Weighted Attacks in Argumentation Frameworks

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Abstract

Recently, (Dunne *et al.* 2009; 2011) have suggested to weight attacks within Dung's abstract argumentation frameworks, and introduced the concept of WAF (Weighted Argumentation Framework). However, they use WAFs in a very specific way for relaxing attacks. The aim of this paper is to explore ways to take advantage of attacks weights within an argumentation process. Two different approaches are considered: The first one extends the proposal by (Dunne *et al.* 2011) and accounts for other aggregation functions than sum in the objective of relaxing attacks. The second one shows how weights can be exploited to strengthen the usual notion of defence, leading to new concepts of extensions.

Introduction

Many authors have argued about the significance of extending Dung's setting for abstract argumentation by weighting arguments, and accordingly several notions of weighted argumentation frameworks have been defined in the literature. Among them are the Preferential Argumentation Frameworks (PAFs) (Amgoud & Cayrol 2002; Amgoud & Vesic 2011) and the Value-based Argumentation Frameworks (VAFs) (Bench-Capon 2002; 2003).

Recently (Dunne *et al.* 2011) introduces the notion of weighted argumentation frameworks (WAFs), which extends Dung's argumentation frameworks by associating a weight (a positive real number) with each attack. Among other things, Dunne et al. (Dunne *et al.* 2011) provide motivations for extending Dung's setting with such weights, explain how weights can be interpreted, and how they might be derived for several domains. They consider general algorithmic and combinatorial properties of the WAF setting, and show that it is more expressive than four existing settings generalizing Dung's one.

Specifically, the authors of (Dunne *et al.* 2011) use WAF for expanding Dung's extensions, which is expected when extensions are trivial ones (i.e., empty sets) since in this case inference trivializes. The process goes through a relaxation of the usual notion of conflict-free sets of arguments: some inconsistencies are tolerated in sets S of arguments, provided that the sum of the weights of attacks between arguments of S does not exceed a given inconsistency budget β. Admissibility is defined in the standard way, and standard semantics are considered leading to various notions of so-called β-extensions which echo Dung's ones (i.e., grounded, preferred, stable extensions are defined).

This is an interesting proposal, but within it, the full benefits which can be obtained by taking weights into account are not explored. To fill the gap, two possible uses of weights in WAFs are considered in the following:

- We generalize the WAF setting by considering other ways to aggregate weights than using summation. Especially, we provide a more general definition of relaxation, which offers the property of leading to non-trivial extensions whatever the semantics.
- We show how weights can be exploited to define new notions of extensions, through a strengthening of the notion of defence.

Thus weights on the attack relation can be used for defining new notions of extensions from existing ones, with two different purposes. On the one hand, by allowing to expand extensions, by relaxing the notion of attack, which is useful when only trivial extensions exist. On the other hand, by defining new extensions, via a strengthening of the notion of defence. Accordingly, the benefits offered by weights are similar to those achieved by a preference relation over arguments (Amgoud & Vesic 2011).

The rest of the paper is organized as follows. In the following section, classical definitions about Dung's theory and the weighted argumentation frameworks proposed in (Dunne *et al.* 2011) are first recalled. In the third section, we present our proposal for a generalized relaxation method, which ensures to derive non-trivial extensions for any argumentation framework whatever the semantics (among Dung's ones). In the fourth section, we describe how to refine the usual notion of defence using weights. The fifth section discusses some other related works. The last section concludes the paper and gives some perspectives for further research.

Preliminaries

Let us start by presenting some basic definitions at work in Dung's theory of abstract argumentation (Dung 1995).

Definition 1 (argumentation frameworks) *A (finite) argumentation framework is a pair* AF $= \langle A, R \rangle$ *where* A *is a (finite) set of so-called arguments and* R *is a binary relation over* A *(a subset of* $A \times A$*), called the attack relation.*

In order to define a notion of extension, a first important notion is the notion of acceptability: an argument a is acceptable with respect to a set of arguments S whenever it is defended by the set, i.e., every argument which attacks a is attacked by an element of S.

