Playing Games with Dynamic Epistemic Logic

Dynamics in Logic II, Lille, 1 March 2012

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based on joint work with:

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Introduction

- Dynamic epistemic logic is concerned with describing actions and other events, and their epistemic pre- and post-conditions

- Which actions will rational agents actually make?

- To answer that, some kind of preferences over epistemic states must be assumed

- Epistemic states eventually depend on the actions chosen by all agents

  ➡ game theoretic scenario

- Games are inherent in epistemic structures!

Dynamic Epistemic Games: Dimensions

- Dimensions:
  - Models/representations of preferences (epistemic goals, ...)
  - Types of actions (public announcements, ...)
  - Simultaneous vs. alternating actions
  - Single action or action sequence
  - Possibility of coalition formation
  - ...
- ... this looks like a **research program**

Today: two types of action

- Today I will discuss a couple of the simplest cases
- Actions:
  - public announcements
  - questions and answers
- with “simple” assumptions about the other dimensions

Part I: Public Announcement Games

Setting

- Simple (or so you may think) setting:
    - Actions = truthful announcements
    - Goals in the form of formulae of epistemic logic (assumed common knowledge)
    - Strategic form games

Knowledge and Games, and Vice Versa

- Much existing work on epistemics in games
- Now: from *knowledge in games* to *games of knowledge*
  - ... and back again

Public Announcement Logic (Plaza, 1989)

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2$$

ϕ_1 is true, and ϕ_2 is true after ϕ_1 is announced

Formally:

$$M = (S, \sim_1, \ldots, \sim_n, V) \qquad \sim_i \text{ equivalence rel. over S}$$

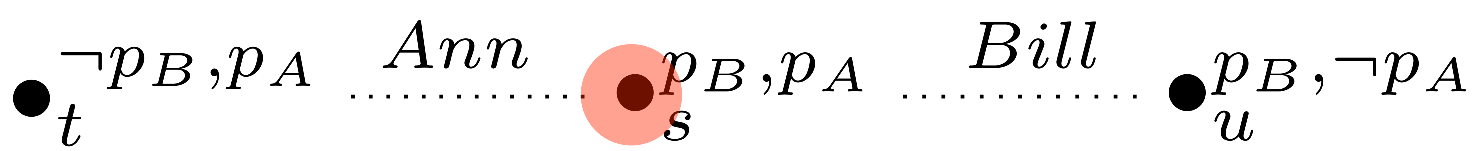
$$M, s \models K_i\phi \quad \Leftrightarrow \quad \forall t \sim_i s \; M, t \models \phi$$

$$M, s \models \langle\phi_1\rangle\phi_2 \quad \Leftrightarrow \quad M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2$$

The model resulting from removing states where ϕ_1 is false

Example

Example



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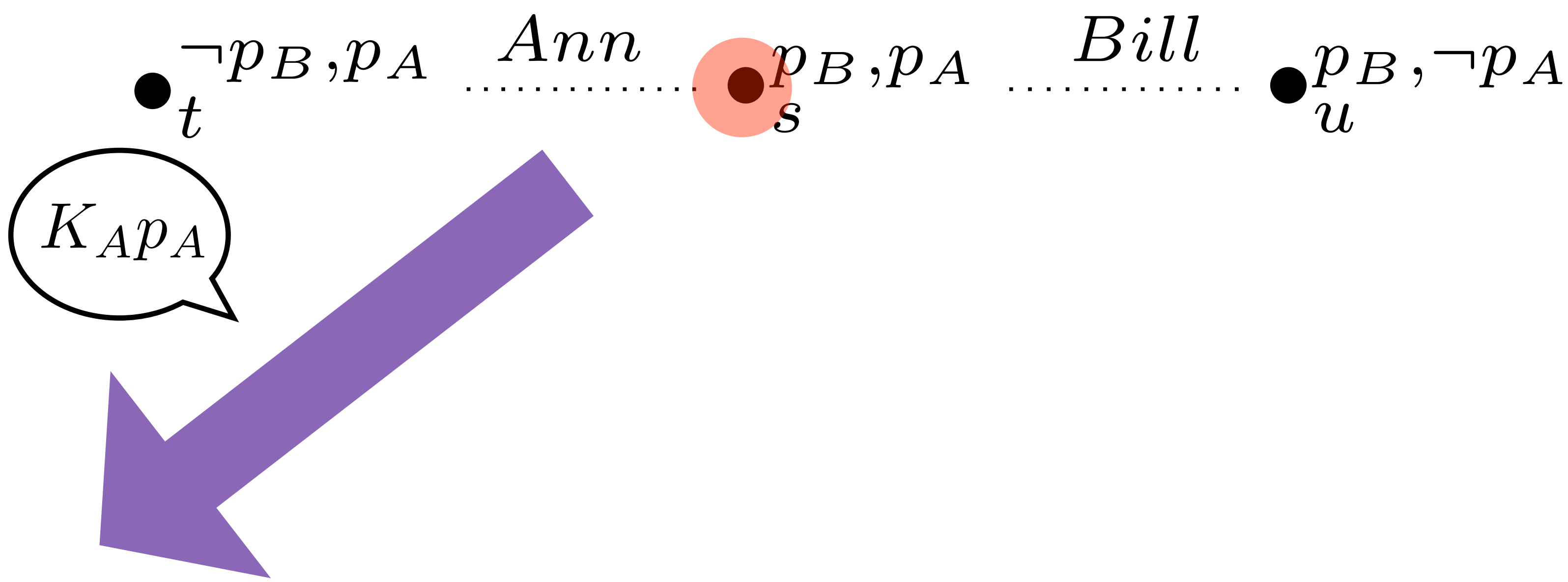

