On the Difference between Merging Knowledge Bases and Combining them

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Abstract

We investigate the logical properties of knowledge base combination operators proposed in the literature. These operators are based on the selection of some maximal subsets of the union of the knowledge bases. We argue that they are not fully satisfactory to merge knowledge bases, since the source of information is lost in the combination process. We show that it is the reason why those operators do not satisfy a lot of logical properties. Then we propose to use more refined selection mechanisms in order to take the distribution of information into account in the combination process. That allows to define merging operators with a more subtle behaviour.

1 INTRODUCTION

In the fields of artificial intelligence and databases, one is often faced with conflicting information coming from several sources. Thus, an important problem in such cases is how to reach a coherent piece of information from these contradictory ones.

For example, if one wants to build an expert system from a group of human experts, it is sensible to code the knowledge of each expert in a knowledge base and then to combine them in a knowledge base that represents the knowledge of the group. This process allows to discover new pieces of knowledge distributed among the sources. For example, if an expert knows that a is true and another one knows that $a \to b$ holds, then the "synthesized" knowledge knows that b is true whereas none of the expert knows it. This is called *implicit knowledge* in [HM92]. However, simply put these knowledge bases together is a wrong way since there could be contradictions between some experts.

Some combination operators have been proposed, see *e.g.* [BKM91, BKMS92]. They are all based on the union of all the knowledge bases and on the selection of some maximal subsets, due to a given order (not necessarily the inclusion).

We study the logical properties of these operators. More exactly we investigate the rationality of these operators through the logical characterization of merging operators stated in [KP98, KP99]. This characterization is useful to classify particular merging methods and to highlight flaws and advantages of each of them.

In particular, an important drawback of combination operators is that the source of each knowledge is lost in the fusion process. We shall call merging operators the fusion operators that take the source of information into account. We propose in this paper a definition of selection functions *à la* AGM [AGM85, Gär88] for combination operators that allows to take into account the source of each piece of information. So we can define operators with a more subtle behaviour.

In order to motivate the need to take the source of information into account, consider the following scenario: Consider that we want to combine the following knowledge bases: $K_1 = K_2 = \{a, b\}$, $K_3 = \{a, b \to c\}$, $K_4 = \{\neg a, d\}$. Then the union of the knowledge bases is $\{a, \neg a, b, b \to c, d\}$. With a combination operator the maxiconsistent sets will be $\{a, b, b \to c, d\}$ and $\{\neg a, b, b \to c, d\}$.With this result we can not decide whether a or $\neg a$ holds. But a is supported by three of the four experts whereas only one supports $\neg a$. So it could be sensible to put a in the resulting knowledge base. Combination operators do not allow to take such arguments into account. We will see how to build merging operators that allow that behaviour.

The rest of the paper is organized as follows: In section 2 we give some definitions and state some notations. In section 3 we give a set of logical properties for merging operators. In section 4 we investigate the logical properties of some combination operators given in [BKMS92]. In section 5 we propose to use selection functions in order to define merging operators taking care of the distribution of information among the sources. Finally we discuss open problems in a concluding section.

2 PRELIMINARIES

Definition 1 *A knowledge base K is a finite set of well-formed first-order formulae.*

Note that a knowledge base is not necessarily closed under consequence relation.

Definition 2 *A knowledge set E is a multi-set of knowledge bases.*

We will note $\sqcup$ the union on multi-sets. By abuse if K is a knowledge base, K will also denote the knowledge set $E = \{K\}$. And if K and K' are two knowledge bases we will denote $K \sqcup K'$ the knowledge set $E = \{K, K'\}$.

For the proofs, we need to define a partition operator $\oplus$. $K = K' \oplus K''$ denotes that $K' \cup K'' = K$ and that $K' \cap K'' = \emptyset$.

Let K and K' be two knowledge bases, $K \wedge K'$ will denote the knowledge base $K \cup K'$. Analogously, if $E = \{K_1, \ldots, K_n\}$, then $E \wedge K = \{K_1 \wedge K, \ldots, K_n \wedge K\}$. We will note $\bigwedge E$ the conjunction of the knowledge bases of E, i.e. $\bigwedge E = K_1 \wedge \ldots \wedge K_n$.

Definition 3 *A knowledge set E is consistent if and only if $\bigwedge E$ is consistent.*

In addition to these basic definitions, we have to define the equivalence between knowledge sets.

Definition 4 *Let E_1, E_2 be two knowledge sets. E_1 and E_2 are equivalent, noted $E_1 \leftrightarrow E_2$, iff there exists a bijection f from $E_1 = \{K_1^1, \ldots, K_n^1\}$ to $E_2 = \{K_1^2, \ldots, K_n^2\}$ such that $f(K) \leftrightarrow K$.*

Let E be a knowledge set, E^n will denote the knowledge set $\underbrace{E \sqcup \ldots \sqcup E}_{n}$.

The result of the combination operators investigated in this paper is a set of knowledge bases. These sets have been called *flocks* by Fagin *et al.* [FKUV86].

Note that flocks and knowledge sets are both sets of knowledge bases (in fact knowledge sets are multi-sets). But the difference is that in the case of knowledge sets, the sets denote different sources of information, whereas in flocks the sets denote alternatives about the result of a combination. Flocks are similar to the extensions in default knowledge bases [Rei80, Eth88]. In order to underline this difference we will note K the elements of a knowledge set and M, P, Q, R the elements of a flock.

In order to investigate the logical properties of combination operators we need to define what are the consequences of a flock. We will adopt a cautious approach that considers, in a sense, flocks as disjunctions of knowledge bases.

Definition 5 *Let $\mathcal{F} = \langle M_1, \ldots, M_n \rangle$ be a flock. If $\mathcal{F} = \emptyset$, then $\mathcal{F}$ is inconsistent and, as usual, $Cn(\mathcal{F})$ is the set of all formulae. Else we define*

$$Cn(\mathcal{F}) = \bigcap_{i=1}^{n} Cn(M_i)$$

We can then define equivalence between flocks:

Definition 6 *Let $\mathcal{F}$ and $\mathcal{F}'$ be two flocks, we say that $\mathcal{F}$ implies $\mathcal{F}'$, noted $\mathcal{F} \vdash \mathcal{F}'$ if $Cn(\mathcal{F}) \supseteq Cn(\mathcal{F}')$. $\mathcal{F}$ and $\mathcal{F}'$ are equivalent, noted $\mathcal{F} \equiv \mathcal{F}'$, if both $\mathcal{F} \vdash \mathcal{F}'$ and $\mathcal{F}' \vdash \mathcal{F}$ hold. Similarly, we define $\mathcal{F} \vdash K$ where K is a knowledge base as $Cn(\mathcal{F}) \supseteq Cn(K)$.*

