# A Comparative Study of Ranking-based Semantics for Abstract Argumentation

**Paper #1599**

**Abstract**

Argumentation is a process of evaluating and comparing a set of arguments. A way to compare them consists in using a ranking-based semantics which rank-order arguments from the most to the least acceptable ones. Recently, a number of such semantics have been proposed independently, often associated with some desirable properties. However, there is no comparative study which takes a broader perspective. This is what we propose in this work. We provide a general comparison of all these semantics with respect to the proposed properties. That allows to underline the differences of behavior between the existing semantics.

## Introduction

Argumentation consists in reasoning with conflicting information based on the exchange and evaluation of interacting arguments. The most popularly way to represent argumentation process was proposed by Dung (1995) with argumentation frameworks modelized by binary graphs, where the nodes represent the arguments, and the edges represent the attacks between them. From these argumentation frameworks, several semantics indicating which sets of arguments, called extensions, are mutually compatible were proposed (see (Baroni, Caminada, and Giacomin 2011) for an overview). However, for applications with a big number of arguments, it can be problematic to have only two levels of evaluations (arguments are either accepted or rejected). For instance, such a limitation can be questionable when using argumentation for debate platforms on the web (see (Leite and Martins 2011) for such a discussion).

In order to fix these problems, a solution consists in using semantics that distinguish arguments not with the classical accepted/rejected evaluations, but with a large number of levels of acceptability. A lot of these semantics, called ranking-based semantics, were proposed in recent years (Amgoud and Ben-Naim 2013; Cayrol and Lagasquie-Schiex 2005; Leite and Martins 2011; Matt and Toni 2008; Gabbay 2012) with, for each semantics, different behaviour and logical properties. However, all these semantics have never been compared between them.

This is what we propose in this work. We study the existing ranking-based semantics in the literature (focusing on the semantics that return a unique ranking between arguments) in the light of the proposed properties. That allows us to underline the differences of behavior between those semantics, and to propose a better reading of the different choices one has on this matter.

The paper is organized as follows. Section 2 presents the relevant background regarding abstract argumentation and ranking-based semantics. Section 3 presents the different properties that have been introduced in the literature, whereas Section 4 formally introduces the existing ranking-based semantics. Note that due to space constraints, we can not recall all the details and justifications of semantics and properties of the literature, but the reader can find them in the corresponding papers. In Section 5 we discuss the different properties and compare the semantics, and Section 6 concludes.



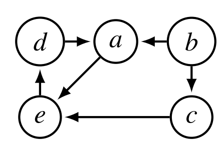
## Preliminaries

In this section, we start by briefly recalling what is a Dung's abstract argumentation framework (Dung 1995), where the exact structure of arguments is unspecified.

**Definition 1.** *An* **argumentation framework (AF)** *is a pair $F = \langle A, R \rangle$ with $A$ a set of arguments and $R$ a binary relation on $A$, i.e. $R \subseteq A \times A$, called the* **attack relation**. *A set of arguments $S \subseteq A$ attacks an argument $b \in A$, if there exists $a \in S$, such that $(a, b) \in R$. We note $Arg(F) = A$.*

Let $\mathbb{AF}$ be the set of all argumentation frameworks. For two AF $F = \langle A, R \rangle$ and $G = \langle A', R' \rangle$, we define the union $F \cup G = \langle A \cup A', R \cup R' \rangle$.

**Example 1.** *Let $F = \langle A, R \rangle$ with $A = \{a, b, c, d, e\}$ and $R = \{(a, e), (b, a), (b, c), (c, e), (d, a), (e, d)\}$.*




We can now introduce some useful notions in order to formalize properties of argumentation frameworks.

**Definition 2.** *Let $F = \langle A, R \rangle$ be an AF and $a, b \in A$. A* **path** *$P$ from $b$ to $a$, noted $P(b, a)$, is a sequence $s = \langle a_0, \ldots, a_n \rangle$ of arguments such that $a_0 = a$, $a_n = b$ and $\forall i \leq n, (a_i, a_{i-1}) \in R$. We denote by $l_P = n$ the*

**length** *of P. A* **defender** *(resp.* **attacker***) of $a$ is an argument situated at the beginning of an even-length (resp. odd-length) path. We denote the multiset of defenders and attackers of $a$ by $R^+_n(a) = \{b \mid \exists P(b,a)\ with\ l_P \in 2\mathbb{N}\}$ and $R^-_n(a) = \{b \mid \exists P(b,a)\ with\ l_P \in 2\mathbb{N}{+}1\}$ respectively. The* **direct attackers** *of $a$ are arguments in $R^-_1(a)$. An argument $a$ is* **defended** *if $R^+_n(a) \neq \emptyset$.*
*A* **defense root** *(resp.* **attack root***) is a non-attacked defender (resp. attacker). We denote the multiset of defense roots and attack roots of $a$ by $BR^+_n(a) = \{b \in R^+_n(a) \mid |R^-_1(b)| = 0\}$ and $BR^-_n(a) = \{b \in R^-_n(a) \mid |R^-_1(b)| = 0\}$ respectively. A path from $b$ to $a$ is a* **defense branch** *(resp.* **attack branch***) if $b$ is a defense (resp. attack) root of $a$. Let us note $BR^+(a) = \bigcup_n BR^+_n(a)$ and $BR^-(a) = \bigcup_n BR^-_n(a)$.*

The *connected components* of an AF are the set of largest subgraphs of AF, denoted by $cc(AF)$, where two arguments are in the same component of AF if and only if there is some path (ignoring the direction of the edges) between them.

