Some Operators for Iterated Revision

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Abstract. We propose a construction that allows to define operators for iterated revision from "classical" AGM revision operators. We call those operators *revision with memory operators.* We show that the operators obtained have nice logical properties. We illustrate this construction with the well-known Dalal revision operator. We also give two new particular revision operators based on the revision operators on OTP proposed by Ryan [20]. His operator do not satisfy a lot of logical properties. The two operators we give based on OTP satisfy all wanted revision properties.

1 Introduction

One of the predominant approaches to model belief change was proposed by Alchourrón, Gärdenfors and Makinson and is known as the AGM framework [1, 10]. The core of this framework is a set of logical properties that a revision operator has to satisfy to guarantee a nice behaviour.

A drawback of AGM definition of revision is that it is a static one, that means that, with this definition of revision operators, one can have a rational one step revision but the conditions for the iteration of the process are very weak. The problem is that AGM postulates state conditions only between the initial knowledge base, the new evidence and the resulting knowledge base. But the way to perform further revisions on the new knowledge base does not depend on the way the old knowledge base was revised.

Numerous proposals have tried to state a logical characterization that adequatly models iterated belief change behaviour [8, 7, 5, 13, 17, 16, 12]. The more famous one seems to be [8]. The main idea that is common to all of those works is that the belief base framework is not sufficient to encompass iterated revision, since one needs some additional information for coding the revision policy of the agent. So the need of *epistemic states* to encode the agent "state of mind" is widely accepted. An epistemic state allows to code agent's beliefs but also to code its relative confidence in alternative possible states of the world. Epistemic states can be represented by several means: pre-orders on interpretations [8, 13], conditionals [5, 8], epistemic entrenchments [21, 16], prioritized belief bases [2, 3], etc. In this paper we will focus on the representation of epistemic states in terms of pre-orders on interpretations.

What we propose in this paper is not yet an other logical characterization, but the definition of a family of operators, that we call *revision with memory operators*, that aims to have good iteration properties.

Dalal-like revision operators are sometimes decry for their *a priori*, *extra-logical* information which represents the distance that they use to order interpretations. We give two operators derived from Ryan's OTP revision operator [20]. We will see that Ryan's operator does not satisfy the wanted logical properties and give two modifications of Ryan OTP revision operator that will. These three operators are interesting since, conversely to Dalal-like operators, there is no *a priori* distance. This information is provided by the formulae themselves in a very natural (syntactical) way.

In section 2 we recall the logical characterization of Darwiche and Pearl. In section 3 we give the definition of revision with memory operators and state the general logical results. Then, in section 4 we provide five examples of operators. Apart from Ryan operator (section 4.3), that is not a revision with memory operator, the four other operators have nice logical properties. Three of them are, as far as we know, new operators. We conclude in section 5 by some general remarks.

2 Iterated Revision Postulates

We give here a formulation of AGM postulates for belief revision *à la* Katsuno and Mendelzon [11]. More exactly we give a formulation of these postulates in terms of epistemic states [8]. The epistemic states framework is an extension of the belief bases one. Intuitively an epistemic state can be seen as a composed information: the beliefs of the agent, plus all information that agent needs about how to perform revision (preference ordering, conditionals, etc.). Then we give the additional iteration postulates proposed by Darwiche and Pearl [8].

2.1 Formal Preliminaries

We will work in the finite propositional case. A belief base φ is a set of formulae, which can be considered as the formula that is the conjunction of its formulae.

The set of all interpretations is denoted $\mathcal{W}$. Let φ be a formula, $Mod(\varphi)$ denotes the set of models of φ, *i.e.* $Mod(\varphi) = \{I \in \mathcal{W} : \ I \models \varphi\}$.

A pre-order $\leq$ is a reflexive and transitive relation, and $<$ is its strict counterpart, i.e $I < J$ if and only if $I \leq J$ and $J \not\leq I$. As usual, $\simeq$ is defined by $I \simeq J$ iff $I \leq J$ and $J \leq I$.

