A framework for iterated revision

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ABSTRACT. We consider in this work the problem of iterated belief revision. We propose a family of belief revision operators called revision with memory operators and we give a logical (both syntactical and semantical) characterization of these operators. They obey what we call the principle of strong primacy of update : when one revises his beliefs by a new evidence, then all possible worlds that satisfy this new evidence become more reliable than those that do not. We show that those operators have a satisfying behaviour concerning the iteration of the revision process. Then we provide four particular operators of this family.
KEYWORDS: Belief revision, Iterated Belief revision.

1 Introduction

Modelling belief change is a central topic in artificial intelligence, psychology and databases. One of the predominant approaches was proposed by Alchourrón, Gärdenfors and Makinson and is known as the AGM framework [AGM85, Gär88]. The main requirement imposed by AGM postulates is the so called principle of *minimal change* saying that we have to keep as much of the old information as possible. Another important requirement is the principle of *primacy of update* that demands the new information to be true in the new knowledge base. A drawback of AGM definition of revision is that it is a static one, in the sense that, with this definition of revision operators, one can have a rational one step revision but the conditions for the iteration of the process are very weak. The problem is that AGM postulates state conditions only between the initial knowledge base, the new evidence and the resulting knowledge base. But the way to perform other revisions on the new knowledge base does not depend on the way the old knowledge base was revised. In particular, conditionals of the old knowledge base can be totally lost in the revision process. We argue that, in order to have a rational behaviour concerning the iteration of the revision process, one has to care about updating conditionals of the old knowledge base. This fact has already been pointed out by Darwiche and Pearl [DP94]. They show that AGM postulates are too weak to ensure a good behaviour concerning the iteration.

What is new in our proposal is that with each new piece of information, which is in general a simple formula, is associated a more complex information and it is this information that is incorporated during the revision process.

Some non-prioritized revisions have been explored recently [Han97, Mak98, FH99, Sch98] rejecting the principle of primacy of update. However, iterated revision can be carried on accepting the principle that new information is much more reliable than older one. In this paper we show how this is possible. We call this policy of giving strong preference to new information over the old one the principle of *strong primacy of update*: when one learns a new evidence in which he has full confidence, all possible worlds that satisfy this new evidence become more credible than the worlds that do not.

We claim that maintaining a knowledge base history allows to ensure a rational behaviour concerning the iteration. The point is that one cannot model an agent belief only by her present beliefs but has to care about how the agent has got her beliefs. So an epistemic state cannot be represented only by a knowledge base. One needs additional information as, for example, the history of the knowledge bases accepted by the agent. Let us consider the two following revisions in which a and b mean *it is raining* and *the grass is wet* respectively: $K_1 = a * (a \rightarrow b)$ and $K_2 = (a \rightarrow b) * a$; we have then the same resulting knowledge base, that is $a \wedge b$. But if we learn later that b is not true, that is *the grass is not wet*, do we have to obtain $K_1 * \neg b = K_2 * \neg b$? We do not think so. According to the principle of primacy of update, in K_1 the information a is less reliable than $a \rightarrow b$ so when we learn $\neg b$ we are willing to accept the falsity of the older information and so to obtain $K_1 * \neg b = \neg a \wedge \neg b$ and, of course *if it is raining then the grass is wet* still holds. Whereas in K_2, it is $a \rightarrow b$ which is less reliable, then learning $\neg b$ will lead to $K_2 * \neg b = a \wedge \neg b$ and in this case *if it is raining then the grass is wet* does not hold. The differences between knowledge bases and epistemic states reside in conditionals. $K_1 * \neg b = \neg a \wedge \neg b$ means that in K_1 the conditional "If $\neg b$ then $\neg a$" holds, whereas in K_2 the conditional is "If $\neg b$ then a".

The paper is organized as follows: in section 2 we recall some previous approaches to iterated revision. In section 3 we define revision operators with memory and give a logical characterization of these operators. We provide some examples of revision operators with memory in section 4. Finally, we conclude with a short discussion.

2 Belief revision

A knowledge base is often defined as a set of formulas not necessarily closed under logical deduction. Nevertheless in this work we will deal with a finite propositional language $\mathcal{L}$ and we will identify a knowledge base with a formula having the same logical consequences.

The set of all interpretations is denoted by $\mathcal{W}$. Let φ be a formula, $Mod(\varphi)$ denotes the set of models of φ, *i.e.* $Mod(\varphi) = \{I \in \mathcal{W} : I \models \varphi\}$. Let M be a set of interpretations, $form(M)$ denotes a formula whose set of models is M. When $M = \{I, J\}$ we will use the notation $form(I, J)$ instead of $form(M)$ for reading convenience. As usual, $\varphi \vdash \psi$ denotes the fact that ψ is a classical consequence of φ.

A pre-order $\leq$ is a reflexive and transitive relation, and $<$ is its strict counterpart, i.e $I < J$ if and only if $I \leq J$ and $J \not\leq I$. As usual, $\simeq$ is defined by $I \simeq J$ iff $I \leq J$ and $J \leq I$.

2.1 AGM Postulates for Epistemic States

We give here a formulation of AGM postulates for belief revision *à la* Katsuno and Mendelzon [KM91b]. More exactly we give a formulation of these postulates in terms of epistemic states [DP97]. The epistemic states framework is an extension of the knowledge bases one. In their formulation, Darwiche and Pearl [DP97] do not define precisely what an epistemic state is. Intuitively an epistemic state can be seen as a composed information: the information that an agent has regarding some scenario (her actual beliefs), plus all information that same agent has about how to perform revision. We will indicate in the next section exactly what we mean by epistemic state.

To each epistemic state Ψ is associated a knowledge base $Bel(\Psi)$ which is a propositional formula and which represents the objective (logical) part of Ψ. The models of Ψ are the models of its associated knowledge base, thus $Mod(\Psi) = Mod(Bel(\Psi))$. Let Ψ be an epistemic state and μ be a sentence denoting the new information. $\Psi \circ \mu$ denotes the epistemic state resulting of the revision of Ψ by μ. For reading convenience we will write respectively $\Psi \vdash \mu$, $\Psi \wedge \mu$, $\Psi \circ \mu \leftrightarrow \Psi \wedge \mu$ and $I \models \Psi$ instead of $Bel(\Psi) \vdash \mu$, $Bel(\Psi) \wedge \mu$, $Bel(\Psi \circ \mu) \leftrightarrow Bel(\Psi) \wedge \mu$ and $I \models Bel(\Psi)$.

Two epistemic states are equivalent, noted $\Psi \equiv \Psi'$, if and only if their objective parts are equivalent formulae, *i.e.* $Bel(\Psi) \leftrightarrow Bel(\Psi')$. Two epistemic states are equal, noted $\Psi = \Psi'$, if and only if they are identical. Thus equality is stronger than equivalence. We will see in section 3 that *equality* can be viewed as a dynamic equivalence whereas *equivalence* has to be considered as a static equivalence.

The operator $\circ$ is said to be a revision operator if it satisfies the following postulates:

(R*1) $\Psi \circ \mu \vdash \mu$

(R*2) If $\Psi \wedge \mu \nvdash \bot$, then $\Psi \circ \mu \leftrightarrow \Psi \wedge \mu$

(R*3) If $\mu \nvdash \bot$, then $\Psi \circ \mu \nvdash \bot$

(R*4) If $\Psi_1 = \Psi_2$ and $\mu_1 \leftrightarrow \mu_2$, then $\Psi_1 \circ \mu_1 \equiv \Psi_2 \circ \mu_2$

(R*5) $(\Psi \circ \mu) \wedge \varphi \vdash \Psi \circ (\mu \wedge \varphi)$

(R*6) If $(\Psi \circ \mu) \wedge \varphi \nvdash \bot$, then $\Psi \circ (\mu \wedge \varphi) \vdash (\Psi \circ \mu) \wedge \varphi$

This is nearly the Katsuno and Mendelzon formulation of AGM postulates, the only differences are that we work with epistemic states instead of knowledge bases and that postulate (R*4) is weaker than its AGM counterpart, in fact it is the only postulate in which we use explicitly the epistemic nature of the knowledge. See [DP97, FH96] for a full motivation of this definition.

A representation theorem, stating how revisions can be characterized in terms of models, holds. In order to give such semantical representation, the concept of faithful assignment on epistemic states is defined.

Definition 1 *A function that maps each epistemic state Ψ to a pre-order $\leq_\Psi$ on interpretations is called a faithful assignment over epistemic states if and only if:*

1. *If $I \models \Psi$ and $J \models \Psi$, then $I \simeq_\Psi J$*
2. *If $I \models \Psi$ and $J \not\models \Psi$, then $I <_\Psi J$*
3. *If $\Psi_1 = \Psi_2$, then $\leq_{\Psi_1} = \leq_{\Psi_2}$*

Now the reformulation of Katsuno and Mendelzon [KM91b] representation theorem in terms of epistemic states is:

Theorem 1 *A revision operator $\circ$ satisfies postulates (R*1-R*6) if and only if there exists a faithful assignment that maps each epistemic state Ψ to a total pre-order $\leq_\Psi$ such that:*

$$Mod(\Psi \circ \mu) = \min(Mod(\mu), \leq_\Psi)$$

Notice that this theorem gives information only on the objective part of the resulting epistemic state.