Definition 7 *Let $\mathcal{F} = \langle M_1, \ldots, M_n \rangle$ and $\mathcal{F}' = \langle M'_1, \ldots, M'_m \rangle$ be two flocks. $\mathcal{F} \vee \mathcal{F}'$ denotes the flock $\langle M_1, \ldots, M_n, M'_1, \ldots, M'_m \rangle$ and $\mathcal{F} \wedge \mathcal{F}'$ denotes the flock $\langle M''_{1,1}, \ldots, M''_{n,m} \rangle$ where $M''_{j,k} = M_j \cup M'_k$.*

So notice that if $\mathcal{F} = \langle M_1, \ldots, M_n \rangle$ and $\mathcal{F}' = \mathcal{F} \vee \langle P_1, \ldots, P_m \rangle$ with $\forall i\ P_i$ inconsistent, then $\mathcal{F} \equiv \mathcal{F}'$. So, when considering a flock, we can focus only on its consistent knowledge bases.

3 IC MERGING OPERATORS

In [KP98, KP99] a set of logical properties for merging operators is stated. We call operators satisfying these postulates Integrity Constraints merging operators (IC merging operators for short). Next we recall those postulates.

Definition 8 *Let E be a knowledge set, let IC be a knowledge base coding the integrity constraints of the merging, and let $\triangle$ be an operator that assigns to each knowledge set E and knowledge base IC a knowledge base $\triangle_{IC}(E)$. $\triangle$ is an IC merging operator if and only if it satisfies the following properties:*

(IC0) $\triangle_{IC}(E) \vdash IC$

(IC1) *If IC is consistent, then $\triangle_{IC}(E)$ is consistent*

(IC2) *If $\bigwedge E$ is consistent with IC, then* $\triangle_{IC}(E) = \bigwedge E \wedge IC$

(IC3) *If* $E_1 \leftrightarrow E_2$ *and* $IC_1 \leftrightarrow IC_2$, *then* $\triangle_{IC_1}(E_1) \leftrightarrow \triangle_{IC_2}(E_2)$

(IC4) *If* $K \vdash IC$ *and* $K' \vdash IC$, *then* $\triangle_{IC}(K \sqcup K') \wedge K \nvdash \bot \Rightarrow \triangle_{IC}(K \sqcup K') \wedge K' \nvdash \bot$

(IC5) $\triangle_{IC}(E_1) \wedge \triangle_{IC}(E_2) \vdash \triangle_{IC}(E_1 \sqcup E_2)$

(IC6) *If* $\triangle_{IC}(E_1) \wedge \triangle_{IC}(E_2)$ *is consistent, then* $\triangle_{IC}(E_1 \sqcup E_2) \vdash \triangle_{IC}(E_1) \wedge \triangle_{IC}(E_2)$

(IC7) $\triangle_{IC_1}(E) \wedge IC_2 \vdash \triangle_{IC_1 \wedge IC_2}(E)$

(IC8) *If* $\triangle_{IC_1}(E) \wedge IC_2$ *is consistent, then* $\triangle_{IC_1 \wedge IC_2}(E) \vdash \triangle_{IC_1}(E)$

Most of these postulates are a generalization of belief revision postulates [AGM85, Gär88, KM91]. (IC0) states that the result of the merging complies with the integrity constraints. (IC1) ensures that, when the integrity constraints are consistent, we always manage to extract a coherent piece of information from the knowledge set. (IC2) says that, if it is possible, the result of the merging is simply the conjunction of the knowledge bases of the knowledge set with the integrity constraints. (IC3) is the principle of irrelevance of syntax. It states that if two knowledge sets are equivalent and two integrity constraints knowledge bases are equivalent, then the result of the merging of each knowledge set under their respective integrity constraints will give two equivalent knowledge bases. The purely "merging" postulates are (IC4),(IC5) and (IC6). (IC4) is what we call the fairness postulate. It ensures that when one merges two knowledge bases, it can not give the preference to one of them. (IC5) and (IC6) correspond to Pareto's conditions in Social Choice Theory [Arr63]. (IC5) states that if a group compromises on a set of alternatives A belongs to, and another group compromises on another set of alternatives which contains also A, then A has to be in the chosen alternatives if we join the two groups. (IC6) states that if a group prefers strictly an alternative A to an alternative B and another group finds A and B equally plausible, then A will be preferred to B if we join the two groups. Finally (IC7) and (IC8) state conditions on the conjunction of integrity constraints. It ensures that the notion of "closeness" is well-behaved. See [KP99] for a full motivation of this set of postulates and for a semantical characterization in terms of family of pre-orders on interpretations.

There are two major subclasses of merging operators, namely majority and arbitration operators. Whereas majority operators try to satisfy the majority of the protagonists, arbitration operators try to satisfy each protagonist to the best possible degree.

A majority merging operator is an IC merging operator that satisfies the following property:

(Maj) $\exists n \;\; \triangle_{IC}(E_1 \sqcup {E_2}^n) \vdash \triangle_{IC}(E_2)$

This postulate expresses the fact that if an opinion has a large audience, it will be the opinion of the group.

An arbitration operator is an IC merging operator that satisfies the following property:

$$\textbf{(Arb)} \quad \left.\begin{array}{l} \triangle_{IC_1}(K_1) \leftrightarrow \triangle_{IC_2}(K_2) \\ \triangle_{IC_1 \leftrightarrow \neg IC_2}(K_1 \sqcup K_2) \leftrightarrow (IC_1 \leftrightarrow \neg IC_2) \\ IC_1 \nvdash IC_2 \\ IC_2 \nvdash IC_1 \end{array}\right\}$$

$$\Rightarrow \quad \triangle_{IC_1 \vee IC_2}(K_1 \sqcup K_2) \leftrightarrow \triangle_{IC_1}(K_1)$$

From a semantical point of view (Arb) ensures that it is the median possible worlds that are chosen, that is if K_1 prefers strictly a world A to a world B and if K_2 prefers strictly A to a world C and if B and C are equally desirable for the merging, then A will be strictly preferred to B and C for the merging (*cf* [KP99]).

Another property, opposed to the majority postulate, we can mention is the *majority independence* which is the following one:

(MI) $\forall n \;\; \triangle_{IC}(E_1 \sqcup {E_2}^n) \leftrightarrow \triangle_{IC}(E_1 \sqcup E_2)$

That very strong property states that the result of the merging is fully independent of the popularity of the views but simply takes into account each different view. But the following results hold [KP98]:

Theorem 1 *(i). There is no IC merging operator satisfying (MI).*
(ii). If an operator satisfies (IC1), then it can't satisfy both of (MI) and (Maj).

