Private Expansion and Revision in Multi-Agent Settings

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Abstract. AGM belief change aims at modeling the evolution of an agent's beliefs about its environment. In many applications though, a set of agents sharing the same environment must be considered. For such scenarios, beliefs about other agents' beliefs must be taken into account. In this work, we study private expansion and revision operators in such a multi-agent setting. More precisely, we investigate the changes induced by a new piece of information made available to one agent in the set. We point out an adaptation of AGM expansion and revision postulates to this setting, and present expansion and revision operators.

1 Introduction

Belief change aims at finding adequate ways to make the beliefs of an agent evolve when she faces new evidence. The main theoretical framework for belief change is AGM (Alchourrón-Gärdenfors-Makinson) theory and its developments [1, 14, 13]. In most works on belief revision, the belief set of the agent consists of beliefs about the environment (the world), and is represented by a set of formulas in classical logic. However, in many applications, an agent is not alone in her environment, but shares it with other agents, who also have beliefs. Beliefs about the beliefs of other agents is an important piece of information, in order to make the best decisions and to perform the best actions. Using beliefs on beliefs of other agents is for instance crucial in game theory [5, 6, 22, 18]. The most common logical tools for representing beliefs on beliefs of other agents are epistemic logics. So belief change in epistemic logics is an important issue. There exist some works on the connections between epistemic logics and belief change theory. However most of them study how to encode belief change operators within models with accessibility relations representing plausibility levels, which guide the revision process [23, 8, 10, 21]. Here, we are interested in another connection between epistemic logics and belief change theory that is closer to the AGM approach. Our objective is to design operators that change the beliefs of the agents in standard KD45$_n$ models. This task is more complicated than in the standard AGM framework, because, in a multi-agent context, the new pieces of evidence can take different forms. For instance, a new piece of evidence can be either observed/transmitted/available to every agent or only to some of them. This kind of issue has already been studied in epistemic logics with announcements, where public and private announcements lead to distinct belief changes [24, 4]. We use the terms "public change" and "private change" in the following. A public change is a change that is produced by a piece of evidence available to every agent. In this case, we are in the standard AGM case, and we can use the standard AGM machinery in order to define adequate belief change operators. A private change is a change that is produced by a piece of evidence available to one agent only. This means that the beliefs of this agent must change, whereas the beliefs of the other ones remain unchanged. In this case, we cannot directly apply AGM operators. Specific operators are required and this is what we present in this work. More precisely the aim of this paper is to define and study a multi-agent belief change setting, where the beliefs of the agents are encoded by a $KD45_n$ model. We consider private change, so a given agent receives some new piece of evidence, and one wants to define the new $KD45_n$ model that represents the new epistemic situation. We consider only objective pieces of evidence, i.e., evidences about the environment (world). The problem of considering change by subjective pieces of evidence, i.e., evidences about the beliefs of other agents, is more difficult and is left for future work. We study both expansion and revision. For each case, we provide a translation of AGM postulates for the multi-agent setting, and some specific operators. The rest of the paper is as follows. First, we give some formal preliminaries about $KD45_n$ models and AGM belief change theory. Then, we translate the AGM postulates for expansion to the multi-agent setting. In the next sections, we present a particular expansion operator, we translate the AGM postulates for revision, and we point out a family of revision operators. Finally we discuss some related works before concluding. For space reasons we cannot give the proofs, they can be found in the corresponding technical report [11].

