States and Time in Modal Action Logic

Camilla Schwind

LIF- CNRS, Luminy, Université de la Méditerranée
163 Avenue de Luminy, Case 901 13288 Marseille Cedex 9, France,
E-mail: schwind@lif.univ-mrs.fr

Abstract. We present a multi-modal action logic with first-order modalities, which contain terms which can be unified with the terms inside the subsequent formulas and which can be quantified. This makes it possible to handle simultaneously time and states. We discuss applications of this language to action theory where it is possible to express many temporal aspects of actions, as for example, beginning, end, time points, delayed preconditions and results, duration and many others.

1 Introduction

Most action theories consider actions being specified by their preconditions and their results. The temporal structure of an action system is then defined by the sequence of actions that occur. A world is conceived as a graph of situations where every link from one node to the next node is considered as an action transition. This yields also a temporal structure of the action space, namely sequences of actions can be considered defining sequences of world states. The action occurs instantantly at one moment and its results are true at the "next" moment.

However, the temporal structure of actions can be much more complex and complicated.

- Actions may have a duration.
- The results may be true before the action is completed or after it is finished.
- Actions may have preconditions which have to have been true during some interval preceding the action occurrence.

In order to represent complex temporal structures, underlying actions' occurrences, we have developed an action logic which allows to handle both states and time simultanuously.

We want to be able to express, for instance, that action a occurs at moment t if conditions $p_1, \ldots p_n$ have been true during the intervals $i_1, \ldots i_k$ all preceding t.

Here we present an approach where it is possible to describe actions in a complex temporal environment. In reality, actions have sometimes a beginning time, a duration, preconditions which may also have temporal aspects. Action results may become true only instants after the end of the action performance. For an example, consider the action of calling an elevator, taking place at instant t_1. Depending on the actual situation this action may cause the elevator to move only many instants later, to stop still later, an so on. The action of pressing the button of a traffic light, in order to get green light to cross the street may result in a switch immediately or after some seconds and in another switch after some more minutes.

In order to represent such issues, we define a modal action logic, $\mathcal{D}al$, where the modalities are terms containing variables which can be quantified. The same variables can occur inside the modalities as well as in the formulas after the modalities, allowing for unification between action term components and logical terms. This language makes it possible to express reasoning on states and the action terms allow to express temporal aspects of the actions.

A similar logic, called term modal logic, has been defined in [1]. In this work, any term of the first-order language can be a modal operator. This makes it possible to quantify over variables naming the accessability relation. In our appraoch, we quanantify over terms which index the accessability relation. It is possible to simulate term modal logic in our logic $\mathcal{D}al$ by having one single action a of arity 1 and by using as action operators any operator $a(t)$ for any term t of the language. The simulation of $\mathcal{D}al$ by term modal logic is not so straightforward, because in our logic action names cannot occur within the underlying first-order language (they are different from all predicate and function symbols of underlying the first-order logic). Another difference of our logic with term modal logic is that our logic includes equality.

Another related formalism is hybrid logic where it is possible to quantify over state variables naming worlds. In hybrid logic however these state variables belong to the object language.

2 The first-order modal action logic $\mathcal{D}al$

The language of first-order action logic is an extension of the language of classical predicate logic, $\mathcal{L}_0$. $\mathcal{L}_0$ consists of a set of variables $x, y, x_1, y_1, \ldots$, a set $\boldsymbol{F}$ of function symbols F, where $|F| \in \omega$ is the arity of F, a set $\boldsymbol{P}$ of predicate symbols P, including $\top$ and $\bot$, where $|P| \in \omega$ is the arity of P[1], an equality symbol $=$, the logical symbols $\neg$, $\wedge$,$\forall$. Terms and formulas are defined as usual and so are $\exists$ and $\vee$. We denote by V_t the set of all terms of $\mathcal{L}_0$. Free occurrence of a variable x in a formula ϕ is defined as usual.

$\mathcal{D}al$ is a multi-modal logic where each modality is a term of the form $a(t_1, \ldots t_n)$ for some *action* name a. We call these terms actions terms. They only occur within modalities. In addition, $\mathcal{D}al$ contains the **S4**-modal operator $\Box$ which will be used to express that a sentence is true in every world of a Kripke model.