Definition 2 (acceptable sets) *Let* AF $= \langle A, R \rangle$ *be an argumentation framework. An argument* $a \in A$ *is* acceptable *with respect to a subset* S *of* A *if and only if for every* $b \in A$ *s.t.* $(b, a) \in R$*, there exists* $c \in S$ *such that* $(c, b) \in R$*. A set of arguments is acceptable with respect to* S *when each of its elements is acceptable with respect to* S*.*

A second important notion is the notion of absence of conflicts. Intuitively, two arguments should not be considered together whenever one of them attacks the other one.

Definition 3 (conflict-free sets) *Let* AF $= \langle A, R \rangle$ *be an argumentation framework. A subset* S *of* A *is* conflict-free *if and only if for every* $a, b \in S$*, we have* $(a, b) \notin R$*.*

Requiring the absence of conflicts and the form of autonomy captured by self-acceptability leads to the notion of admissible set:

Definition 4 (admissible sets) *Let* AF $= \langle A, R \rangle$ *be an argumentation framework. A subset* S *of* A *is* admissible *for* AF *if and only if* S *is conflict-free and acceptable with respect to* S*.*

The significance of the concept of admissible sets is reflected by the fact that every extension of an argumentation framework under the standard semantics introduced by Dung (i.e., grounded, preferred, and stable) is an admissible set, satisfying some additional criteria:

Definition 5 (extensions) *Let* AF $= \langle A, R \rangle$ *be an argumentation framework and let* $S \subseteq A$*.*

- S *is* a preferred extension *of* AF *if and only if it is maximal (with respect to set inclusion) among the set of admissible sets for* AF *.*
- S *is* a stable extension *of* AF *if and only if* S *is conflict-free and* $\forall a \in A \setminus S$*,* $\exists b \in S$ *such that* $(b, a) \in R$*.*

A more skeptical semantics is based on the characteristic function $\mathcal{F}_{\text{AF}}$ of AF :

Definition 6 (characteristic function) *The* characteristic function $\mathcal{F}_{\text{AF}}$ *of an argumentation framework* AF $=$ $\langle A, R \rangle$ *is defined as follows:* $\mathcal{F}_{\text{AF}} : 2^A \to 2^A$
$\mathcal{F}_{\text{AF}}(S) = \{a \mid a \text{ is acceptable with respect to } S\}$*.*

Definition 7 (grounded extensions) *Let* AF $= \langle A, R \rangle$ *be an argumentation framework. The* grounded extension *of* AF *is the least fixed point of* $\mathcal{F}_{\text{AF}}$ *.*

Dung has shown that every argumentation framework has a (unique) grounded extension and at least one preferred extension, while it may have zero, one or many stable extensions.

These extensions are linked up as follows:

Proposition 1 ((Dung 1995)) *Let* AF *be an argumentation framework. Every preferred (resp. stable) extension of* AF *contains the grounded extension of* AF *.*

Given a semantics σ (grounded, preferred, stable, etc.), $\mathcal{E}_\sigma(\langle A, R \rangle)$ denotes the set of all σ-extensions of $\langle A, R \rangle$.

Now given a set of extensions for a given semantics, one has to make precise the arguments which can be inferred. This calls for an inference relation:

Definition 8 (inference relations) *We note* AF $\mid\!\sim S$ *where* AF $= \langle A, R \rangle$ *is an argumentation framework and* $S \subseteq A$*, to state that* S *is a consequence of* AF *under the inference relation* $\mid\!\sim$*.*

Such inference relations are defined from the extensions of $\langle A, R \rangle$ for a given semantics σ. Two most common inference relations are the skeptical one and the credulous one:

Definition 9 (skeptical and credulous inference) *We denote by* $\mid\!\sim^{\forall,\sigma}$ *the skeptical inference relation defined from the semantics* σ*.* AF $\mid\!\sim^{\forall,\sigma} a$ *if* a *belongs to all* σ*-extensions, i.e.,* AF $\mid\!\sim^{\forall,\sigma} a$ *iff* $\forall E \in \mathcal{E}_\sigma(\langle A, R \rangle),\ a \in E$

We denote by $\mid\!\sim^{\exists,\sigma}$ *the credulous inference relation defined from the semantics* σ*.* AF $\mid\!\sim^{\exists,\sigma} a$ *if* a *belongs to at least one* σ*-extensions, i.e.,* AF $\mid\!\sim^{\forall,\sigma} a$ *iff* $\exists E \in \mathcal{E}_\sigma(\langle A, R \rangle),\ a \in E$

In the following we will focus on skeptical inference, that is the more cautious one.