4 COMBINATION OPERATORS

Baral, Kraus, Minker and Subrahmanian proposed in [BKM91, BKMS92] several theory merging operators, these operators are based on a selection of maxiconsistent subsets in the union of the knowledge bases of the knowledge set.

Once the union of the knowledge bases is settled, the problem is to find a coherent information from an inconsistent knowledge base. Thus, such a definition is very close to Brewka's preferred subtheories [Bre89] and to the work of Benferhat *et al.* on entailment in inconsistent databases [BCD+93, BDP97, BDL+98].

Definition 9 *Let* MAXCONS(K, IC) *be the set of maximal (with respect to inclusion) consistent subsets of*

$K \wedge IC$ *which contain* IC*, i.e.* $\text{MAXCONS}(K, IC)$ *is the set of all* M *such that*

- $M \subseteq K \wedge IC$,
- $IC \subseteq M$,
- *if* $M \subset M' \subseteq K \wedge IC$*, then* $M' \vdash \bot$.

Let $\text{MAXCONS}(E, IC) = \text{MAXCONS}(\bigwedge E, IC)$*. We will use the subscript* $\text{MAXCONS}_{card}(E, IC)$ *when the maximality of the sets is with respect to cardinality.*

Let's define the following operators:

Definition 10 *Let* E *be a knowledge set and* IC *be a knowledge base:*

$\triangle^{C1}{}_{IC}(E) = \text{MAXCONS}(E, IC)$
$\triangle^{C3}{}_{IC}(E) = \{M : M \in \text{MAXCONS}(E, \top)$ *and* $M \wedge IC$ *consistent*$\}$
$\triangle^{C4}{}_{IC}(E) = \{M : M \in \text{MAXCONS}_{card}(E, IC)\}$
$\triangle^{C5}{}_{IC}(E) = \{M \wedge IC : M \in \text{MAXCONS}(E, \top)$ *and* $M \wedge IC$ *consistent* $\}$ *if this set is non empty and* IC *otherwise.*

The $\triangle^{C1}{}_{IC}(E)$ operator takes as result of the combination the set of maximal consistent subsets of $E \wedge IC$ which contain the constraints IC. The $\triangle^{C3}{}_{IC}(E)$ operator computes first the set of maximal consistent subsets of E, and then selects those that are consistent with the constraints. The $\triangle^{C4}{}_{IC}(E)$ operator selects the set of consistent subsets of $E \wedge IC$ which contain the constraints IC and that are maximal with respect to cardinality.

The operators $\triangle^{C1}{}_{IC}(E)$, $\triangle^{C3}{}_{IC}(E)$ and $\triangle^{C4}{}_{IC}(E)$ correspond respectively to the operators $Comb1(E, IC)$, $Comb3(E, IC)$ and $Comb4(E, IC)$ in [BKMS92]. The $\triangle^{C5}$ operator is a slight modification of $\triangle^{C3}$ in order to grasp more logical properties.

In the following theorems we investigate the logical properties of the operators defined above.

Theorem 2 *The* $\triangle^{C1}$ *operator satisfies (IC0), (IC1), (IC2), (IC4), (IC5), (IC7), and (MI). It does not satisfy (IC3), (IC6), (IC8) and (Maj).*

Proof: (IC0), (IC1) and (IC2) are satisfied by definition of the $\triangle^{C1}$ operator.

$\triangle^{C1}$ satisfies (IC4) because the stronger following property is satisfied:

If $K \vdash IC$ then $\triangle^{C1}{}_{IC}(K \sqcup K') \wedge K \nvdash \bot$.

Since K is a consistent subset of $K \wedge K' \wedge IC$ and by hyp. $K \vdash IC$, then there exists a maxiconsistent subset of $K \wedge K' \wedge IC$ that contains K. And then $\triangle^{C1}{}_{IC}(K \sqcup K') \wedge K \nvdash \bot$.

(IC5) holds. It is trivially true when $\triangle_{IC}(E_1) \wedge \triangle_{IC}(E_2)$ is not consistent. And if $\triangle_{IC}(E_1) \wedge \triangle_{IC}(E_2)$ is consistent, let P_i be the elements of $\text{MAXCONS}(E_1 \sqcup E_2, IC)$, Q_i the elements of $\text{MAXCONS}(E_1, IC)$, and R_i the elements of $\text{MAXCONS}(E_2, IC)$. To show that (IC5) holds it is enough to prove that if $Q_j \wedge R_k$ is consistent then $\exists i \; P_i = Q_j \wedge R_k$. First put $L = IC \cup \bigwedge E_1 \cup \bigwedge E_2$, $Q_j \wedge R_k$ is a consistent (with IC) subset of L, and as P_i is a maxiconsistent (with IC) subset of L, then $\exists i \; P_i \supseteq Q_j \wedge R_k$. We claim that $P_i \subseteq Q_j \wedge R_k$ holds. Because otherwise we have $P_i \supset Q_j \wedge R_k$. Then we can decompose $P_i = P_1 \wedge P_2$ with $P_1 = P_i \cap (\bigwedge E_1 \wedge IC)$ and $P_2 = P_i \cap (\bigwedge E_2 \wedge IC)$. And then we have that $P_1 \supset Q_j$ or $P_2 \supset R_k$. So either Q_j or R_k is not a maximum consistent subset of E_1 or E_2 respectively. Contradiction.

$\triangle^{C1}$ does not satisfy (IC6). Consider the following example: $K_1 = \{a \rightarrow c, e \rightarrow \neg c, b \rightarrow \neg c\}$, $K_2 = \{a, e\}$, $K_3 = \{b\}$ and $IC = \top$. Then $\triangle^{C1}(K_1 \sqcup K_2) \wedge \triangle^{C1}(K_3)$ is consistent but $\triangle^{C1}(K_1 \sqcup K_2 \sqcup K_3) \nvdash \triangle^{C1}(K_1 \sqcup K_2) \wedge \triangle^{C1}(K_3)$.

(IC7) is satisfied. When $\triangle_{IC_1}(E) \wedge IC_2$ is not consistent (IC7) is trivial. Otherwise let P_i be the elements of $\text{MAXCONS}(E, IC_1)$ and Q_i the elements of $\text{MAXCONS}(E, IC_1 \wedge IC_2)$. It is enough to prove that if $P_i \wedge IC_2$ is consistent then $\exists Q_j$ such that $P_i \wedge IC_2 = Q_j$. Let $P_i \wedge IC_2$ be a consistent subset of $\bigwedge E \wedge IC_1 \wedge IC_2$ then there exists a maxiconsistent subset Q_j that contains $P_i \wedge IC_2$, so $\exists j \; P_i \subseteq Q_j$. Moreover, we have that $P_i \wedge IC_2 \supseteq Q_j$, otherwise $P_i \wedge IC_2 \subset Q_j$, and from this it is easy to see that $P_i \subset Q_j \cap (\bigwedge E \wedge IC_1)$. So P_i is not maximum. Contradiction.