2 Preliminaries

We consider a propositional language L_0 built up from a finite set of propositional variables P and the usual connectives. $\bot$ and $\top$ represent respectively contradiction and tautology. Let K be a belief set (i.e., a deductively closed set of formulas) and let φ be a formula. $K + \varphi$ denotes the expansion of K by φ, which is the new belief set obtained by adding φ to K. And $K * \varphi$ denotes the revision of K by φ. Alchourrón, Gärdenfors and Makinson [1, 14] pointed out some postulates for the expansion and revision of belief sets. These postulates logically encode the constraints expected on the behaviour of expansion/revision operators. Several representation theorems in terms of maximal consistent sets [1], plausibility relations on formulas [14], or plausibility relations on worlds exist [17], allowing to define operators with the expected properties. We are interested here in a framework with several agents, each of them having her own beliefs about the state of the world and about the beliefs of the other agents.. This requires the use of epistemic logic. Formally, let $A = \{1, \ldots, n\}$ be a finite set of agents. We consider the language L containing the propositional language L_0 plus one belief operator B_i for each agent $i \in A$. In addition, we sometimes use B_i^k to abbreviate a sequence of k operators B_i (i.e., $\mathrm{B}_i^0\varphi$ abbreviates φ and $\mathrm{B}_i^{k+1}\varphi$ abbreviates $\mathrm{B}_i\mathrm{B}_i^k\varphi$, for $k \geq 0$.) A formula of the form $\mathrm{B}_i\varphi$ is read 'agent i believes that φ is true'. Formulas in L_0 are also called objective formulas, while subjective formulas are formulas which are not objective. In order to give the right interpretation to our formulas, especially, to the operators B_i, we use the standard system $KD45_n$ for n-agent doxastic logic [12]. Such a system consists of the set of formulas in L that can be derived using some axioms and inference rules. The same set of validities can be captured using a semantic approach. The most common one is based on Kripke models.

Definition 1 (Kripke Model). *A Kripke model is a tuple $\langle W, R, V \rangle$ where $W \neq \emptyset$ is a set of possible worlds, $R = \{R_i \mid i \in A\}$, with R_i a binary accessibility relation* *for agent i that is serial, transitive and Euclidean, and $V : W \to 2^P$ is a valuation function. For each world $w \in W$, $V(w)$ is the set of propositional variables which are true at w. A pointed Kripke model is a pair (M, w), where $M = \langle W, R, V\rangle$ is a Kripke model and $w \in W$ is the real world.*

$R_i(w)$ denotes the set of possible worlds that are accessible from w for agent i, that is, $R_i(w) = \{w' \mid (w, w') \in R_i\}$. We note $(M, w) \models \varphi$ the fact that the formula φ is satisfied at the world w in the model M. This notion is defined using the usual satisfaction relation such that $(M, w) \models \mathrm{B}_i\varphi$ iff $\forall w' \in W$ if $(w, w') \in R_i$ then $(M, w') \models \varphi$. We use $\|\varphi\|_M$ to denote the set of possible worlds of M that satisfy φ, that is, $\|\varphi\|_M = \{w : w \in W$ and $(M, w) \models \varphi\}$. Two pointed Kripke models may satisfy the same set of formulas, and are then considered equivalent. It is known that if two pointed Kripke models are bisimilar[1] (noted $(M, w) \leftrightarroweq (M', w')$), then they are equivalent. A pointed KD45$_n$ model (M, w) represents a set of n belief sets $K_i^{(M,w)}$, one for each agent $i \in A$, where $K_i^{(M,w)} = \{\varphi \mid (M, w) \models B_i\varphi\}$. We also define the objective belief set of agent i (i.e., what i believes about the state of the world). This is the set $O_i^{(M,w)} = K_i^{(M,w)} \cap L_0$. In the following, for simplicity reasons, we make the assumption that the new piece of evidence is a consistent formula. Making a change by an inconsistent formula is allowed by AGM postulates, but is not of much interest in practical applications. Furthermore, the axiom **D** forbids inconsistent beliefs.