2.2 Darwiche and Pearl Postulates

As we have said in the introduction, a strong limitation of revision postulates is that they impose very weak constraints on the iteration of the revision process. Darwiche and Pearl [DP94, DP97] proposed postulates for iterated revision. The aim of these postulates is to keep as much as possible of conditional beliefs of the old knowledge base. So, besides postulates (R*1-R*6), a revision operator has to satisfy:

(C1) If $\alpha \vdash \mu$, then $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$

(C2) If $\alpha \vdash \neg\mu$, then $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$

(C3) If $\Psi \circ \alpha \vdash \mu$, then $(\Psi \circ \mu) \circ \alpha \vdash \mu$

(C4) If $\Psi \circ \alpha \nvdash \neg\mu$, then $(\Psi \circ \mu) \circ \alpha \nvdash \neg\mu$

These postulates can be explained as follows: (C1) states that if two pieces of information arrive and if the second implies the first, the second alone would give the same knowledge base. (C2) says that when two contradictory pieces of information arrive, the second alone would give the same knowledge base. (C3) states that an information should be retained after revising by a second information such that, when revising the current knowledge base by it, the first one holds. (C4) says that no evidence can contribute to its own denial.

Darwiche and Pearl provide a representation theorem for these postulates.

Theorem 2 *Suppose that a revision operator satisfies (R*1-R*6). The operator satisfies (C1-C4) if and only if the operator and its corresponding faithful assignment satisfy:*

(CR1) *If* $I \models \mu$ *and* $J \models \mu$, *then* $I \leq_\Psi J$ *iff* $I \leq_{\Psi\circ\mu} J$

(CR2) *If* $I \models \neg\mu$ *and* $J \models \neg\mu$, *then* $I \leq_\Psi J$ *iff* $I \leq_{\Psi\circ\mu} J$

(CR3) *If* $I \models \mu$ *and* $J \models \neg\mu$, *then* $I <_\Psi J$ *only if* $I <_{\Psi\circ\mu} J$

(CR4) *If* $I \models \mu$ *and* $J \models \neg\mu$, *then* $I \leq_\Psi J$ *only if* $I \leq_{\Psi\circ\mu} J$

In [DP94] the set of postulates (C1-C4) has first been given as a complement to usual AGM postulates. Freund and Lehmann [FL94] have shown that (C2) is inconsistent with AGM postulates. Furthermore Lehmann [Leh95] has shown that (C1) plus AGM postulates imply (C3) and (C4). In [DP97] Darwiche and Pearl have rephrased their postulates (and AGM ones) in terms of epistemic states instead of knowledge bases. In this way, they have removed these contradictions.

2.3 Boutilier natural revision

Before the work of Darwiche and Pearl, Boutilier proposed in [Bou93, Bou96] a *natural revision* operator which aims to have good iteration properties. This principle is called *absolute minimization* by Darwiche and Pearl and can be characterized by [Bou93]:

(CB) If $\Psi \circ \varphi \vdash \neg\mu$, then $(\Psi \circ \varphi) \circ \mu \equiv \Psi \circ \mu$

Which gives the following condition on the assignment: [DP97]

Theorem 3 *Suppose that a revision operator $\circ$ satisfies (R*1-R*6). $\circ$ satisfies CB if and only if the operator and its corresponding faithful assignment satisfy (CBR):*

(CBR) *If $I \not\models \Psi \circ \mu$ and $J \not\models \Psi \circ \mu$, then $I \leq_\Psi J$ iff $I \leq_{\Psi\circ\mu} J$*

So this operator can be seen as accomplishing a minimal change in the pre-order associated with the epistemic state. When one learns a new piece of information, one looks at the minimal models of this information according to the pre-order of the old epistemic state. The pre-order of the new epistemic state is exactly the same as the old one except that the minimal models of the new information are the new minimal elements of the pre-order.

Darwiche and Pearl [DP97] have shown that applying this principle leads to questionable results. However, natural revision even seems to be a simple and meaningful operator.

2.4 Lehmann ranked revision

Lehmann [Leh95] proposed postulates for revision operators that ensure good iteration properties. Its postulates are based on revision sequences. Let Ψ and Ψ' be two revision sequences, i.e. $\Psi = \varphi_1 \circ \ldots \circ \varphi_n$, and let φ and μ be two sentences:

(I1) Ψ is a consistent theory

(I2) $\Psi \circ \varphi \vdash \varphi$

(I3) If $\Psi \circ \varphi \vdash \mu$, then $\Psi \vdash \varphi \rightarrow \mu$

(I4) If $\Psi \vdash \mu$, then $\Psi \circ \Psi' \equiv \Psi \circ \mu \circ \Psi'$

(I5) If $\varphi \vdash \mu$, then $\Psi \circ \mu \circ \varphi \circ \Psi' \equiv \Psi \circ \varphi \circ \Psi'$

(I6) If $\Psi \circ \varphi \not\vdash \neg\mu$, then $\Psi \circ \varphi \circ \mu \circ \Psi' \equiv \Psi \circ \varphi \circ \varphi \wedge \mu \circ \Psi'$

(I7) $\Psi \wedge \varphi \vdash \Psi \circ \neg\varphi \circ \varphi$

Contrary to Darwiche and Pearl postulates, these ones are not an extension of AGM postulates, but Lehmann claims that they capture the spirit of AGM postulates [Leh95]. He gives a semantical characterization in terms of widening ranking models.

We will see in the following that the behaviour of Lehmann's operators is very far from the other proposals quoted in this paper and from revision with memory operators. In particular we will show through some examples that revision with memory operators satisfy very few of Lehmann's postulates.

2.5 Spohn proposal

Spohn [Spo87] proposed to model the epistemic state of an agent by an *Ordinal Conditional Function* (OCF), which can be considered as a ranking on interpretations.

Definition 2 *An Ordinal Conditional Function κ is a function from the set of possible worlds $\mathcal{W}$ to the class of ordinals such that at least some world is assigned 0.*

The ordinal $\kappa(I)$ associated to a possible world I can be viewed as the *degree of disbelief* of this world in the epistemic state represented by the OCF.

This function on possible worlds can straightforwardly be lifted to formulae in the following way:

$$\kappa(\varphi) = \min_{I \models \varphi} \kappa(I)$$

Thus a formula φ is believed in an epistemic state represented by κ if $\kappa(\varphi) = 0$. Otherwise the degree of disbelief of φ is $\kappa(\varphi)$.

This notion allows also to make a distinction between believed formulae. Let's define the *degree of firmness* of a formula in the following way:

φ is believed with firmness α relatively to the OCF κ if and only if

- either $\kappa(\varphi) = 0$ and $\alpha = \kappa(\neg\varphi)$,
- or $\kappa(\varphi) > 0$ and $\alpha = -\kappa(\varphi)$.

The revision on an OCF is called a (φ, α)*-conditionalization of* κ, where κ is the current OCF, φ is the new evidence and α is the *degree of firmness* of this new evidence. Intuitively, this degree of firmness represents the confidence in the new information, the higher the value, the more reliable the new evidence. The result of the conditionalization is a new OCF where the new evidence is believed with firmness α.

Definition 3 *Let κ be an OCF, φ be a formula, α be an ordinal, the (φ, α)-conditionalization of κ is the OCF $\kappa_{\varphi,\alpha}$ such that:*

$$\kappa_{\varphi,\alpha}(I) = \begin{cases} \kappa(I) - \kappa(\varphi) & \text{if } I \models \varphi \\ \kappa(I) - \kappa(\neg\varphi) + \alpha & \text{if } I \models \neg\varphi \end{cases}$$

Where $a - b$ represents the unique ordinal c such that $b + c = a$.

It is clear that Spohn operators satisfy AGM postulates (if $\alpha \neq 0$) as well as Darwiche and Pearl ones. Furthermore, the additional ordinal information allows to define more subtle notions than the AGM formalism [Gär88]. But the major flaw of this proposal is that it needs a degree of certainty for the incoming information. For some applications this measure is given by the system but for most of them we simply do not know the degree of certainty of new evidences. Moreover, it is difficult to appreciate the distinction between such degrees: is there any sense to make a distinction between a new evidence with a degree of certainty of 100 and another one with a degree of certainty of 101 ?

For the cases where we can not attach such a numerical information to the new evidence it seems that we have only two solutions in the OCF spirit. Spohn presents these two solutions as two limiting cases and criticizes this two approaches. He proposes his OCF as a median choice. It turns out that one of this limiting cases corresponds to Boutilier's natural revision operator. But the other case has never been investigated more deeply. In fact, our *basic revision operator with memory* corresponds to the other limiting case (actually it is a particular conditionalization) and what follows in this paper is an investigation of the properties of this family of operators. We will first define what we call revision operators with memory and then we will address Spohn criticisms.