Action terms The language for action operators consists of

- a set $\boldsymbol{A}$ of *action symbols* $a_1, a_2, \ldots$ where $|a| \in \omega$ is the arity of a and such that $\boldsymbol{A} \cap \boldsymbol{P} = \emptyset$

Action terms are built from action symbols and terms of $\mathcal{L}_0$.

[1] There are two predicate symbols of arity 0 which can be identified with $\top$ and $\bot$. Sometimes $\top$ and $\bot$ are also considered as empty conjunction and disjunction, hence as logical symbols.

- if a is an action symbol of arity n and $t_1, \dots t_n$ are terms of $\mathcal{L}_0$, then $a(t_1, \dots t_n)$ is an action term.

An action term is called *grounded* if no variable occurs free in it. The set of grounded action terms is denoted by $\boldsymbol{A}_t$.

Action operators If a and $a_1, a_2, \dots a_n$ are action terms, then

- $[a]$ is an action operator
- $[a_1; a_2; \dots a_n]$ is an action operator
- For $n = 0$, the corresponding action operator is noted $[\varepsilon]$

$[\varepsilon]$ is the "empty" action operator. We use it in order to define the initial state of a system.

Modal operator $\Box$ is the standard modal operator (**S4**)

We use $\Box$ for describing general laws (constraints) which hold in all states of a system.

An action operator is called *grounded* if all the action terms occurring in it are grounded.

Example: $[a]$, $[a(c_1, c_2, c_3)]$ are grounded, $[a(x, c_2, y)]$ is not grounded.

$\mathcal{D}al$ formulas

- Every first-order formula (of $\mathcal{L}_0$) is a $\mathcal{D}al$ formula.
- If ϕ is a $\mathcal{D}al$ formula and $[A]$ is an action operator, then $[A]\phi$ is a $\mathcal{D}al$ formula.
- If ϕ is a $\mathcal{D}al$ formula and $\Box$ is the modal operator, then $\Box\phi$ is a $\mathcal{D}al$ formula.
- If ϕ is a $\mathcal{D}al$ formula and x is a variable, then $\forall x \phi$ is a $\mathcal{D}al$ formula.

Instantiation If ϕ is a formula and t is a term, then ϕ^x_t is the formula obtained from ϕ by replacing every free occurrence of x by t. If t is the name of an element of a set $\mathcal{O}$ then ϕ^x_t is called $\mathcal{O}$-instance of ϕ.

Example:

$$[a(x, c)](\neg\phi(c, x) \vee \psi(x))^x_{c_1} = [a(c_1, c)](\neg\phi(c, c_1) \vee \psi(c_1))$$
$$[a_1; a_2; a_3(c, y)]P(c, y)^y_{c_3} = [a_1; a_2; a_3(c, c_3)]P(c, c_3)$$

A formula is called *grounded* if there is no variable occurring free in it. The notion of free occurrence of a variable within a formula is extended to $\mathcal{D}al$ -formulas as follows: If a variable occurs free within an action term A then it occurs free in $[A]\phi$.

2.1 Semantical Characterization of $\mathcal{D}al$

$\mathcal{D}al$ formulas are semantically characterized by Kripe structures, i.e. sets of worlds where each world is a classical structure. Since action operators are indexed by terms of the language, we have a different action operator $[a(t_1, \dots t_n)]$ for any grounded tuple of terms $t_1, \dots t_n$ of the first-order language $\mathcal{L}_0$. And consequently, we have a different transition relation for each of these action operators. The semantics of $\mathcal{D}al$ is defined as follows:

A $\mathcal{D}al$ structure is a Kripke-type structure, such that the transition relation between worlds depends on grounded action terms.

A $\mathcal{D}al$ structure is a tuple $\mathcal{M} = (\mathcal{W}, \{\mathcal{S}_w : w \in \mathcal{W}\}, \mathcal{A}, \mathbf{R}, \tau)$, where

- $\mathcal{W}$ is a set of *worlds*
- for every $w \in \mathcal{W}$, $\mathcal{S}_w = (\mathcal{O}, \mathcal{F}_w, \mathcal{P}_w)$ is a classical structure, where $\mathcal{O}$ is the set of individual objects (the same set in all worlds), $\mathcal{F}_w$ is a set of functions over $\mathcal{O}$ and $\mathcal{P}_w$ is a set of predicates over $\mathcal{O}$.
- $\mathcal{A}$ is a set of *action functions*, for $f \in \mathcal{A}$, $f : \mathcal{W} \times \underbrace{\mathcal{O} \times \ldots \times \mathcal{O}}_{n} \longrightarrow 2^{\mathcal{W}}, n \in \omega$.