3 Private Expansion

Our goal in this section is to provide an extension of the AGM postulates to a multi-agent setting. We focus on private expansion operators: only one agent increases her beliefs, on a private announcement, the beliefs of other agents as well as the higher order beliefs remain unchanged. Let us denote the result of the private expansion of the model (M, w) by the objective formula φ for agent a as the model $(M, w) +_a \varphi = (M', w') = (\langle W', R', V'\rangle, w')$. The AGM postulates for expansion can be rewritten as follows:

(E_n0) $V'(w') = V(w)$
(E_n1) If $(M, w) \not\models \mathrm{B}_a\neg\varphi$ then $(M, w) +_a \varphi \in$ KD45$_n$
(E_n2) $(M, w) +_a \varphi \models \mathrm{B}_a\varphi$
(E_n3) $(M, w) \models \mathrm{B}_i\psi$ iff $(M, w) +_a \varphi \models \mathrm{B}_i\psi$, for $i \neq a$
(E_n4) If $(M, w) \not\models \mathrm{B}_a\neg\varphi$ then $(M, w) \models \mathrm{B}_a^k\mathrm{B}_i\psi$ iff $(M, w) +_a \varphi \models \mathrm{B}_a^k\mathrm{B}_i\psi$, for $i \neq a$ and $k \geq 1$
(E_n5) If $(M, w) \models \mathrm{B}_a\psi$ then $(M, w) +_a \varphi \models \mathrm{B}_a\psi$
(E_n6) If $(M, w) \models \mathrm{B}_a\varphi$ then $(M, w) +_a \varphi \leftrightarroweq (M, w)$
(E_n7) If $(M_1, w_1) \models \mathrm{B}_i\psi$ implies $(M_2, w_2) \models \mathrm{B}_i\psi$ then $(M_1, w_1) +_a \varphi \models \mathrm{B}_i\chi$ implies $(M_2, w_2) +_a \varphi \models \mathrm{B}_i\chi$
(E_n8) For all (M', w'), if (M', w') satisfies (E_n1)–(E_n7) then $(M, w) +_a \varphi \models \mathrm{B}_a\psi$ implies $(M', w') \models \mathrm{B}_a\psi$

Most of these postulates are a translation of AGM ones for KD45$_n$ models. The other ones mostly translate the fact that the only things that change are the beliefs of agent a about the state of the world. (E_n0) says that the true world does not change: as usual

[1] for the definition, see [9]

in belief revision the world does not change,[2] it is only the beliefs of the agents that evolve. (E_n1) says that, in the event that new piece of information does not contradict the beliefs of the agent, after the private expansion, the model remains KD45$_n$. Indeed, when the expansion is done by a formula that contradicts the beliefs of the agent, the result infringes the axiom **D** for the agent. The model is therefore no longer KD45$_n$. In fact, it may happen that the model is not KD45$_n$ if the agent a makes an expansion by a formula that contradicts her current beliefs. (E_n2) is the success postulate. It states that after the private expansion by φ, the agent a believes φ. Postulate (E_n3) states that the beliefs of all agents except a do not change. Postulate (E_n4) states that the beliefs of the agent a about other agents do not change. These two postulates can be seen as an adaptation of Parikh relevant revision postulates in this multi-agent setting [19]. Postulates (E_n5) and (E_n6) ensure that if φ is already believed by agent a then the private expansion does not change anything, so the resulting model is bisimilar to the initial one. Postulate (E_n7) is the translation of the monotonicity property. It states that, if a model allows more inferences than another one, then the expansion of the first one allows more inferences than the expansion of the second one. Postulate (E_n8) is the minimality postulate. It states that the result of the expansion of the model by φ is a minimal belief change. These postulates imply that:

Proposition 1. *There is a unique (up to modal equivalence) private expansion operator satisfying* (E_n0)–(E_n8).

The following proposition shows that our private expansion operator is closely related to the AGM expansion operator.

Proposition 2. *Let* $+_a$ *be the pivate expansion operator for* a *satisfying postulates* (E_n0)–(E_n8)*. The* $+$ *operator defined by* $O_a^{(M,w)} + \varphi = O_a^{(M,w)+_a\varphi}$ *is the AGM expansion operator (i.e., it satisfies (K+1)–(K+6) [1]).*

4 A Private Expansion Operator

Let us now give a constructive definition of the private expansion operator characterized in the previous section. In the remainder of this paper, we use as a notation for the newly created worlds (due to expansion or revision) v_w^e. This notation means that the world v_w^e is a "copy" of the world w (this copy is essential to avoid losing the higher-order beliefs of the agent who performs the expansion or the revision of her beliefs) and having the valuation e.