(IC8) is not satisfied. We use the counterexample to (IC6) slightly modified: $K = \{a \rightarrow c, e \rightarrow \neg c, b \rightarrow \neg c\}$, $IC_1 = \{a, e\}$ and $IC_2 = \{b\}$. Then $\triangle^{C1}{}_{IC_1}(K) \wedge IC_2$ is consistent but $\triangle^{C1}{}_{IC_1 \wedge IC_2}(K) \nvdash \triangle^{C1}{}_{IC_1}(K) \wedge IC_2$.

■

Theorem 3 *The* $\triangle^{C3}$ *operator satisfies (IC4), (IC5), (IC7), (IC8), (MI). It does not satisfy (IC0), (IC1), (IC2), (IC3), (IC6) and (Maj).*

Proof: $\triangle^{C3}$ does not satisfy (IC0) since the result is only consistent with IC. It does not satisfy (IC1) since when there is no maxiconsistent consistent with IC the disjunction is empty and the result is $\bot$. And it does not satisfy (IC2), we have instead: If $\bigwedge E$ is consistent with IC, then $\triangle^{C3}{}_{IC}(E) = \bigwedge E$.

The proofs of (IC4) and (IC5) for $\triangle^{C3}$ are similar to the ones for $\triangle^{C1}$.

Table 1: Properties of combination operators

	IC0	IC1	IC2	IC3	IC4	IC5	IC6	IC7	IC8	MI	Maj
$\triangle^{C1}$	✓	✓	✓	–	✓	✓	–	✓	–	✓	–
$\triangle^{C3}$	–	–	–	–	✓	✓	–	✓	✓	✓	–
$\triangle^{C4}$	✓	✓	✓	–	–	–	–	✓	✓	✓	–
$\triangle^{C5}$	✓	✓	✓	–	✓	✓	–	✓	✓	✓	–

$\triangle^{C3}$ does not satisfy (IC6). There is the same counterexample than for $\triangle^{C1}$.

(IC7) and (IC8) are satisfied directly. If ${\triangle^{C3}}_{IC_1}(E) \wedge IC_2$ is not consistent (IC7) is trivial. Otherwise since by definition ${\triangle^{C3}}_{IC_1}(E) = \{Q : Q \in \text{MAXCONS}(E, \top)$ and $Q \wedge IC_1$ consistent$\}$, then ${\triangle^{C3}}_{IC_1}(E) \wedge IC_2 \equiv \{Q : Q \in \text{MAXCONS}(E, \top)$ and $Q \wedge IC_1 \wedge IC_2$ consistent$\}$ (this set is non empty by hypothesis) what is by definition ${\triangle^{C3}}_{IC_1 \wedge IC_2}(E)$. ■

Theorem 4 *The $\triangle^{C4}$ operator satisfies (IC0), (IC1), (IC2), (IC7), (IC8) and (MI). It does not satisfy (IC3), (IC4), (IC5), (IC6) and (Maj).*

Proof: (IC0), (IC1) and (IC2) are satisfied by definition.

$\triangle^{C4}$ does not satisfy (IC4): Let's take $IC = \{a, b\}$, $K = \{a, b, c, d\}$, $K' = \{a, b, c \rightarrow \neg d, d \rightarrow \neg c\}$. Then ${\triangle^{C4}}_{IC}(K \sqcup K') = \{\{a, b, c, c \rightarrow \neg d, d \rightarrow \neg c\}, \{a, b, d, c \rightarrow \neg d, d \rightarrow \neg c\}\}$, so ${\triangle^{C4}}_{IC}(K \sqcup K') \wedge K' \nvdash \bot$ and ${\triangle^{C4}}_{IC}(K \sqcup K') \wedge K \vdash \bot$.

(IC5) does not hold. Consider the following example: Let $K_1 = \{a\}$, $K_2 = \{\neg a \wedge b\}$, $K_3 = \{\neg a \wedge c\}$. Then with $E_1 = K_1 \sqcup K_2$, $E_2 = K_1 \sqcup K_3$ and $IC = \top$, we have ${\triangle^{C4}}_{IC}(E_1) \wedge {\triangle^{C4}}_{IC}(E_2) \nvdash {\triangle^{C4}}_{IC}(E_1 \sqcup E_2)$.

(IC6) is not satisfied. Consider the following example: Let $K_1 = \{a\}$, $K_2 = \{\neg a \wedge (b \vee c)\}$, $K_3 = \{a, a \wedge z\}$, $K_4 = \{\neg a \wedge \neg b, \neg a \wedge \neg c\}$. Then with $E_1 = K_1 \sqcup K_2$, $E_2 = K_3 \sqcup K_4$ and $IC = \top$, we have ${\triangle^{C4}}_{IC}(E_1) \wedge {\triangle^{C4}}_{IC}(E_2)$ consistent but ${\triangle^{C4}}_{IC}(E_1 \sqcup E_2) \nvdash {\triangle^{C4}}_{IC}(E_1) \wedge {\triangle^{C4}}_{IC}(E_2)$.

(IC7) and (IC8) are satisfied. When ${\triangle^{C4}}_{IC_1}(E) \wedge IC_2$ is not consistent (IC7) and (IC8) are satisfy straightforwardly. So assume that ${\triangle^{C4}}_{IC_1}(E) \wedge IC_2$ is consistent. Let P be an element of $\text{MAXCONS}_{card}(E, IC_1 \wedge IC_2)$. And let Q be an element of $\text{MAXCONS}_{card}(E, IC_1)$ consistent with IC_2. We want to show that $Q \cup IC_2 \in \text{MAXCONS}_{card}(E, IC_1 \wedge IC_2)$ and that P can be rewritten $P_1 \oplus P_2$ with $P_1 \in \text{MAXCONS}_{card}(E, IC_1)$ and $P_2 \subseteq IC_2$. This is enough to show that ${\triangle^{C4}}_{IC_1}(E) \wedge IC_2 \leftrightarrow {\triangle^{C4}}_{IC_1 \wedge IC_2}(E)$. Let's define $A_1 = \bigwedge E \cup IC_1$ and $A_2 = IC_2 \setminus A_1$. Then we can split $P = P_1 \oplus P_2$ with $P_1 = P \cap A_1$ and $P_2 = P \cap A_2$. Similarly let's define $Q \cup IC_2 = Q_1 \oplus Q_2$ such that $Q_1 = (Q \cup IC_2) \cap A_1$ and $Q_2 = (Q \cup IC_2) \cap A_2$. As $IC_2 \subseteq P$ and $IC_2 \subseteq Q \cup IC_2$, by construction it is easy to see that $P_2 = A_2 = Q_2$. In terms of cardinalities $|P| = |P_1| + |P_2|$ and $|Q \cup IC_2| = |Q_1| + |Q_2|$. But we have that $|P_2| = |Q_2|$. As P is in $\text{MAXCONS}_{card}(E, IC_1 \wedge IC_2)$ and $Q \cup IC_2 \subseteq E \cup IC_1 \cup IC_2$, then $|P| \geq |Q \cup IC_2|$. So $|P_1| \geq |Q_1|$. Similarly as $Q_1 = Q$ is in $\text{MAXCONS}_{card}(E, IC_1)$ and $P_1 \subseteq E \cup IC_1$, then $|P_1| \leq |Q_1|$. From this it is easy to see that both $|P| = |Q \cup IC_2|$ and $|Q| = |P_1|$ hold. So $Q \cup IC_2$ is in $\text{MAXCONS}_{card}(E, IC_1 \wedge IC_2)$, and P_1 is in $\text{MAXCONS}_{card}(E, IC_1)$. ■