3 Revision operators with memory

As we have already said, an epistemic state is intuitively a complex information: a knowledge base denoting the objective (logical) part of this epistemic state, plus an additional information, that we call *conditional set*, representing the preferences of the agent, i.e. what he is willing to accept. This additional information can take different forms, such as a pre-order over possible worlds, a set of conditional sentences, an epistemic entrenchment [NFPS96], a revision history, etc. In order to formalize these ideas we will consider a very simple syntactic definition of epistemic state.

Essentially, an epistemic state is a list of formulas (a revision history) together with its objective part. For simplicity reasons we will only consider consistent knowledge bases. So from now on all formulae considered will be consistent formulae. Nevertheless with some little (but technical) changes we can treat the case of $\bot$.

Formally the epistemic states are constructed from the consistent formulas in the following way:

Definition 4 *1.* $[\top]$ *is an epistemic state.*

2. If Ψ *is an epistemic state and* φ *is a consistent formula, then* $[\Psi \cdot \varphi]$ *is an epistemic state*

3. Each epistemic state is built by a finite number of applications of the previous two rules.

4. To each epistemic state Ψ *we associate a formula* $\pi(\Psi)$*, the objective part (the knowledge base) of* Ψ*.*

The set of epistemic states will be denoted E.

We will denote $[\varphi_1 \cdot \ldots \cdot \varphi_n]$ the epistemic state $[\ldots[[\top \cdot \varphi_1] \cdot \varphi_2] \ldots \cdot \varphi_n]$. When $\Psi = [\varphi_1 \cdot \ldots \cdot \varphi_n]$ and $\Psi' = [\varphi'_1 \cdot \ldots \cdot \varphi'_n]$, then $[\Psi \cdot \Psi']$ will denote the epistemic state $[\varphi_1 \cdot \ldots \cdot \varphi_n \cdot \varphi'_1 \cdot \ldots \cdot \varphi'_n]$.

We will now consider a notion of equality between epistemic states, that extends the notion of identity between two epistemic states. In fact, this definition highlights the fact that the equality between epistemic states is not simply a syntactical identity, but it means that the two cognitive states are the same, since different ways can lead to the same cognitive state. Equality can be considered as a dynamic equivalence. Whereas equivalence denotes simply the fact that statically the knowledge bases of two epistemic states are logically equivalent, equality states an extensional equivalence, that is, all revision starting from these two epistemic states will lead to epistemic states with equivalent knowledge bases. So, more precisely, we have the following definition:

Definition 5 $\Psi_1 = \Psi_2$ *iff for all* φ, $[\Psi_1 \cdot \varphi] \leftrightarrow [\Psi_2 \cdot \varphi]$

We are going to consider interpretations of epistemic states. An interpretation is a function $I : E \longrightarrow E^I$ together with a function $\pi^I : E^I \longrightarrow \mathcal{L}$. The elements of E^I are the concrete realizations of epistemic states (for instance, a concrete epistemic state can be encoded by several means, such as a pre-order over possible worlds, an epistemic entrenchment [NFPS96], etc.). We ask that the interpretations are compatible with the equality notion of Definition 5, that is

$$\forall \varphi \; (\pi^I(I([\Psi_1 \cdot \varphi])) \leftrightarrow \pi^I(I([\Psi_2 \cdot \varphi]))) \;\; \Rightarrow \;\; I(\Psi_1) = I(\Psi_2)$$

We ask too that π^I and π are compatible, *i.e.*

$$\pi^I(I(\Psi)) = \pi(\Psi)$$

The operators $\circ$ we will consider will be functions from $E^I \times \mathcal{L}^*$ into E^I, where $\mathcal{L}^*$ is the set of consistent formulas. We ask that

$$I(\Psi) \circ \varphi = I([\Psi \cdot \varphi])$$

Before proposing a set of postulates for revision operators with memory, we first give an example of a construction of those operators from a given faithful assignment in order to show the motivation of the logical characterization. Note that, among other things, this construction will show how to build an operator with memory from a given classical AGM revision operator.

We assign to each knowledge base φ a total pre-order $\leq_\varphi$, which represents the models of the knowledge base (at the lowest level) and the credibility *a* *priori* of "alternative worlds". It can be viewed as the epistemic state the agent would have if she has no prior knowledge and learns this evidence. This assignment can be interpreted as the agent's revision policy.

We impose very few constraints on this assignment, assuming it is faithful, that is:

Definition 6 *A function that maps each consistent knowledge base φ to a pre-order $\leq_\varphi$ on interpretations is called a faithful assignment over knowledge bases if and only if:*

1. *If $I \models \varphi$ and $J \models \varphi$, then $I \simeq_\varphi J$*
2. *If $I \models \varphi$ and $J \not\models \varphi$, then $I <_\varphi J$*
3. *If $\varphi_1 \equiv \varphi_2$, then $\leq_{\varphi_1} = \leq_{\varphi_2}$*

Note that two definitions of faithful assignment are available, one working on epistemic states (Definition 1), and the other one working on knowledge bases (Definition 6). This assignment on knowledge bases will induce an assignment on epistemic states in the following way:

Definition 7 *Consider a faithful assignment that maps each consistent knowledge base φ to a total pre-order $\leq_\varphi$. Let $\Psi = [\varphi_1 \cdot \ldots \cdot \varphi_n]$ be an epistemic state. We define $\leq_\Psi$ as:*

- *If $n = 1$, then $\leq_\Psi = \leq_{\varphi_1}$*
- *Otherwise, $I \leq_\Psi J$ if $I <_{\varphi_n} J$ or*
  $I \simeq_{\varphi_n} J$ and $I \leq_{\varphi_1 \cdot \ldots \cdot \varphi_{n-1}} J$

The pre-order $\leq_\Psi$ is called the conditional set of Ψ. The objective part of Ψ, denoted $\pi(\Psi)$, is the sentence (up to logical equivalence) such that $Mod(\pi(\Psi)) = \min(\mathcal{W}, \leq_\Psi)$

This faithful assignment over epistemic states allows us to define an interpretation in the following way: E^I is the set of total pre-orders over valuations. The function $I : E \longrightarrow E^I$ is defined by putting $I(\Psi) = \leq_\Psi$. The function $\pi^I : E^I \longrightarrow \mathcal{L}$ is defined by $\pi^I(\leq) = \varphi$ where φ is a formula such that $Mod(\varphi) = \min(\mathcal{W}, \leq)$.

It is easy to see that the following theorem holds:

Theorem 4 *If the function that assigns to each knowledge base φ a pre-order $\leq_\varphi$ is a faithful assignment (over knowledge bases),then the function that assigns to each epistemic state Ψ a pre-order $\leq_\Psi$ as defined in Definition 7 is a faithful assignment (over epistemic states).*

Corollary 5 *Let $\leq_\Psi$ be a faithful assignment over epistemic states as in Definition 7. Let $\circ$ be a revision operator such that $Mod(\Psi \circ \mu) = \min(Mod(\mu), \leq_\Psi)$. Then $\circ$ satisfies (R*1-R*6).*

Notice that this corollary gives only properties about the knowledge base resulting of the revision, that is, the objective part of the resulting epistemic state. Nevertheless, the revision process has to specify the full new epistemic state and we need some additional logical properties in order to characterize this kind of operators. Thus, we define revision with memory operators as the operators satisfying the axioms of the following definition.

Definition 8 *Let $\langle E^I, I, \pi^I \rangle$ be an interpretation. Let $\circ : E^I \times \mathcal{L} \longrightarrow E^I$ an operator. $\circ$ is said to be a revision operator with memory if and only if the following conditions hold*[1]*:*

(H0) $\pi[\top] \leftrightarrow \top$

(H1) $[\Psi \cdot \varphi] \vdash \varphi$

(H2) *If* $\Psi \wedge \varphi$ *is satisfiable, then* $[\Psi \cdot \varphi] \equiv \Psi \wedge \varphi$

(H3) $[\Psi \cdot \varphi]$ *is satisfiable*

(H4) *If* $\Psi_1 = \Psi_2$ *and* $\varphi_1 \leftrightarrow \varphi_2$*, then* $[\Psi_1 \cdot \varphi_1] = [\Psi_2 \cdot \varphi_2]$

(H5) $[\Psi \cdot \varphi] \wedge \mu \vdash [\Psi \cdot \varphi \wedge \mu]$

(H6) *If* $[\Psi \cdot \varphi] \wedge \mu$ *is satisfiable, then* $[\Psi \cdot \varphi \wedge \mu] \vdash [\Psi \cdot \varphi] \wedge \mu$

(H7) $[\Psi \cdot \Psi'] \equiv [\Psi \cdot \pi([\Psi'])]$

(H0) states that the knowledge base associated with the empty history is $\top$ and that the epistemic state corresponding to an empty history is the same as the one occurring when we know only tautologies. Notice that, in the presence of the other postulates, (H0) implies that $[\varphi] = [\top \cdot \varphi]$. (H1-H6) are exactly postulates (R*1-R*6) except for (H4) which is stronger than (R*4). Postulate (H7) is a sort of associativity of the constructor of epistemic states. It expresses the strong confidence in the new information. This will be seen more clearly from the semantic point of view below.