Definition 2. ***Expansion of*** (M, w_0) ***by*** φ ***for agent*** a***.***
Let $(M, w_0) = (\langle W, R, V\rangle, w_0)$ *be a KD45*$_n$ *pointed model, and* φ *be a consistent objective formula (i.e.,* $\varphi \in L_0$*). We define the private expansion of* (M, w_0) *by* φ *for agent* a *as* $(M, w_0) +_a \varphi = (\langle W', R', V'\rangle, w'_0)$*, such that:*

- $E = \{V(w) \mid w \in R_a(w_0) \cap \|\varphi\|_M\}$
- $W' = W \cup W^\varphi \cup \{w'_0\}$ *where*
  - $W^\varphi = \bigcup_{w \in R_a(w_0)} W_w^\varphi$ *and* $W_w^\varphi = \bigcup_{e \in E} \{v_w^e\}$
- $R'_a = R_a \cup R_a^\varphi \cup R_a^0$ *where*
  - $R_a^\varphi = \{(w_1^\varphi, w_2^\varphi) \mid w_1^\varphi, w_2^\varphi \in W^\varphi\}$

[2] When the world evolves, one has to use update [16, 15].

  - $R_a^0 = \{(w'_0, w^\varphi) \mid w^\varphi \in W^\varphi\}$
- $R'_i = R_i \cup R_i^{\overrightarrow{\varphi}} \cup R_i^0$, *for* $i \neq a$, *where*
  - $R_i^{\overrightarrow{\varphi}} = \{(v_w^e, w') \mid wR_iw'$ *and* $v_w^e \in W^\varphi\}$, *for* $i \neq a$
  - $R_i^0 = \{(w'_0, w) \mid (w_0, w) \in R_i\}$, *for* $i \neq a$
- $V'(w) = V(w)$, *for* $w \in W$
- $V'(v_w^e) = e$, *for* $v_w^e \in W^\varphi$
- $V'(w'_0) = V(w_0)$

When the agent a expands her beliefs, the model must change in order to represent these new beliefs, but the beliefs of other agents should remain unchanged. The new set of possible worlds W' contains all possible worlds of the initial model plus a new real world w'_0 and a set of worlds W^φ representing the new beliefs of a. The set W^φ contains a copy of each world in $R_a(w_0)$ which does not contradict φ. The new accessibility relation R'_a contains the initial relation R_a and the set R_a^0. The set R_a^0 consists of pairs (w'_0, w^φ) where $w^\varphi \in W^\varphi$, thus modifying the beliefs of the agent performing the expansion. The set R_a^φ consists of the pairs $(w_1^\varphi, w_2^\varphi) \in W^\varphi$. The worlds in W^φ thus form a clique, because they are equally plausible for the agent performing the expansion. Each accessibility relation R'_i, for $i \neq a$, contains the initial relation R_i and the sets R_i^0 and $R_i^{\overrightarrow{\varphi}}$. The set R_i^0 consists of all pairs (w'_0, w) such that $(w_0, w) \in R_i$, thus preserving the beliefs of agents not performing expansion and higher-order beliefs of all agents. The set $R_i^{\overrightarrow{\varphi}}$ consists of pairs (v_w^e, w'), where $v_w^e \in W^\varphi$ such that $(w, w') \in R_i$, thus keeping higher-order beliefs of the agent performing the expansion. We can now show that:

Proposition 3. *The operator $+$ satisfies (E_n0)–(E_n8).*

As a direct consequence of Proposition 1, we know that this operator is the unique private expansion operator. Let us now illustrate the behaviour of this private expansion operator on a simple example.