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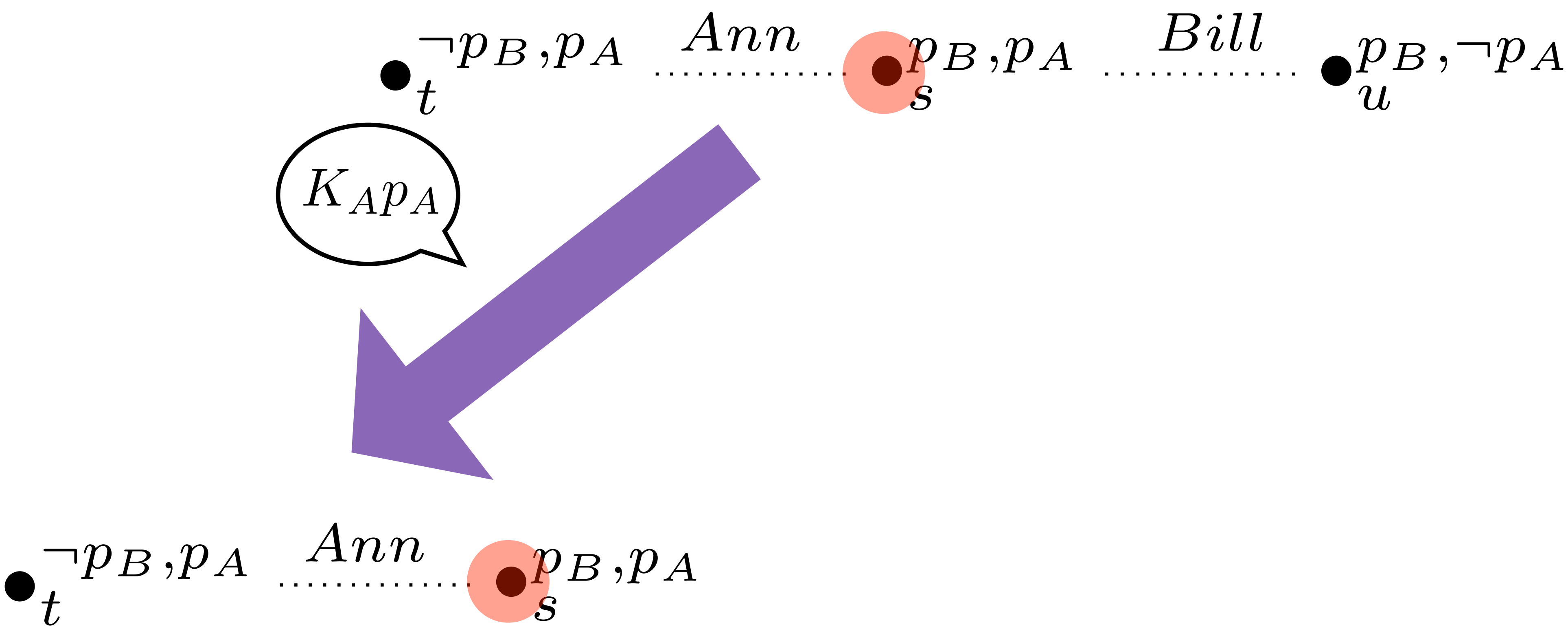