  Action functions will characterize the action operators (every action symbol of arity n in $\boldsymbol{A}$ will be associeted with an action function of arity $n+1$).
- $\mathbf{R} \subseteq \mathcal{W} \times \mathcal{W}$ is a binary accessability relation on $\mathcal{W}$, which will characterize the modal operater $\Box$. We will write $R(w) = \{w' : (w, w') \in R\}$.
- τ is a valuation, $\tau = (\tau_0, \tau_1, \tau_2, \tau_3)$, where τ_0 is a function assigning objects from $\mathcal{O}$ to terms. In order to speak about objects from $\mathcal{O}$, we introduce into the language, for every $o \in \mathcal{O}$, an o-place function symbol (denoted equally o, for simplicity).
  τ_1 is a function assigning, for every world $w \in \mathcal{W}$, functions (from $\mathcal{F}_w$) to function symbols (from $\boldsymbol{F}$), of the same arity,
  $\tau_1 : \mathcal{W} \times \boldsymbol{\mathcal{F}} \longrightarrow \mathcal{F}$ such that $|\tau_1(w, F)| = |F|$.
  τ_2 is a function assigning, for every world $w \in \mathcal{W}$, predicates to predicate symbols of the same arities,
  $\tau_2 : \mathcal{W} \times \boldsymbol{\mathcal{P}} \longrightarrow \mathcal{P}$, such that $|\tau_2(w, P)| = |P|$.
  τ_3 is a function assigning action functions to action symbols,
  $\tau_3 : \boldsymbol{A} \longrightarrow \mathcal{A}$, such that $|\tau_3(a)| = |a| + 1$
- $\tau_3(a)(w, \tau_0(t_1), \ldots, \tau_0(t_n)) \subseteq R(w)$. If a world can be reached from w by the execution of action $a(t_1, t_2, \ldots, t_n)$ then it is accessible (via the relation R).

τ_0, τ_1, τ_2 and τ_3 define the valuation τ as follows:

- If $F(t_1, t_2, \ldots, t_m)$ is a term then
  $\tau_0(w, F(t_1, t_2, \ldots, t_m)) = \tau_1(w, F)(\tau_0(w, t_1), \tau_0(w, t_2), \ldots, \tau_0(w, t_m))$.
- if P is an n-ary predicate symbol and $t_1, t_2, \ldots, t_n$ are free object variables then
  $\tau(w, Pt_1, t_2, \ldots, t_n) = \top$ iff $(\tau_0(t_1), \ldots \tau_0(t_n)) \in \tau_2(w, P)$
- $\tau(w, t_1 = t_2) = \top$ iff $\tau_0(w, t_1) = \tau_0(w, t_2)$
- $\tau(w, \neg\phi) = \top$ iff $\tau(w, \phi) = \bot$
- $\tau(w, \phi \wedge \psi) = \top$ iff $\tau(w, \phi) = \tau(w, \psi) = \top$
- $\tau(w, \forall x \phi) = \top$ iff for every $o \in \mathcal{O}$[2] $\tau(w, \phi^x_o) = \top$
- $\tau(w, [a(t_1, t_2, \ldots, t_n)]\phi) = \top$ iff for every $w' \in \tau_3(a)(w, \tau_0(t_1), \ldots, \tau_0(t_n))$, $\tau(w', \phi) = \top$
- $\tau(w, \Box\phi) = \top$ iff for every $w' \in R(w)$, $\tau(w', \phi) = \top$

[2] Note that we added every $o \in \mathcal{O}$ as a new term (the name of o) to the language.

