# Comparing Variants of Strategic Ability

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March 1, 2012, Dynamics in Logic II

# LAMAS: Logical Aspects of Multi-Agent Systems

**LAMAS 2012:** 5th of June 2012 at AAMAS2012 (Valencia, Spain)

**http://icr.uni.lu/lamas2012/**

# Outline

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# Introduction

- Strategic logics: ATL, coalition logic, stit
- Basic issue: "Can agent $a$ (coalition $A$) bring about $\varphi$?"
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- Basic issue: "Can agent $a$ (coalition $A$) bring about $\varphi$?"
- Semantic variants of ATL: encapsulate various notions of ability
- **We study the relationship between standard variants of ATL on the level of valid sentences**
- Surprisingly, nobody has studied it before

# Motivation

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- Thus, by comparing validity sets we compare logics in the traditional sense
- Validities capture general properties of games under consideration
- If two variants of ATL generate the same valid sentences then the underlying notions of ability induce the same kind of games
- First step towards devising algorithms for satisfiability checking

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**$\langle\langle A \rangle\rangle \Phi$: coalition $A$ has a collective strategy to enforce $\Phi$**

$\rightsquigarrow$ $\Phi$ can include temporal operators: $\bigcirc$ (next), $\Diamond$ (sometime in the future), $\Box$ (always in the future), $\mathcal{U}$ (strong until)

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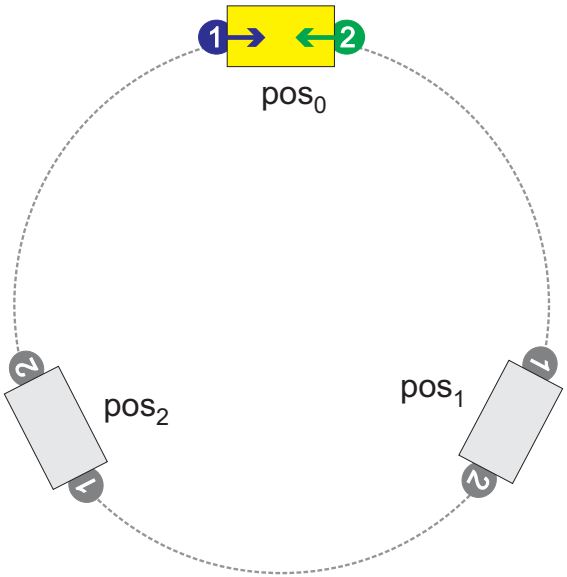
- $\langle\langle robber \rangle\rangle \Diamond$open:
  "The robber has a strategy to eventually get the vault open no matter how the other agents act"

- $\langle\langle bank \rangle\rangle \Box \neg$robbery:
  "The bank can protect itself against being robbed"

# Example: Robots and Carriage



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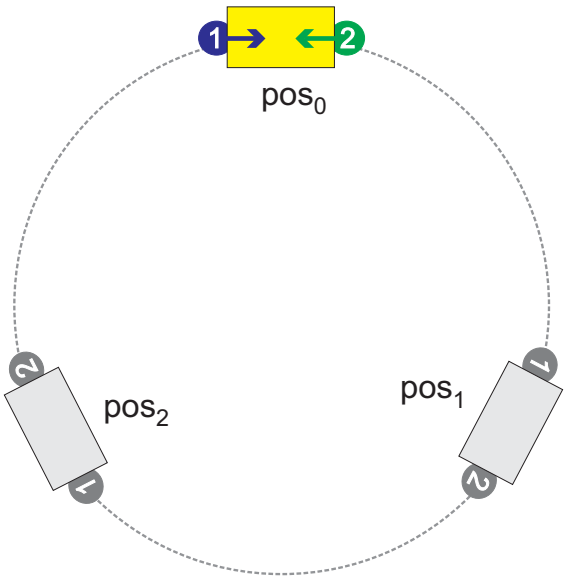