To each epistemic state Ψ is associated a belief base $Bel(\Psi)$ which is a propositional formula and which represents the objective (logical) part of Ψ. The models of Ψ are the models of its associated belief base, thus $Mod(\Psi) = Mod(Bel(\Psi))$. Let Ψ be an epistemic state and μ be a sentence denoting the new information. $\Psi \circ \mu$ denotes the epistemic state resulting of the revision of Ψ by μ. For reading convenience we will write respectively $\Psi \vdash \mu$, $\Psi \wedge \mu$ and $I \models \Psi$ instead of $Bel(\Psi) \vdash \mu$, $Bel(\Psi) \wedge \mu$ and $I \models Bel(\Psi)$.

Two epistemic states are equivalent, noted $\Psi \equiv \Psi'$, if and only if their objective parts are equivalent formulae, *i.e.* $Bel(\Psi) \leftrightarrow Bel(\Psi')$. Two epistemic states are equal, noted $\Psi = \Psi'$, if and only if they are identical. Thus equality is stronger than equivalence. In fact *equivalence* denotes a static equivalence, since after a belief change, the two epistemic states can lead to very different ones, whereas *equality* denotes a dynamic equivalence between epistemic states, since all sequences of belief change perform on these two epistemic states will lead to two equal epistemic states[1].

2.2 AGM Postulates for Epistemic States

Let Ψ be an epistemic state and μ and φ be formulae. An operator $\circ$ that maps an epistemic state Ψ and a formula μ to an epistemic state $\Psi \circ \mu$ is said to be a revision operator on epistemic states if it satisfies the following postulates [8]:

(R*1) $\Psi \circ \mu \vdash \mu$
(R*2) If $\Psi \wedge \mu \nvdash \bot$, then $\Psi \circ \mu \leftrightarrow \Psi \wedge \mu$
(R*3) If $\mu \nvdash \bot$, then $\Psi \circ \mu \nvdash \bot$
(R*4) If $\Psi_1 = \Psi_2$ and $\mu_1 \leftrightarrow \mu_2$, then $\Psi_1 \circ \mu_1 \equiv \Psi_2 \circ \mu_2$
(R*5) $(\Psi \circ \mu) \wedge \varphi \vdash \Psi \circ (\mu \wedge \varphi)$
(R*6) If $(\Psi \circ \mu) \wedge \varphi \nvdash \bot$, then $\Psi \circ (\mu \wedge \varphi) \vdash (\Psi \circ \mu) \wedge \varphi$

This is nearly the Katsuno and Mendelzon formulation of AGM postulates [11], the only differences are that we work with epistemic states instead of belief bases and that postulate (R*4) is weaker than its AGM counterpart. See [8] for a full motivation of this definition.

A representation theorem, stating how revisions can be characterized in terms of pre-orders on interpretations, holds. In order to give such semantical representation, the concept of faithful assignment on epistemic states is defined.

Definition 1. *A function that maps each epistemic state Ψ to a pre-order $\leq_\Psi$ on interpretations is called a faithful assignment over epistemic states if and only if:*

1. *If $I \models \Psi$ and $J \models \Psi$, then $I \simeq_\Psi J$*
2. *If $I \models \Psi$ and $J \not\models \Psi$, then $I <_\Psi J$*
3. *If $\Psi_1 = \Psi_2$, then $\leq_{\Psi_1} = \leq_{\Psi_2}$*

Now the reformulation of Katsuno and Mendelzon [11] representation theorem in terms of epistemic states is:

Theorem 1 *A revision operator $\circ$ satisfies postulates (R*1-R*6) if and only if there exists a faithful assignment that maps each epistemic state Ψ to a total pre-order $\leq_\Psi$ such that:*

$$Mod(\Psi \circ \mu) = \min(Mod(\mu), \leq_\Psi)$$

Notice that this theorem gives information only on the objective part of the resulting epistemic state.