In Dung's framework (Dung 1995), the *acceptability* of an argument depends on its membership to some sets, called extensions. Another way to select a set of acceptable arguments is to *rank* arguments from the most to the least acceptable ones. Ranking-based semantics aim at determining such a ranking between arguments.

**Definition 3.** *A* **ranking-based semantics** *$\sigma$ associates to any argumentation framework AF = $\langle A, R\rangle$ a ranking $\succeq^\sigma_{AF}$ on A, where $\succeq^\sigma_{AF}$ is a preorder (a reflexive and transitive relation) on A. $a \succeq^\sigma_{AF} b$ means that $a$ is at least as acceptable as $b$.*

When there is no ambiguity about the argumentation framework in question, we will use $\succeq^\sigma$ instead of $\succeq^\sigma_{AF}$.

Finally, we need to introduce the notion of lexicographical order in order to define some ranking-based semantics.

**Definition 4.** *A* **lexicographical order** *between two vectors of real number $V = \langle V_1, \ldots, V_n\rangle$ and $V' = \langle V'_1, \ldots, V'_n\rangle$, is defined as $V \succeq_{lex} V'$ iff $\exists i \leq n$ s.t. $V_i \geq V'_i$ and $\forall j \in \{1, 2 \ldots, i-1\}, V_j = V'_j$.*

## Properties

Let us recall the logical properties proposed in the literature. Unless stated explicitly, all the properties are defined for a ranking-based semantics $\sigma$, $\forall AF \in \mathbb{AF}$ and $\forall a, b \in Arg(AF)$.

**Definition 5.** *An* **isomorphism** *$\gamma$ between two argumentation frameworks AF = $\langle A, R\rangle$ and AF' = $\langle A', R'\rangle$ is a bijective function $\gamma : A \to A'$ such that $\forall x, y \in A$, $(x, y) \in R$ iff $(\gamma(x), \gamma(y)) \in R'$. With a slight abuse of notation, we will note $AF' = \gamma(AF)$.*

**Abstraction.** The ranking on $A$ should be defined only on the basis of the attacks between arguments.
**(Abs)** Let $AF, AF' \in \mathbb{AF}$. For any isomorphism $\gamma$ s.t. $AF' = \gamma(AF)$, we have $a \succeq^\sigma_{AF} b$ iff $\gamma(a) \succeq^\sigma_{AF'} \gamma(b)$

**Independance.** The ranking between two arguments $a$ and $b$ should be independent of any argument that is neither connected to $a$ nor to $b$.

**(In)** $\forall AF' \in cc(AF), \forall a, b \in Arg(AF')$,
$$a \succeq^\sigma_{AF'} b \Rightarrow a \succeq^\sigma_{AF} b$$

We may have expectations regarding the best and worst arguments that we may find in an AF:
**Void Precedence.** A non-attacked argument is ranked strictly higher than any attacked argument.
**(VP)** $R^-_1(a) = \emptyset$ and $R^-_1(b) \neq \emptyset \Rightarrow\ a \succ^\sigma b$

**Self-Contradiction.** A self-attacking argument is ranked lower than any argument that does not attack itself.
**(SC)** $(a, a) \notin R$ and $(b, b) \in R\ \Rightarrow\ a \succ^\sigma b$

The following local properties are concerned with the direct attackers, or defenders, of arguments:
**Cardinality Precedence.** The greater the number of direct attackers for an argument, the weaker the level of acceptability of this argument.
**(CP)** $|R^-_1(a)| < |R^-_1(b)| \Rightarrow\ a \succ^\sigma b$

**Quality Precedence.** The greater the acceptability of one direct attacker for an argument, the weaker the level of acceptability of this argument.
**(QP)** $\exists c \in R^-_1(b)$ s.t. $\forall d \in R^-_1(a),\ c \succ^\sigma d \Rightarrow\ a \succ^\sigma b$

Before defining the next properties, we need to introduce a relation that compares sets of arguments on the basis of their rankings (Amgoud and Ben-Naim 2013):