Theorem 5 *The $\triangle^{C5}$ operator satisfies (IC0), (IC1), (IC2), (IC4), (IC5), (IC7), (IC8), (MI). It does not satisfy (IC3), (IC6) and (Maj).*

Proof: The proofs are mainly the same that for theorem 3 except (IC0), (IC1) and (IC2) that are now satisfied by definition. ■

We sum up the previous results in Table 1. The symbol ✓ (respectively –) in a square means that the corresponding operator satisfies (resp. does not satisfy) the corresponding postulate. By construction all the operators satisfy (MI). We can also note that none of these operators satisfies (IC6). We will see in the next section how to build merging operators with more logical properties.

None of the operators we study in this paper satisfies (IC3) since they are all syntax sensitive. We can illustrate this on the following example. Consider three knowledge bases $K_1 = \{a, b\}$, $K_2 = \{a \wedge b\}$, and $K_3 = \{\neg b\}$. Let $E_1 = K_1 \sqcup K_3$ and $E_2 = K_2 \sqcup K_3$ be two knowledge sets. The maxiconsistent subsets of E_1 are $\{a, b\}$ and $\{a, \neg b\}$, the ones of E_2 are $\{a \wedge b\}$ and $\{\neg b\}$. So each maxiconsistent of E_1 implies a, whereas it is not the case for E_2. So, although the knowledge bases K_1 and K_2 are logically equivalent, with the syntactical operators studied in this paper, the result of the fusion of E_1 will imply a, whereas it will not be the case with the fusion of E_2.

5 SELECTION FUNCTION AND MERGING OPERATORS

The combination operators do not take into account the "individual side" of the merging, since the source of information doesn't matter in the combination process. They simply put all the pieces of information together and then select some maximal consistent subsets. So, with this approach, it is not possible to try to reach the best consensus between protagonists. We can not for example select only the maxiconsistent sets that fit the majority of agents. In the same way we can not try to arbitrate these views, that is to satisfy all the protagonists to the best possible degree. The idea in this section is to use a selection function to choose among the maxiconsistent subsets, those that best fit a "merging criterion".

The motivation to define such selection functions comes from AGM revision framework [AGM85, Gär88]. The $\triangle^{C1}$ operator corresponds to full meet contraction function [AM82, Gär88] that has been shown to be unsatisfactory for a revision since it drops too much information. Partial meet contraction functions, defined from selection functions by choosing only some of the maxiconsistents, have been shown to have a less drastic behaviour.

In this section, we will examine some selection merging operators. The idea of this kind of operators is to give the preference to the maxiconsistents that are closer to the agents' view. The differences between these operators lie firstly in the definition of the "distance" between a maxiconsistent and a knowledge base, and secondly in the aggregation of these results to define the "distance" between a maxiconsistent and a knowledge set.

We will focus on operators defined from the $\triangle^{C1}$ operator and investigate their logical properties. But the following methods can also be used with the other combination operators.

5.1 DRASTIC MAJORITY OPERATOR

The "distance" between knowledge bases we will consider here is drastic. We will set this distance to 0 if the conjunction of these two knowledge bases is consistent and 1 otherwise.

Definition 11 *Consider a knowledge set E and a knowledge base M.*

- $dist_{\mathrm{D}}(M,K) = \begin{cases} 0 & \text{if } M \wedge K \text{ consistent} \\ 1 & \text{otherwise} \end{cases}$
- $dist_{\mathrm{D}}(M,E) = \sum_{K \in E} dist_{\mathrm{D}}(M,K)$
- $\triangle^{\mathrm{D}}_{IC}(E) = \{M \in \triangle^{C1}{}_{IC}(E) : dist_{\mathrm{D}}(M,E) = \min_{M_i \in \triangle^{C1}{}_{IC}(E)} (dist_{\mathrm{D}}(M_i,E))\}$

So this selection function chooses among the maxiconsistents those that are consistent with a maximum of knowledge bases.

It is easy to see that when a maxiconsistent is consistent with a knowledge base it contains this knowledge base. So it amounts to choose the maxiconsistents that contain a maximum of knowledge bases.

Theorem 6 *The $\triangle^{\mathrm{D}}_{IC}$ operator satisfies (IC0), (IC1), (IC2), (IC4), (IC5), (IC7) and (Maj). It does not satisfy (IC3), (IC6), (IC8) and (MI).*

Proof: (IC0), (IC1) and (IC2) are straightforwardly satisfied.

$\triangle^{\mathrm{D}}$ satisfies (IC4). Since either K is consistent with K' and then (IC4) holds trivially, or K is not consistent with K'. So there is no maxiconsistent consistent with the two knowledge bases. There exists a maxiconsistent that contains K. So $\triangle^{\mathrm{D}}_{IC}(K \sqcup K') \wedge K \nvdash \perp$.