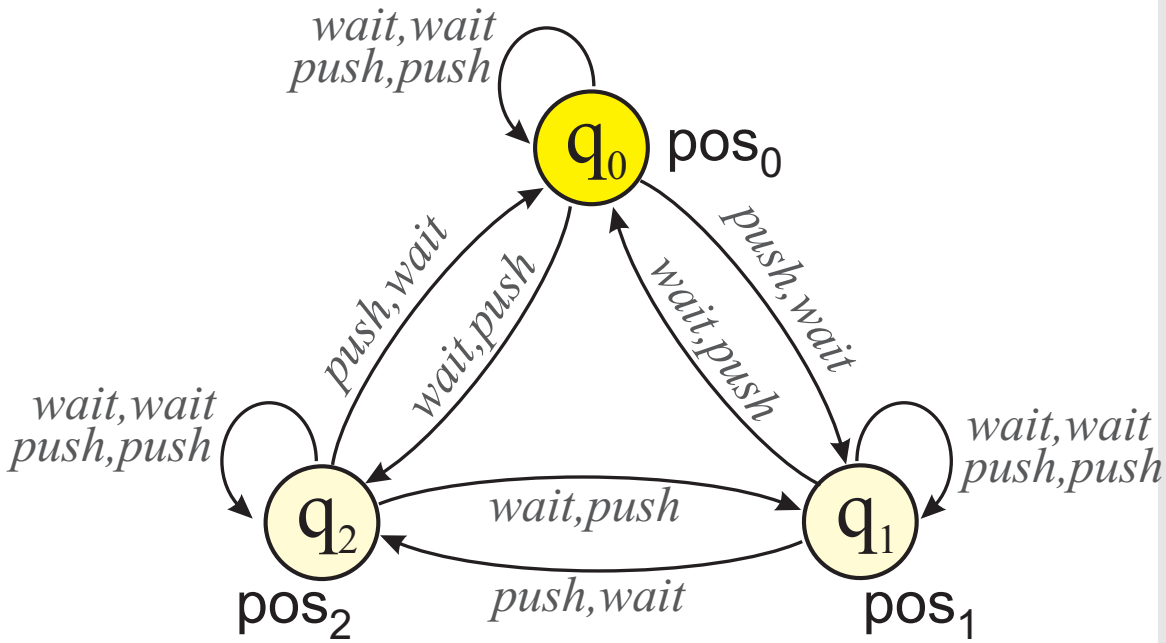

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## Semantics of ATL

$M, q \models \langle\langle A \rangle\rangle \Phi$ iff there is a collective strategy $s_A$ such that, for every path $\lambda$ that may result from execution of $s_A$ from $q$ on, we have that $M, \lambda \models \Phi$.

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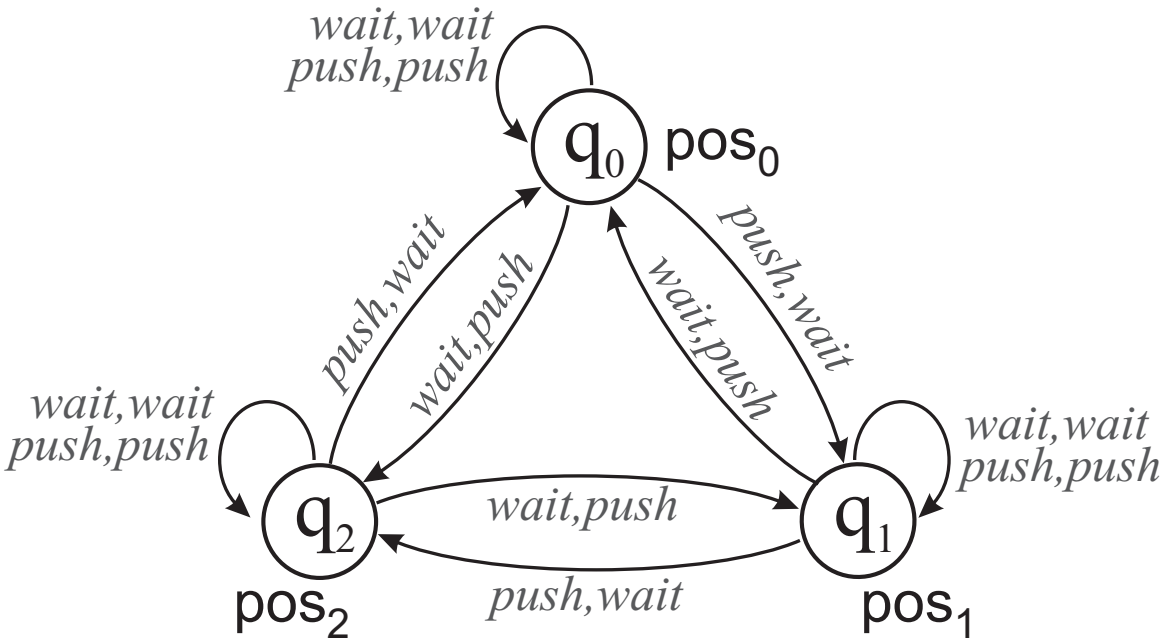