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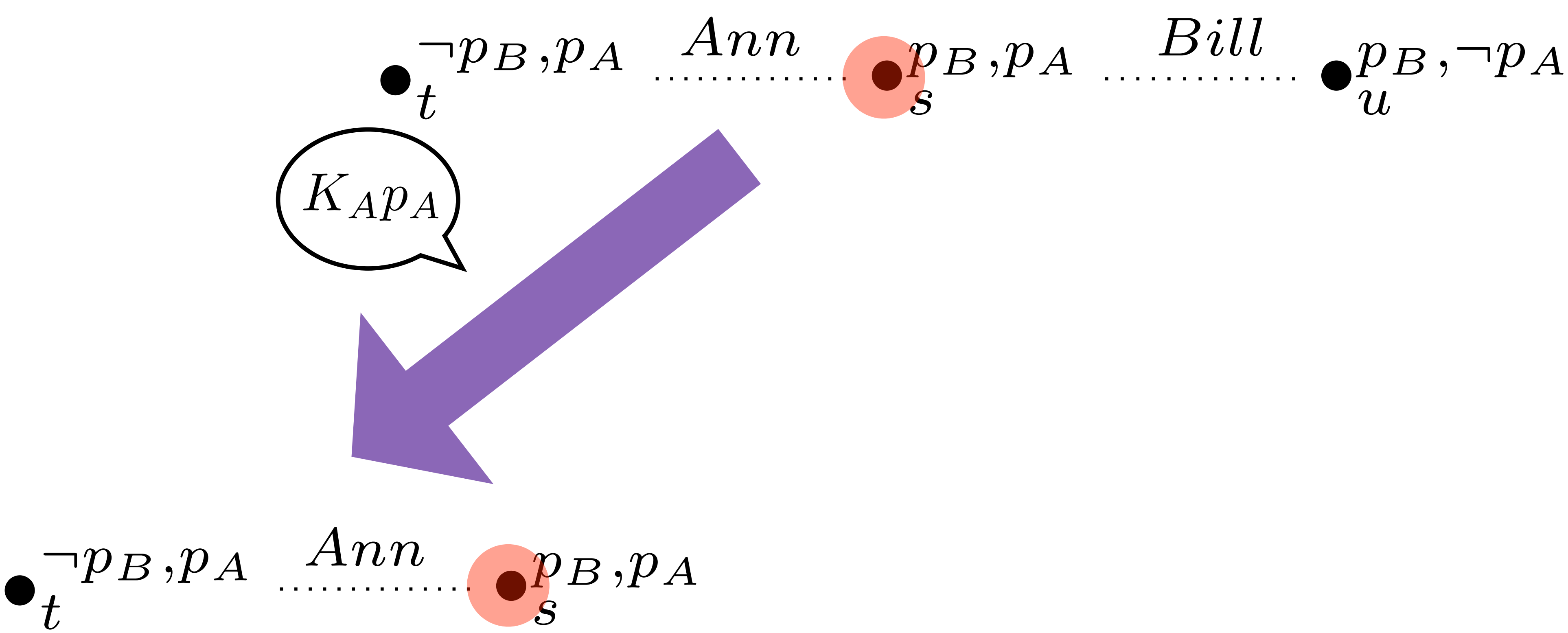

