Propositional Merging and Judgment Aggregation: Two Compatible Approaches?

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Abstract. There are two theories of aggregation of logical formulae: merging and judgment aggregation. In this work we investigate the relationships between these theories; one of our objectives is to point out some correspondences/discrepancies between the associated rationality properties.

1 INTRODUCTION

Merging [6, 5] is a way to aggregate contradictory belief bases (or goal bases) coming from a group of agents, in order to obtain a collective belief (or goal) base. Merging operators have been defined and studied as an extension of AGM belief revision theory [4, 2].

Judgment aggregation (JA) has been introduced in political philosophy and social choice theory [9, 8]. The aim of judgment aggregation is to make collective yes/no judgments on several (possibly logically related) issues, from the judgments given on each issue by the members of a group.

Clearly enough, merging and JA do not coincide, since they do not have the same inputs and outputs, as illustrated in the following figure. Thus, merging takes as input a profile of n propositional bases K_i, a formula μ representing some integrity constraints on the result of the merging process,[4] and outputs an (aggregated/collective) base $\Delta(K_1, \dots, K_n)$. JA takes as input a profile P of n individual judgments γ_i on an agenda X, i.e., a set of m propositional formulae φ_k (considered as binary questions); a judgment γ_i is a vector of m binary values, so that $\gamma_i(\varphi_k) = 1$ precisely when agent i answer to φ_k is yes. A judgment aggregation correspondance $\gamma(P)$ outputs a set of (aggregated/collective) judgments γ_{P_i} on the same agenda.

$$
\begin{tikzcd}
K_1, \dots, K_n \arrow[r, "\Delta"] \arrow[d, leftrightarrow, "p_C"'] & \Delta(K_1, \dots, K_n) \arrow[d, leftrightarrow, "p_C"'] \\
\gamma_1, \dots, \gamma_n \arrow[r, "\gamma"] & \gamma(\gamma_1, \dots, \gamma_n)
\end{tikzcd}
$$

Accordingly, JA can be seen as an aggregation issue based on partial information, i.e., only the agents' judgments on the questions φ_k are available, while in a merging process, the whole bases are considered. Thus, in order to compare both methods on a fair basis with respect to the informational contents, one needs to provide all the information merging use to the judgment aggregation method. This can be done easily by choosing the set of all interpretations (i.e. all complete formulae) as the agenda.

For space reasons we can not provide a full formal background on belief merging and judgment aggregation; the reader can refer to [6] for belief merging and to [3] for judgment aggregation. The logical properties we consider in the following are reported in these papers.

2 MERGING VS. JUDGMENT AGGREGATION

In the following we assume that the agenda is the set of all interpretations (i.e. all complete formulae) $X = \{\omega_1, \dots, \omega_m\}$. The connection between merging and JA illustrated on the previous figure takes advantage of a decision policy p_C that denotes the answer $\gamma_i(\omega_j)$ agent i provides to question ω_j of the agenda, i.e. $p_C(K_i) = \gamma_i$ and:

- $\gamma_i(\omega_j) = 1$ if $\omega_j \models K_i$ and $\gamma_i(\omega_j) = 0$ otherwise.

We also define the inverse operation p^{-1} as follows: $p_C^{-1}(\gamma_i) = K_i$ such that the set of models $[K]$ of K is $\{\omega_j \mid \gamma_i(\omega_j) = 1\}$.

Thanks to the decision policy p_C we can associate a merging operator Δ with a resolute JA correspondence γ, and a resolute JA correspondence γ with a merging operator Δ, as follows:

Definition 1
- *Given an integrity constraint μ, a merging operator Δ and a profile $E = \{K_1, \dots, K_n\}$, we note $P = \{p_C(K_1), \dots, p_C(K_n)\}$ and we define $\forall \omega \models \mu$, $\gamma_{P_\omega}(\omega) = 1$ iff $\omega \models \Delta_\mu(E)$. $\gamma(P) = \{\gamma_{P_\omega} \mid \omega \models \mu\}$.*
- *Given a non-empty set of interpretations $[\mu]$, a resolute judgment aggregation correspondence γ and a profile $P = \{\gamma_1, \dots, \gamma_n\}$ of judgments on $[\mu]$, we note $E = \{p_C^{-1}(\gamma_1), \dots, p_C^{-1}(\gamma_n)\}$ and $[\Delta_\mu(E)] = \{\omega \in [\mu] \mid \exists \gamma_P \in \gamma(P)$ s.t. $\gamma_P(\omega) = 1\}$.*

To study the links between merging and judgment aggregation properties, in the following we enumerate the standard IC merging properties and see what are their corresponding properties in judgment aggregation.

(IC0) By construction of Δ, (IC0) is satisfied, and this postulate does not impose any constraint on the corresponding γ.

(IC1) **Proposition 1** *Δ satisfies (IC1) iff γ satisfies collective rationality.*

(IC2) Let us define an additional property for JA methods, namely consensuality:

Definition 2 *A judgment profile $P = (\gamma_1, \dots, \gamma_n)$ is* consensual *for a given agenda $X = \{\varphi_1, \dots, \varphi_m\}$ when there exists φ_j such that $\gamma_i(\varphi_j) = 1$ for all i.*

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[4] Integrity constraints are omitted in the figure (i.e., $\mu = \top$).