Now we state a theorem regarding representation of these operators with memory in terms of families of pre-orders on interpretations. This gives a more constructive definition of those operators. We define first what a conservative assignment is:

[1] These conditions, written in syntactical manner, have to be satisfied in the usual semantical way. Thus, for instance, the operator satisfies (H7) iff $\pi^I(I([\Psi \cdot \Psi'])) \leftrightarrow \pi^I(I([\Psi \cdot \pi([\Psi'])]))$.

Definition 9 *A function that maps each epistemic state Ψ to a total pre-order $\leq_\Psi$ on interpretations is called a conservative assignment if and only if:*

1. *If $I \models \Psi$ and $J \models \Psi$, then $I \simeq_\Psi J$*
2. *If $I \models \Psi$ and $J \not\models \Psi$, then $I <_\Psi J$*
3. *If $\Psi_1 = \Psi_2$, then $\leq_{\Psi_1} = \leq_{\Psi_2}$*
4. *If $\Psi = [\varphi]$ then $\min(\mathcal{W}, \leq_\Psi) = Mod(\varphi)$*
5. *If $I <_{[\varphi]} J$, then $I <_{[\Psi \cdot \varphi]} J$*
6. *If $I \simeq_{[\varphi]} J$, then $I \leq_{[\Psi \cdot \varphi]} J$ iff $I \leq_\Psi J$*

The pre-orders defined in definition 7 satisfy these properties. Conversely, when one has a conservative assignment one recovers straightforwardly the lexicographical order defined in definition 7 when one starts from the faithful assignment $\varphi \mapsto \leq_{[\varphi]}$.

In particular, this implies that a conservative assignment is completely determined by a unique faithful assignment over formulae.

We can strengthen the properties on the assignment by demanding the following property:

7. If $I \not\models \varphi$ and $J \not\models \varphi$, then $I \leq_{[\Psi \cdot \varphi]} J$ iff $I \leq_\Psi J$

This condition is the condition (CR2) of Darwiche and Pearl (cf. section 2.2). We call such an assignment a strong conservative assignment.

We have the following representation theorem:

Theorem 6 *Let $\langle E^I, I, \pi^I \rangle$ be an interpretation. Let $\circ : E^I \times \mathcal{L} \longrightarrow E^I$ an operator. The operator $\circ$ satisfies postulates (H0-H7) if and only if there exists a conservative assignment that maps each epistemic state Ψ to a total pre-order $\leq_\Psi$ such that:*

$$Mod(\pi^I(I(\Psi) \circ \varphi)) = \min(Mod(\varphi), \leq_\Psi)$$

The proof of this theorem is given in the Appendix. However, let us notice that, by the conditions imposed to operators, the left hand side member of the previous equality can be written as $Mod([\Psi \cdot \varphi])$.

This theorem has two important consequences. But before we state them, let us introduce some useful notation.

Definition 10 *Let $\preceq_1$ and $\preceq_2$ be two partial pre-orders over valuations. The lexicographical partial pre-order associated to $\preceq_1$ and $\preceq_2$, denoted $\preceq_{lex(\leq_1, \leq_2)}$, is defined by*

$$I \preceq_{lex(\preceq_1, \preceq_2)} J \textit{ iff } (I <_1 J) \textit{ or } (I \simeq_1 J \textit{ and } I \preceq_2 J)$$

In particular, when $\preceq_1$ and $\preceq_2$ are total pre-orders we have that $\preceq_{lex(\leq_1,\leq_2)}$ is a total pre-order. Notice that the total pre-order associated with an epistemic state in the Definition 7 is the (reversed) lexicographical order on the sequence of pre-orders associated with knowledge bases.

Now, we can state the first consequence of theorem 6: it shows how to construct revision operators with memory when the interpretation of epistemic states are pre-orders over valuations. More precisely, we have the following result:

Corollary 7 *Let E^I be the set of total pre-orders over valuations. Let $I : E \longrightarrow E^I$ and $\pi^I : E^I \longrightarrow \mathcal{L}$ be an interpretation. Then $\circ : E^I \times \mathcal{L}^* \longrightarrow E^I$ is a revision operator with memory iff there is a faithful assignment over consistent knowledge bases, $\varphi \rightarrow \leq_\varphi$, such that the following conditions hold:*

(i) $I([\varphi]) = \leq_\varphi$

(ii) $I([\Psi \cdot \varphi]) = \leq_{lex(\leq_\varphi, I(\Psi))}$

(iii) $I(\Psi) \circ \varphi = I([\Psi \cdot \varphi])$

(iv) $\pi^I(\leq) = \min(\mathcal{W}, \leq)$

This corollary states that revision operators built from a lexicographical order as done in Definition 7 are exactly characterized by postulates (H0-H7). So, the conditions on pre-orders are captured by these logical properties.

We will call the interpretations and the revision operators of corollary 7 the *standard* interpretations and the *standard* revision operators with memory.

The other consequence of Theorem 6 is that the standard revision operators with memory are in some sense universal. More precisely we have the following:

Corollary 8 *Let $I : E \longrightarrow E^I$ and $\pi^I : E^I \longrightarrow \mathcal{L}$ be an interpretation. Let $\circ : E^I \times \mathcal{L}^* \longrightarrow E^I$ be a revision operator with memory. Then there exists a standard revision operator with memory $\circ' : E^{I'} \times \mathcal{L}^* \longrightarrow E^{I'}$ such that*

$$\pi^I(I(\Psi) \circ \varphi) = \pi^{I'}(I(\Psi) \circ' \varphi)$$

What this corollary says is that any revision operator with memory can be simulated by a standard revision operator at the level of the objective part.

Now, we will look at some logical properties of this family of revision operators.

Theorem 9 *If an operator satisfies (H0-H6), then (H7) is equivalent to the following postulates:*

(H'7) *If $[\varphi \cdot \mu] \equiv \mu$, then $[\Psi \cdot \varphi \cdot \mu] \equiv [\Psi \cdot \mu]$*

(H'8) *If $\Psi' \vdash \mu$, then $[\Psi \cdot \Psi'] \vdash \mu$*

The proof of this theorem is given in the Appendix. It is also interesting to note that postulates (H0-H7) imply the following property:

(C) If $\varphi \wedge \mu$ is satisfiable, then $[\Psi \cdot \varphi \cdot \mu] \equiv [\Psi \cdot \varphi \wedge \mu]$

(C) states that when one revises successively by two consistent pieces of information, it amounts to revise by their conjunction. It is close to a postulate proposed by Nayak and al. [NFPS96] called *Conjunction*, but (C) is weaker than *Conjunction*, since it requires only the equivalence of the two resulting epistemic states, not the equality. Thus, the two epistemic states have the same objective part but can have different conditional sets.

The following two theorems are easy to prove:

Theorem 10 *A revision operator with memory satisfies postulates (C1), (C3) and (C4). It does not, in general, satisfy (C2).*

We have in fact the following representation theorem:

Theorem 11 *A revision operator satisfies postulates (H0-H7) and (C2) if and only if there exists a strong conservative assignment (i.e. a conservative assignment satisfying condition 7) which maps each epistemic state Ψ to a total pre-order $\leq_\Psi$ such that:*

$$Mod([\Psi \cdot \varphi]) = \min(Mod(\varphi), \leq_\Psi)$$

This theorem states that a subclass of revision operators with memory satisfy (C2), but we will see in the next section that there is a unique revision with memory operator (the basic one) that satisfies these requirements.

Postulate (C2) has been shown to be inconsistent with AGM Postulates by Freund and Lehmann [FL94]. So Darwiche and Pearl slightly modify AGM Postulates in [DP97] and then remove the contradiction. We claim that (C2) is not always desirable. The next example shows that satisfying this postulate leads to counterintuitive results.

Example 1 *Consider a circuit containing an adder and a multiplier. In this example we have two atomic propositions,* adder_ok *and* multiplier_ok*, denoting respectively the fact that the adder and the multiplier are working. We have initially no information about this circuit ($\Psi = \top$) and we learn that the adder and the multiplier are working ($\mu =$ adder_ok $\wedge$ multiplier_ok). Then someone tells us that the adder is not working ($\alpha = \neg$ adder_ok). There is, then, no reason to "forget" that the multiplier is working, which is imposed by (C2) : $\alpha \models \neg\mu$ so by (C2) we have $[\Psi \cdot \mu \cdot \alpha] \equiv [\Psi \cdot \alpha] \equiv \alpha$*

So, in some cases, postulate (C2) induces exactly the same kind of bad behaviour it tries to prevent.

4 Some revision operators with memory

We give in this section four standard operators with memory, the first one is called *Basic memory operator* since it corresponds to the simplest faithful assignment we can define. We show that even with such a basic operator, an agent can build complex preference orderings. We prove that this is the sole revision operator with memory that satisfies (C2). Then we give the *Dalal memory operator*, showing the behaviour of operators with memory when the pre-orders are more complex. Finally, we show how to generalize Ryan revision with OTP (Ordered Theory Presentation) [Rya94] in order to obtain two more revision operators with memory.

