; Cantwell, J.; and Falappa, M. A. 2001. Credibility limited revision. *Journal of Symbolic Logic* 66(4):1581–1596.

Hansson, S. O. 1997a. Semi-revision. *Journal of Applied Non-Classical Logics* 7(1-2):151–175.

Hansson, S. O. 1997b. Special issue on non-prioritized belief revision. *Theoria* 63(1-2).

Hansson, S. O. 1997c. What's new isn't always best. *Theoria* 1–13. Special issue on non-prioritized belief revision.

Jin, Y., and Thielscher, M. 2007. Iterated belief revision, revised. *Artificial Intelligence* 171:1–18.

Katsuno, H., and Mendelzon, A. O. 1991. Propositional knowledge base revision and minimal change. *Artificial Intelligence* 52:263–294.

Katsuno, H., and Mendelzon, A. O. 1992. On the difference between updating a knowledge base and revising it. In *Belief Revision*. Cambridge University Press. 183–203.

Konieczny, S., and Pino Pérez, R. 2002. Merging information under constraints: a logical framework. *Journal of Logic and Computation* 12(5):773–808.

Konieczny, S., and Pino Pérez, R. 2008. Improvement operators. In *Proceedings of the Eleventh International Conference on Principles of Knowledge Representation And Reasoning (KR'08)*, 177–187.

Konieczny, S.; Medina Grespan, M.; and Pino Pérez, R. 2010. Taxonomy of improvement operators and the problem of minimal change. In *Proceedings of the Twelfth International Conference on Principles of Knowledge Representation And Reasoning (KR'10)*, 161–170.

Liberatore, P., and Schaerf, M. 1998. Arbitration (or how to merge knowledge bases). *IEEE Transactions on Knowledge and Data Engineering* 10(1):76–90.

Makinson, D. 1997. Screened revision. *Theoria* 63(1-2):14–23.

Schwind, N.; Inoue, K.; Bourgne, G.; Konieczny, S.; and Marquis, P. 2016. Is promoting beliefs useful to make them accepted in networks of agents? In *Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence (IJCAI'16)*, 1237–1243.