Consensuality. γ *satisfies consensuality iff for any agenda* $X = \{\varphi_1, \ldots, \varphi_m\}$ *and any consensual judgment profile* $P = (\gamma_1, \ldots, \gamma_n)$ *for it, we have* $\gamma_P(\varphi_j) = 1$ *iff* $\gamma_i(\varphi_j) = 1$ *for all* i.
Proposition 2 Δ *satisfies (IC2) iff* γ *satisfies consensuality.*

(IC3) **Proposition 3** Δ *satisfies (IC3) iff* γ *satisfies anonymity.*

(IC4) (IC4) is the only IC postulate for which we have only an implication (and not an equivalence). This is because (IC4) considers only the special case of two bases, whereas neutrality is defined for more general profiles.
Proposition 4 *If* γ *satisfies neutrality,*[5] *then* Δ *satisfies (IC4).*

(IC5) Let us now define two additional properties for JA methods, based on the consistency condition that exists for voting methods [12, 1]. They correspond respectively to properties (IC5) and (IC6).

Weak consistency. For any two judgment profiles P and P' and any $\varphi \in X$, if $\gamma_P(\varphi) = 1$ and $\gamma_{P'}(\varphi) = 1$, then $\gamma_{P \sqcup P'}(\varphi) = 1$.

Consistency. For any two judgment profiles P and P'. If there is $\varphi \in X$, s.t. $\gamma_P(\varphi) = 1$ and $\gamma_{P'}(\varphi) = 1$, then for every $\psi \in X$, if $\gamma_{P \sqcup P'}(\psi) = 1$ then $\gamma_P(\psi) = 1$ and $\gamma_{P'}(\psi) = 1$.

Quite surprisingly these conditions have not been considered as standard ones for JA methods[6].
Proposition 5 Δ *satisfies (IC5) iff* γ *satisfies weak consistency*

(IC6) **Proposition 6** Δ *satisfies (IC6) iff* γ *satisfies consistency*

(IC7) For (IC7) and (IC8) we need two additional properties for JA methods. The first property is the translation in the JA setting of the well-known Sen's property α from social choice theory [10] (also known as Chernoff Condition).

Sen's property α**.** Let P be a judgment profile and X be an agenda s.t. $\varphi \in X$ and $\gamma_P(\varphi) = 1$ on X. Suppose $\varphi \in X' \subset X$, then $\gamma_P(\varphi) = 1$ on X'.

Proposition 7 Δ *satisfies (IC7) iff* γ *satisfies Sen's property* α.

(IC8) For (IC8) one needs the translation to JA of another property due to Sen:

Sen's property β**.** Let P be a judgment profile and X be an agenda s.t. $\varphi_1, \varphi_2 \in X$, $\gamma_P(\varphi_1) = 1$ and $\gamma_P(\varphi_2) = 1$ on X. Suppose $X \subset Y$. Then $\gamma_P(\varphi_1) = 1$ on Y iff $\gamma_P(\varphi_2) = 1$ on Y.

Proposition 8 *If* γ *satisfies Sen's property* α *and Sen's property* β*, then* Δ *satisfies (IC8). If* Δ *satisfies (IC8), then* γ *satisfies Sen's property* β.

Notice that there is no direct correspondence between (IC8) and Sen's property β, we need also Sen's property α to obtain (IC8).

The following proposition summarizes the results:

Proposition 9
- *If* γ *satisfies collective rationality, consensuality, anonymity, neutrality, weak consistency, consistency, Sen's property* α *and Sen's property* β*, then* Δ *is an IC merging operator (it satisfies (IC0-IC8)).*
- *If* Δ *is an IC merging operator (it satisfies (IC0-IC8)), then* γ *satisfies collective rationality, consensuality, anonymity, weak consistency, consistency, Sen's property* α *and Sen's property* β.

Let us also stress that a JA operator cannot satisfy both consensuality and majority preservation:

[5] We consider here the JA standard notion of neutrality [9], and not the one proposed in [3].

[6] We are only aware of [7, 11] that gives the consistency condition (but called it separability).

Proposition 10 *Consensuality and majority preservation cannot be satisfied together.*

Surprisingly, unanimity and consensuality are not logically connected:

Proposition 11 *Consensuality does not imply unanimity and unanimity does not imply consensuality.*

Now that the connections between the postulates satisfied by Δ and those satisfied by the corresponding γ have been made precise, a key question is to determine whether one can find existing JA operators satisfying all JA postulates above. Interestingly, the answer is positive: the ranked majority methods proposed in [3] .[7] Especially this is the case of the ranked majority judgment aggregation methods $\gamma^{RM_\oplus}$ with $\oplus = \Sigma$ or $\oplus = leximax$ (or more generally with any $\oplus$ satisfying strict non-decreasingness) – all these operators coincide when the agenda X is the set of all interpretations.

Proposition 12 *Let the agenda* X *be the set of all interpretations. For any* $\oplus$ *satisfying strict non-decreasingness,* $\gamma^{RM_\oplus}$ *satisfies collective rationality, collective completeness, anonymity, neutrality, unanimity, consensuality, weak consistency and consistency, Sen's properties* α *and* β*. It satisfies neither independence nor majority preservation.*

3 CONCLUSION

In this paper we have sketched some relationships between propositional merging operators and judgment aggregation ones in the full information case (when the agenda contains all possible interpretations). We have also obtained some results in the general case, which cannot be reported here but are left for the long version of this paper.

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[7] Some operators of [7] are recovered as special cases of ranked majority methods (see [3]), so they also satisfy these properties.