**Definition 6.** *Let $\succeq_S$ be a ranking on a set of arguments A. For any $S_1, S_2 \subseteq A$, $S_1 \succeq_S S_2$ is a* **group comparison** *iff there exists an injective mapping f from $S_2$ to $S_1$ such that $\forall a \in S_2, f(a) \succeq a$. And $S_1 \succ_S S_2$ is a strict group comparison iff $S_1 \succeq_S S_2$ and ($|S_2| < |S_1|$ or $\exists a \in S_2, f(a) \succ a$).*

**Counter-Transitivity.** If the direct attackers of $b$ are at least as numerous and acceptable as those of $a$, then $a$ is at least as acceptable as $b$.
**(CT)** $R^-_1(b) \succeq_S R^-_1(a) \Rightarrow a \succeq^\sigma b$

**Strict Counter-Transitivity.** If CT is satsified and either the direct attackers of $b$ are strictly more numerous or acceptable than those of $a$, then $a$ is strictly more acceptable than $b$.
**(SCT)** $R^-_1(b) \succ_S R^-_1(a) \Rightarrow a \succ^\sigma b$

**Defense Precedence.** For two arguments with the same number of direct attackers, a defended argument is ranked higher than a non-defended argument.
**(DP)** $|R^-_1(a)| = |R^-_1(b)|, R^+_2(a) \neq \emptyset$ and $R^+_2(b) = \emptyset$
$$\Rightarrow\ a \succ^\sigma b$$

**Definition 7.** *Let $AF = \langle A, R\rangle$ and $a \in A$. The* **defense** *of $a$ is* **simple** *iff every defender of $a$ attacks exactly one direct attacker of $a$. The defense of $a$ is* **distributed** *iff every direct attacker of $a$ is attacked by at most one argument.*

**Distributed-Defense Precedence.** The best defense is when each defender attacks a distinct attacker.
**(DDP)** $|R^-_1(a)| = |R^-_1(b)|$ and $|R^+_2(a)| = |R^+_2(b)|$, if the defense of $a$ is simple and distributed and the defense of $b$

is simple but not distributed, then $a \succ^\sigma b$

The following properties check if some change in an AF can improve or degrade the ranking of one argument. These properties have been proposed informally by Cayrol and Lagasquie-Schiex (2005), in the context of their semantics. We propose a formalization that generalize them for any argumentation frameworks. We first define the addition of a defense/attack branch to an argument.

**Definition 8.** *Let $AF = \langle A, R\rangle$, $a \in A$. The* **defense branch added to** *$a$, denoted $P_+(a) = \langle A', R'\rangle$, with $A \cap A' = \{a\}$, is the sequence $\langle x_0, \dots, x_n\rangle$ with $x_0 = a$, $x_1, \dots, x_n \in A'$, $n \in 2\mathbb{N}$ such that $\forall i \leq n$, $(x_i, x_{i-1}) \in R'$. The* **attack branch added to** *$a$, denoted $P_-(a)$ is defined similarly except that the sequence is of odd length (i.e. $n \in 2\mathbb{N} + 1$).*

The following properties are defined $\forall AF, AF^\gamma \in \mathbb{AF}$ such that exists an isomorphism $\gamma$ with $AF^\gamma = \gamma(AF)$, and $\forall a \in Arg(AF)$. We use $AF^\gamma$ as a clone of $AF$.

**Strict addition of Defense Branch.** Adding a defense branch to any argument improves its ranking.

**($\oplus$DB)** $AF \cup AF^\gamma \cup P_+(\gamma(a)) \Rightarrow \gamma(a) \succ^\sigma a$

**Addition of Defense Branch.** It could make sense to treat differently non-attacked arguments. So in (Cayrol and Lagasquie-Schiex 2005), this property is defined in a more specific way: adding a defense branch to any attacked argument improves its ranking.

**(+DB)** $AF \cup AF^\gamma \cup P_+(\gamma(a)),\ |R_1^-(a)| \neq 0 \Rightarrow \gamma(a) \succ^\sigma a$

**Increase of Attack branch.** Increasing the length of an attack branch of an argument improves its ranking.

**($\uparrow$AB)** $b \in BR^-(a)$, $AF \cup AF^\gamma \cup P_+(\gamma(b)) \Rightarrow \gamma(a) \succ^\sigma a$

**Addition of Attack Branch.** Adding an attack branch to any argument degrades its ranking.

**(+AB)** $AF \cup AF^\gamma \cup P_-(\gamma(a)) \Rightarrow a \succ^\sigma \gamma(a)$

**Increase of Defense branch.** Increasing the length of a defense branch of an argument degrades its ranking.

**($\uparrow$DB)** $b \in BR^+(a)$, $AF \cup AF^\gamma \cup P_+(\gamma(b)) \Rightarrow a \succ^\sigma \gamma(a)$

One can find the properties Abs, In, VP, DP, CT, SCT, CP, QP and DDP in (Amgoud and Ben-Naim 2013), the properties In, VP and SC in (Matt and Toni 2008) and the property VP in (Cayrol and Lagasquie-Schiex 2005).