Let ϕ be a formula and $x_1, x_2, \ldots, x_n$ be the free object variables occurring in ϕ. Then $\tau(w, \phi) = t$ iff for every tuple $t_1, t_2, \ldots, t_n$ of ground terms,

$$\tau(w, \phi^{x_1, x_2, \ldots, x_n}_{t_1, t_2, \ldots, t_n}) = t$$

A formula ϕ is called valid in state $w \in \mathcal{W}$ of a $\mathcal{D}al$ -structure $\mathcal{M}$ iff $\tau(w, \phi) = \top$. This is denoted by $\mathcal{M}, w \models \phi$. We also say then that ϕ is satisfiable. A formula ϕ is called valid in a $\mathcal{D}al$ - structure $\mathcal{M}$ with the set of states $\mathcal{W}$, iff ϕ is valid in every $w \in \mathcal{W}$. We denote that by $\mathcal{M} \models \phi$. A formula ϕ is called $\mathcal{D}al$ - valid iff ϕ is valid in every $\mathcal{D}al$ - structure. This is denoted by $\models_{\mathcal{D}al} \phi$. We suppress the index $\mathcal{D}al$, whenever it is clear from the context, in which system we are.

Remark 1 $[a]\bot$ *is satisfiable and we have* $\tau(w, [a]\bot) = \top$ *iff* $\tau_3(a)(w, \tau_0(t_1), \ldots, \tau_0(t_n)) = \emptyset$

2.2 Axioms and inference rules of $\mathcal{D}al$

In addition to the axioms and inference rules of classical first - order logic and those of the system K, which rule all action operators including $[\varepsilon]$, and those of the system and $\mathcal{S}_4$, which rule the operator $\Box$, we have the following axioms and inference rules, (where $[A]$, $[A_1]$ and $[A_2]$ are arbitrary action operators):

$[A1]$ $[A_1; A_2]\alpha \leftrightarrow [A_1][A_2]\alpha$
$[A2]$ $\Box\alpha \rightarrow [A]\alpha$
$[A3]$ $[\varepsilon]\alpha \rightarrow \alpha$
$[A4]$ $\forall x\alpha \rightarrow \alpha^x_c$ for any term c of $\mathcal{L}$,
$[A5]$ $\forall x[X]\alpha \leftrightarrow [X]\forall x\alpha$ for any modal operator X, with no occurrence of x

$[R1]$ From α infer $\Box\alpha$
$[R2]$ From $\alpha \rightarrow \beta$ infer $\alpha \rightarrow \forall x\beta$ provided x has no free occurrence in α

$\vdash_{\mathcal{D}al}$ is defined as usual, such that $\vdash_{\mathcal{D}al} \phi$ for any instance ϕ of one of the axioms; and $\vdash_{\mathcal{D}al} \psi$, whenever ψ can be inferred from ϕ, for any ϕ, such that $\vdash_{\mathcal{D}al} \phi$ by use of one of the inference rules. Again, we suppress the index $\mathcal{D}al$, whenever it is clear from the context, in which system we are.

$A2$ says that a formula which is true "in all worlds of a model" will still be true after the occurrence of actions $[A]$. $A3$ asserts the "emptyness" of the action operator $[\varepsilon]$: if formula α is true after the occurrence of ε it is already true. $A5$ is the well-known Barcan formula (and its contrapositive). We need $A5$ because we choose the same set of objects for every world. Objects cannot disappear neither new objects can appear.

2.3 Soundness, Completeness, Decidability

The $\mathcal{D}al$ -logic is sound and complete:

Theorem 1 $\vdash_{\mathcal{D}al} \phi$ *if and only if* ϕ *is* $\mathcal{D}al$ *- valid (*$\models_{\mathcal{D}al} \phi$*)*

The soundness proof is easy and the completeness proof goes along the lines of completeness proofs for modal logics by construction of a canonical model. The proof, which can be found in the full paper [13], bears several modifications according to the specific language which allows to quantify over terms occurring within modal operators.

$\mathcal{D}al$ is a first order language and therefore undecidable in the general case. But for action logics, we will make use of a decidable subset of $\mathcal{D}al$.

$\mathcal{D}al$ is very close to term modal logic introduced by [1]. Term modal logic allows terms in general as modalities, whereas our action logic only admits action terms. Moreover $\mathcal{D}al$ contains the **S4** modal operator $\Box$ which is not part of term modal logic.

Decidable subsets of first-order logic can "yield" decidable subsets of $\mathcal{D}al$: the subset of $\mathcal{D}al$ without function symbols and existential quantifiers can be shown to be decidable by the finite model property. We omit the proof for space limit.