Let us now turn to the Weighted Argumentation Frameworks as defined in (Dunne *et al.* 2011):

Definition 10 (weighted argumentation framework) *A* Weighted Argumentation Framework *(*WAF*) is a triple* WAF $= \langle A, R, w \rangle$ *where* $\langle A, R \rangle$ *is a Dung-style abstract argumentation framework, and* $w : A \times A \to \mathbb{N}$ *is a function assigning a natural number*[1] *to each attack (i.e.* $w(a, b) > 0$ *iff* $(a, b) \in R$*), and a null value otherwise (*$w(a, b) = 0$ *iff* $(a, b) \notin R$*).*

In (Dunne *et al.* 2011) the weight function is defined as a real value function. In most situations natural numbers are enough, and this simplifies some of the definitions to come. Another point in the definition also differs from the one given in the original definition: the fact that in (Dunne *et al.* 2011) the weight function is defined only for attacks (i.e., $w : R \to \mathbb{R}_*^+$). (Dunne *et al.* 2011) discusses the possibility to assign a 0 weight to an attack (but conclude that *"alllowing* 0*-weight attacks is perhaps counter-intuitive"*), following this view, we consider 0-weight for an ordered pair of arguments (a, b) as meaning "no attack" from a to b.

Let WAF $= \langle A, R, w \rangle$ be a weighted argumentation framework, $\widehat{\text{WAF}}$ denotes the corresponding standard argumentation framework, forgetting the weights, i.e., $\widehat{\text{WAF}} = \langle A, R \rangle$.

[1]We let $\mathbb{N}$ denote the natural numbers greater than or equal to 0, $\mathbb{N}_*$ denotes the natural numbers strictly greater than 0, and $\mathbb{R}_*^+$ denotes the real numbers strictly greater than 0.

Relaxing Extensions

Let us now give our definition of extension relaxation of a WAF , that generalizes the one proposed in (Dunne *et al.* 2011). Let us first define the notion of aggregation function to be considered here and in the remaining sections:

Definition 11 (aggregation functions) *An* aggregation function *is a mapping*[2] *$\oplus$ from $\mathbb{N}^n$ to $\mathbb{N}$, which satisfies:*

- *if $x_i \geq x'_i$, then* $\oplus(x_1, \ldots, x_i, \ldots, x_n) \geq \oplus(x_1, \ldots, x'_i, \ldots, x_n)$ **(non-decreasingness)**
- $\oplus(x_1, \ldots, x_n) = 0$ *if* $\forall i,\ x_i = 0$ **(minimality)**
- $\oplus(x) = x$ **(identity)**

We focus on sum and on max for simplicity in the forthcoming examples, but several other aggregation functions could be considered as well (e.g., leximin, leximax, etc.).

Definition 12 (relaxing attacks using $\oplus$) *Let* WAF $= \langle A, R, w\rangle$ *be a weighted argumentation framework, and $S \subseteq A$. Let us define the aggregation of the weights of the attacks in a set S as:*

$$w_{\oplus}(S, w) = \oplus_{(a,b)\in S} w(a, b)$$

The function $Sub(R, w, \beta)$, that returns the set of subsets of R whose total aggregated weight does not exceed β is defined as:

$$Sub(R, w, \beta) = \{S \mid S \subseteq R \text{ and } w_{\oplus}(S, w) \leq \beta\}$$

Then we can define the $\sigma^{\beta}_{\oplus}$-extensions of a WAF .

Definition 13 ($\sigma^{\beta}_{\oplus}$-extensions) *Given a weighted argumentation framework* WAF $= \langle A, R, w\rangle$*, a semantics σ, and an aggregation function $\oplus$, the set of* $\sigma^{\beta}_{\oplus}$-extensions *of the* WAF *, denoted by $\mathcal{E}^{\oplus,\beta}_{\sigma}(\langle A, R, w\rangle)$, is defined as:* $\mathcal{E}^{\oplus,\beta}_{\sigma}(\langle A, R, w\rangle) = \{E \in \mathcal{E}_{\sigma}(\langle A, R \setminus S\rangle) \mid S \in Sub(R, w, \beta)\}$.

The definition proposed in (Dunne *et al.* 2011) considers only sum as aggregation function for the weights of the WAF . The point is that other aggregation functions can prove more sensible than sum in some contexts.