(IC5) holds for $\triangle^{\mathrm{D}}$. Let Q_i the elements of $\triangle^{\mathrm{D}}_{IC}(E_1)$, and R_i the elements of $\triangle^{\mathrm{D}}_{IC}(E_2)$. Note that if Q_i is consistent with $K \in E_1$, then $K \subseteq Q_i$ and then if $Q_i \wedge R_j$ is consistent, so $Q_i \wedge R_j \wedge K$ is consistent. So if $K \in E_1$, then $dist_{\mathrm{D}}(Q_i \wedge R_j, K) = dist_{\mathrm{D}}(Q_i, K)$. And similarly for $K \in E_2$, $dist_{\mathrm{D}}(Q_i \wedge R_j, K) = dist_{\mathrm{D}}(R_j, K)$. So if $Q_i \wedge R_j$ is consistent, then $dist_{\mathrm{D}}(Q_i, E_1) + dist_{\mathrm{D}}(R_j, E_2) = dist_{\mathrm{D}}(Q_i \wedge R_j, E_1 \sqcup E_2)$. Since we know that $\triangle^{C1}$ satisfies (IC5), we have that $Q_i \wedge R_j \in \textsc{maxcons}(E_1 \sqcup E_2, IC)$. It remains to show that $dist_{\mathrm{D}}(Q_i \wedge R_j, E_1 \sqcup E_2)$ is minimal. If it is not the case $\exists P \in \triangle^{\mathrm{D}}_{IC}(E_1 \sqcup E_2)$ such that $dist_{\mathrm{D}}(P, E_1 \sqcup E_2) < dist_{\mathrm{D}}(Q_i \wedge R_j, E_1 \sqcup E_2)$. So either $dist_{\mathrm{D}}(P, E_1) < dist_{\mathrm{D}}(Q_i \wedge R_j, E_1)$ or $dist_{\mathrm{D}}(P, E_2) < dist_{\mathrm{D}}(Q_i \wedge R_j, E_2)$ hold. Suppose w.l.g. that $dist_{\mathrm{D}}(P, E_1) < dist_{\mathrm{D}}(Q_i \wedge R_j, E_1)$, then $dist_{\mathrm{D}}(P \cap (E_1 \cup IC), E_1) < dist_{\mathrm{D}}(Q_i, E_1)$. So $Q_i \notin \triangle^{\mathrm{D}}_{IC}(E_1)$. Contradiction.

$\triangle^{\mathrm{D}}$ does not satisfy (IC6) and (IC8). The counterexamples used for $\triangle^{C1}$ hold here too.

The proof that (IC7) holds for $\triangle^{\mathrm{D}}$ is exactly the same that in theorem 2, since add integrity constraints IC_2 does not change the "score" of each maxiconsistent. ■

This operator satisfies as many basic properties as the $\triangle^{C1}$ operator. But it satisfies (Maj) instead of (MI), so it can be used to merge knowledge bases, since it takes the distribution of information into account.

Furthermore, the complexity of this operator is not much higher than the one of $\triangle^{C1}$ since we only add inclusion tests.

5.2 CARDINALITY OPERATORS

The previous operator is very rough, because the evaluation of a maxiconsistent is a drastic one: a maxiconsistent is ever good or bad for a knowledge base. Therefore, we can expect a more subtle way to evaluate a maxiconsistent. Such a problem has already been addressed in the literature. For example in the case of database update, Fagin *et al.* [FUV83, FKUV86] proposed a notion of *fewer change*:

Definition 12 *Let K_1, K_2 and K be knowledge bases.*

1. *K_1 has fewer insertions than K_2 with respect to K if $K_1 \setminus K \subset K_2 \setminus K$.*
2. *K_1 has fewer deletions than K_2 with respect to K if $K \setminus K_1 \subset K \setminus K_2$.*
3. *K_1 has fewer change than K_2 with respect to K if K_1 has fewer deletions than K_2, or K_1 and K_2 have the same deletions and K_1 has fewer insertions than K_2.*

The problem with Fagin *et al.* definition of fewer change is that it gives only a partial order. Here we need a total order in order to aggregate the "individual" preferences into "social" preferences.

Furthermore, Fagin *et al.* give more importance to deletions than to insertions. Even if it can be justified, for update, by the wish to keep as many as possible of the formulae of the old knowledge, it seems to contradict the "smallest change" requirement, since a set that accomplishes no deletions but adds thousands of formulae, would be considered better than a set that accomplishes one deletion and no insertion. So, from a merging point of view, it seems that we have to give the same importance to deletions as to insertions. This leads to the following operators.

5.2.1 The Symmetrical Difference Operator

The following operator is defined from a distance that denotes the cardinality of the symmetrical difference between the knowledge base and the maxiconsistent sets.

Definition 13 *Consider a knowledge set E and a knowledge base M.*

- $dist_S(M, K) = |K \setminus M| + |M \setminus K|$
- $dist_S(M, E) = \sum_{K \in E} dist_S(M, K)$
- $\triangle_{IC}^{S,\Sigma}(E) = \{M \in \triangle^{C1}{}_{IC}(E) : dist_S(M, E) = \min_{M_i \in \triangle^{C1}{}_{IC}(E)} (dist_S(M_i, E))\}$

So the selected maxiconsitent sets are those that have the least differences (in terms of number of formulae) with the knowledge bases.

The following theorem states the logical properties satisfied by this operator.

Theorem 7 *The $\triangle_{IC}^{S,\Sigma}$ operator satisfies (IC0), (IC1), (IC2), (IC4), (IC7), (IC8) and (Maj). It does not satisfy (IC3), (IC5), (IC6) and (MI).*

Proof: (IC0), (IC1) and (IC2) are directly satisfied by definition.

(IC4) is satisfied by $\triangle^{S,\Sigma}$. It follows from the following property: $\forall M \ K \cap K' \subseteq M \subseteq K \cup K' \ dist_S(M, K \sqcup K') = |K| + |K'| - 2|K \cap K'|$. Then if $\triangle_{IC}^{S,\Sigma}(K \sqcup K') \wedge K \nvdash \perp$, that is K is consistent with a maxiconsistent M. It implies that $K \subseteq M$, so by the above property $dist_S(M, K \sqcup K') = |K| + |K'| - 2|K \cap K'|$. But K' is a consistent subset of $K \cup K'$, so there exists an element $M' \in \triangle^{C1}(K \sqcup K')$ such that $K' \subseteq M'$. By the same property we get $dist_S(M', K \sqcup K') = dist_S(M, K \sqcup K')$, so $M' \in \triangle_{IC}^{S,\Sigma}(K \sqcup K')$. So $\triangle_{IC}^{S,\Sigma}(K \sqcup K') \wedge K' \nvdash \perp$.

(IC5) does not hold for $\triangle^{S,\Sigma}$. Consider the three knowledge bases $K_1 = \{a, a \rightarrow b \wedge c\}$, $K_2 = \{\neg c\}$ and $K_3 = \{a \rightarrow \neg c\}$. And define $E_1 = K_1 \sqcup K_2$, $E_2 = K_3$ and $IC = \top$.

(IC6) does not hold for $\triangle^{S,\Sigma}$. The counterexample for $\triangle^{C1}$ holds here too.