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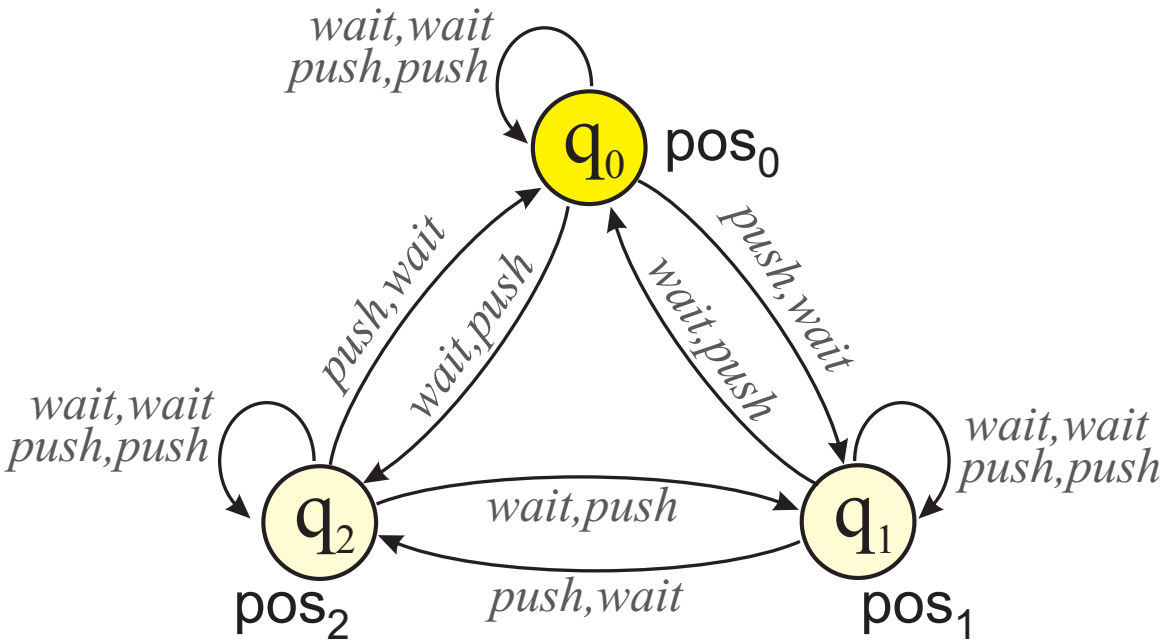


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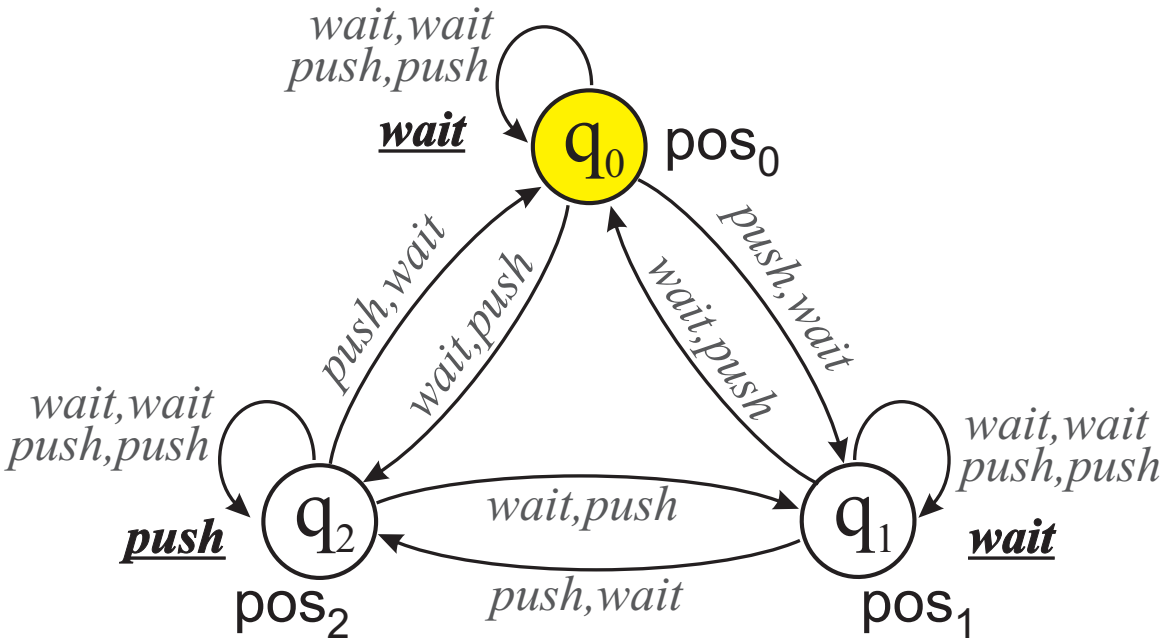