$$M, s \models \langle K_A p_A \rangle K_B p_A$$

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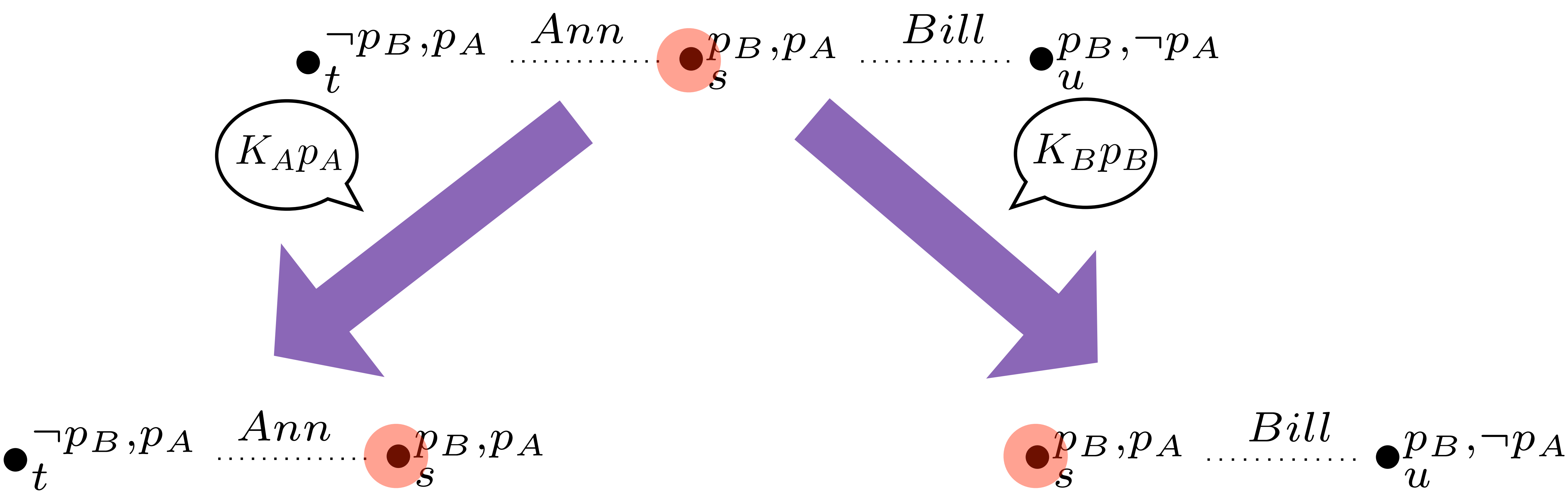

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Setting

- Assume that agents:
  - have incomplete information about the world;
  - have goals in the form of formulae of epistemic logic (common knowledge);
  - only make truthful announcements;
  - choose announcements independently;
  - act rationally

Epistemic Goal Structures

Definition 1 (Epistemic Goal Structure) *An (n-player) epistemic goal structure (ECG) is a tuple*

$$\langle M, \gamma_1, \ldots, \gamma_n \rangle$$

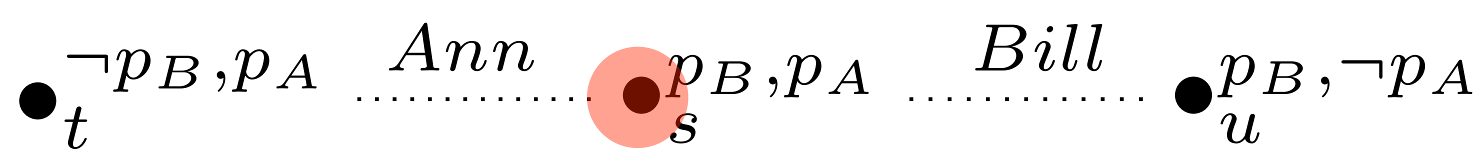
where M is an epistemic structure, and $\gamma_i \in \mathcal{L}_{pal}$ is the goal formula *for agent i. A* pointed ECG *is a tuple*

$$\langle M, s, \gamma_1, \ldots, \gamma_n \rangle$$

where s a state in M.

Example

$$\langle M, s, \gamma_1, \ldots, \gamma_n \rangle$$



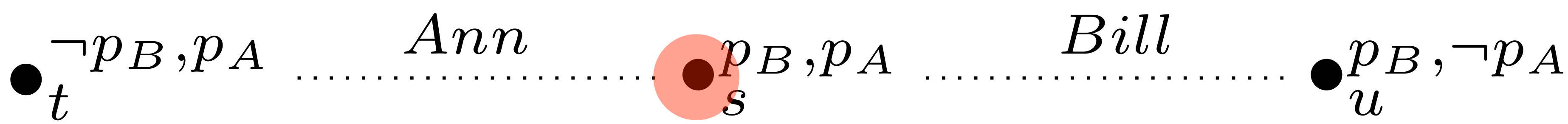
$$\gamma_{Ann} = (K_B p_A \vee K_B \neg p_A) \rightarrow (K_A p_B \vee K_A \neg p_B)$$
$$\gamma_{Bill} = (K_A p_B \vee K_A \neg p_B) \rightarrow (K_B p_A \vee K_B \neg p_A)$$

From ECG to Public Announcement Game

$$\langle M, s, \gamma_1, \dots, \gamma_n \rangle$$

- Strategies: $A_i = \{\phi_i : M, s \models K_i \phi_i\}$
- Payoffs: $u_i(\langle \phi_1, \dots, \phi_n \rangle) = \begin{cases} 1 & M, s \models \langle K_1\phi_1 \wedge \cdots \wedge K_n\phi_n \rangle \gamma_i \\ 0 & \text{otherwise} \end{cases}$