[1] note that $\Psi = \Psi'$ implies $\Psi \equiv \Psi'$.

2.3 Darwiche and Pearl Postulates

A strong limitation of AGM revision postulates is that they impose very weak constraints on the iteration of the revision process. Darwiche and Pearl [7, 8] proposed postulates for iterated revision. The aim of these postulates is to keep as much as possible of conditional beliefs[2] of the old belief base. So, besides postulates (R*1-R*6), a revision operator has to satisfy:

(C1) If $\varphi \vdash \mu$, then $(\Psi \circ \mu) \circ \varphi \equiv \Psi \circ \varphi$
(C2) If $\varphi \vdash \neg\mu$, then $(\Psi \circ \mu) \circ \varphi \equiv \Psi \circ \varphi$
(C3) If $\Psi \circ \varphi \vdash \mu$, then $(\Psi \circ \mu) \circ \varphi \vdash \mu$
(C4) If $\Psi \circ \varphi \nvdash \neg\mu$, then $(\Psi \circ \mu) \circ \varphi \nvdash \neg\mu$

These postulates can be explained as follows: (C1) states that if two pieces of information arrive and if the second implies the first, the second alone would give the same belief base. (C2) says that when two contradictory pieces of information arrive, the second alone would give the same belief base. (C3) states that an information should be retained after revising by a second information such that, when revising the current belief base by it, the first one holds. (C4) says that no piece of information can contribute to its own denial.

3 Building memory operators from "classical" AGM ones

A "classical" AGM revision operator is equivalent to a faithful assignment over belief bases as stated in the following theorem [11].

Definition 2. *A function that maps each* **belief base** φ *to a pre-order* $\leq_\varphi$ *on interpretations is called a faithful assignment over belief bases if and only if:*

1. *If* $I \models \varphi$ *and* $J \models \varphi$*, then* $I \simeq_\varphi J$
2. *If* $I \models \varphi$ *and* $J \not\models \varphi$*, then* $I <_\varphi J$
3. *If* $\varphi_1 \leftrightarrow \varphi_2$*, then* $\leq_{\varphi_1} = \leq_{\varphi_2}$

Theorem 2 *A revision operator* $\circ$ *satisfies "classical" AGM postulates (R1-R6)*[3] *if and only if there exists a faithful assignment (over belief bases) that maps each belief base* φ *to a total pre-order* $\leq_\varphi$ *such that:*

$$Mod(\varphi \circ \mu) = \min(Mod(\mu), \leq_\varphi)$$

So one can define a revision operator directly by defining the corresponding faithful assignment over belief bases. It is the case for most distance-based revision operators such as Dalal operator for example [6, 11].

More precisely we say that a revision operator $\circ$ is defined from a distance d iff the following conditions hold:

- d is a distance, that is d is a function $d : \mathcal{W} \times \mathcal{W} \mapsto \mathbb{R}^+$ that satisfies: $d(I, J) = d(J, I)$ and $d(I, J) = 0$ iff $I = J$.

[2] a conditional belief can be expressed as "if μ would be the case, then φ must be true"

[3] it is the same set of postulates than (R*1-R*6) but expressed for belief bases instead of belief states (*cf* [11]).

- Then the distance between an interpretation I and a belief base φ is defined as: $d(I,\varphi) = \min\{d(I,J) : J \models \varphi\}$
- This distance induces a faithful assignment: $I \leq_\varphi J$ iff $d(I,\varphi) \leq d(J,\varphi)$
- And the revision operator is defined by $Mod(\varphi \circ \mu) = \min(Mod(\mu), \leq_\varphi)$

One can check that the assignment obtained like this is a faithful assignment and thus that all operators defined in this way satisfy AGM postulates. It can also be easily checked that operators defined in this way do not satisfy a lot of iterated revision postulates.