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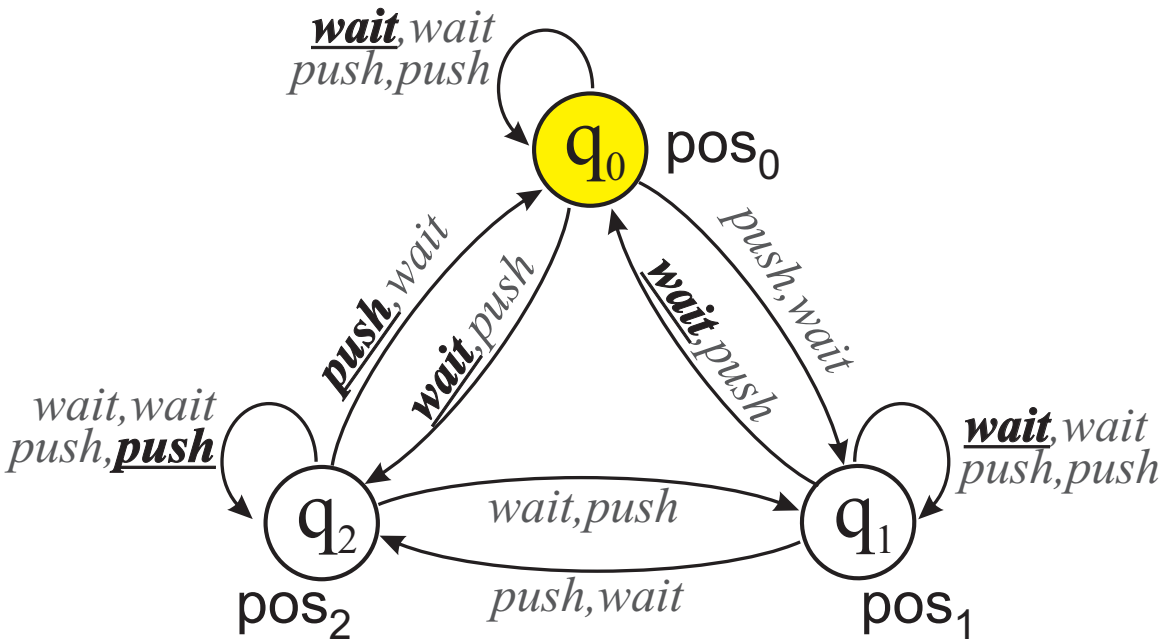


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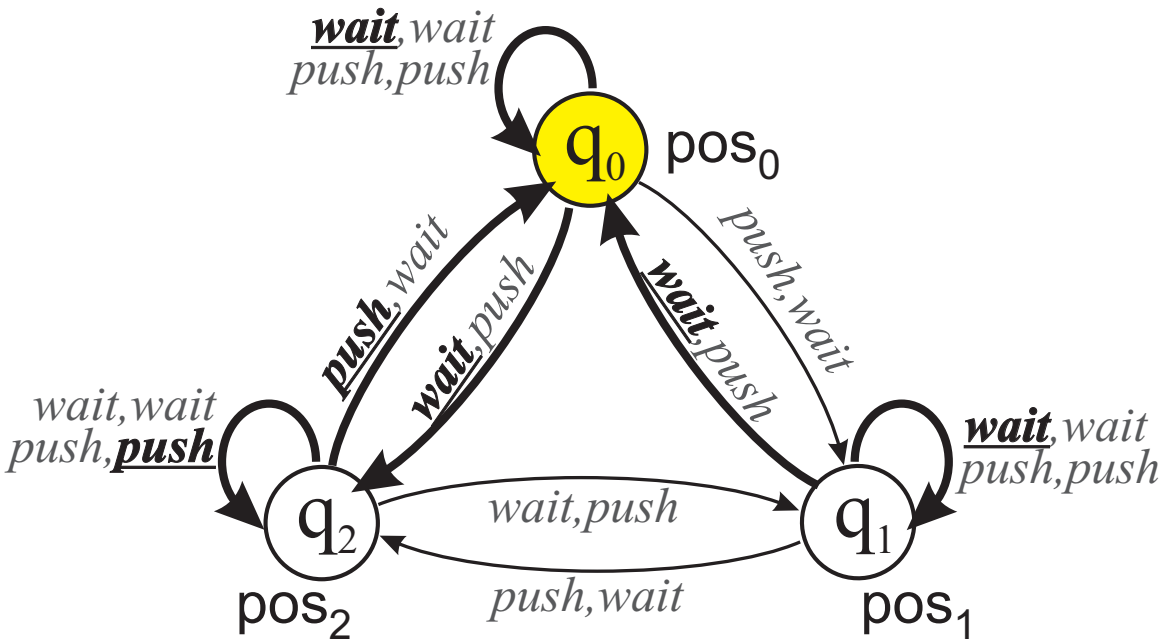