4.1 Basic memory operator

Let us define the assignment that maps each consistent formula to a pre-order in the following way:

Definition 11 *Let φ be a consistent formula and I, J two valuations.*

$I \leq^b_\varphi J$ *if and only if* $I \models \varphi$ *or*

$I \not\models \varphi$ *and* $J \not\models \varphi$

So we have what we shall call a basic order, which is a two-level order (at most), with the models of φ at the lower level and the other worlds at the higher level.

It is easy to show that the assignment that maps each consistent knowledge base φ to a pre-order $\leq^b_\varphi$ is a faithful assignment. We can now use corollary 7 to define a standard operator.

Definition 12 *The basic memory revision operator, $\circ^b$, is the standard operator defined after the faithful assignment $\varphi \rightarrow \leq^b_\varphi$ using the characterization of corollary 7. In particular:*

$$I(\Psi) \circ^b \varphi = \leq_{lex(\leq^b_\varphi, I(\Psi))}, \qquad \textit{and}$$

$$Mod(\pi^I(I(\Psi) \circ^b \varphi)) = \min(Mod(\varphi), \leq^b_\Psi)$$

Even with this basic order on knowledge bases, one can build very complex epistemic states. This is due to revision history. We illustrate the behaviour of this operator through some simple examples.

Example 2 *Consider a language $\mathcal{L}$ with only two propositional letters a and b. We will denote interpretations simply by the truth assignment, i.e* 10 *denotes the interpretation mapping a to True and b to False. Two interpretations are equivalent, with respect to the pre-order, if they appear at the same level. An* *interpretation I is better than another interpretation J ($I \leq J$) if it appears at a lower level. Let us see some examples of epistemic states:*

$$\leq^b_{[a\cdot b]} = \begin{array}{c}00\\10\\01\\11\end{array} \qquad \leq^b_{[a\wedge b]} = \begin{array}{c}00\ 01\ 10\\11\end{array} \qquad \leq^b_{[a\wedge b\cdot a]} = \begin{array}{c}00\ 01\\10\\11\end{array} \qquad \leq^b_{[a\wedge b\cdot a\cdot\neg b]} = \begin{array}{c}01\\11\\00\\10\end{array}$$

$$\leq^b_{[a\wedge b\cdot\neg b]} = \begin{array}{c}01\\11\\00\ 10\end{array} \qquad \leq^b_{[a\cdot b\cdot\neg(a\wedge b)]} = \begin{array}{c}11\\00\\10\\01\end{array} \qquad \leq^b_{[a\cdot a\wedge b\cdot\neg(a\wedge b)]} = \begin{array}{c}11\\00\ 01\\10\end{array}$$

With these examples, one can show that revision operators with memory do not satisfy Lehmann's postulates (I4), (I5) and (I6). Let us take $[a \wedge b \cdot a \cdot \neg b] \not\equiv [a \wedge b \cdot \neg b]$ as a counterexample for (I4) and (I5). Also, we have $[a \cdot b \cdot \neg(a \wedge b)] \not\equiv [a \cdot\ a \wedge b \cdot \neg(a \wedge b)]$ as a counterexample for (I6). The basic operator satisfies (I7) since, as we show below, it satisfies (C2) (see section 4.2 for a counterexample for (I7)).

For each ordering, there exists a revision history leading to the ordering. Sufficiently long sequences of different (non-equivalent) formulas lead to linear pre-orders, that is, pre-orders where there is no pair of equivalent interpretations. So, with such pre-orders, the objective part of the resulting epistemic states are complete formulae. This can be viewed as a learning process: when an agent has a long revision history he keeps preferences over possible worlds due to his past experience. Note that Darwiche and Pearl operators lead to the same kind of linear orders whereas, conversely, Lehmann operators lead to basic orders, that is, pre-orders in which all the non-model interpretations are equivalent. This is why revision with memory operators do not satisfy most of Lehmann's postulates. It seems that no iterated revision process can avoid belonging to one of these two limiting cases. So the question is to know if one case is better than the other. We consider that the former is preferable to the latter since, as we noticed above, a linear order as a limiting case can be interpreted as a learning process whereas, conversely, basic orders seem to denote a forgetting attitude. When you reach such pre-orders all *severe* revisions, *i.e.* revisions by an information that is not consistent with the current knowledge, amount only to this new information, that is, *if* $\Psi \vdash \neg\mu$ *then* $\Psi \circ \mu = \mu$.

The order paired with each epistemic state by the basic order is a strong conservative assignment. It is easy to show then that the sole revision operator with memory that satisfies the conditions of the strong conservative assignment is the basic operator:

Theorem 12 *The sole revision operator with memory that satisfies (H0-H7) and (C2) is the basic memory revision operator.*

In [Spo87] Spohn criticized this revision method by outlining three flaws of this approach. First, this operator is not reversible, that is, if you know the current epistemic state and the last new evidence incorporated, there is no means to recover the old epistemic state. It is true in Spohn formalism and in the one presented here. But in [BDP99, BKPP99, BKPP00] epistemic states are coded by polynomials and the basic operator is a "multiplication" on polynomials. Given this representation, the basic operator is reversible.

Second, this operator is not commutative, that is, given φ_1 and φ_2 two logically independent propositions, the epistemic state resulting of the accommodation of these two propositions is not the same if φ_1 arrives before φ_2 or if φ_2 arrives before φ_1 or if the two propositions arrive together. According to Spohn (Nayak and al. [NFPS96] give a similar property), if two propositions do not contradict each other, then the resulting epistemic state does not have to depend on the order of accommodation of these evidences. Evidently, revision with memory operators do not obey this requirement since they give a high primacy to the last evidence. However, a weak form of this requirement is satisfied since these operators verify property (C), which expresses this idea on the objective part of the resulting epistemic states.

Thirdly, Spohn underlines that the hypothesis that being informed about φ makes all the possible worlds satisfying φ more reliable than the possible worlds satisfying $\neg\varphi$ is a strong one. It is true that this assumption is strong, which is why we call it *strong primacy of update.* But, on one hand, the study of the properties of this operators was missing and seeing this basic operator as a particular case of the more general *revision with memory operators* helps to justify such an approach. On the other hand, when we dispose of no additional information attached to the new evidence that could help to define the revision as in the Spohn approach, the two possibilities that provide a minimal set of rational properties seem to be Boutilier's natural revision operator and revision with memory operators.

Finally, we can note that Liberatore has shown [Lib97] that several problems are computationally simpler for the basic memory operator than for the other iterated belief revision proposals (including Boutilier's natural revision [Bou93], Lehmann's ranking revision [Leh95] and Williams' transmutations [Wil94]).

4.2 Dalal memory operator

We use in this section the Hamming's distance between interpretations and then the Dalal's distance between an interpretation and a formula [Dal88], defined in the following way:

Definition 13 *Let I and J be two interpretations, the Hamming's distance $dist(I, J)$ is defined as the number of propositional letters the two interpretations differ on. Let φ be a consistent knowledge base, the Dalal's distance* *between I and φ is:*

$$d(I, \varphi) = \min_{J \models \varphi}(dist(I, J))$$

Let's define the assignment that maps each knowledge base to a pre-order in the following way:

Definition 14 *Let φ be a knowledge base and I, J two interpretations.*

$$I \leq^d_\varphi J \text{ if and only if } d(I, \varphi) \leq d(J, \varphi)$$

So we have a pre-order, with the models of φ at the lowest level and the other worlds in the higher levels.

We use Corollary 7 to build an operator from the assignment $\leq^d_\varphi$.

Definition 15 *The Dalal memory revision operator, $\circ^d$, is the standard operator defined after the faithful assignment $\varphi \rightarrow \leq^d_\varphi$ using the characterization of Corollary 7. In particular:*

$$I(\Psi) \circ^d \varphi = \leq_{lex(\leq^d_\varphi, I(\Psi))}, \qquad \text{and}$$

$$Mod(\pi^I(I(\Psi) \circ^d \varphi)) = \min(Mod(\varphi), \leq^d_\Psi)$$

From theorem 12 we know that the Dalal memory operator does not satisfy (C2). This can be easily shown through Example 1. Let $\Psi = \top$, $\mu = adder\_ok \wedge multiplier\_ok$ and $\alpha = \neg adder\_ok$ (in the following the two propositional letters denote respectively $adder\_ok$ and $multiplier\_ok$).

$\leq^d_\Psi =$ 00 01 10 11

$\leq^d_\mu =$
00
01 10
11

$\leq^d_\alpha =$
10 11
00 01

$\leq^d_{[\Psi \cdot \alpha]} =$
10 11
00 01

$\leq^d_{[\Psi \cdot \mu \cdot \alpha]} =$
11
10
00
01

So $\alpha \vdash \neg\mu$ but $[\Psi \cdot \mu \cdot \alpha] \equiv \neg adder\_ok \wedge multiplier\_ok$ whereas $[\Psi \cdot \alpha] \equiv \neg adder\_ok$.