Now we will give a construction that allows, from a given faithful assignment (*i.e.* from a given "classical" revision operator), to define an other revision operator that satisfy AGM postulates but also most of iterated revision postulates.

First, let us notice that an epistemic state can be represented by a total pre-order on interpretations as suggested by theorem 1 and by several related works (*cf e.g* [8, 3]). So, with this particular representation, that is if we identify the epistemic state Ψ with a pre-order $\leq_\Psi$, the belief base $Bel(\Psi)$ is simply the formula whose models are minimal for the pre-order, that is $Bel(\Psi) = \min(\mathcal{W}, \leq_\Psi)$. And the other interpretations are ordered according to their relative plausibility for the agent. For example $I \leq_\Psi J$ means that the agent that is in the epistemic state Ψ consider I as more plausible than J. It is this preferential information that can be used to encompass the iterated revision behaviour, by considering revision operators as functions that maps a pre-order (epistemic state) and a formula (new information) into a new pre-order (epistemic state). This idea is the mainstay in most of iterated revision works [21, 8, 16].

So using this representation by means of pre-orders on interpretations and theorem 1 we will define a familly of revision operators as follows:

Definition 3. *Suppose that we dispose of a function that maps each belief base φ to a pre-order $\leq_\varphi$. Then we define the epistemic state (the pre-order) $\Psi \circ \varphi$ result of the revision of Ψ by the new information φ as:*

$$I \leq_{\Psi\circ\varphi} J \text{ iff } I <_\varphi J \text{ or } I \simeq_\varphi J \text{ and } I \leq_\Psi J$$

Then one can check that:

Theorem 3 *If the function that maps each belief base φ to a total pre-order $\leq_\varphi$ is a faithful assignment over belief bases, then the revision operator on epistemic states defined in definition 3 satisfies postulates (R*1-R*6). We will call* revision operators with memory *those operators.*

So with definition 3, one can start from any epistemic state (total pre-order over interpretations) and carry on iterated revisions. A particular epistemic state we can mention is the "empty" epistemic state, where the agent has no belief and no preferential information, that is such that $\forall I, J\ I \simeq J$. We will note Ξ this epistemic state. So the objective part of this epistemic state is $Bel(\Xi) = \top$. It can be considered as the epistemic state generalisation of $\top$ for the belief base framework, since they are both neutral elements for the corresponding operators: $\Psi \circ \Xi = \Psi$ (as $\varphi \circ \top = \varphi$ in the belief base framework). One can consider that all agents start with this epistemic sate (we will consider this in the examples).

In fact the family defined is more specific than that, since there are more properties that are satisfied by those operators:

(H4) If $\Psi_1 = \Psi_2$ and $\mu_1 \leftrightarrow \mu_2$, then $\Psi_1 \circ \mu_1 = \Psi_2 \circ \mu_2$
(C) If $\varphi \wedge \mu$ is satisfiable, then $\Psi \circ \varphi \circ \mu \equiv \Psi \circ (\varphi \wedge \mu)$

(H4) is a strenghten of (R*4). (C) states that when one revises successively by two consistent pieces of information, it amounts to revise by their conjunction. It is close to a postulate proposed by Nayak and al. [17] called *Conjunction*, but (C) is weaker than *Conjunction*, since it requires only the equivalence of the two resulting epistemic states, not the equality. See [12] for a full logical characterization of revision with memory operators.

Concerning iteration postulates stated by Darwiche and Pearl [8]:

Theorem 4 *Revision operators with memory satisfy postulates (C1),(C3) and (C4).*

It can be also easily checked that (C2) is satisfied by a unique operator with memory, since it demands (in the presence of the other revision postulates), that the pre-order associated to a belief base by the faithful assignment on belief base used in definition 3 is a two-level pre-order with the models of the belief base at the lowest level and the counter-models at the higher one. This operator will be presented in the next section.

So most of our revision with memory operators do not satisfy (C2). But we do not consider this as a drawback. We rather think that it is (C2) that is not fully satisfactory.