3 Temporal Action Theories

Here we present one application of the formalism introduced above. This application is related to the formal description of actions by modal logic. Frequenly in action theories "time points" are identified with states (or worlds of a Kripke model). This yields a discrete conception of systems where actions can occur in a state of the system producing as a result a "next" state. But we argue that in real world systems both notions co-exist: time and state. Frequenly, we conceive an action as occurring instantly and producing its results at some "next" instant. But at the same time, we have an underlying idea of *time*, measured eventually by a clock even when we do not systematically need to refer to this time axis. For some scenarios it is necessary to take into account different (and various) temporal a spects of actions, simply because there may be termporally delayed preconditions or results of actions.

We can modelize these temporal aspects of actions using $\mathcal{D}al$. The modal logic allows to define action operators as modalities much like in [3, 12]. The first order logic is used to formulate actions at a more general level. Here, we show an example where in addition to the relative representation of time by the modal operators, it is possible to add assertions about time points or intervals.

Subsequently, we use a restricted subset $\mathcal{D}al$, whith no positive occurrence of existential quantifiers and no negative ocurrence of universal quantifiers.

We presuppose a time axis, $\mathcal{T}$, linearly ordered (dense or continuous or discrete). Given a $\mathcal{D}al$ -structure, we will define a transitive relation on the set of states, $\mathcal{W}$, which will be related to the order on $\mathcal{T}$.

Definition 1 *Let* $\mathcal{M} = (\mathcal{W}, \{\mathcal{S}_w : w \in \mathcal{W}\}, \mathcal{A}, \mathbf{R}, (\tau_0, \tau_1, \tau_2, \tau_3))$ *, be a* $\mathcal{D}al$ *-model. Then* $w \prec_0 w'$ *iff* $\exists a \in \mathcal{A}$ *of arity* n *and there are terms* $t_1, \ldots, t_n$*, such that* $w' \in \tau_3(a)(w, \tau_0(t_1), \ldots, \tau_0(t_n))$*. Let be* $\preceq$ *the reflexive and transitive closure of* $\prec_0$*.*

Intuitively, this means that $w \prec w'$ if we can possibly "reach" w' from w by performing actions $a_1, a_2, \ldots, a_n$. Obviously, $\preceq$ is transitive and reflexive. Since we want to "link" worlds of $\mathcal{W}$ to time points in $\mathcal{T}$, which is ordered, $\prec$ must also be antisymmetric. The temporal entrenchment of the states is defined by a homomorphism $time : \mathcal{W} \longrightarrow \mathcal{T}$ from $\mathcal{W}$ into $\mathcal{T}$, where $w \preceq w'$ implies $time(w) \leq time(w')$. Using this construction, action operators can be defined admitting complex temporal structures, including beginning and ending instants and a duration, which can be 0, when the result is immediate. The preconditions and results of actions can be defined to occur at freely determinable time instants before or after the instant when the action occurs. When an action a occurs in the state w, $time(w)$ gives us the time point at which a occurs. If the duration of the action is Δ, the time point of the resulting state w' is $time(w') = time(w) + \Delta$.

In this particular framework, we define

- *Action terms* as binary action predicates $a(t, d, \overrightarrow{x})$, where t denotes the instant on which a occurs and d denotes the duration of a, i.e. the interval on $\mathcal{T}$ after which the results of a will hold. , $\overrightarrow{x}$ is the sequence of other variables denoting the other entities or objects involved in the action occurrence.
  To give an example, let $\mathcal{T} = \{1, \ldots, 24\}$ be discrete and finite, denoting the hours during one day. Then action $move(t, 3, TGV, Marseille, Paris)$ is the action "train TGV goes from Marseille to Paris, the duration being 3 hours".
- *Action axioms*. An action axiom is characterized by a precondition $\pi(t, \overrightarrow{x})$ and a result $\rho(t + d, \overrightarrow{x})$, where π is a $\mathcal{D}al$ formula describing all preconditions of action a and ρ is a conjunction of litterals describing the results of a.
  To continue the previous example, the action execution axiom of the move-action is $at(t, x, y) \rightarrow [move(t, d, x, y, z)]at(t + d, x, z)$ (and can be instantiated to $at(6, TGV, Marseille) \rightarrow [move(6, 3, TGV, Marseille, Paris)]at(9, TGV, Paris)$), which means: if x is at y at instant t, then, after moving from y to z, x is at z at instant $t + d$.