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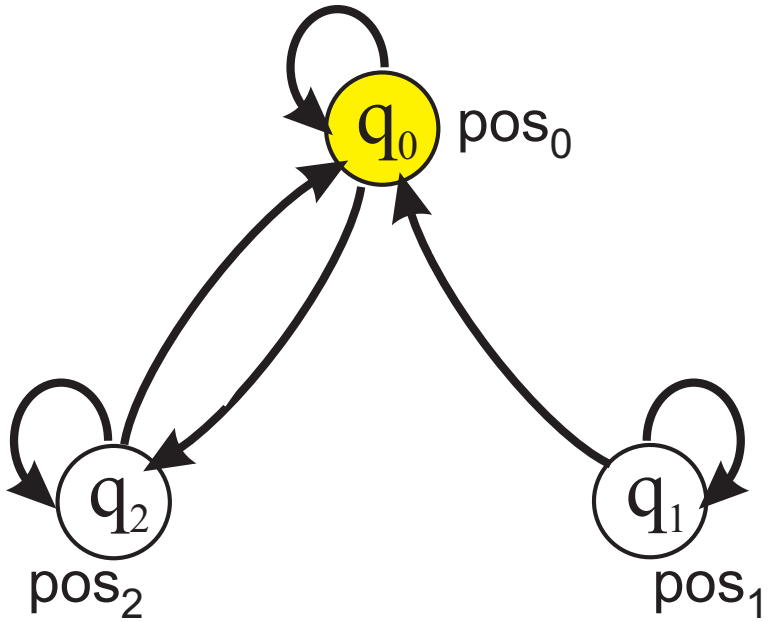


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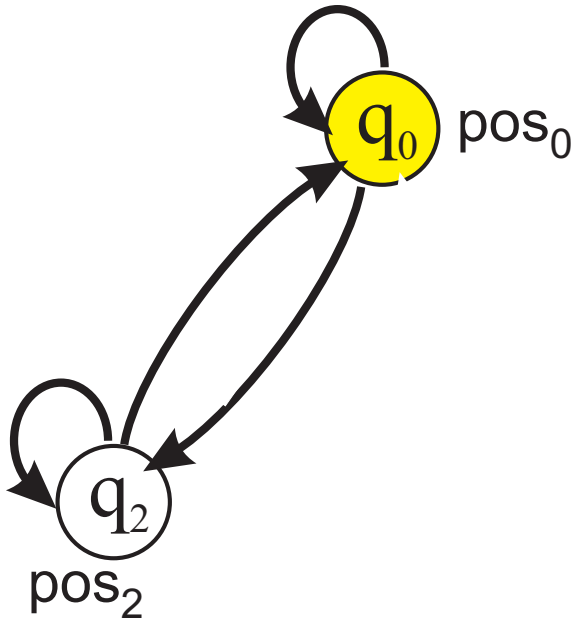


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# Semantic Variants of ATL

- Basic semantics of ATL assumes perfect information - not very realistic
- Semantic variants for more realistic cases defined in (Jamroga 2003), (Jonker 2003), (Schobbens 2004), (Jamroga & van der Hoek 2004), (Agotnes 2004), ...
- Encapsulate different assumptions about agents and abilities

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Success of strategies:

- Objectively **($i_o$)** vs. subjectively successful strategies **($i_s$)**

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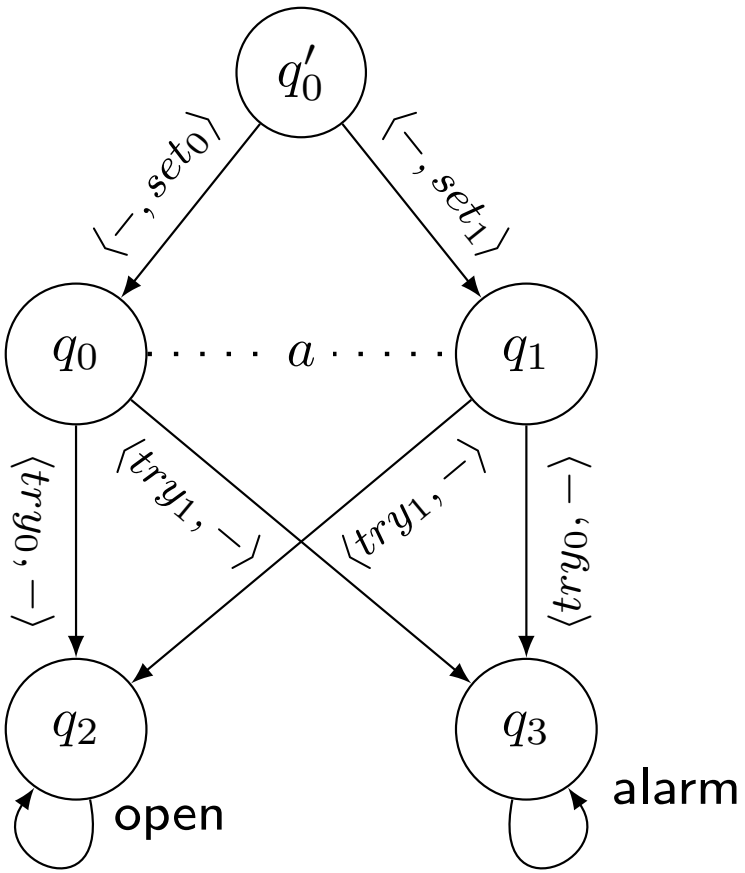
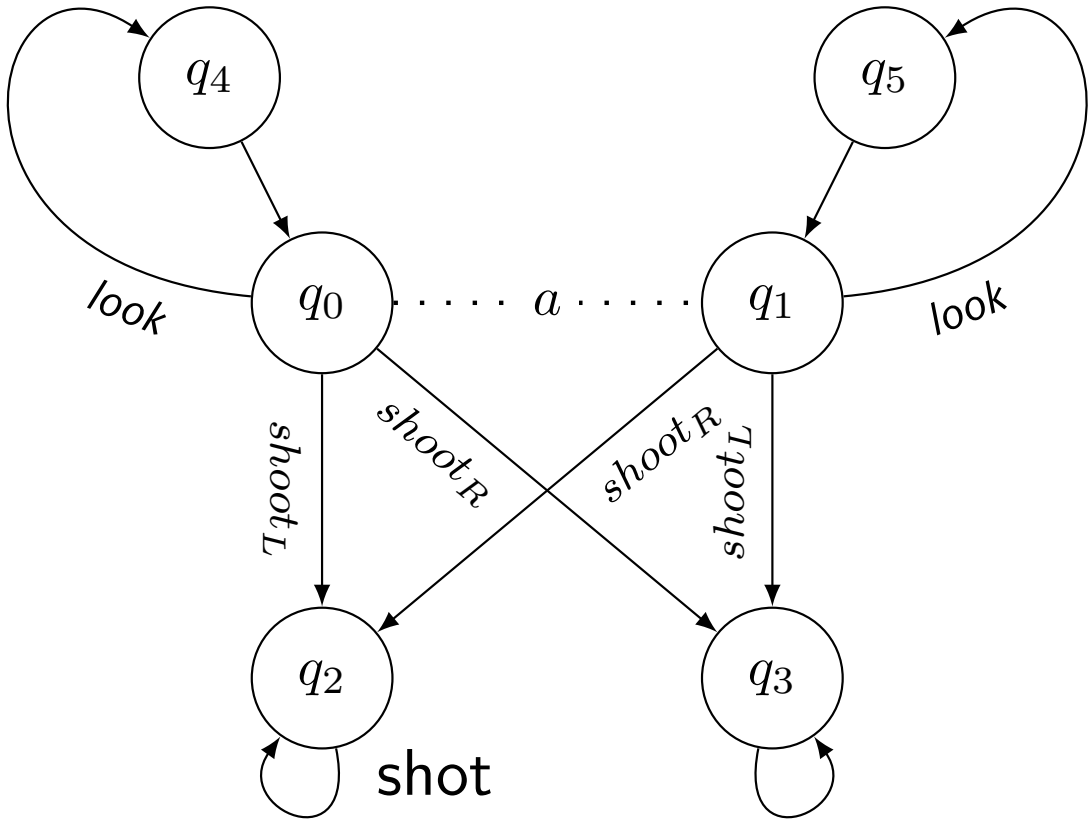


# Example: Poor Duck Problem



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- Validity $\rightleftharpoons$ general properties of games

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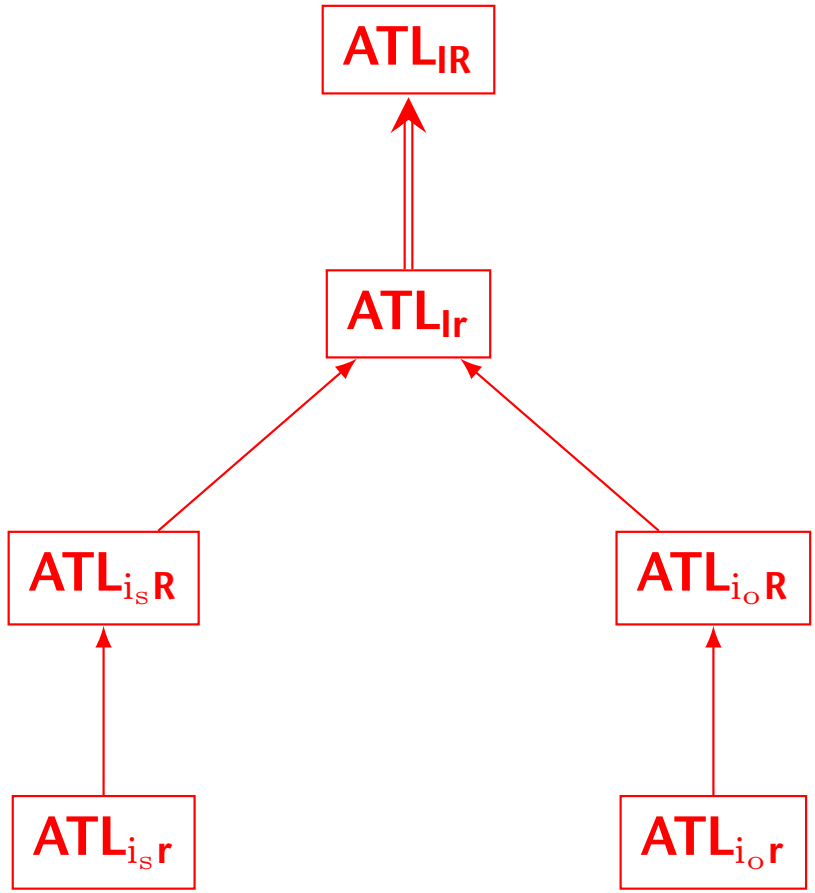
# Validities in Variants of ATL: Subsumption Graph