Example



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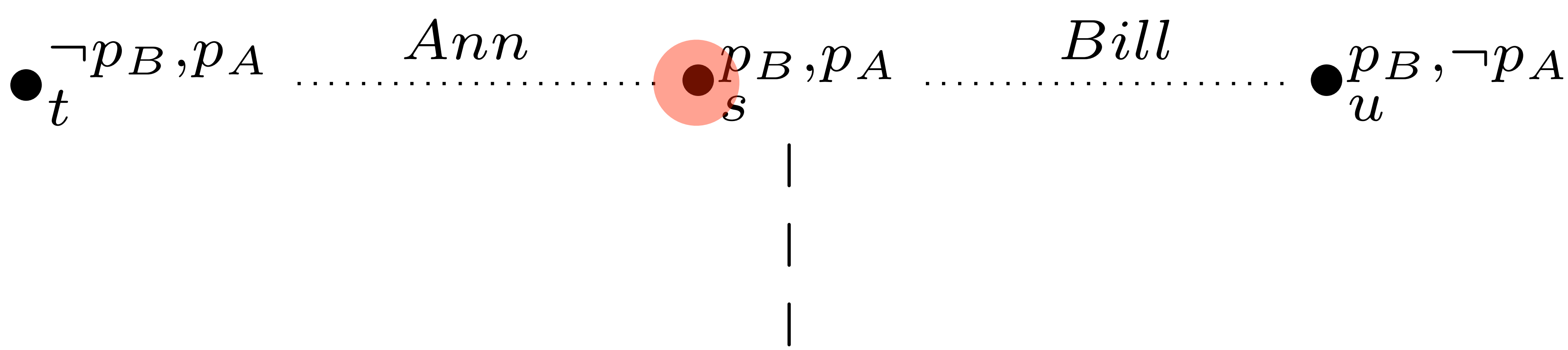

	$\top$	$\mathbf{p_B}$
$\top$	11	10
$\mathbf{p_A}$	01	11

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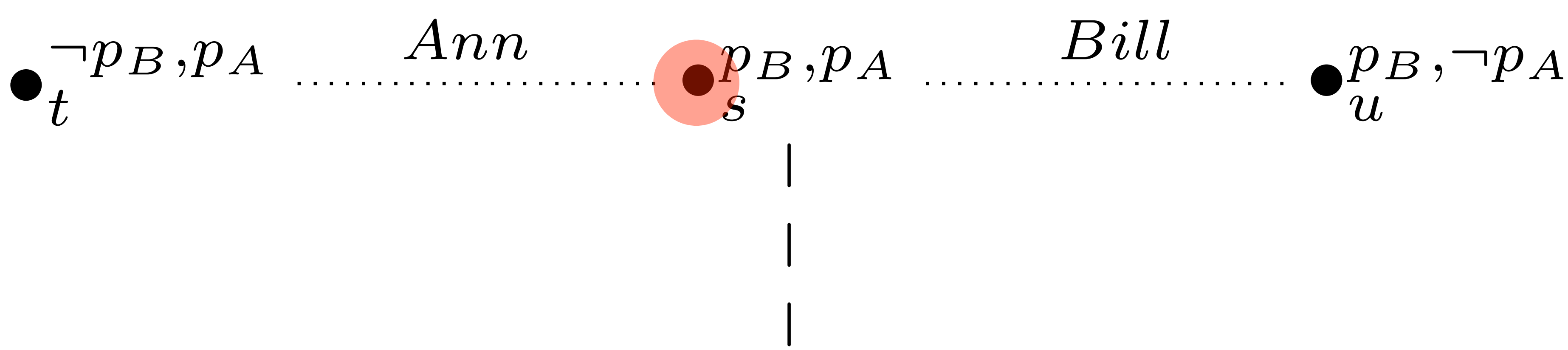

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"Representative" strategies;
all other are equivalent