Remark 1 *The Dalal memory operator does not satisfy (I7)*

For example, $\top \wedge \neg(a \wedge b) \nvdash [\top \cdot a \wedge b \cdot \neg(a \wedge b)]$.

4.3 Ryan OTP operator

Mark Ryan has proposed to apply his *Ordered Presentations of Theories* (or OTP) to belief revision [Rya94]. Very roughly, an OTP is a multi-set of formulae equipped with a partial pre-order. This pre-order represents the relative reliability of the sources of each formula. To give the definition of OTP is not a subject of this work, the interested reader can see *e.g* [Rya91, Rya92]. We will simply introduce the notions needed to define the OTP revision operator.

We will see that Ryan's proposal does not satisfy the desired properties of revision with memory operators; we will then give two modifications of Ryan OTP revision operator that satisfy the required properties. These operators are interesting since, as in the two previous examples, there are no *a priori* pre-orders. Furthermore this information is provided by the formula itself in a very natural (syntactical) way.

First we have to define what the monotonicities of a formula are.

Definition 16 *Let I be an interpretation and p be a propositional letter, then $I^{[p]}$ (respectively $I^{[\neg p]}$) denotes the interpretation that is identical to I on each propositional letter except (maybe) on the propositional letter p that is assigned to true (resp. false).*

Definition 17 *Let φ be a consistent formula and p be a propositional letter.*

1. *φ is* monotonic in p *if $I \models \varphi$ implies that $I^{[p]} \models \varphi$.*
2. *φ is* anti-monotonic in p *if $I \models \varphi$ implies that $I^{[\neg p]} \models \varphi$.*

The set of symbols in which φ is monotonic (resp. anti-monotonic) is noted φ^+ (resp. φ^-). These two sets are called the monotonicities of φ. If $\varphi \leftrightarrow \bot$, then $\varphi^+ = \varphi^- = \emptyset$.

After this definition, Ryan defines an inference relation that he named *natural entailment.*

Definition 18 *φ naturally entails μ, written $\varphi \mid\!\sim_N \mu$, if $\varphi \vdash \mu$, $\varphi^+ \subseteq \mu^+$ and $\varphi^- \subseteq \mu^-$.*

This relation has some nice properties, in particular and conversely to classical entailment, it does not allow to add irrelevant disjuncts in the conclusions (for example $p \not\mid\!\sim_N p \vee q$). See [Rya91, Rya92] for more details.

Finally, the preference relation associated with a formula φ is given by the set of natural consequences that the interpretations satisfy, that is:

Definition 19 *Let φ be a formula, and I, J two interpretations, the relation $\preceq_\varphi$ is defined as: $I \preceq_\varphi J$ if for each μ such that $\varphi \mid\!\sim_N \mu$ it holds ($J \models \mu \Rightarrow I \models \mu$).*

So an interpretation is better than another if it satisfies more natural consequences. Note that the relation $\preceq_\varphi$ is a partial pre-order.

In order to define the OTP revision operator we use the technique of Definition 7.

Definition 20 *Let $E^{I^{OTP}}$ be the set of partial pre-orders over valuations. The OTP revision operator, $\circ^{OTP} : E^I \times \mathcal{L}^* \longrightarrow E^I$ and the OTP interpretation $I^{OTP} : E \longrightarrow E^{I^{OTP}}$, $\pi^{I^{OTP}} : E^{I^{OTP}} \longrightarrow \mathcal{L}^*$ are defined starting from the assignment $\varphi \rightarrow \preceq_\varphi$ in the following way:*

(i) $I^{OTP}([\varphi]) = \preceq_\varphi$

(ii) $I^{OTP}([\Psi \cdot \varphi]) = \preceq_{lex(\preceq_\varphi,\, I^{OTP}(\Psi))}$

(iii) $I^{OTP}(\Psi) \circ^{OTP} \varphi = I^{OTP}([\Psi \cdot \varphi])$

(iv) $\pi^{I^{OTP}}(\preceq) = \min(\mathcal{W}, \preceq)$

In particular $Mod(\pi^{I^{OTP}}([\Psi \cdot \varphi])) = \min(Mod(\varphi), I^{OTP}(\Psi))$.

Because the starting assignment takes partial pre-orders as values, the operator $\circ^{OTP}$ does not satisfy all the postulates. More precisely, we have the following result:

Theorem 13 *The OTP revision operator satisfies postulates (H0), (H1), (H3), (H4), (H5) and (H7), but does not satisfy (H2) and (H6).*

A counter-example to (H2) and (H6), given in [Rya94], is the following:

Let $\varphi_1 = p \vee q \vee r$, $\varphi_2 = \neg p \wedge \neg q \wedge \neg r$ and $\varphi_3 = (p \leftrightarrow q) \wedge \neg r$. Then for (H2), take $\Psi = [\varphi_1 \cdot \varphi_2]$ and $\varphi = \varphi_3$. Then $Mod(\pi^{I^{OTP}}(I^{OTP}(\Psi))) = \{011, 101, 110\}$ and $Mod(\varphi) = \{000, 001, 010, 100, 110, 111\}$, so $Mod(\pi^{I^{OTP}}(\Psi) \wedge \varphi) = \{110\}$ whereas $Mod(\pi^{I^{OTP}}([\Psi \cdot \varphi])) = \{110, 001\}$. The same counter-example holds for (H6) also by putting $\Psi = \varphi_1$, $\varphi = \varphi_2$ and $\mu = \varphi_3$.

These two violations of the rationality postulates seem to be very awkward. Especially (H2) seems hardly debatable. We will next see how we can modify Ryan's definition in order to satisfy these properties.

The easiest way to modify the OTP revision operator in order to obtain revision with memory operators is to "complete" the $\preceq_\varphi$ partial pre-orders to total pre-orders. This can be achieved by two means.

Closure of the pre-order

First, following the construction of the rational closure of a conditional knowledge base [LM92], we can figure out a lazy deformation of the pre-order, that is, the deformation that transforms the partial pre-order in a total pre-order with a minimal effort.

Definition 21 *Let $\rho_\varphi(I)$ be the "distance from I to φ" in the following sense:*

1. *If $I \in \min(\mathcal{W}, \preceq_\varphi)$ then $\rho_\varphi(I) = 0$,*
2. *Otherwise $\rho_\varphi(I) = a$, where a is the length of the longest chain of strict inequalities $I_0 \prec_\varphi \ldots \prec_\varphi I_n$ with $I_0 \in \min(\mathcal{W}, \preceq_\varphi)$ and $I_n = I$.*

This "distance" gives a total pre-order on interpretations:

Definition 22 *$I \leq_\varphi^{\mathrm{OTP}_1} J$ if and only if $\rho_\varphi(I) \leq \rho_\varphi(J)$.*

Finally, we define a revision operator as usual with the characterization of Corollary 7.

Definition 23 *The OTP_1 revision operator, $\circ^{OTP_1}$, is the standard operator defined after the faithful assignment $\varphi \rightarrow \leq_\varphi^{OTP_1}$ using the characterization of Corollary 7. In particular:*

$$I(\Psi) \circ^{OTP_1} \varphi = \leq_{lex(\leq_\varphi^{OTP_1}, I(\Psi))}, \qquad \textit{and}$$

$$Mod(\pi^I(I(\Psi) \circ^{OTP_1} \varphi)) = \min(Mod(\varphi),\ I(\Psi))$$

We illustrate this principle of "minimal effort" with an example: Let $\varphi = (\neg a \vee \neg b) \wedge \neg c$ be a knowledge base.

The left hand side presents the partial pre-order $\preceq_\varphi$. Arrows $I \leftarrow J$ denote $I \prec_\varphi J$ (for reading convenience we do not represent transitivity, reflexivity and the equivalence between minimal interpretations). The right hand side presents the $\leq_\varphi^{\mathrm{OTP}_1}$ corresponding pre-order. It is clear that if $I \prec_\varphi J$ then $I <_\varphi^{\mathrm{OTP}_1} J$. Thus the only interpretation that is not straightforwardly placed is 110. The "minimal effort" is being illustrated here as follows: the first place where can be placed 001 is at the second level, so it is the chosen level. Conversely, for the interpretation 011 for example, the first "acceptable" level is the third one because there is an interpretation (001) that is strictly better than 011 which is occupying the second level.

As a consequence of Corollary 7 and the fact that $\varphi \mapsto \leq_\varphi^{OTP_1}$ is a faithful assignment, we have the following result

Theorem 14 *The OTP_1 operator is a revision with memory operator.*

Using cardinalities

A second way to define a total pre-order from an OTP revision operator is to interpret it differently. The idea of the $\preceq_\varphi$ order, defining OTP revision operator, is that an interpretation I is better than another J for a knowledge base φ if I satisfies all the natural consequences that J satisfies. In other terms I is better than J if I satisfies more natural consequences than J. Following this idea we can then focus uniquely on the number of natural consequences satisfied.