To this set of properties from the literature we want to add some other important properties.

**Total.** A total relation between arguments is returned.

**(Tot)** $a \succeq^\sigma b$ *or* $b \succeq^\sigma a$

The next property states that all the non-attacked arguments should have the same ranking.

**Non-attacked Equivalence.** All the non-attacked argument have the same rank.

**(NaE)** $R_1^-(a) = \emptyset$ *and* $R_1^-(b) = \emptyset \Rightarrow a \succeq^\sigma b$ *and* $b \succeq^\sigma a$

The last property describes the behavior adopted by a semantics concerning the notion of defense, and can be viewed as some kind of compatibility with usual Dung's semantics. The idea is that a defended argument is always better than an attacked argument.

**Attack vs Full defense.** An argument without any attack branch is ranked higher than an argument only attacked by one non-attacked argument.

**(AvsFD)** $AF$ is acyclic, $|BR^-(a)| = 0, |R_1^-(b)| = 1$ and $|R_2^+(b)| = 0 \Rightarrow a \succ^\sigma b$

Let us now check the incompatibility between pairs of properties. An incompatibility between CP and QP has already been proved by Amgoud and Ben-Naim (2013)[1].

**Proposition 1.** *For every ranking-based semantics, the following pairs of properties are not compatible :*

- *CP and AvsFD*
- *CP and +DB*
- *VP and $\oplus$DB*

# Existing Ranking-based Semantics

## Categoriser

Besnard and Hunter (2001) propose a *categoriser* function which assigns a value to each argument, given the value of its direct attackers.

**Definition 9** (Besnard and Hunter 2001)**.** *The* **categoriser function** *$Cat : A \to ]0, 1]$ is defined as:*

$$Cat(a) = \begin{cases} 1 & \text{if } R_1^-(a) = \emptyset \\ \frac{1}{1+\sum_{c \in R_1^-(a)} Cat(c)} & \text{otherwise} \end{cases}$$

**Definition 10.** *The* **ranking-based semantics Categoriser** *associates to any AF = $\langle A, R\rangle$ a ranking $\succeq_{AF}^{Cat}$ on $A$ such that $\forall a, b \in A, a \succeq_{AF}^{Cat} b$ iff $Cat(a) \geq Cat(b)$.*

**Example 1** (continued)**.** *$Cat(a) \approx 0.38$ , $Cat(b) = 1$ , $Cat(c) = 0.5$, $Cat(d) \approx 0.65$ and $Cat(e) \approx 0.53$ . So we obtain the ranking : $b \succ^{Cat} d \succ^{Cat} e \succ^{Cat} c \succ^{Cat} a$.*

This semantics take into account only the value of the direct attackers to compute the strength of an argument. This is why the argument $e$ in the previous example, which is attacked twice but by arguments that are attacked by a not-attacked argument, is ranked higher than the argument $c$, which is attacked just once, but by a stronger argument.

**Proposition 2.** *The ranking-based semantics Categoriser satisfies[2] Abs, In, VP, DP, CT, SCT, $\uparrow$AB, $\uparrow$DB, +AB, Tot and NaE. The other properties are not satisfied.*

## Social Abstract Argumentation Framework

Leite and Martins (2011) introduce an extension of Dung's abstract argumentation frameworks that include social voting on the arguments: the Social Abstract Argumentation Frameworks (SAF). They also propose a family of semantics where a model is a solution to the equation system[3] with one

[1]The Proofs of all the propositions are provided in the appendix file on the AAAI submission system.

[2]The properties Abs, In, VP, DP, CT, SCT have already been checked by Pu et al. (2014).

[3]An equational approach was also proposed by Gabbay (2012). This method returns multiple solutions, and thus several rankings for one AF. This is why we do not consider this method in this paper.

equation for each argument, based on its social support and its direct attackers. In order to compare SAFs with the existing ranking-based semantics, we chose to ignore the social support of arguments by giving them the same value, and to only focus on the attacks.

**Definition 11.** *Let $F = \langle A, R\rangle$ be an AF and $S = \langle L, \tau, \curlywedge, \curlyvee, \neg\rangle$ be a (well-behaved) SAF semantic. The total mapping $M_S : A \to L$ is a* **social model** *of $F$ under semantics $S$ such that $\forall a \in A$:*

$$M_S(a) = \tau(a) \curlywedge \neg \curlyvee \{M(a_i) : a_i \in R_1^-(a)\}, \text{ where}$$