Example 1 *Let* WAF $= \langle A, R, w\rangle$ *with* $A = \{a, b, c\}$, $R = \{(a, b), (b, c), (c, a)\}$ *and* $w : (a, b) \rightarrow 3; (b, c) \rightarrow 4; (c, a) \rightarrow 5$.[3]

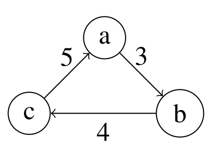

Figure 1: The digraph of WAF

On this example, the grounded extension of the associated classical argumentation framework $\widehat{\text{WAF}}$ *is the empty set:* $\mathcal{E}_{gr}(\widehat{\text{WAF}}) = \{\emptyset\}$*. Interestingly, if relaxed extensions are considered, non-trivial skeptical inferences can be drawn because new extensions are generated, thus* $\mathcal{E}^{\Sigma,3}_{gr} = \mathcal{E}^{\max,3}_{gr} = \{\emptyset, \{b, a\}\}$.

In this example, if the weights are interpreted as numbers of votes for each attack from a set of five agents, then the approaches for merging/aggregating argumentation frameworks using the majority graph (Coste-Marquis *et al.* 2007; Bonzon & Maudet 2011) keep the whole graph $\widehat{\text{WAF}}$ as a result, the grounded extension of which being $\emptyset$. Accordingly, the approach via attacks relaxation, that allows to obtain $\{b, a, \}$ as an extension, can be advocated as an interesting method for merging argumentation frameworks. We keep nevertheless a detailed study of merging weighted argumentation frameworks for future work.

In contrast to what happens in Dung's setting, we observe on this example that several grounded extensions may exist when relaxed extensions are considered. In particular it may happen that the empty set belongs to a set of relaxed extensions, leading the corresponding skeptical inference relation to trivialize. This calls for the following generalized version of skeptical inference:

Definition 14 (non-trivial extensions) *A set of extensions is* non-trivial *if it contains at least one non-empty extension. Given a set of extensions $\mathcal{E}$, the set of* non-empty extensions *of $\mathcal{E}$ is defined as:*

$$\overline{\mathcal{E}} = \begin{cases} \mathcal{E} \setminus \{\emptyset\} & \text{if } \exists E \in \mathcal{E}, E \neq \emptyset \\ \mathcal{E} & \text{otherwise} \end{cases}$$

We take advantage of $\overline{\mathcal{E}}$ in order to obtain non-trivial skeptical inferences when empty extensions exist:

Definition 15 (non-trivial skeptical inference) $\overline{\mid\!\sim}^{\forall,\sigma}$ *denotes the non-trivial skeptical inference relation for the semantics σ.* WAF $\overline{\mid\!\sim}^{\forall,\sigma} a$ *if a belongs to all non-trivial σ-extensions, i.e.,* WAF $\overline{\mid\!\sim}^{\forall,\sigma} a$ *iff* $\forall E \in \overline{\mathcal{E}_{\sigma}(\text{WAF})},\ a \in E$.

This definition does not add much in the case of standard argumentation frameworks, since for most semantics in the classical setting, if the empty set is one of the extensions, it is the only one. However, it proves useful when weighted argumentation frameworks are considered since the latter property does not hold any longer in this case.

Coming back to the definition of relaxed extensions, it can be observed that all relaxations are not equally valuable in the general case. For instance, choosing too large values for β is not interesting, since there always exists a value of β for which all the attacks will be relaxed, leading to the full set of arguments A as the unique extension whatever the semantics.

Proposition 2 *Let* WAF *be a weighted argumentation framework. Then there is a β such that* $\mathcal{E}^{\oplus,\beta}_{\sigma}(\text{WAF})$ *is non-trivial.*

The most interesting value for β is the least one leading to a non-empty extension (for the semantics under consideration). This problem of minimality of β is evoked in (Dunne *et al.* 2011), and the complexity of the associated decision

[2] Strictly speaking, it is a family of relations, one for each n.