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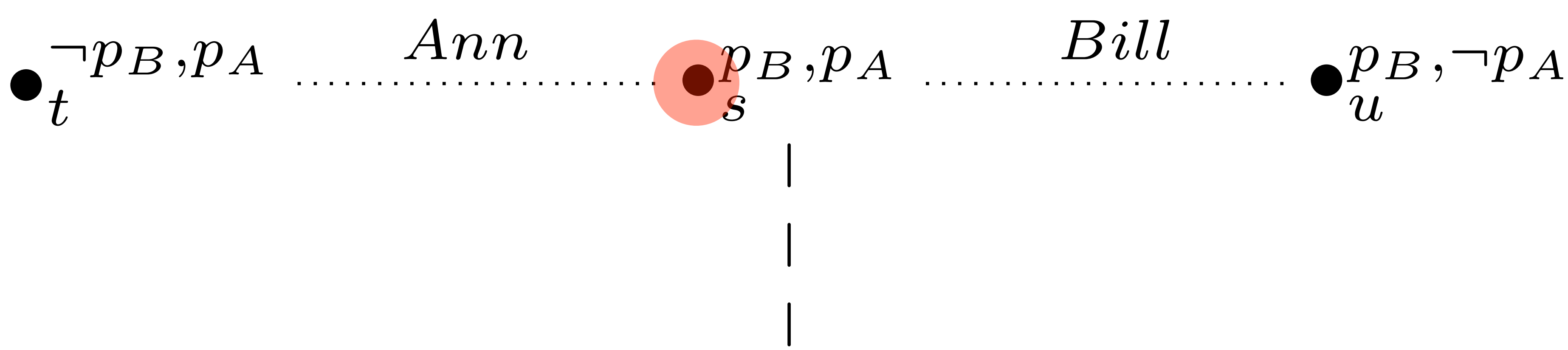

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Nash eq.

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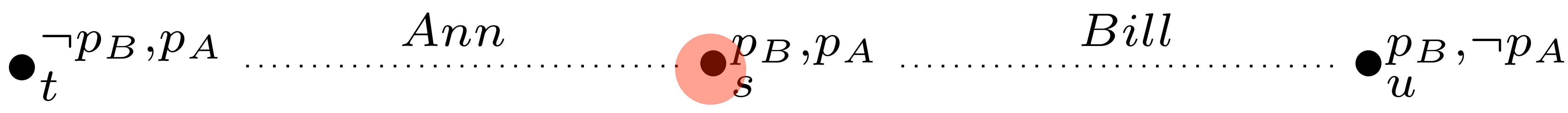
State games



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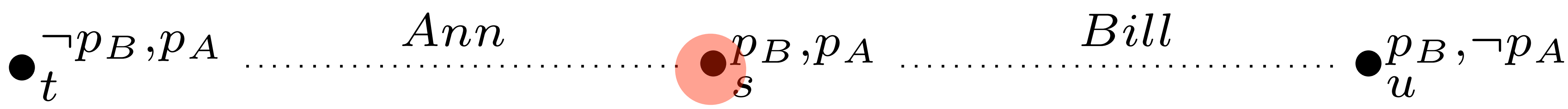

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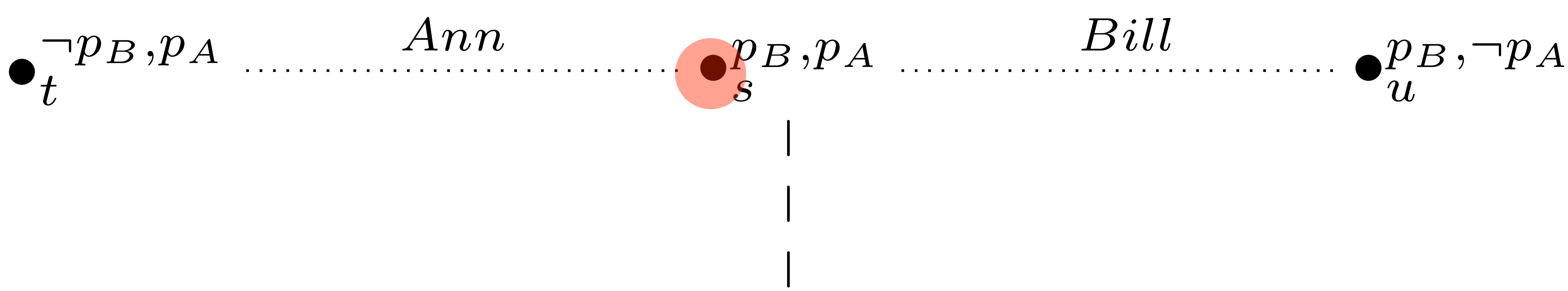
- Similarities to *Boolean Games* (Harrenstein, et al.)