- *$L$ is a totally ordered set with top $\top$ and bottom $\bot$ elements, containing all possible valuations of an argument;*
- *$\tau : A \to L$ is an attenuation factor. $\tau$ is monotonic w.r.t. the first argument and antimonotonic w.r.t the second argument;*
- *$\curlywedge : L \times L \to L$ combines the initial score with the score of direct attackers. $\curlywedge$ is continuous, commutative, associative, monotonic w.r.t. both arguments and $\top$ is its identity element;*
- *$\curlyvee : L \times L \to L$ aggregates the score of direct attackers. $\curlyvee$ is continuous, commutative, associative, monotonic w.r.t. both arguments and $\bot$ is its identity element;*
- *$\neg : L \to L$ restricts the value of the attacked argument. $\neg$ is antimonotonic, continuous, $\neg\bot = \top$, $\neg\top = \bot$ and $\neg\neg a = a$.*

One possible (well-behaved) SAF semantic proposed in (Leite and Martins 2011) is the *simple product semantic* $SP_\epsilon = \langle [0, 1], \tau_\epsilon, \curlywedge, \curlyvee, \neg\rangle$ where $\tau_\epsilon = \frac{1}{1+\epsilon}$ (with $\epsilon > 0$, to ensure the uniqueness of the semantics), $x_1 \curlywedge x_2 = x_1 \times x_2$ (Product T-Norm), $x_1 \curlyvee x_2 = x_1 + x_2 - x_1 \times x_2$ (Probabilistic Sum T-CoNorm) and $\neg x_1 = 1 - x_1$.

**Definition 12.** *The* **ranking-based semantics SAF** *associates to any $AF = \langle A, R\rangle$ a ranking $\succeq^{SAF}_{AF}$ on $A$ such that $\forall a, b \in A$, $a \succeq^{SAF}_{AF} b$ iff $M_S(a) \geq M_S(b)$.*

**Example 1** (cont.)**.** *With $\epsilon = 0.1$, we obtain $M_{\mathrm{SP}_\epsilon}(a) \approx 0.07$ , $M_{\mathrm{SP}_\epsilon}(b) \approx 0.91$ , $M_{\mathrm{SP}_\epsilon}(c) \approx 0.08$, $M_{\mathrm{SP}_\epsilon}(d) \approx 0.20$ and $M_{\mathrm{SP}_\epsilon}(e) \approx 0.78$. We obtain the ranking: $b \succ^{SAF} e \succ^{SAF} d \succ^{SAF} c \succ^{SAF} a$.*

As for the Categoriser semantics, the strength of attackers is more important than their numbers, and thus $e$ is preferred to $c$. However the impact of a defense branch on an argument is weaker with SAF than with Categoriser.

**Proposition 3.** *SAF satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot and NaE. Other properties are not satisfied.*

## Discussion-based semantics

The Discussion-based semantics (Amgoud and Ben-Naim 2013) compares arguments by counting the number of paths ending to them. If some arguments are equivalent (they have the same number of direct attackers), the size of paths is recursively increased until a difference is found.

**Definition 13.** *Let $F = \langle A, R\rangle$ be an AF, $a \in A$, and $i \in \mathbb{N}$.*

$$Dis_i(a) = \begin{cases} -|R_i^+(a)| & \text{if } i \text{ is odd} \\ |R_i^-(a)| & \text{if } i \text{ is even} \end{cases}$$

*The* **discussion count of** *$a$ is denoted $Dis(a) = \langle Dis_1(a), Dis_2(a), \dots\rangle$.*

**Definition 14.** *The* **ranking-based semantics Dbs** *associates to any AF = $\langle A, R\rangle$ a ranking $\succeq^{Dbs}_{AF}$ on $A$ such that $\forall a, b \in A$, $a \succeq^{Dbs}_{AF} b$ iff $Dis(b) \succeq_{lex} Dis(a)$.*

**Example 1** (cont.)**.**

| *step* | *a* | *b* | *c* | *d* | *e* |
|---|---|---|---|---|---|
| *1* | *2* | *0* | *1* | *1* | *2* |
| *2* | *-1* | *0* | *0* | *-2* | *-3* |

*Using the lexicographical order, we obtain the following ranking: $b \succ^{Dbs} d \succ^{Dbs} c \succ^{Dbs} e \succ^{Dbs} a$*

The number of attacker is here more important than their strength, thus $c$ is here stronger than $e$.

**Proposition 4.** *Dbs satisfies Abs, In, VP, DP, CT, SCT, CP, ↑AB, ↑DB, +AB, Tot and NaE. The other properties are not satisfied.*

## Burden-based semantics

The Burden-based semantics (Amgoud and Ben-Naim 2013) assigns, at each step $i$, a Burden number to every argument, that depends on the Burden numbers of its direct attackers.

**Definition 15.** *Let $F = \langle A, R\rangle$ be an AF, $a \in A$ and $i \in \mathbb{N}$.*

$$Bur_i(a) = \begin{cases} 1 & \text{if } i = 0 \\ 1 + \sum_{b \in R_1^-(a)} \frac{1}{Bur_{i-1}(b)} & \text{otherwise} \end{cases}$$

*The* **Burden number of** *$a$ is denoted $Bur(a) = \langle Bur_0(a), Bur_1(a), \dots\rangle$.*

Two arguments are lexicographically compared on the basis of their Burden numbers.