(IC7) and (IC8) hold for $\triangle^{S,\Sigma}$. When $\triangle_{IC_1}^{S,\Sigma}(E) \wedge IC_2$ is not consistent (IC7) and (IC8) are satisfied straightforwardly. So assume that $\triangle_{IC_1}^{S,\Sigma}(E) \wedge IC_2$ is consistent. Let P be an element of $\triangle_{IC_1 \wedge IC_2}^{S,\Sigma}(E)$, and let Q be an element of $\triangle_{IC_1}^{S,\Sigma}(E)$ consistent with IC_2. Let's define $A_1 = \bigwedge E \wedge IC_1$ and $A_2 = IC_2 \setminus A_1$. Then we can split $P = P_1 \oplus P_2$ with $P_1 = P \cap A_1$ and $P_2 = P \cap A_2$. Similarly let's define $Q \cup IC_2 = Q_1 \oplus Q_2$ such that $Q_1 = (Q \cup IC_2) \cap A_1$ and $Q_2 = (Q \cup IC_2) \cap A_2$. As $IC_2 \subseteq P$ and $IC_2 \subseteq Q \cup IC_2$, by construction it is easy to see that $P_2 = A_2 = Q_2$. In terms of distances, let $K \in E$, $dist_S(P, K) = dist_S(P_1, K) + |P_2 \setminus K| - |P_2 \cap K|$ and $dist_S(Q \cup IC_2, K) = dist_S(Q_1, K) + |Q_2 \setminus K| - |Q_2 \cap K|$. We have that $|P_2 \setminus K| - |P_2 \cap K| = |Q_2 \setminus K| - |Q_2 \cap K|$. As P is in $\triangle_{IC_1 \wedge IC_2}^{S,\Sigma}(E)$ and $Q \cup IC_2 \subseteq E \cup IC_1 \cup IC_2$, then $dist_S(P, K) \leq dist_S(Q \cup IC_2, K)$. Similarly as $Q_1 = Q$ is in $\triangle_{IC_1}^{S,\Sigma}(E)$ and $P_1 \subseteq E \cup IC_1$, then $dist_S(P_1, K) \geq dist_S(Q, K)$. From this it is easy to see that both $dist_S(P, K) = dist_S(Q \cup IC_2, K)$ and $dist_S(Q, K) = dist_S(P_1, K)$ hold. So $Q \cup IC_2$ is in $\triangle_{IC_1 \wedge IC_2}^{S,\Sigma}(E)$, and P_1 is in $\triangle_{IC_1}^{S,\Sigma}(E)$. ■

This operator doesn't seem to have a lot of logical properties. In particular, it does not satisfy the very impor-

Table 2: Properties of the merging operators based on $\triangle^{C1}$

	IC0	IC1	IC2	IC3	IC4	IC5	IC6	IC7	IC8	MI	Maj
$\triangle^{C1}$	✓	✓	✓	–	✓	✓	–	✓	–	✓	–
$\triangle^{d}$	✓	✓	✓	–	✓	✓	–	✓	–	–	✓
$\triangle^{S,\Sigma}$	✓	✓	✓	–	✓	–	–	✓	✓	–	✓
$\triangle^{\cap,\Sigma}$	✓	✓	✓	–	–	✓	✓	✓	✓	–	✓

tant (IC5) and (IC6) postulates, whereas this is mainly those two postulates that deal with "aggregation" properties of the merging operators.

But instead of focusing on the differences between the knowledge bases and the maxiconsistent sets, we can focus on what is common in those sets. These two approaches are very close in spirit but the second one is more interesting from a logical point of view.

5.2.2 The Intersection Operator

This operator is defined from a distance that denotes the cardinality of the intersection between the knowledge base and the maxiconsistent sets.

Definition 14 *Consider a knowledge set E and a knowledge base M.*

- $dist_\cap(M,K) = |K \cap M|$
- $dist_\cap(M,E) = \sum_{K\in E} dist_\cap(M,K)$
- $\triangle^{\cap,\Sigma}_{IC}(E) = \{M \in \triangle^{C}{}_{IC}(E) : dist_\cap(M,E) = \max_{M_i \in \triangle^{C1}{}_{IC}(E)} (dist_\cap(M_i,E))\}$

So the selected maxiconsistent sets are those that fit the knowledge bases on a maximum of formulae.

Theorem 8 *The $\triangle^{\cap,\Sigma}_{IC}$ operator satisfies (IC0), (IC1), (IC2), (IC5), (IC6), (IC7), (IC8) and (Maj). It does not satisfy (IC3), (IC4) and (MI).*

Proof: (IC0), (IC1) and (IC2) are straightforwardly satisfied.

(IC4) is not satisfied. Consider the following example: $K = \{a,b\}$ and $K' = \{\neg a \wedge \neg b\}$. Then $\triangle^{\cap,\Sigma}_{\top}(K \sqcup K') = K$ and $\triangle^{\cap,\Sigma}_{\top}(K \sqcup K') \wedge K' \vdash \bot$.

(IC5) holds for $\triangle^{\cap,\Sigma}$. The result is straightforward if $\triangle^{\cap,\Sigma}_{IC}(E_1) \wedge \triangle^{\cap,\Sigma}_{IC}(E_2)$ is not consistent. Otherwise there exists Q_i an element of $\triangle^{\cap,\Sigma}_{IC}(E_1)$ and R_j an element of $\triangle^{\cap,\Sigma}_{IC}(E_2)$ such that $Q_i \wedge R_j$ is consistent. Notice that if $K_l \in E_1$, then $dist_\cap(Q_i \wedge R_j, K_l) = dist_\cap(Q_i, K_l)$. Because if it is not the case, that is $dist_\cap(Q_i \wedge R_j, K_l) > dist_\cap(Q_i, K_l)$ then there exists a formula $\alpha \in \bigwedge E_1$ such that $\alpha \notin Q_i$ and $Q_i \cup \alpha$ is consistent. So Q_i is not a maxiconsistent. Contradiction. And similarly if $K_l \in E_2$, then $dist_\cap(Q_i \wedge R_j, K_l) = dist_\cap(R_j, K_l)$. So if $Q_i \wedge R_j$ is consistent, then $dist_\cap(Q_i \wedge R_j, E_1 \sqcup E_2) = dist_\cap(Q_i, E_1) + dist_\cap(R_j, E_2)$. From properties of $\triangle^{C1}$ we know that $Q_i \wedge R_j$ is in $\textsc{maxcons}(E_1 \cup E_2, IC)$. It remains to show that $dist_\cap(Q_i \wedge R_j, E)$ is maximum. If it is not the case $\exists P \in \triangle^{\cap,\Sigma}_{IC}(E_1 \sqcup E_2)$ such that $dist_\cap(P, E_1 \sqcup E_2) > dist_\cap(Q_i \wedge R_j, E_1 \sqcup E_2)$. So either $dist_\cap(P, E_1) > dist_\cap(Q_i \wedge R_j, E_1)$ or $dist_\cap(P, E_2) > dist_\cap(Q_i \wedge R_j, E_2)$ hold. Suppose w.l.g. that $dist_\cap(P, E_1) > dist_\cap(Q_i \wedge R_j, E_1)$, then $dist_\cap(P \cap (E_1 \cup IC), E_1) > dist_\cap(Q_i, E_1)$. So $Q_i \notin \triangle^{\cap,\Sigma}_{IC}(E_1)$. Contradiction.