Example 1. Consider the KD45$_n$ model (M, w_0) of Figure 1. In this situation, agent 1 believes $\neg p$ and she believes that agent 2 also believes $\neg p$. Agent 2 believes $\neg p \wedge \neg q$, and she believes that agent 1 believes $\neg p$. After the expansion by q, agent 1 must believe $\neg p \wedge q$. The obtained model (M', w'_0) is reported as well on Figure 1. The world having the valuation $\neg p \wedge q$ has to be duplicated in order to keep the higher-order beliefs of agent 1. Contrastingly, the beliefs of agent 2 remain unchanged, so in particular she still believes that agent 1 believes $\neg p$.

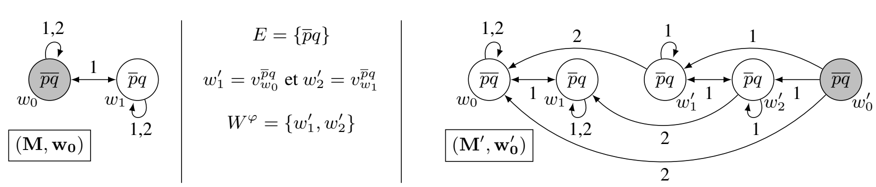

Fig. 1. $(M, w_0) +_1 q$

5 Private Revision

Let us turn now to the definition of private revision operators. These operators behave like expansion when there is no inconsistency between the beliefs of the agent and the new piece of evidence, but, unlike expansion, do not trivialize when this is not the case.

Let us denote the result of the private revision of the model (M, w) by the objective formula φ for agent a to be the model $(M, w) \star_a \varphi = (M', w') = (\langle W', R', V' \rangle, w')$. The AGM postulates for revision can be rewritten as follows:

(R_n0) $V'(w') = V(w)$
(R_n1) $(M, w) \star_a \varphi \in \text{KD45}_n$
(R_n2) $(M, w) \star_a \varphi \models \mathrm{B}_a\varphi$
(R_n3) $(M, w) \models \mathrm{B}_i\psi$ iff $(M, w) \star_a \varphi \models \mathrm{B}_i\psi$, for $i \neq a$
(R_n4) $(M, w) \models \mathrm{B}_a^k\mathrm{B}_i\psi$ iff $(M, w) \star_a \varphi \models \mathrm{B}_a^k\mathrm{B}_i\psi$, for $i \neq a$
(R_n5) If $(M, w) \star_a \varphi \models \mathrm{B}_i\psi$ then $(M, w) +_a \varphi \models \mathrm{B}_i\psi$
(R_n6) If $(M, w) \not\models \mathrm{B}_a\neg\varphi$, then $(M, w) +_a \varphi \leftrightarroweq (M, w) \star_a \varphi$
(R_n7) If $(M^1, w^1) \leftrightarroweq (M^2, w^2)$ and $\models \varphi \equiv \psi$, then $(M^1, w^1) \star_a \varphi \leftrightarroweq (M^2, w^2) \star_a \psi$
(R_n8) If $(M, w) \star_a (\varphi \wedge \psi) \models \mathrm{B}_i\chi$ then $((M, w) \star_a \varphi) +_a \psi \models \mathrm{B}_i\chi$
(R_n9) If $(M, w) \star_a \varphi \not\models \mathrm{B}_a\neg\psi$, then $((M, w) \star_a \varphi) +_a \psi \models \mathrm{B}_i\chi$ implies $(M, w) \star_a (\varphi \wedge \psi) \models \mathrm{B}_i\chi$.

(R_n1) ensures that the model obtained after a revision is still a KD45$_n$ model. (R_n2) is the success postulate, it states that φ is believed by a after the revision. (R_n3) states that the beliefs of all agents except a do not change. (R_n4) states that the beliefs of the agent a about other agents do not change. These two postulates can be seen as an adaptation of Parikh relevant revision postulates in this multi-agent setting [19]. (R_n5) and (R_n6) state that when the new piece of evidence is consistent with the beliefs of the agent, revision is just expansion. (R_n7) is an irrelevance of syntax postulate, stating that if two formulas are logically equivalent, then they lead to the same revision results. (R_n8) and (R_n9) state when the revision by a conjunction can be obtained by a revision followed by an expansion. Let us now show that the revision operators satisfying those postulates are conservative extensions of the usual AGM belief revision operators:

Proposition 4. *Let $\star_i$ be an revision operator satisfying postulates (R_n0)–(R_n9). The $\star$ operator defined as $O_i^{(M,w)} \star \varphi = O_i^{(M,w)\star_i\varphi}$ is an AGM revision operator (i.e., it satisfies (K*1)–(K*8) [1]).*

6 A Family Of Private Revision Operators

Let us now define a family of private revision operators. These operators are defined similarly to the expansion operator of the previous section, but in the cases when the new piece of evidence is inconsistent with the current beliefs of the agent they use a classical AGM belief revision operator $\circ$ in order to compute the new beliefs of the agent.

Definition 3. ***Revision of (M, w_0) by φ for agent a.***
Let $(M, w_0) = (\langle W, R, V \rangle, w_0)$ be a KD45$_n$ model, let φ be a consistent objective formula (i.e., $\varphi \in L_0$), and let $\circ$ be an AGM revision operator. We define the private revision of (M, w_0) by φ for agent a (with revision operator $\circ$) as $(M, w_0) \star_a^\circ \varphi = (\langle W', R', V' \rangle, w_0')$, such that:

- *if $R_a(w_0) \cap \|\varphi\|_M \neq \emptyset$*
  - *then $E = \{V(w) \mid w \in R_a(w_0) \cap \|\varphi\|_M\}$*

- else $E = \{e \mid e \subseteq P$ and $e \models O_a^{(M,w_0)} \circ \varphi\}$
- $W' = W \cup W^{\varphi} \cup \{w'_0\}$ where
  - $W^{\varphi} = \bigcup_{w \in R_a(w_0)} W_w^{\varphi}$ and $W_w^{\varphi} = \bigcup_{e \in E} \{v_w^e\}$
- $R'_a = R_a \cup R_a^{\varphi} \cup R_a^0$ where
  - $R_a^{\varphi} = \{(w_1^{\varphi}, w_2^{\varphi}) | w_1^{\varphi}, w_2^{\varphi} \in W^{\varphi}\}$
  - $R_a^0 = \{(w'_0, w^{\varphi}) | w^{\varphi} \in W^{\varphi}\}$
- $R'_i = R_i \cup R_i^{\overrightarrow{\varphi}} \cup R_i^0$ for $i \neq a$, where
  - $R_i^{\overrightarrow{\varphi}} = \{(v_w^e, w') | wR_iw', v_w^e \in W^{\varphi}\}$ for $i \neq a$
  - $R_i^0 = \{(w'_0, w) | (w_0, w) \in R_i\}$ for $i \neq a$
- $V'(w) = V(w)$ for $w \in W$
- $V'(v_w^e) = e$ for $v_w^e \in W^{\varphi}$
- $V'(w'_0) = V(w_0)$

The construction of the revised model is similar to the construction of the expanded model discussed earlier. Only the new set of worlds W^{φ} is different: if the new information φ is considered possible by agent a, she performs an expansion, otherwise, each of the worlds of the new set W^{φ} has as valuation a (propositional) model of the new information φ.

Let us now show that these operators exhibit the expected logical properties:

Proposition 5. *The operators $\star_a^{\circ}$ satisfy (R_n0)–(R_n9).*

Let us now illustrate the behaviour of these private revision operators on a simple example.

Example 2. We consider the model (M, w_0) of Figure 2, where agent 1 believes $\neg x \wedge \neg y$ and believes that agent 2 believes $x \wedge y$. Agent 2 believes $x \wedge y$ and believes that agent 1 believes $x \leftrightarrow y$. After the revision by $x \wedge y$, agent 1 must believe $x \wedge y$. Whereas the beliefs of agent 2 remain unchanged. The obtained model (M', w'_0) is reported as well in Figure 2. In this example, agent 1 uses Dalal's AGM revision operator $\circ_D$ [17]. We can observe that the revised model obtained using Definition 3 may not be minimal. Nevertheless, a minimal model can be obtained via a bisimulation contraction. Here, this leads to the model (M'', w'_0).