**Definition 16.** *The* **ranking-based semantics Bbs** *associates to any $AF = \langle A, R\rangle$ a ranking $\succeq^{Bbs}_{AF}$ on $A$ such that $\forall a, b \in A$, $a \succeq^{Bbs}_{AF} b$ iff $Bur(b) \succeq_{lex} Bur(a)$.*

**Example 1** (cont.)**.**

| *step* | *a* | *b* | *c* | *d* | *e* |
|---|---|---|---|---|---|
| *1* | *3* | *1* | *2* | *2* | *3* |
| *2* | *2.5* | *1* | *2* | *1.33* | *1.83* |

*Using the lexicographical order, we obtain the following ranking: $b \succ^{Bbs} d \succ^{Bbs} c \succ^{Bbs} e \succ^{Bbs} a$*

As on this example, Dbs and Bbs often return the same result. The main difference between those semantics is that Bbs satisfies DDP, so examples related to that kind of structures lead to distinct results.

**Proposition 5.** *Bbs satisfies Abs, In, VP, DP, CT, SCT, CP, DDP, ↑AB, ↑DB, +AB, Tot and NaE. The other properties are not satisfied.*

## Valuation with tuples

This semantics (Cayrol and Lagasquie-Schiex 2005) takes account of all the ancestors branches of an argument (defender and attacker) store in tupled values :

**Definition 17.** *Let $F = \langle A, R\rangle$ be an AF and $a \in A$. Let $v_p(a)$ be the (ordered) tuple of even integers representing the lengths of all the defense branches of $a$, i.e. $v_p(a)$ is the smallest ordered tuple such that $|BR_n^+(a)| = x \Rightarrow n \in_x v_p(a)$, where $\in_x$ means "appears at least x times". Similarly* *let $v_i(a)$ be the (ordered) tuple of odd integers representing the lengths of all the attack branches of $a$, i.e. $v_i(a)$ is the smallest ordered tuple such that $|BR^-_n(a)| = x \Rightarrow n \in_x v_i(a)$. A* **tupled value** *for $a$ is the pair $v(a) = [v_p(a), v_i(a)]$.*

When cycles exist in the AF, some tuples can be infinite. To calculate them, this method requires a highly involved process, that turn cyclical graphs into infinite acyclic graphs. We thus only consider this approach for acyclic graph, and denote it by $Tuples^*$.

Once the tupled value of each argument has been computed, one can compare them. To do so one has to compare the length of attack/defense branches and, in case of a tie, to compare the values inside each tuples (see Algorithm 1).

**Algorithm 1:** $Tuples^*$

**Input**: $v(a), w(b)$ two tupled values of arguments $a$ and $b$
**Output**: A ranking $\succeq^{\mathrm{T}}$ between $a$ and $b$

**begin**
  **if** $v = w$ **then** $a \succeq^{\mathrm{T}} b$ and $b \succeq^{\mathrm{T}} a$;
  **else**
    **if** $|v_i| = |w_i|$ **and** $|v_p| = |w_p|$ **then**
      **if** $v_p \preceq_{lex} w_p$ **and** $v_i \succeq_{lex} w_i$ **then** $a \succ^{\mathrm{T}} b$;
      **else**
        **if** $v_p \succeq_{lex} w_p$ **and** $v_i \preceq_{lex} w_i$ **then** $a \prec^{\mathrm{T}} b$;
        **else** $a \not\succeq^{\mathrm{T}} b$ and $a \not\preceq^{\mathrm{T}} b$;
    **else**
      **if** $|v_i| \geq |w_i|$ **and** $|v_p| \leq |w_p|$ **then** $a \prec^{\mathrm{T}} b$;
      **else**
        **if** $|v_i| \leq |w_i|$ **and** $|v_p| \geq |w_p|$ **then** $a \succ^{\mathrm{T}} b$;
        **else** $a \not\succeq^{\mathrm{T}} b$ and $a \not\preceq^{\mathrm{T}} b$;

Let us remark that two arguments can be incomparable. It is the case, for example, if an argument has strictly more attack branches and more defense branches than another one. Consequently, this semantics returns a partial ranking between arguments.

As example 1 contains a cycle, we can not compute Tuples$^*$ on this running example.

**Proposition 6.** *The ranking-based semantics Tuples$^*$ satisfies Abs, In, VP, +DB, ↑AB, ↑DB, +AB, NaE and AvsFD. The other properties are not satisfied.*

## Matt & Toni

Matt and Toni (2008) compute the strength of an argument using a two-person zero-sum strategic game. This game confronts two players, a proponent and an opponent of a given argument, where the strategies of the players are sets of arguments. For an $AF = \langle A, R\rangle$ and $a \in A$, the sets of strategies for the proponent and opponent are $S_P(a) = \{P \mid P \subseteq A, a \in P\}$ and $S_O = \{O \mid O \subseteq A\}$ respectively.