In fact, in [7] the set of postulates (C1-C4) has first been given as a complement to usual “classical” AGM postulates. Freund and Lehmann [9] have shown that (C2) is inconsistent with those postulates. Furthermore Lehmann [13] has shown that (C1) plus AGM postulates imply (C3) and (C4). In [8] Darwiche and Pearl have rephrased their postulates (and AGM ones) in terms of epistemic states instead of belief bases, and thus have removed these logical contradictions.

But we do not think that it is enough to requalify (C2) and we think that satisfy (C2) can lead to counterintuitive results. Consider the following example:

Example 1 *Consider a circuit containing an adder and a multiplier. In this example we have two atomic propositions,* adder_ok *and* multiplier_ok*, denoting respectively the fact that the adder and the multiplier are working. We have initially no information about this circuit ($\Psi = \Xi$) and we learn that the adder and the multiplier are working ($\mu =$ adder_ok $\wedge$ multiplier_ok). Then someone tells us that the adder is not working ($\varphi = \neg$ adder_ok). There is, then, no reason to “forget” that the multiplier is working, which is imposed by (C2): $\varphi \models \neg\mu$ so by (C2) we have $\Psi \circ \mu \circ \varphi \equiv (\Psi \circ \varphi) \equiv \varphi$.*

This example is a slight modification of an example given in [8]. So, in some cases, postulates (C2) induces exactly the same kind of bad behaviour it tries to prevent.

4 Some revision with memory operators

4.1 Basic memory operator

Let us define the assignment that maps each belief base to a pre-order in the following way:

Definition 4. $I \leq^b_\varphi J$ *if and only if* $I \models \varphi$ *or*
$I \not\models \varphi$ *and* $J \not\models \varphi$

So we have what we shall call a basic order, which is a two-level order (at most), with the models of φ at the lower level and the other worlds at the higher level.

Definition 5. *The* basic memory operator *is the memory operator obtained from this assignment (i.e. the operator obtained by definitions 4 and 3).*

Even with this basic order on belief bases, one can build very complex epistemic states. This is due to revision memory. We illustrate the behaviour of this operator through some simple examples.

Example 2 *Consider a language* $\mathcal{L}$ *with only two propositional letters* a *and* b. *We will denote interpretations simply by the truth assignment, i.e* 10 *denotes the interpretation mapping* a *to true and* b *to false. Two interpretations are equivalent, with respect to the pre-order, if they appear at the same level. An interpretation* I *is better than another interpretation* J *(*$I \leq J$*) if it appears at a lower level. Let us see some examples of epistemic states:*

$$\leq^b_{\Xi\circ a\circ b} = \begin{array}{c} 00 \\ 10 \\ 01 \\ 11 \end{array} \quad \leq^b_{\Xi\circ a\wedge b} = \begin{array}{c} 00\ 01\ 10 \\ 11 \end{array} \quad \leq^b_{\Xi\circ(a\wedge b)\circ a} = \begin{array}{c} 00\ 01 \\ 10 \\ 11 \end{array} \quad \leq^b_{\Xi\circ(a\wedge b)\circ a\circ\neg b} = \begin{array}{c} 01 \\ 11 \\ 00 \\ 10 \end{array}$$

$$\leq^b_{\Xi\circ(a\wedge b)\circ\neg b} = \begin{array}{c} 01 \\ 11 \\ 00\ 10 \end{array} \quad \leq^b_{\Xi\circ a\circ b\circ\neg(a\wedge b)} = \begin{array}{c} 11 \\ 00 \\ 10 \\ 01 \end{array} \quad \leq^b_{\Xi\circ a\circ(a\wedge b)\circ\neg(a\wedge b)} = \begin{array}{c} 11 \\ 00\ 01 \\ 10 \end{array}$$

The assignment defined is a faithful assignment on belief bases, with theorems 3 and 4, it is easy to show that:

Theorem 5 *The only revision operator with memory that satisfies (R*1-R*6) and (C1-C4) is the basic memory revision operator.*

This operator has been already studied in the litterature under different particular representations: in [16] with epistemic entrenchments, in [2] with polynomials and syntactic belief bases. Finally, we can note that Liberatore has shown [15] that several problems are computationally simpler for the basic memory operator than for the other iterated belief revision proposals (including Boutilier's natural revision [4], Lehmann's ranking revision [13] and Williams' transmutations [21]).