Definition 24 $I \leq_\varphi^{\mathrm{OTP}_2} J$ *if and only if* $card(\{\mu \mid \varphi \mid\!\sim_N \mu, \text{ and } J \models \varphi\}) \leq card(\{\mu \mid \varphi \mid\!\sim_N \mu, \text{ and } I \models \varphi\})$.

Then we can define an operator using this pre-order and the characterization of Corollary 7.

Definition 25 *The OTP_2 revision operator, $\circ^{OTP_2}$, is the standard operator defined after the faithful assignment $\varphi \rightarrow \leq_\varphi^{OTP_2}$ using the characterization of Corollary 7. In particular:*

$$I(\Psi) \circ^{OTP_2} \varphi = \leq_{lex(\leq_\varphi^{OTP_2}, I(\Psi))}, \qquad \text{and}$$

$$Mod(\pi^I(I(\Psi) \circ^{OTP_2} \varphi)) = \min(Mod(\varphi),\ I(\Psi))$$

This definition is also a "completion" of the $\preceq_\varphi$ pre-order since if $I \preceq_\varphi J$, then $I \leq_\varphi^{\mathrm{OTP}_2} J$. And, as a consequence of Corollary 7 and the fact that $\varphi \mapsto \leq_\varphi^{OTP_2}$ is a faithful assignment, we have the following result

Theorem 15 *The OTP_2 operator is a revision with memory operator.*

Conclusion

We have shown in this paper the connection between the problem of the iteration of the revision process and revision histories. Thus, revision history is a means to warrant rational iteration properties. A representation theorem is provided, showing that one can impose in logical terms conditions on the set of epistemic states.

We have compared our approach to iterated revision with previous related works. In particular, we have shown that the only revision with memory operator that satisfies postulate (C2) proposed by Darwiche and Pearl is the basic memory operator. This illustrates the fact that (C2) is a very strong requirement.

We have adopted here a drastic strategy, applying the principle of strong primacy of update. Adopting the principle of primacy of update as a reasonable requirement, surely leads to finding the possible worlds that satisfy this new information more reliable than those that do not satisfy it.

The ontology for this family of operators is the one given by this principle of strong primacy of update. Consider an agent that learns successively some pieces of evidence about some world. All these evidences are observations about this world and the agent has full confidence in newer evidence. Moreover, when a new piece of evidence arrives, it increases the confidence in possible worlds that best fit this new evidence according to the agent's revision policy. Then, when a new piece of information arrives, the agent reconsiders the relative reliability of all possible worlds according to her past experience and to the new piece of information.

The principle of primacy of update is often criticized, since it can not be accepted in all circumstances. Sometimes we have more confidence in our current beliefs than in the new information. Some non-prioritized revisions, denying this principle, have been explored recently [Han97, Mak98, FH99, Sch98]. If we accept this point of view, our framework can be used in an amazing way by reversing the revision arguments, i.e. by revising the new information by the old epistemic state ($\mu \circ \varphi = [\varphi \cdot \mu]$). Thus we obtain a family of "revision" operators that give a strong primacy to the current knowledge. Then, the new information is not accepted as true in the new epistemic state, but its confidence is increased. For example, if two possible worlds are equally plausible for an epistemic state, and if the new information is satisfied by only one of these possible worlds, then this world will be considered more reliable than the other one in the new epistemic state. So we can imagine applications using alternatively these two definitions of revision operators, according to the relative reliability of the current beliefs and the new information.

We would like to finish these remarks by raising the following question: how to build a contraction operator having good properties with respect to iteration? Indeed, this question hides another one: which are the desirable properties of the contraction in the process of iteration? Concerning the first question, let us remark that Harper's identity is not enough to define a contraction operator at the level of epistemic states.

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Appendix: Proofs

Proof of theorem 6 : [2]

In order to simplify the notation we identify $I(\Psi)$, $I(\Psi \circ \varphi)$, $\pi^I(I(\Psi))$ with Ψ, $[\Psi \cdot \varphi]$, $\pi(\Psi)$ respectively.

(Only if) Let $[\,\cdot\,]$ be an operator that satisfies (H0-H7). Let us define an assignment that associates to each epistemic state Ψ a pre-order $\leq_\Psi$ defined by $\forall I, J \in \mathcal{W}$, $I \leq_\Psi J$ if and only if $I \models [\Psi \cdot form(I,J)]$. We prove that $\leq_\Psi$ is a total pre-order, that the assignment is a conservative assignment, and that $Mod([\Psi \cdot \varphi]) = \min(Mod(\varphi), \leq_\Psi)$.

First we show that $\leq_\Psi$ is a total pre-order :

Reflexivity: From (H1) and (H3) we have that $\forall I$ $[\Psi \cdot form(I)] \equiv form(I)$, that is $I \leq_\Psi I$.

Transitivity: Assume that both of $I \leq_\Psi J$ and $J \leq_\Psi L$ hold. We show that $I \leq_\Psi L$.

Case 1: $[\Psi \cdot form(I,J,L)] \wedge form(I,L)$ is not consistent. Then $[\Psi \cdot form(I,J,L)] \equiv form(J)$. So $[\Psi \cdot form(I,J,L)] \wedge form(I,J) \equiv form(J)$. By (H5) and (H6) it follows that $[\Psi \cdot form(I,J)] \equiv form(J)$, so by definition $I \not\leq_\Psi J$. Contradiction.

Case 2: $[\Psi \cdot form(I,J,L)] \wedge form(I,L)$ is consistent. Suppose towards a contradiction that $I \not\leq_\Psi L$, that is, from (H1) and (H3), $[\Psi \cdot form(I,L)] \equiv form(L)$. By (H5) and (H6) we have $[\Psi \cdot form(I,J,L)] \wedge form(I,L) \equiv [\Psi \cdot form(I,L)] \equiv form(L)$. So $I \not\models [\Psi \cdot form(I,J,L)]$. From (H1) and (H3) we have that either $[\Psi \cdot form(I,J,L)] \equiv form(J,L)$ or $[\Psi \cdot form(I,J,L)] \equiv form(L)$. In the first case we get $[\Psi \cdot form(I,J,L)] \wedge form(I,J) \equiv form(J)$, by (H5) and (H6) we conclude $[\Psi \cdot form(I,J)] \equiv form(J)$, that is $I \not\leq_\Psi J$. Contradiction. In the second case we get $[\Psi \cdot form(I,J,L)] \wedge form(J,L) \equiv form(L)$, by (H5) and (H6) we conclude $[\Psi \cdot form(J,L)] \equiv form(L)$, that is $J \not\leq_\Psi L$. Contradiction.

Totality: $\forall I, J \in \mathcal{W}$, from (H1) we have that $[\Psi \cdot form(I,J)] \vdash form(I,J)$ and from (H3) that $[\Psi \cdot form(I,J)] \nvdash \bot$, so either $I \models [\Psi \cdot form(I,J)]$ or $J \models [\Psi \cdot form(I,J)]$, that is $I \leq_\Psi J$ or $J \leq_\Psi I$.

Now we verify the conditions of the conservative assignment:

1. If $I \models \Psi$ and $J \models \Psi$, then by (H2) we have $[\Psi \cdot form(I,J)] \equiv form(I,J)$. That is, by definition, $I \leq_\Psi J$ and $J \leq_\Psi I$, so $I \simeq_\Psi J$.
2. If $I \models \Psi$ and $J \not\models \Psi$, then by (H2) we have $[\Psi \cdot form(I,J)] \equiv form(I)$.

[2] The proof that postulates (H1-H6) corresponds to conditions 1-3 on the assignment is mainly the same as Katsuno and Mendelzon one [KM91a]

That is, by definition, $I \leq_\Psi J$ and $J \not\leq_\Psi I$, so $I <_\Psi J$.

3. Straightforward from (H4).
4. If $\Psi = [\varphi]$, then from (H0) and (H2) we get that $\Psi \equiv \varphi$, then by conditions 1 and 2 we get that $\min(\mathcal{W}, \leq_\Psi) = Mod(\varphi)$.
5. If $I <_{[\varphi]} J$, then by definition $[\varphi \cdot form(I,J)] \equiv form(I)$ and from (H7) $[\Psi \cdot \varphi \cdot form(I,J)] \equiv [\Psi \cdot Bel([\varphi \cdot form(I,J)])]$. Furthermore from (H1) and (H3) we get that $[\Psi \cdot Bel([\varphi \cdot form(I,J)])] \equiv form(I)$, so we have $[\Psi \cdot \varphi \cdot form(I,J)] \equiv form(I)$. That is by definition $I <_{[\Psi \cdot \varphi]} J$.
6. If $I \simeq_{[\varphi]} J$, then by definition $[\varphi \cdot form(I,J)] \equiv form(I,J)$. So $[\Psi \cdot Bel([\varphi \cdot form(I,J)]) \equiv [\Psi \cdot form(I,J)]$. From (H7) we have $[\Psi \cdot \varphi \cdot form(I,J)] \equiv [\Psi \cdot Bel([\varphi \cdot form(I,J)])]$, that gives $[\Psi \cdot \varphi \cdot form(I,J)] \equiv [\Psi \cdot form(I,J)]$. That is directly by definition: $I \leq_{[\Psi \cdot \varphi]} J$ if and only if $I \leq_\Psi J$.