The general form of an action law is

$$(*)\ \Box(\pi(t_1, \overrightarrow{x_1}) \rightarrow [a(t, d, \overrightarrow{x_2})]\rho(t_2, \overrightarrow{x_3})), where\ \overrightarrow{x_1} \cup \overrightarrow{x_2} \subseteq \overrightarrow{x_3}$$

Note that it is then possible to derive one action law

$$\Box(\pi(t_1, \overrightarrow{x_1}) \rightarrow [a(t, d, \overrightarrow{x_2})]l(t_2, \overrightarrow{x_3}))$$

from axiom (*) for each of the litterals occurring within ρ.

We define action systems much as in [3, 4] or [2].

An *action system* is a tuple $(\Pi, Frame, \mathcal{C})$ where Π is a set of action laws, and of *causal laws*, $\mathcal{C}$ is a set of constraints (or general laws) and $Frame$ provides a classification of the atoms of the language as frame fluents and non-frame fluents. As in [3, 4] we use a completion construction in order to solve the frame problem.

Causal laws have the general form $\Box(\alpha \wedge [a]\gamma \rightarrow [a]f)$ or $\Box(\beta \wedge [a]\delta_j \rightarrow [a]\neg f)$, where f is an atom (a fluent) and α, β, γ and δ are first-order formulas (not containing modalities).

$Frame$ is a set of pairs (f, a), where $a \in \boldsymbol{A}$ is an action symbol and f is an atom. Our action system relies on solutions to the frame problem similar to those described in [3, 4]. In this paper a completion construction is defined which, given a domain description, introduces frame axioms for all frame fluents in the style of the successor state axioms in situation calculus [10]. This completion construction applies only to action laws and causal laws and not to the constraints.

Let be Π contain action and causal laws which both have the general form

$$\Box(\alpha_i \wedge [a]\gamma_i \rightarrow [a]f) \qquad \Box(\beta_j \wedge [a]\delta_j \rightarrow [a]\neg f),$$

where $\alpha_i, \beta_j, \gamma_i, \delta_j$ are arbitrary (non-temporal) formulas and some of the conjuncts in the antecedents may be missing (in the case of action laws). This is an enumeration of the action laws for action a.

The completion of Π is the set of formulas $Comp(\Pi)$ containing, for all actions a and fluents f such that $(f, a) \in Frame$, the following axioms:

$$\Box(\langle a\rangle\top \rightarrow ([a]f \leftrightarrow \bigvee_i(\alpha_i \wedge [a]\gamma_i) \vee (f \wedge \neg[a]\neg f))) \tag{1}$$

$$\Box(\langle a\rangle\top \rightarrow ([a]\neg f \leftrightarrow \bigvee_j(\beta_j \wedge [a]\delta_j) \vee (\neg f \wedge \neg[a]f))) \tag{2}$$

Notice that, for each action a and fluent f which is nonframe with respect to a, i.e. $(f, a) \notin Frame$, axioms (1) and (2) above are not added in $Comp(\Pi)$. As in [10], these laws express that a fluent f (or its negation $\neg f$) holds either as a consequence of some action a or some causal law, or by persistency, since f (or $\neg f$) held in the state before the ocurrence of a and $\neg f$ (or f) is not a result of a. The occurrences of $\langle a\rangle\top$ assure that there is a succeeding state after action a occurred (formula $\langle a\rangle\top$ is true if there is a resulting state after occurrence of action a).

From the two axioms above we can derive the following axioms, which are similar, in their structure to Reiter's successor state axioms [10]:

$$\Box(\langle a\rangle\top \rightarrow ([a]f \leftrightarrow (\textstyle\bigvee_i(\alpha_i \wedge [a]\gamma_i)) \vee (f \wedge \textstyle\bigwedge_j(\neg\beta_j \vee \neg[a]\delta_j))))$$
$$\Box(\langle a\rangle\top \rightarrow ([a]\neg f \leftrightarrow (\textstyle\bigvee_j(\beta_j \wedge [a]\delta_j)) \vee (\neg f \wedge \textstyle\bigwedge_i(\neg\alpha_i \vee \neg[a]\gamma_i))))$$

The construction above is similar to the one that we have introduced in [3], though there are some differences for the fact that in [3], we have adopted a different formalization of causal laws by using the next operator. Also, in the present paper, causal laws are more general than in [3], as they refer (in their antecedent) to the values of fluents in the current state as well as in the next state.