[3] For avoiding heavy notations, we refrained from enumerating the pairs of arguments which are given a null weight by the weight function.

problem is studied. Our opinion is that these relaxed extensions for a least value of β are in fact the very aim of the construction, so we define the corresponding extensions in formal terms:

Definition 16 (**$\sigma^{\oplus}$-extensions**) *Given a weighted argumentation framework* WAF $= \langle A, R, w\rangle$*, a semantics* σ*, and an aggregation function* $\oplus$*, the set of* $\sigma^{\oplus}$-extensions *of the* WAF *, denoted by* $\mathcal{E}^{\oplus}_{\sigma}(\langle A, R, w\rangle)$ *is defined as*

$$\mathcal{E}^{\oplus}_{\sigma}(\langle A, R, w\rangle) = \mathcal{E}^{\oplus,\beta}_{\sigma}(\langle A, R, w\rangle)$$

where:

- $\mathcal{E}^{\oplus,\beta}_{\sigma}(\langle A, R, w\rangle)$ *is non-trivial,*
- *There is no* $\beta' < \beta$ *s.t.* $\mathcal{E}^{\oplus,\beta'}_{\sigma}(\langle A, R, w\rangle)$ *is non-trivial.*

To illustrate the need of other aggregation functions than sum, let us stress first that using $\max$ allows to define an equivalent qualitative version of the relaxed extension (with $\max$ as aggregation function one can simply view the set of weights as a set with a total pre-order, instead of a set of numerical values). This is interesting for applications where only a pre-order is available on such "qualitative weights".

As to the generality of the WAF setting, it is first obvious that Dung's setting can be recovered as a specific case of the WAF one. This is also the case for the PAF setting (Amgoud & Cayrol 2002; Amgoud & Vesic 2011). Indeed, in (Dunne *et al.* 2011) Dunne et al. show an inclusion of the grounded extension of a PAF in the set of the sum-based grounded relaxed extensions of a corresponding WAF :

Proposition 3 (**(Dunne *et al.* 2011)**) *Let* $\langle A, R, \succ\rangle$ *be a* PAF*. There is a weight function* $w : A \to \mathbb{R}^{+}_{*}$ *and an inconsistency budget* β *for which* $\mathcal{E}^{\mathrm{PAF}}_{gr}(\langle A, R, \succ\rangle) \subseteq \mathcal{E}^{\Sigma,\beta}_{gr}(\langle A, R, w\rangle)$.

More generally, using $\max$ as aggregation function, we can recover exactly (under weak conditions) the extensions of a PAF (whatever the semantics) as the relaxed extensions of a corresponding WAF:

Proposition 4 *Let* $\langle A, R, \succ\rangle$ *be a PAF.*

- *Let* $R' = R \cup \{(a, a) \mid a \in A\}$
- *Let* w_p *be as follows:*
  - *If* $(a, b) \notin R'$*, then* $w_p(a, b) = 0$*,*
  - *If* $(a, a) \in R$*, then* $w_p(a, a) = 2$*,*
  - *If* $(a, a) \notin R$*, then* $w_p(a, a) = 1$*,*
  - *If* $(a, b) \in R'$ *and* $b \succ a$*, then* $w_p(a, b) = 1$*,*
  - *If* $(a, b) \in R'$ *and* $b \not\succ a$*, then* $w_p(a, b) = 2$.

If $\overline{\mathcal{E}^{PAF}_{\sigma}(\langle A, R, \succ\rangle)} \neq \emptyset$*, then*

$$\mathcal{E}^{PAF}_{\sigma}(\langle A, R, \succ\rangle) = \mathcal{E}^{\max}_{\sigma}(\langle A, R', w_p\rangle).$$

Refining Defence

In this section, we show how weights can be used to refine the usual notion of defence. Intuitively, a defence will be considered to hold if and only if it is strong enough, when compared to the strength of the corresponding attack. As in the PAF framework (Amgoud & Vesic 2011), this amounts to selecting some defences among the standard ones. Formally, we need to define a refined notion of acceptability:

Definition 17 (**$\oplus$-acceptability**) *Let* WAF $= \langle A, R, w\rangle$ *be a weighted argumentation framework,* $S \subseteq A$ *a subset of arguments and* $a \in A$ *an argument. Let* $b \in A$ *be an argument such that* $(b, a) \in R$ *and* $\oplus$ *be an aggregation function. We note* $S_{ab} \subseteq S$ *the subset of arguments defending* a *against* b *(i.e.,* $S_{ab} = \{c \in S \mid (c, b) \in R\}$*). The value* $w_{\oplus_{S_{a\to b}}}$ *is defined as:* $w_{\oplus_{S_{a\to b}}} = \oplus_{c \in S_{ab}} w(c, b)$.