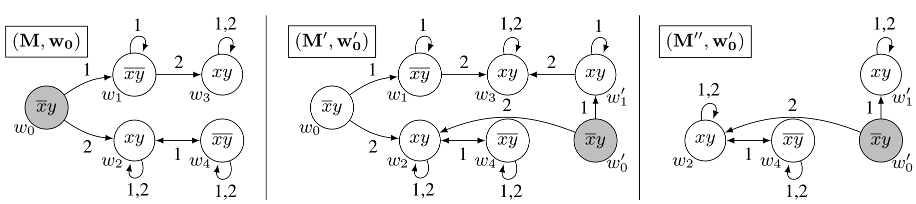

Fig. 2. $(M'', w'_0) \leftrightarroweq (M, w_0) \star_1^{\circ_D} (x \wedge y)$

Our approach to private revision can be encoded in a formalism called dynamic epistemic logic [7]. To provide such an encoding, we need event models with assignments, as proposed in [25]. The idea is, for a given formula φ, to create a specific event model such that its execution simulates the revision by φ. An event model is a structure $N = \langle S, T, \mathrm{pre}, \mathrm{pos}\rangle$, where S is a non-empty set of possible events; $T = \{T_i : i \in A\}$, where T_i is a binary accessibility relation for agent i; $\mathrm{pre} : S \to L$ is a function that returns, for each possible event $s \in S$, a formula in L representing its pre-condition; and $\mathrm{pos} : S \to (P \to \{\top, \bot\})$ is a function that returns, for each possible event $s \in S$, its post-condition. The post-condition is an assignemnt of propositional variables to $\top$ or $\bot$. Thus, pos is used to reset the valuations after the execution of the events. A pointed event model is a pair (N, s), where $s \in S$ is the actual event. The product of (M, w) by (N, s) is a new pointed model $(M^N, w.s)$ where $M^N = \langle W^N, R^N, V^N\rangle$, $W^N = \{w.s \mid M, w \models \mathrm{pre}(s)\}$, $R^N = \{(w.s, w'.s') : (w, w') \in R_a \text{ and } (s, s') \in T_a\}$ and $V^N(w) = \{p \mid \mathrm{pos}(w)(p) = \top\}$.

In the sequel, we show that the revision of Definition 3 is equivalent to a specific model product. More precisely, $(M, w_0) \star_a^\circ \varphi$ and $(M^{N^{\star_a^\circ}}, w.s_0)$ are bisimilar, where:

- $S = \{s_0, s_\top\} \cup \{s_w^e \mid v_w^e \in W^\varphi\}$
- $T_a = \{(s_0, s_w^e) \mid v_w^e \in W^\varphi\} \cup \{(s_{w_1}^{e_1}, s_{w_2}^{e_2}) \mid v_{w_1}^{e_1}, v_{w_2}^{e_2} \in W^\varphi\} \cup \{(s_\top, s_\top)\}$
- $T_i = \{(s_0, s_\top), (s_\top, s_\top)\} \cup \{(s_w^e, s_\top) \mid v_w^e \in W^\varphi\}$, for $i \neq a$
- $\mathrm{pre}(s_0) = \bigwedge_{p \in V(w_0)} p \wedge \bigwedge_{p \in P \setminus V(w_0)} \neg p$
- $\mathrm{pre}(s_w^e) = \bigwedge_{p \in V(w)} p \wedge \bigwedge_{p \in P \setminus V(w)} \neg p$
- $\mathrm{pre}(s_\top) = \top$
- $\mathrm{pos}(s_w^e)(p) = \begin{cases} \top, & \text{if } e \models p \\ \bot, & \text{if } e \not\models p \end{cases}$
- $\mathrm{pos}(s_0) = \mathrm{pos}(s_\top) = \emptyset$

The event model here is somewhat similar to the one we could make for expansion. A main difference is that the clique of possible events s_w^e is replaced by a single possible event s_φ with $\mathrm{pre}(s_\varphi) = \varphi$ and $\mathrm{pos}(s_\varphi) = \emptyset$.