State games



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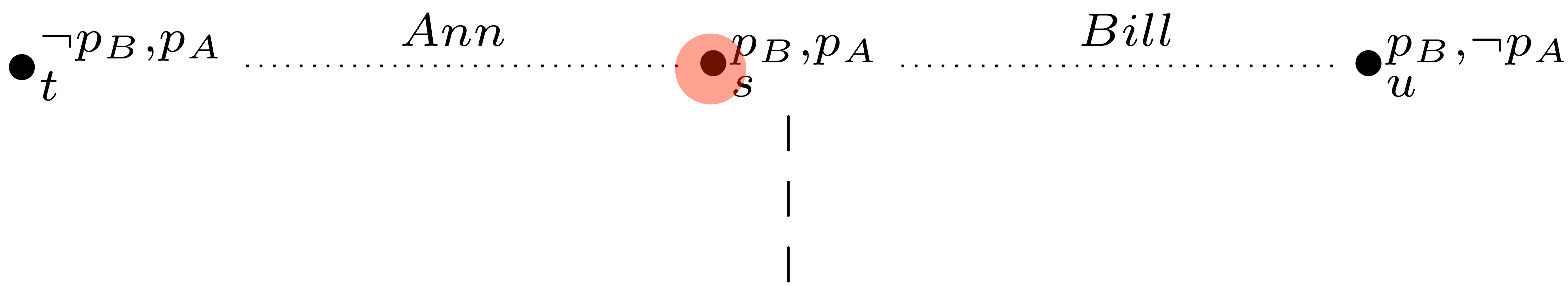

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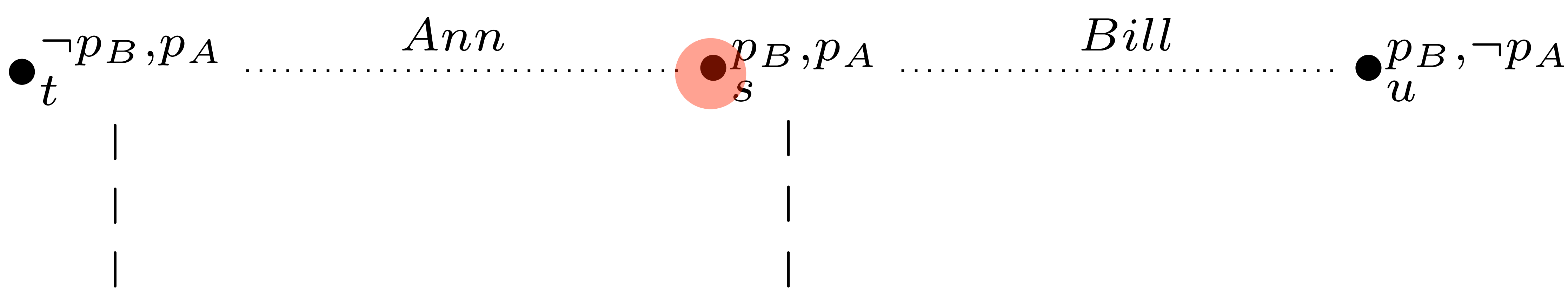

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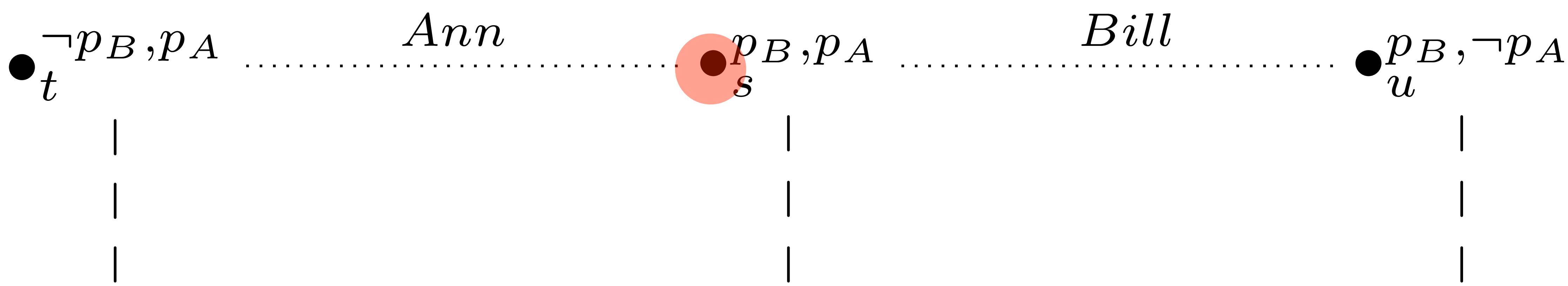