4.2 Dalal memory operator

We use in this section the Hamming distance between interpretations[4] and then the Dalal distance between an interpretation I and a belief base φ is defined as $d(I, \varphi) = \min_{J \models \varphi}(dist(I, J))$.

Let's define the assignment that maps each belief base to a pre-order in the following way:

Definition 6. *$I \leq_{\varphi}^{d} J$ if and only if $d(I, \varphi) \leq d(J, \varphi)$.*

So we have a pre-order with the models of φ at the lowest level and the other worlds in the higher levels.

Definition 7. *The* Dalal memory operator *is the memory operator obtained from this assignment (i.e. the operator obtained by definitions 6 and 3).*

We can show on a toy example that this operator differs from classical Dalal revision operator [6, 11]. Let a and b be two propositional letters and consider for example the sequence $\Psi = \Xi \circ a \circ b \circ \neg(a \wedge b)$. The classical Dalal operator gives $Bel(\Psi) = (a \wedge \neg b) \vee (\neg a \wedge b)$. Whereas Dalal memory operator gives $Bel(\Psi) = (\neg a \wedge b)$. This behaviour seems more natural since at the penultimate step we learnt that b was true, and it is normal to keep some credit for this evidence in the following step. It is in this way, that our operators use revision "memory".

4.3 Ryan OTP operator

Mark Ryan has proposed to apply his *Ordered Presentations of Theories* (or OTP) to belief revision [20]. Very roughly, an OTP is a multi-set of formulae equipped with a partial pre-order. This pre-order represents the relative reliability of the sources of each formula. So, using a linear order, one can express the fact that the new information is more reliable than older ones and thus can simulate iterated revisions. To give the definition of OTP is not a subject of this work, the interested reader can see *e.g* [19]. We will simply introduce the notions needed to define the OTP revision operator.

First we have to define what the monotonicities of a formula are.

Definition 8. *Let I be an interpretation and p be a propositional letter, then $I^{[p]}$ (respectively $I^{[\neg p]}$) denotes the interpretation that is identical to I on each propositional letter except (maybe) on the propositional letter p that is assigned to true (resp. false).*

Definition 9. *Let φ be a consistent formula and p be a propositional letter.*

1. *φ is* monotonic in p *if $I \models \varphi$ implies that $I^{[p]} \models \varphi$.*
2. *φ is* anti-monotonic in p *if $I \models \varphi$ implies that $I^{[\neg p]} \models \varphi$.*

[4] the Hamming distance between two interpretations is the number of propositional letters on which the two interpretations differ

The set of symbols in which φ is monotonic (resp. anti-monotonic) is noted φ^+ (resp. φ^-). If $\varphi \leftrightarrow \bot$, then $\varphi^+ = \varphi^- = \emptyset$.

After this definition, Ryan defines an inference relation that he named *natural entailment*.

Definition 10. *φ naturally entails μ, written $\varphi \mid\!\sim_N \mu$, if $\varphi \vdash \mu$, $\varphi^+ \subseteq \mu^+$ and $\varphi^- \subseteq \mu^-$.*

This relation has some nice properties, in particular it does not allow to add irrelevant disjuncts in the conclusions (for example $p \not\mid\!\sim_N p \vee q$). See [19] for more details.