An argument is $\oplus$-acceptable *with respect to* S *if and only if* $\forall b \in A, (b, a) \in R \Rightarrow w_{\oplus_{S_{a\to b}}} \geq w(b, a)$.

So an argument is $\oplus$-acceptable if for each attack against a, the aggregated weight of the defence of a is greater than the weight of the corresponding attack.

Definition 18 (**$\oplus$-admissibility**) *Let* WAF $= \langle A, R, w\rangle$ *be a weighted argumentation framework. A subset of arguments* $S \in A$ *is* $\oplus$-admissible *iff* S *is conflict-free and* $\forall a \in S$*,* a *is* $\oplus$*-acceptable with respect to* S.

Let us illustrate these definitions on a example:

Example 2 *Let* WAF $= \langle A, R, w\rangle$ *with* $A = \{a, b, c\}$*,* $R = \{(a, b), (b, c)\}$ *and* $w : (a, b) \to 2; (b, c) \to 3$*. In the corresponding classical argumentation framework* $\widehat{\text{WAF}} = \langle A, R\rangle$*,* a *defends* c *against* b *so* $\mathcal{E} = \{a, c\}$ *is the preferred, stable and grounded extension and we have:* $\widehat{\text{WAF}} \mid\!\sim^{\forall,\sigma} E$ *for every semantics (stable, preferred or grounded). However,*

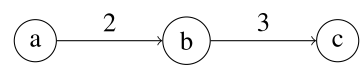

Figure 2: The digraph of WAF

in the weighted argumentation framework WAF *,* a *does not defend* c *any longer against* b*.* c *is not* $\oplus$*-acceptable with respect to* $\{c\}$*, for any aggregation function* $\oplus$.

We can then redefine classical semantics using this notion of $\oplus$-admissibility:

Definition 19 (**$\oplus$-acceptability semantics**) *Let* WAF $= \langle A, R, w\rangle$ *be a weighted argumentation framework and* $S \in A$ *a subset of arguments.*

- S *is a* $\oplus$*-complete extension iff* S *is conflict-free and every argument* $a \in A$ *which is* $\oplus$*-acceptable with respect to* S *belongs to* S*;*
- S *is a* $\oplus$*-grounded extension iff* S *is a* $\oplus$*-complete extension minimal with respect to set inclusion;*
- S *is a* $\oplus$*-preferred extension iff* S *is a* $\oplus$*-admissible set maximal with respect to set inclusion;*
- S *is a* $\oplus$*-stable extension iff* S *is a* $\oplus$*-admissible set and, for every argument* $b \notin S$*, there exists an argument* $a \in S$ *such that* $(a, b) \in R$.

An interesting issue is to determine how classical extensions (i.e., when weights are not taken into account) and their $\oplus$ versions are linked. This first result is trivial (as every $\oplus$-admissible subset of a WAF $= \langle A, R, w\rangle$ is also an admissible set of the corresponding $\widehat{\text{WAF}} = \langle A, R\rangle$):

Proposition 5 *The set of the* $\oplus$*-admissible sets of a* WAF $= \langle A, R, w\rangle$ *is included into the set of the admissible sets of the corresponding* $\widehat{\text{WAF}} = \langle A, R\rangle$.

Then it is easy to derive the following connections between classical extensions and $\oplus$-extensions.

Proposition 6 *Let* WAF $= \langle A, R, w\rangle$ *be a weighted argumentation framework and* $\hat{\text{WAF}} = \langle A, R\rangle$ *the corresponding classical argumentation framework.*

- *Every $\oplus$-preferred extension of* WAF *is included into a preferred extension of* $\hat{\text{WAF}}$.
- *Every $\oplus$-stable extension of* WAF *is also a stable extension of* $\hat{\text{WAF}}$.
- *Every $\oplus$-complete extension of* WAF *is included into a complete extension of* $\hat{\text{WAF}}$.
- *The $\oplus$-grounded extension of* WAF *is included into the grounded extension of* $\hat{\text{WAF}}$.