# Summary in Plain Words

In terms of general properties of games we get the following:

- Perfect information is a (strict) special case of imperfect information
- Perfect recall games are (strict) special case of memoryless games
- Information type has more impact than type of recall
- Properties of objective and subjective abilities of agents are incomparable

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- $\neg\langle\langle \emptyset \rangle\rangle \Diamond \neg p \leftrightarrow \langle\langle \mathbb{A}\mathrm{gt} \rangle\rangle \Box p$
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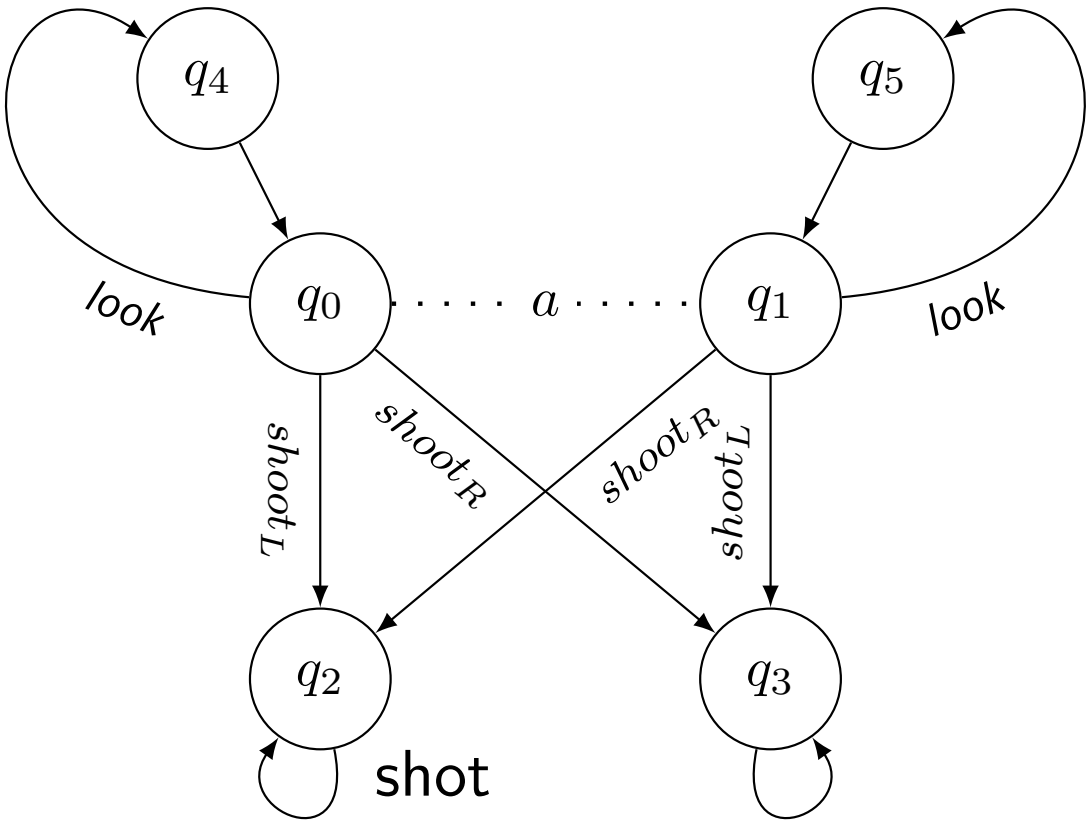