Finally, the preference relation associated with a formula φ is given by the set of natural consequences that the interpretations satisfy, that is:

Definition 11. *Let φ be a formula, and I, J two interpretations, the relation $\preceq_\varphi$ is defined as: $I \preceq_\varphi J$ if for each μ such that $\varphi \mid\!\sim_N \mu$, $(J \models \mu \Rightarrow I \models \mu)$ holds.*

So an interpretation is better than another if it satisfies more natural consequences. Note that the relation $\preceq_\varphi$ is a partial pre-order.

Definition 12. *The* Ryan operator *is the operator obtained from this assignment (i.e. the operator obtained by definitions 11 and 3).*

Because the starting assignment takes partial pre-orders as values, Ryan operator does not satisfy all the postulates. More precisely, one has the following result [20]:

Theorem 6 *The Ryan revision operator satisfies postulates (R*1), (R*3), (R*4), and (R*5), but does not satisfy (R*2) and (R*6).*

A counter-example to (R*2) and (R*6), given in [20], is the following:

Let $\varphi_1 = p \vee q \vee r$, $\varphi_2 = \neg p \wedge \neg q \wedge \neg r$ and $\varphi_3 = (p \leftrightarrow q) \wedge \neg r$. Then for (R*2), take $\Psi = \Xi \circ \varphi_1 \circ \varphi_2$ and $\varphi = \varphi_3$. Then $Mod(\Psi) = \{011, 101, 110\}$ and $Mod(\varphi) = \{000, 001, 010, 100, 110, 111\}$, so $Mod(\Psi \wedge \varphi) = \{110\}$ whereas $Mod(\Psi \circ \varphi) = \{110, 001\}$. The same counter-example holds for (R*6) also by putting $\Psi = \Xi \circ \varphi_1$, $\varphi = \varphi_2$ and $\mu = \varphi_3$.

These two violations of the rationality postulates seem to be very awkward. Especially (R*2) seems hardly debatable. The question is: how can we modify Ryan's definition in order to satisfy these properties ? In fact, the easiest way to modify this operator in order to obtain revision with memory operators is to "complete" the $\preceq_\varphi$ partial pre-orders in order to obtain total pre-orders. This can be achieved by two means that give the two following operators.

4.4 Closure of the pre-order

First, following the construction of the rational closure of a conditional belief base [14] (see also Pearl's System Z [18]), we can figure out a lazy deformation of the pre-order, that is, the deformation that transforms the partial pre-order in a total pre-order with a minimal effort.

Definition 13. *Let $\rho_\varphi(I)$ be the "distance from I to φ" in the following sense:*

1. *If $I \in \min(\mathcal{W}, \preceq_\varphi)$ then $\rho_\varphi(I) = 0$,*
2. *Otherwise $\rho_\varphi(I) = a$, where a is the length of the longest chain of strict inequalities $I_0 \prec_\varphi \ldots \prec_\varphi I_n$ with $I_0 \in \min(\mathcal{W}, \preceq_\varphi)$ and $I_n = I$.*

This "distance" gives a total pre-order on interpretations:

Definition 14. *$I \leq_\varphi^{\mathrm{OTP}_1} J$ if and only if $\rho_\varphi(I) \leq \rho_\varphi(J)$.*

We illustrate this principle of "minimal effort" with an example: Let $\varphi = (\neg a \vee \neg b) \wedge \neg c$ be a belief base.