4 Example

The following example is due to Lewis [6] and has been discussed by Halpern and Pearl in [5] in the framework of a theory of causation. Interestingly, this example defines actions with a complex temporal structure.

Billy and Suzanne throw rocks at a bottle. Suzanne throws first and her rock arrives first. The bottle shatters. When Billy's rock gets to where the bottle used to be, there is *nothing there but flying shards of glass. Without Suzanne's throw, the impact of Billy's rock on the intact bottle would have been one of the final steps in the causal chain from Billy's throw to the shattering of the bottle. But, thanks to Suzanne's preempting throw, that impact never happens.*

In our formulation, we focalize on the temporal structure of the throw action. We consider that the action occurs along a continuous (or dense) time axis, $[0, \infty[$. We define one action term for "throw", T, and two predicates H for "hits" and BB for "the bottle is broken". The action term $T(t, d, p)$ means that "person p throws a stone to a bottle at instant t and the result of the action (the stone hits its target) occurs at instant $t + d$". The formula $H(t, p)$ means that "the stone thrown by person p hits the bottle at instant t and formula $BB(t)$ means that the bottle is broken at instant t. The intended result of the action is to hit the bottle, but this result can only be achieved if the bottle is still at the intended place and nothing else has been happened to it, namely if it is not broken in the meantime. In this example it is not enough to have the precondition that the bottle is there and not broken at the instant of throwing, but it must be non-broken at the moment when the action is to be completed, just before it is to be hit. Therefore the action law for "throw" has a precondition which must hold after the instant when the action occurs.

Example 1 *The following set of laws represents the framework of this story:*

$$\begin{array}{l}(1)\ \Box(\neg BB(t+d) \rightarrow [T(t,d,p)]H(t+d,p))\\ (2)\ \Box(H(t,p) \rightarrow BB(t+d_1))\\ (3)\ \Box(BB(t) \rightarrow \forall t'(t < t' \rightarrow BB(t')))\\ (4)\ [\varepsilon]\neg BB(0)\end{array}$$

(1) is the action law for successful execution of the throw action, (2) describes the impact of hitting the bottle (d_1 is infinitesimally small) and the general law (3) says that a broken bottle remains broken "forever" [3].

Several scenarios can happen within this framework. Here we discuss the scenario where Suzanne throws at instant 0 and Billy throws some instant later[4].

$$\begin{array}{l}(5) < T(0, d_s, suzy) > \top\\ (6) < T(t_1, d_b, billy) > \top\end{array}$$

Three cases can be distinguished:

1. The moment when the bottle can be hit (and broken) after Suzanne's throw ($d_s + d_1$) occurs **before** Billy's stone could possibly hit the bottle $t_1 + d_b$.

[3] In this example we focus on the temporal relations between the different instants of throwing (by Suzanne and by Billy), so we neglected other preconditions, as for example having a stone, heavy enough, but not too heavy, having members enabling the person to throw, seeing the object to aim, etc. The throw action defined here is highly abstracted for the purpose of our temporal action theory.

[4] In order to express that action $a$occurs, we write $[a]\top$, which simply means that action a occurs (even when nothing can be said about its results). It is always possible to throw a stone at a bottle, even if the intended result of hitting cannot be achieved.

(7) $d_s + d_1 < t_1 + d_b$
(8) $\Box(BB(d_s + d_1) \to BB(t_1 + d_b))$ from (3) and (7)
(9) $\neg BB(d_s)$ by persistency from (4)[5]
(10) $[T(0, d_s, suzy)]H(d_s, suzy)$ from (1) and (9)
(11) $[T(0, d_s, suzy)]BB(d_s + d_1)$ from (2), (10), K for the action modality and (A2)
(12) $[T(0, d_s, suzy)]BB(t_1 + d_b)$ from (11), (8), K and (A2)

In this scenario, the law $\neg BB(t_1+d_b) \to [T(t_1, d_b, billy)]H(t_1+d_b, billy)$ cannot be used to derive $T(t_1, d_b, billy)]H(t_1 + d_b, billy)$ because $BB(t_1 + d_b)$ holds after Suzanne's throw (12). Billy's stone cannot hit the bottle, because it is already broken when his stone could hit it and we have just $[T(t_1, d_b, billy)]\top$ ((6), Billy has thrown).