Finally we show that $Mod([\Psi \cdot \varphi]) = \min(Mod(\varphi), \leq_\Psi)$. First for the inclusion $Mod([\Psi \cdot \varphi]) \subseteq \min(Mod(\varphi), \leq_\Psi)$ assume that $I \models [\Psi \cdot \varphi]$ and suppose towards a contradiction that $I \notin \min(Mod(\varphi), \leq_\Psi)$. So we can find a $J \models \varphi$ such that $J <_\Psi I$. Then $I \not\models [\Psi \cdot form(I,J)]$. As $[\Psi \cdot \varphi] \wedge form(I,J)$ is consistent, from (H5) and (H6) we have $[\Psi \cdot \varphi] \wedge form(I,J) \equiv [\Psi \cdot form(I,J)]$. But $I \not\models [\Psi \cdot form(I,J)]$, so $I \not\models [\Psi \cdot \varphi]$. Contradiction.

Conversely, assume that $I \in \min(Mod(\varphi), \leq_\Psi)$. We want to show that $I \models [\Psi \cdot \varphi]$. As $I \in \min(Mod(\varphi), \leq_\Psi)$, then for each J such that $J \models \varphi$ we have $I \leq_\Psi J$. That is $I \models [\Psi \cdot form(I,J)]$. As $[\Psi \cdot \varphi] \wedge form(I,J)$ is consistent, from (H5) and (H6) we have $[\Psi \cdot \varphi] \wedge form(I,J) \equiv [\Psi \cdot form(I,J)]$. And as $I \models [\Psi \cdot form(I,J)]$, it follows that $I \models [\Psi \cdot \varphi]$.

(If) Let us consider a conservative assignment that maps each epistemic state Ψ to a total pre-order $\leq_\Psi$ and define an operator $[\cdot]$ by putting $Mod([\Psi \cdot \varphi]) = \min(Mod(\varphi), \leq_\Psi)$, which is enough to capture the whole epistemic state $[\Psi \cdot \varphi]$ because of definition 5. We want to show that $[\cdot]$ satisfies (H0-H7).

(H0) First we have straighforwardly by condition 4 that if $\Psi = [\top]$, then $\min(\mathcal{W}, \leq_\Psi) = Mod(\top)$. From this and condition 1 and 2 we get $Bel([\top]) = \top$. It remains to show that $[] = [\top]$, which is true from definition 5 if for all φ $[\varphi] \equiv [\top \cdot \varphi]$. By definition we have $Mod([\top \cdot \varphi]) = \min(Mod(\varphi), \leq_{[\top]})$ and from the first part of (H0) we know that for all $I, J \in \mathcal{W}$ $I \simeq_{[\top]} J$, so $Mod([\top \cdot \varphi]) = Mod(\varphi)$. It remains to show that $Mod([\varphi]) = Mod(\varphi)$ and we conclude by transitivity. By definition $Mod([\varphi \cdot \top]) = \min(\mathcal{W}, \leq_{[\varphi]})$ and by condition 4 that gives $Mod([\varphi \cdot \top]) = Mod(\varphi)$. By condition 6, since for all $I, J \in \mathcal{W}$ $I \simeq_{[\top]} J$, we conclude that $I \leq_{[\varphi \cdot \top]} J$ iff $I \leq_{[\varphi]} J$. From this it is easy to see that $[\varphi \cdot \top] = [\varphi]$. Then we obtain $Mod([\varphi]) = Mod(\varphi)$, as required.

(H1) By definition $Mod([\Psi \cdot \varphi]) \subseteq Mod(\varphi)$, so $[\Psi \cdot \varphi] \vdash \varphi$.

(H2) Assume that $\Psi \wedge \varphi$ is consistent. We want to show that $\min(Mod(\varphi), \leq_\Psi) = Mod(\Psi \wedge \varphi)$. First note that if $I \models \Psi$ then from conditions 1 and 2 $I \in \min(\mathcal{W}, \leq_\Psi)$. So if $I \models \Psi \wedge \varphi$ then $I \in \min(Mod(\varphi), \leq_\Psi)$. So $\min(Mod(\varphi), \leq_\Psi) \supseteq Mod(\Psi \wedge \varphi)$. For the other inclusion consider $I \in \min(Mod(\varphi), \leq_\Psi)$.

Suppose towards a contradiction that $I \not\models \Psi \wedge \varphi$. Since $I \not\models \Psi$ by condition 2 we have that $\forall J \models \Psi\ J <_\Psi I$. In particular $\forall J \models \Psi \wedge \varphi\ J <_\Psi I$. So $I \notin \min(Mod(\varphi), \leq_\Psi)$. Contradiction.
(H3) If φ is consistent, then $Mod(\varphi) \neq \emptyset$ and, as we have a finite number of interpretations , we have no infinite descending chains of strict inequalities , so $\min(Mod(\varphi), \leq_\Psi) \neq \emptyset$. So $[\Psi \cdot \varphi]$ is consistent.
(H4) Direct from condition 3.
(H5) Let us take $I \models [\Psi \cdot \varphi] \wedge \mu$. Since $I \models [\Psi \cdot \varphi]$ we have $\forall J \models \varphi\ I \leq_\Psi J$. In particular for all J such that $J \models \varphi \wedge \mu$, $I \leq_\Psi J$ holds. Then $I \models [\Psi \cdot \varphi \wedge \mu]$.
(H6) Assume that $[\Psi \cdot \varphi] \wedge \mu$ is consistent, so $\exists J \models [\Psi \cdot \varphi] \wedge \mu$. Consider $I \models [\Psi \cdot \varphi \wedge \mu]$ and suppose towards a contradiction that $I \not\models [\Psi \cdot \varphi]$. Then $J <_\Psi I$ and since $J \models \varphi \wedge \mu$, then $I \notin \min(Mod(\varphi \wedge \mu), \leq_\Psi)$. That is, $I \not\models [\Psi \cdot \varphi \wedge \mu]$. Contradiction.
(H7) First we state the following fact, the proof of which is quite easy.

Fact: $I \models Bel(\Psi)$ iff $I \in \min(\mathcal{W}, \leq_\Psi)$.

From the fact, we have that $I \models Bel([\Psi \cdot \Psi'])$ if and only if $I \models \min(\mathcal{W}, \leq_{[\Psi \cdot \Psi']})$. But $I \in \min(\mathcal{W}, \leq_{[\Psi \cdot \Psi']})$ can be rewritten using conditions 5 and 6 in $I \in \min(\min(\mathcal{W}, \leq_{\Psi'}), \leq_\Psi)$. Using again the fact, we have $\min(\min(\mathcal{W}, \leq_{\Psi'}), \leq_\Psi) = \min(Bel(\Psi'), \leq_\Psi)$, and by definition, $\min(Bel(\Psi'), \leq_\Psi) = Bel([\Psi \cdot Bel(\Psi')])$. So we have that $I \models Bel([\Psi \cdot \Psi'])$ if and only if $I \models Bel([\Psi \cdot Bel(\Psi')])$.

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Proof of theorem 9 :

(H7) implies directly (H'7), and (H7) and (H1) imply (H'8). To show that (H'7) and (H'8) imply (H7) we will show that (H'7) and (H'8) correspond to condition 5 and 6 on the conservative assignment and then we will deduce the implication using the representation theorem. Let $[\cdot]$ be an operator that satisfies (H0-H6) and (H'7-H'8). Let's define an assignment such that for each epistemic state Ψ we define a pre-order $\leq_\Psi$ by putting $\forall I, J \in \mathcal{W}$, $I \leq_\Psi J$ if and only if $I \models [\Psi \cdot form(I, J)]$. The proof is the same as the one of the previous theorem, it remains only to show that conditions 5 and 6 on the conservative assignment hold:

5. If $I <_{[\varphi]} J$, then by definition $[\varphi \cdot form(I, J)] \equiv form(I)$ and from (H'8) if $[\varphi \cdot form(I, J)] \vdash form(I)$, then $[\Psi \cdot \varphi \cdot form(I, J)] \vdash form(I)$. Furthermore from (H3) we get that $[\Psi \cdot \varphi \cdot form(I, J)] \nvdash \bot$, so we have $[\Psi \cdot \varphi \cdot form(I, J)] \equiv form(I)$. That is by definition $I <_{[\Psi \cdot \varphi]} J$.
6. If $I \simeq_{[\varphi]} J$, then by definition $[\varphi \cdot form(I, J)] \equiv form(I, J)$. So from (H'7) as $[\varphi \cdot form(I, J)] \equiv form(I, J)$, then $[\Psi \cdot \varphi \cdot form(I, J)] \equiv [\Psi \cdot form(I, J)]$. That is by definition: $I \leq_{[\Psi \cdot \varphi]} J$ if and only if $I \leq_\Psi J$.

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