(IC6) holds. Let P be an element of $\triangle^{\cap,\Sigma}_{IC}(E_1 \sqcup E_2)$. And decompose $P = P_1 \cup P_2$ with $P_1 = P \cap (IC \cup E_1)$ and $P_2 = P \cap (IC \cup E_2)$. If $P_1 \notin \triangle^{\cap,\Sigma}_{IC}(E_1)$, then exists $Q \in \triangle^{\cap,\Sigma}_{IC}(E_1)$ and $R \in \triangle^{\cap,\Sigma}_{IC}(E_2)$ such that $Q \wedge R$ is consistent and by definition $dist_\cap(Q, E_1) > dist_\cap(P_1, E_1)$ and $dist_\cap(R, E_2) \geq dist_\cap(P_2, E_2)$. So with a similar argument than for (IC5) we have that $dist_\cap(Q, E_1 \sqcup E_2) > dist_\cap(P, E_1 \sqcup E_2)$. So $P \notin \triangle^{\cap,\Sigma}_{IC}(E_1 \sqcup E_2)$. Contradiction.

(IC7) and (IC8) hold. The proof is similar to the one of $\triangle^{S,\Sigma}$. When $\triangle^{\cap,\Sigma}_{IC_1}(E) \wedge IC_2$ is not consistent (IC7) and (IC8) are satisfied straightforwardly. So assume that $\triangle^{\cap,\Sigma}_{IC_1}(E) \wedge IC_2$ is consistent. Let P be an element of $\triangle^{\cap,\Sigma}_{IC_1 \wedge IC_2}(E)$. And let Q be an element of $\triangle^{\cap,\Sigma}_{IC_1}(E)$ consistent with IC_2. Let's define $A_1 = \bigwedge E \cup IC_1$ and $A_2 = IC_2 \setminus A_1$. Then we can split $P = P_1 \oplus P_2$ with $P_1 = P \cap A_1$ and $P_2 = P \cap A_2$. Similarly let's define $Q \cup IC_2 = Q_1 \oplus Q_2$ such that $Q_1 = (Q \cup IC_2) \cap A_1$ and $Q_2 = (Q \cup IC_2) \cap A_2$. As $IC_2 \subseteq P$ and $IC_2 \subseteq Q \cup IC_2$, by construction it is easy to see that $P_2 = A_2 = Q_2$. In terms of distances $dist_\cap(P,K) = dist_\cap(P_1,K) + dist_\cap(P_2,K)$ and $dist_\cap(Q,K) = dist_\cap(Q_1,K) + dist_\cap(Q_2,K)$. But we have that $dist_\cap(P_2,K) = dist_\cap(Q_2,K)$. As P is in $\triangle^{\cap,\Sigma}_{IC_1 \wedge IC_2}(E)$ and $Q \cup IC_2 \subseteq E \cup IC_1 \cup IC_2$, then $dist_\cap(P,K) \geq dist_\cap(Q \cup IC_2, K)$. Similarly as $Q_1 = Q$ is in $\triangle^{\cap,\Sigma}_{IC_1}(E)$ and $P_1 \subseteq E \cup IC_1$, then $dist_\cap(P_1,K) \leq dist_\cap(Q,K)$. From this it is easy to see that both $dist_\cap(P,K) = dist_\cap(Q \cup IC_2, K)$ and $dist_\cap(Q,K) = dist_\cap(P_1,K)$ hold. So $Q \cup IC_2$ is in $\triangle^{\cap,\Sigma}_{IC_1 \wedge IC_2}(E)$, and P_1 is in $\triangle^{\cap,\Sigma}_{IC_1}(E)$. ■

The properties of merging operators defined in this section from selection functions are summed up in Table 2. So it is clear that one can get more logical properties with suitable selection functions.

All the operators we have defined in this section satisfy (Maj) (so they do not satisfy (MI)) and therefore are much more satisfactory than $\triangle^{C1}$ as merging operators.

We can note, in particular, that the intersection operator $\triangle^{\cap,\Sigma}$ satisfies almost all the properties of IC merging operators. It's very hard for a syntactical operator (*i.e.* for an operator working on knowledge bases that are not closed under logical consequences) to satisfy ($IC3$). So the sole "missing" property is (IC4).

Remark that the only operator that satisfies as many properties as $\triangle^{\cap,\Sigma}$ is the $\triangle^{C5}$ operator. But the behaviour of $\triangle^{C5}$ is (over-)simpler than the one of $\triangle^{\cap,\Sigma}$. Furthermore the postulate not satisfied by $\triangle^{\cap,\Sigma}$ is (IC4), whereas the one that $\triangle^{C5}$ does not satisfy is (IC6). But failing to satisfy (IC6) is worse that not satisfy (IC4) since, in fact, (IC5) and (IC6) are the conditions that purely deal with the aggregation problem. Their semantical counterparts [KP99] are seen as essential conditions for aggregation methods in Social Choice Theory (*cf e.g.* [Sen79]).

Finally, as noted at the beginning of this section, we can apply those methods (or other ones) to the other combination operators ($\triangle^{C3}$, $\triangle^{C4}$ and $\triangle^{C5}$) in order to improve their behaviour in the same way.

6 CONCLUSION

We have studied in this paper the logical properties of combination operators. We have shown that, due to the irrelevance of the distribution of information for the combination process, those operators do not have a good behaviour concerning the merging.

Then, we have shown that the use of selection functions can improve the logical properties of combination operators. In particular the intersection operator $\triangle^{\cap,\Sigma}$ satisfies almost all the postulates of IC merging operators.

We have only used a utilitarist aggregation method by adding the different distances when calculating the distance between a maxiconsistent and a knowledge set. So we have only defined majority operators. It could be interesting to see if one obtains similar results with an egalitarian method *à la* leximin (*cf* the $\triangle^{GMax}$ operator in [KP99]), leading to arbitration operators.

Another interesting work could be to find the general properties that selection methods have to verify in order to satisfy the postulates, as done in belief revision with *transitively relational partial meet contraction functions* [AGM85, Gär88]. That can give a new representation theorem for IC merging operators.

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