**Definition 18.** *Let $F = \langle A, R\rangle$ be an AF and $X, Y \subseteq A$. The* **set of attacks from** $X$ **to** $Y$ *is defined by $Y_F^{\leftarrow X} = \{(a, b) \in X \times Y \mid (a, b) \in R\}$. The* **degree of acceptability** *of $P$ w.r.t $O$ is given by $\phi(P, O) = \frac{1}{2}[1 + f(|O_F^{\leftarrow P}|) - f(|P_F^{\leftarrow O}|)]$ where $f(n) = \frac{n}{n+1}$.*

**Definition 19.** *Let $F = \langle A, R\rangle$ be an AF. The* **rewards of** $P$*, denoted by $r_F(P, O)$, are defined by :*

$$r_F(P,O) = \begin{cases} 0 & \textit{iff } \exists a, b \in P, (a,b) \in R, \\ 1 & \textit{iff } |P_F^{\leftarrow O}| = 0, \\ \phi(P,O) & \textit{otherwise} \end{cases}$$

Proponent and opponent choose mixed strategies, according to some probability distributions, respectively $p = (p_1, p_2, \ldots, p_m)$ and $q = (q_1, q_2, \ldots, q_n)$, with $m = |S_P|$ and $n = |S_O|$. For each argument $a \in A$, the proponent's expected payoff $E(a, p, q)$ is then given by $E(a, p, q) = \sum_{j=1}^{n}\sum_{i=1}^{m} p_i q_j r_{i,j}$. Finally the value of the zero-sum game for an argument $a$, denoted by $s(a)$, is $s(a) = \max_p \min_q E(a, p, q)$.

**Definition 20.** *The* **ranking-based semantics M&T** *associates to any AF = $\langle A, R\rangle$ a ranking $\succeq_{AF}^{M\&T}$ on $A$ such that $\forall a, b \in A$, $a \succeq_{AF}^{M\&T} b$ iff $s(a) \geq s(b)$.*

**Example 1** (cont.)**.** *One obtains $s(a) \approx 0.17$ , $s(b) = 1$ , $s(c) = 0.25$, $s(d) = 0.25$ and $s(e) = 0.5$ and the following preorder: $b \succ^{M\&T} e \succ^{M\&T} c \simeq^{M\&T} d \succ^{M\&T} a$.*

On this example, we can see that once again the strength of attackers is more important than their numbers ($e$ is ranked higher than $d$).

**Proposition 7.** *The ranking-based semantics M&T satisfies Abs, In, VP, +AB, SC, Tot, NaE and AvsFD. Other properties are not satisfied.*

## Discussion

As it can be easily checked on the running example, all these proposed ranking semantics have distinct behaviors (the ranking obtained is different for each semantics): this justifies the need of some axiomatic work. Our work initiates this study, by checking properties that have been proposed in the papers that introduce the different semantics. Our analysis is applied to existing semantics, but any new semantics could be inspected through the same lens.[4] Table 1 summarizes the properties satisfied by the ranking semantics we consider in this paper. We also checked what are the properties satisfied by the usual Dung's Grounded semantics, that gives some hints on the compatibility of these properties with classical semantics (note that, in this case, this is a degenerate ranking semantics with only two levels):

**Proposition 8.** *The grounded semantics satisfies Abs, In, CT, QP, Tot, NaE, AvsFD. Other properties are not satisfied.*

A cross $\times$ means that the property is not satisfied, symbol ✓ means that the property is satisfied, symbol $-$ means that the property can not be applied to the semantics (because the semantics is not compatible with the constraint given by the rule), and the shaded cells highlight the results already proved in the literature.

As a general comment, one can check in the table that SAF, Cat, Dbs and Bds share a lot of properties. Tuples$^*$ and M&T are also similar (they are the only ones that satisfy AvsFD). This seems at first sight to draw two general classes of semantics.

[4]For instance, the semantics very recently proposed in (Grossi and Modgil 2015).

| Properties | SAF | Cat | Dbs | Bbs | Tuples* | M&T | Grounded |
|---|---|---|---|---|---|---|---|
| Abs | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| In | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| VP | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | × |
| DP | ✓ | ✓ | ✓ | ✓ | × | × | × |
| CT | ✓ | ✓ | ✓ | ✓ | × | × | ✓ |
| SCT | ✓ | ✓ | ✓ | ✓ | × | × | × |
| CP | × | × | ✓ | ✓ | × | × | × |
| QP | × | × | × | × | × | × | ✓ |
| DDP | × | × | × | ✓ | × | × | × |
| SC | × | × | × | × | - | ✓ | × |
| ⊕DB | × | × | × | × | × | × | × |
| +DB | × | × | × | × | ✓ | × | × |
| ↑AB | ✓ | ✓ | ✓ | ✓ | ✓ | × | × |
| ↑DB | ✓ | ✓ | ✓ | ✓ | ✓ | × | × |
| +AB | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | × |
| Tot | ✓ | ✓ | ✓ | ✓ | × | ✓ | ✓ |
| NaE | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| AvsFD | × | × | × | × | ✓ | ✓ | ✓ |