Fig. 1. Closure of the pre-order

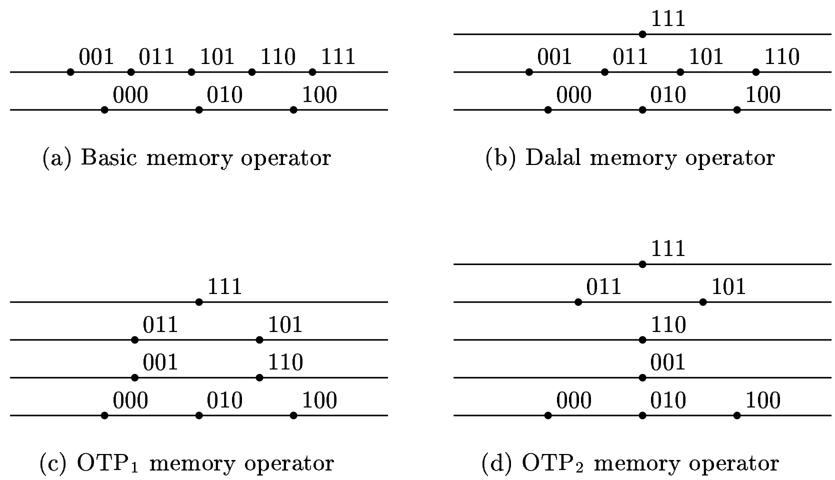
The left hand side presents the partial pre-order $\preceq_\varphi$. Arrows $I \leftarrow J$ denote $I \prec_\varphi J$ (for reading convenience we do not represent transitivity, reflexivity and the equivalence between minimal interpretations). The right hand side presents the $\leq_\varphi^{\mathrm{OTP}_1}$ corresponding pre-order. It is clear that if $I \prec_\varphi J$ then $I <_\varphi^{\mathrm{OTP}_1} J$. Thus the only interpretation that is not straightforwardly placed is 110. The "minimal effort" is being illustrated here as follows: the first place where can be placed 001 is at the second level, so it is the chosen level. Conversely, for the interpretation 011 for example, the first "acceptable" level is the third one because there is an interpretation (001) that is strictly better than 011 which is occupying the second level.

It is easy to show that the function that maps each belief base φ to a total pre-order $\leq_\varphi^{\mathrm{OTP}_1}$ is a faithful assignement. Then we build our memory operator as usual:

Definition 15. *The* OTP_1 memory operator *is the memory operator obtained from this assignment (i.e. the operator obtained by definitions 14 and 3).*

4.5 Using cardinalities

A second way to define a total pre-order from Ryan revision operator is to interpret it differently. The idea of the $\preceq_\varphi$ order, defining Ryan operator, is that an interpretation I is better than another J for a belief base φ if I satisfies all the natural consequences that J satisfies. In other terms I is better than J if I satisfies more natural consequences than J. Following this idea we can then focus uniquely on the number of natural consequences satisfied.



(a) Basic memory operator

(b) Dalal memory operator

(c) OTP_1 memory operator

(d) OTP_2 memory operator

Fig. 2. Behaviour differences between revision with memory operators

Definition 16. *$I \leq_{\varphi}^{\mathrm{OTP}_2} J$ if and only if $card(\{\mu \mid \varphi \mid\!\sim_N \mu$, and $J \models \varphi\}) \leq card(\{\mu \mid \varphi \mid\!\sim_N \mu$, and $I \models \varphi\})$.*

This definition is also a "completion" of the $\preceq_{\varphi}$ pre-order since if $I \preceq_{\varphi} J$, then $I \leq_{\varphi}^{\mathrm{OTP}_2} J$.

Then, as usual:

Definition 17. *The* OTP_1 memory operator *is the memory operator obtained from this assignment (i.e. the operator obtained by definitions 14 and 3).*

5 Conclusion

We will end by showing that the four revision with memory operators defined are differents. To show that, it is enough to show that the corresponding faithful assignments are differents. We will show that on the formula $\varphi = (\neg a \vee \neg b) \wedge c$. In figure 2 one can check that the four pre-orders obtained are different.

We have proposed in this paper a method to build revision operators that have interesting properties for iterated revision from any classical AGM operator.

This family of operators exhibits the fact that Darwiche and Pearl's (C2) postulate is certainly too demanding.

We have also introduced two new operators based on Ryan revision operator [20]. An open question is to know if those operators can be recovered from the definition of a classical distance-based revision operator.

We have mainly deal in this paper with the generic construction of iterated revision operators from classical AGM operators, but a full logical characterization of revision with memory operators can be found in [12].

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