2. Billy's stone hits the bottle, which breaks, **before** Suzanne's stone could possibly hit the bottle.

(13) $t_1 + d_b + d_1 < d_s$
(14) $\Box(BB(t_1 + d_b + d_1) \to BB(d_s))$ from (3) and (13)
(15) $\neg BB(t_1 + d_b)$ by persistency from (4), see (9)
(16) $[T(t_1, d_b, billy)]H(t_1 + d_b, billy)$ from (1) and (15)
(17) $[T(t_1, d_b, billy)]BB(t_1 + d_b + d_1)$ from (2), (16), K and (A2)
(18) $[T(t_1, d_b, billy)]BB(d_s)$ from (14), (17), K and (A2)

Here, Suzanne's stone, which could hit the bottle at instant d_s, will not hit it since we have $BB(d_s)$ and therefore the precondition $\neg BB(d_s)$ is not more true. The law $\Box(\neg BB(d_s) \to [T(0, d_s, suzy)]H(d_s, suzy))$ cannot be used to derive $[T(0, d_s, suzy)]H(d_s, suzy)$ because $BB(t_1 + d_b + d_1)$ holds after Billy's throw (17). All we have is $[T(0, d_s, suzy)]\top$ (Suzanne throws).

3. Suzanne's and Billy's stone hit the bottle precisely at the same moment.

(19) $t_1 + d_b = d_s$
(20) $\neg BB(t_1 + d_b) \wedge \neg BB(d_s)$ by persistency from (4), see (9)
(21) $[T(0, d_s, suzy)]H(d_s, suzy)$ like (10)
(22) $[T(t_1, d_b, billy)]H(t_1 + d_b, billy)$ as (16)
(23) $[T(0, d_s, suzy)]BB(d_s + d_1)$ from (21)
(24) $[T(t_1, d_b, billy)]BB(t_1 + d_s b + d_1)$ from (22))

In this case, both stones hit the bottle which breaks as a result of Suzanne's throw and Billy's throw.

5 Conclusion and Related Work

Modal logic approaches to action theories define a space of states but cannot handle time, neither explicitly not implicitly [2, 7]. In situation calculus [11, 8] reasoning about time was not foreseen, properties change discretely and actions do not have durations. Remember that in situation calculus, there is a starting state, s_0 and for any action a

and state s, $do(a,s)$ is a resulting state of s. One can consider that the set of states is given by $\{s : \exists a_1 \dots a_n s = do(a_n, do(a_{n-1}, \dots do(a_1, s_0)))\}$. Hence the temporal structure of situation calculus is discrete and branching and does not allow for actions of different duration neither for preconditions or results which become true during the action execution or later after the action is ended.

Javier Pinto has extended situation calculus in order to integrate time [9]. He conserves the framework of situation calculus and introduces a notion of time. Intuitively, every situation s has a starting time and an ending time, where $end(s,a) = start(do(a,s))$ meaning that situation s ends when the succeeding situation $do(a,s)$ is reached. The end of the situation s is the same time point as the beginning of the next situation resulting from the occurrence of action a in s. The obvious asymmetry of the $start$ and end functions is due to the fact that the situation space has the form of a tree whose root is the beginning state s_0. Thus, every state has a unique preceding state but eventually more that one succeeding state.

Paolo Tereziani proposes in [14] a system that can handle temporal constraints between events and temporal constraints between instants of events.

In this present article, we have introduced a new modal logic formalism which can handle simultaneously states and time. We did not address here the problem of the persistency of facts over time (or over the execution of actions), because we wanted to focus on the modal temporal formalism. We have adopted a solution similar to the one presented in [12], i.e. “weak” frame laws are nonmonotonically added to the theory. But this solution is a bit more complicated in the case of our first-order action logic presented in this paper, because we need to restrict ourselves to a decidable subset of $\mathcal{D}al$.

Concerning the implementation, we use a labelled analytic tableaux approach including an abductive mechanism for the weak persistency laws, which will be described in more detail in a following paper.

Further work is carried out in two directions:

1. Concerning the logical formalism we want to make a more complete study of the undecidability and complexity issues. We want to study decidability for subclasses corresponding to well known first-order decidability classes.
2. Concerning applications, we will apply this formalism to planning problems where a hybrid approach (states and time) can be very powerful. The idea is to infer temporal constraints from a $\mathcal{D}al$ specification in order to create a plan for a problem. This will yield a hybrid approach to planning and scheduling

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