It turns out that $\oplus$-extensions relate one another in the same way as the classical extensions do:

Proposition 7

- *Every weighted argumentation framework has a unique $\oplus$-grounded extension.*
- *Every weighted argumentation framework has at least one $\oplus$-preferred extension.*
- *Every $\oplus$-stable extension is a $\oplus$-preferred extension.*
- *Every $\oplus$-preferred extension is a $\oplus$-complete extension.*

Other Related Work

(Martinez, Garcia, & Simari 2008) introduce and study another extension of Dung's setting in which different types of attacks can be considered and their respective strengths can be compared. Classical admissibility of an argument a can then be generalized to take account of the relative strength of the attacker of a and the corresponding defenders (for each attacker). The setting pointed out in (Martinez, Garcia, & Simari 2008) achieves a high level of generality so that WAFs are specific AFVs (argumentation frameworks with varied strength attack). Indeed, for a given WAF, there are as many attack types in the corresponding AFV as there are different weights $w((a, b))$ for $(a, b) \in A \times A$, and the relation R used to compare attack strengths amounts to the standard total ordering over numbers. However, there is some price to be paid for such a generality. Thus, since R is just required to be reflexive, it can be the case that an argument a belongs to all admissible sets which are maximal w.r.t. $\subseteq$ without belonging to a top-admissible set, which questions the interest of top-admissibility in this case.[4] Furthermore, (Martinez, Garcia, & Simari 2008) do not investigate the benefits which may result from a *collective defence* of an argument (they only consider individual defence), so that an argument a cannot be considered as acceptable in an AFV when none of its defenders alone has sufficient strength to do it, but there are in some sense sufficiently many such defenders. Accordingly, no aggregation of attack strengths is considered in the AFV setting. This makes a very significant difference with our approach.

[4]This may occur when there is a cycle of strict "preferences" between defenders of a.

Conclusion

In this paper we explored the benefits which can be gained by weighting the attack relation in Dung's abstract argumentation frameworks. We proposed two different ways of improvement. The first one consists in generalizing the relaxation process proposed in (Dunne *et al.* 2011), which ensures to draw non-trivial inferences from any argumentation framework. The second one consists in refining the defense notion, taking the attack weights into account. Basically we ignore the attacks against an argument that are not strong enough given the aggregation of the weights of its defence. Weights in a WAF can also be used to select some extensions of the corresponding AF. We will develop this aspect in a forthcoming paper.

References

Amgoud, L., and Cayrol, C. 2002. A reasoning model based on the production of acceptable arguments. *Annals of Mathematics and Artificial Intelligence* 34(1–3):197–215.

Amgoud, L., and Vesic, S. 2011. Two roles of preferences in argumentation frameworks. In *Proceedings of the Eleventh European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'11)*, volume 6717 of *Lecture Notes on Artificial Intelligence*, 86–97.

Bench-Capon, T. 2002. Value Based Argumentation Frameworks. *Proceedings of the Ninth International Workshop on Non-Monotonic Reasoning (NMR'02)* 444–453.

Bench-Capon, T. 2003. Persuasion in Practical Argument Using Value-based Argumentation Frameworks. *Journal of Logic and Computation* 13(3):429–448.

Bonzon, E., and Maudet, N. 2011. On the Outcomes of Multiparty Persuasion. In *Proceedings of the Tenth International Conference on Autonomous Agents and Multiagent Systems (AAMAS'11)*, 47–54.

Coste-Marquis, S.; Devred, C.; Konieczny, S.; Lagasquie-Schiex, M.-C.; and Marquis, P. 2007. On the merging of Dung's argumentation systems. *Artificial Intelligence* 171:730–753.

Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77(2):321–358.

Dunne, P.; Hunter, A.; McBurney, P.; Parsons, S.; and Wooldridge, M. 2009. Inconsistency Tolerance in Weighted Argument Systems. In *Proceedings of the Eighth International Conference on Autonomous Agents and Multiagent Systems (AAMAS'09)*, 851–858.

Dunne, P.; Hunter, A.; McBurney, P.; Parsons, S.; and M. Woolridge, M. 2011. Weighted Argumentation Systems: Basic Definitions, Algorithms and Complexity Results. *Artificial Intelligence* 175(2):457–486.

Martinez, D.; Garcia, A.; and Simari, G. 2008. An Abstract Argumentation Framework with Varied Strength Attacks. In *Proceedings of the Eleventh International Conference on Knowledge Representation and Reasoning (KR'08)*, 135–144.