Proposition 6. $((M, w_0) \star_a^\circ \varphi) \leftrightarroweq (M^{N^{\star_a^\circ}}, w_0.s_0)$.

7 Related Work

As explained in the introduction, there are some works on the connections between epistemic logics and belief change theory, but most of them study how to encode belief change operators within an epistemic model [23, 8, 10, 21]. Basically the problem is to try to perform belief revision within the epistemic model. Contrastingly, we study in this work how to perform belief revision (and expansion) on a KD45$_n$ model, representing the beliefs of a group of agents. In the same vein, in [20] the authors study what they call revision of KD45$_n$ models due to communication between agents: some agents (publicly) announce (part of) their beliefs. Their model is closer to expansion than to true revision, and concerns only subjective beliefs. In [15] the authors study action progression in multi-agent belief structures. Their work is mainly about the effects of actions using update, but they also briefly mention the problem of revision by objective formulas. Their construction is related to the one we point out, but they do not study the properties of the operators they considered. Finally the closest work to our own one is the study of private expansion and revision made by Aucher [3, 2, 4]. The difference is that Aucher considers an internal model of the problem, i.e., a model of the situation viewed from each agent, so he does not use a KD45$_n$ model for modeling the system, but one internal model by agent. He uses a notion of multi-agent possible worlds in order to compute the result of the revision, so the result of the revision is a set of such multi-agent worlds, whereas in this work we work with KD45$_n$ models, and we obtain a unique KD45$_n$ model as result of a revision. It is easy to find a translation between internal models and KD45$_n$ models, so one can look at the technical details between the expansion and revision operators we present in this work and the one proposed (on internal models) by Aucher [3, 2, 4]. Concerning expansion, it turns out that the two operations are equivalent (that is not surprising since we proved that there is only one rational expansion operator). First, note that it is possible to obtain an internal model I_M for agent $a \in N$ from any KD45$_n$ model (M, w_0). Indeed, it suffices to consider the set formed of models (M^k, w^k) generated from each w^k such that $w^k \in R_a(w_0)$. Similarly, it is possible to obtain an internal event model I_N for agent $a \in A$ from the event model (N, s_0). Now, it is easy to see that the internal model for a obtained from the product of (M, w_0) by (N, s_0) is the same as the product of I_M by I_N. Concerning revision the situation is different. Aucher allows revision by subjective formulas and compute distances between the corresponding (epistemic) models. We are interested here only by revision with objective formulas. In this particular case Aucher's revision does not allow the agent concerned by the private revision to choose, among the models of the objective formulas, the ones that are the most plausible. This is problematic since it is one of the main goals of belief revision to make such a selection. We can do that thanks to the underlying AGM revision operators in the definition of the private revision operator. So our private revision result implies (usually strictly) the result given by Aucher's revision.

8 Conclusion

In this paper we investigate the problem of belief change in a multi-agent context. More precisely we study private expansion and revision of KD45$_n$ models by objective formulas. We present a set of postulates for expansion and revision close to the classical AGM ones for the single agent case. We also define specific expansion and revision operators and show that they satisfy the properties pointed out. As future work we plan to consider different extensions of this work. The first issue to be considered is the problem of private change by subjective formulas. For expansion the method will be quite similar to the one we described here for objective formulas. But for revision the subjective case is both more complicated and richer than the revision by objective formulas, due to the minimality of change requirement. In fact some interesting metrics can be defined and used to define minimal change for revision. Another issue we want to address is group change. The idea is that the new evidence is not given privately to only one agent, but to a group of agents. This case straightforwardly includes private change and public change as special cases. So it is clearly the most general framework. Interaction between the agents adds interesting additional problems, since each agent of the group will have to revise her beliefs about the beliefs of the other agents of the group receiving the same observation.

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