Table 1: Properties satisfy by the studied ranking semantics

As for properties, we see that the properties Abs, In and VP are satisfied by all the ranking semantics. This is expected, since these properties really seem necessary for a good ranking semantics. NaE is also satisfied by all semantics. This is also a very basic requirement for a ranking semantics and we also view it as mandatory. NaE mainly says that the non-attacked arguments are all equivalent. This is a kind of compatibility principle with usual Dung's semantics, and it says that only your attackers should impact your ranking, not the arguments you attack.[5] Another property that we consider as a requirement is the Tot property, which is in line with the idea of "ranking" semantics. It would be necessary if one want to use these semantics in real applications. This is a drawback of Tuples*. An interesting question is to know if it is possible to refine Tuples*, i.e. to define a semantics close to Tuples*, but that is computable for argumentation frameworks with cycles, and that satisfies Tot. A last property satisfied by all semantics is +AB, which states that adding an attack branch towards an argument degrades its ranking. This also seems to be a perfectly natural requirement for ranking semantics: the more you are attacked, the worse you are.

Overall, this gives us a set of 6 properties that should be satisfied by any ranking semantics: Abs, In, VP, NaE, Tot and +AB. One can note that Abs, In, NaE and Tot are satisfied by the grounded semantics, so they are compatible with usual Dung's semantics. VP and +AB are not satisfied by the grounded semantics, because it only has two levels of evaluation (accepted/rejected), and these two properties really introduces graduality in the evaluation.

A very discriminating property is AvsFD, which states that an argument that is (only) attacked once is worse than an argument that have any number of attacks that all belong to defense branches. We think this requirement is very natural, and it seems to be a good candidate as being a basic property. Nonetheless one can check that SAF, Cat, Dbs and Bds do not satisfy it, whereas Tuples* and M&T do. So this property can be seen as a kind of boundary between two sub-classes of ranking properties.

This distinction is interesting since it allows to introduce a discussion on the semantics of ranking semantics, and to argumentation semantics in general. The question relates to the status of the missing information in argumentation systems. Basically this can be summed up as follows: do the absence of attack between $a$ and $b$ means that *I know that there is no attack between* $a$ *and* $b$, or does it means that *I do not know if there is an attack between* $a$ *and* $b$[6]. If one adopts the first interpretation, then AvsFD should be satisfied, since we know that an argument is defended, and the other one attacked. If one adopts the second one, then AvsFD can be violated, since it may appear later that the attacked argument could be finally defended, while the defended arguments, with numerous attack paths, meaning that it is really disputed, can loose some of its defenses.

Interestingly, while all semantics agree axiomatically as which arguments should be the best in a system (VP), there is no consensus regarding the *worst* arguments. SC is very interesting in that respect, but as can be observed none of the semantics comply with it, except the one of Matt and Toni.

Properties related to 'change' are very appealing. A sensible meta-property could be to state that the 'response' of a semantics to a change should be the same, whatever the current state of the argument system. As can be seen with +DB and ⊕DB, this is not case: if one accepts that non-attacked arguments should be the best, it cannot be the case that adding a defense branch always improve the situation.

Finally, 'local' properties (CP, QP, DP, (S)CT), just looking at direct attackers (or defenders), make choice which can be justified in some situations, but which seem hardly general (and sometimes impossible to reconcile with some more global properties, as our Prop. 1 shows).

One last comment is that SAF and Cat satisfy the same set of properties, whereas they have quite different definitions. This mean that at least one property is lacking in order to discriminate these two operators.

## Conclusion

In this work we proposed a comparative study of existing ranking-based semantics. It turns out that the existing ranking-based semantics exhibit quite different behaviours and satisfy different properties. We propose to take as basic properties for ranking-based semantics Abs, In, VP, NaE, Tot and +AB. We also put forward AvsFD that discriminates two subclasses of semantics.

There is still work needed on the topic. First to propose other ranking-based semantics. But it is also important to find other logical properties, and to try to characterize classes of semantics with respect to these properties.

## References

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[5] Note that it could make sense to make a distinction between arguments that attack a lot of arguments and the ones that do not — so to violate NaE, in particular. This could be considered as some kind of power index. But this is not the aim of ranking semantics.

[6] In (Coste-Marquis et al. 2007) it is proposed to add a third relation between arguments in order to allow these two cases.

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