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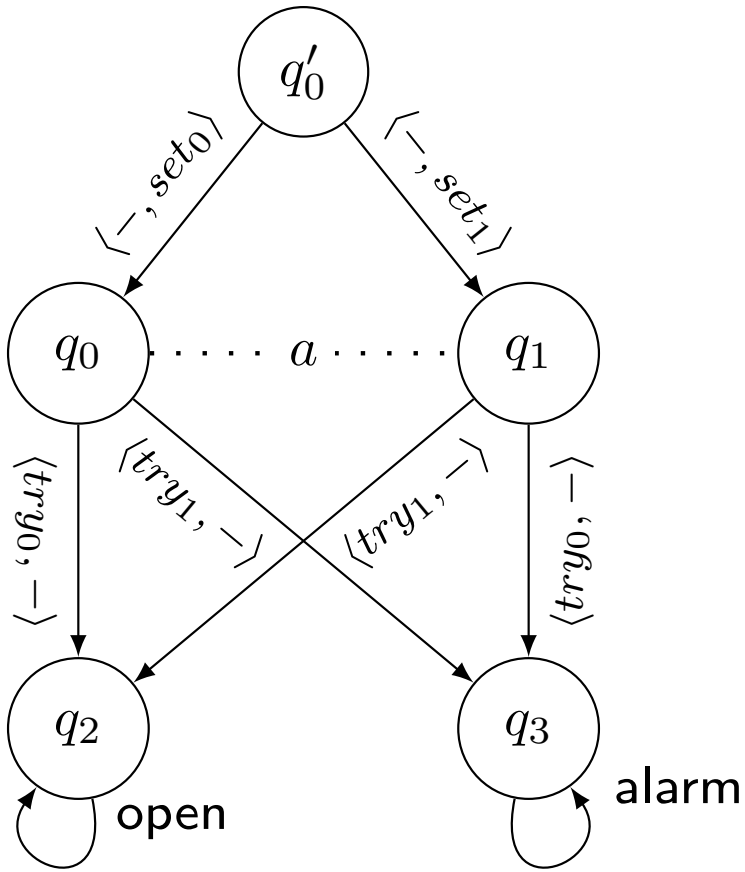

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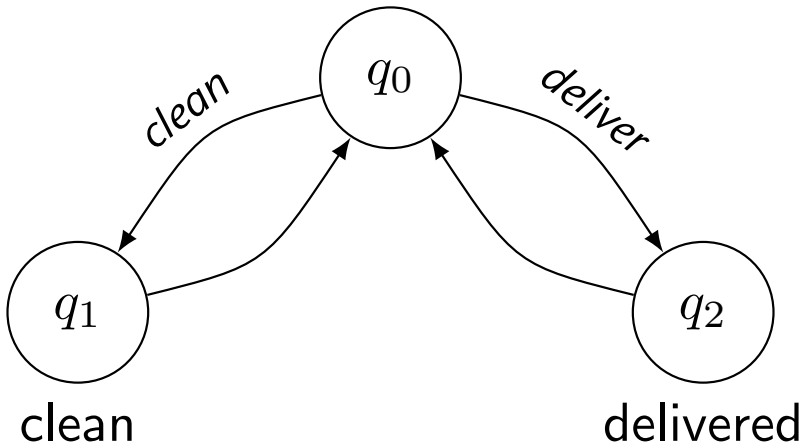

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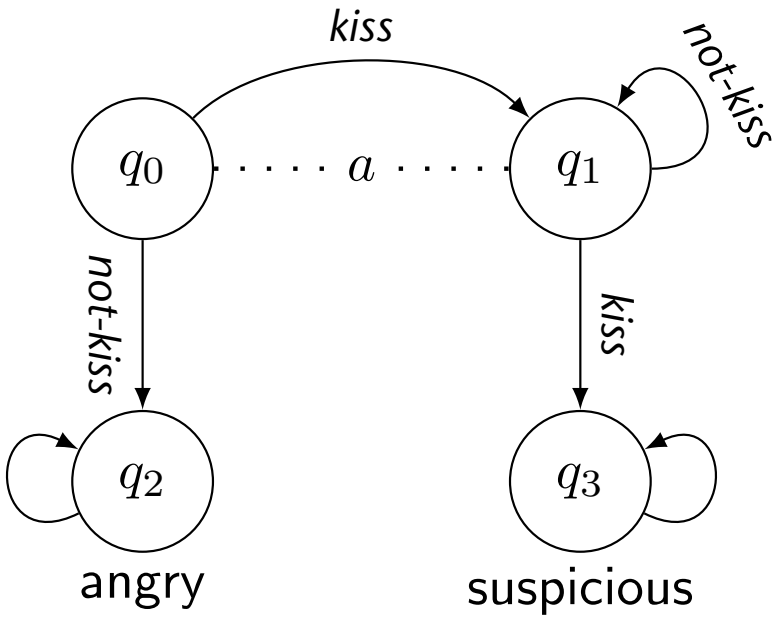

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- **Non-validities more important than the inclusion results**