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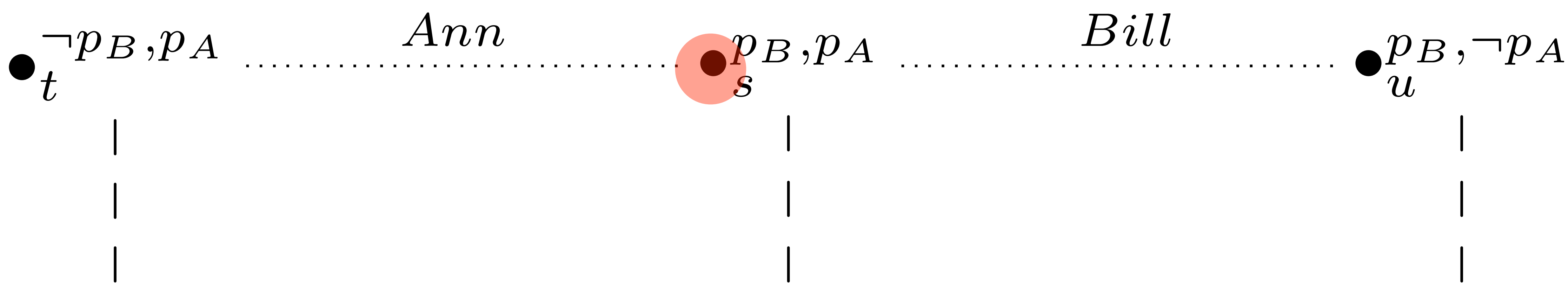
Solution concepts

- Public announcement games is a particular type of strategic games with imperfect information
- Intimate connection between information, strategies and payoff
- What are reasonable solution concepts?
- Let us consider some possibilities

Weakly dominant strategies

- It might be that there is a dominant strategy, but that the agent does not know it
- In the case that the agent knows that there is a dominant strategy, it might be that:
  - The agent has a weakly dominant strategy de dicto: there is a weakly dominant strategy in every state she considers possible
  - The agent has a weakly dominant strategy de re: there is a strategy which is weakly dominant in every state she considers possible

Weakly dominant strategies



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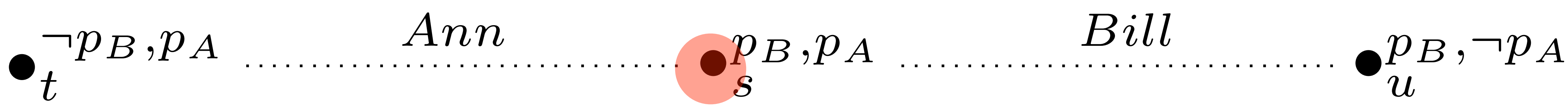

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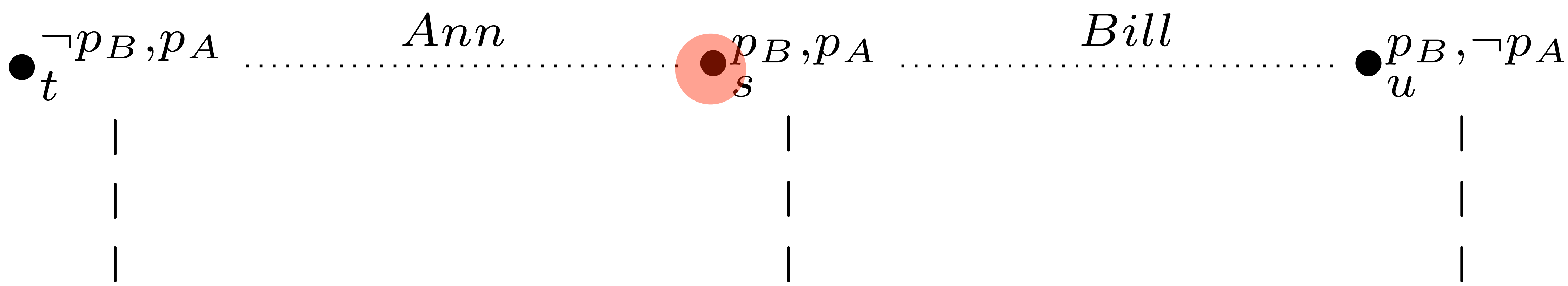

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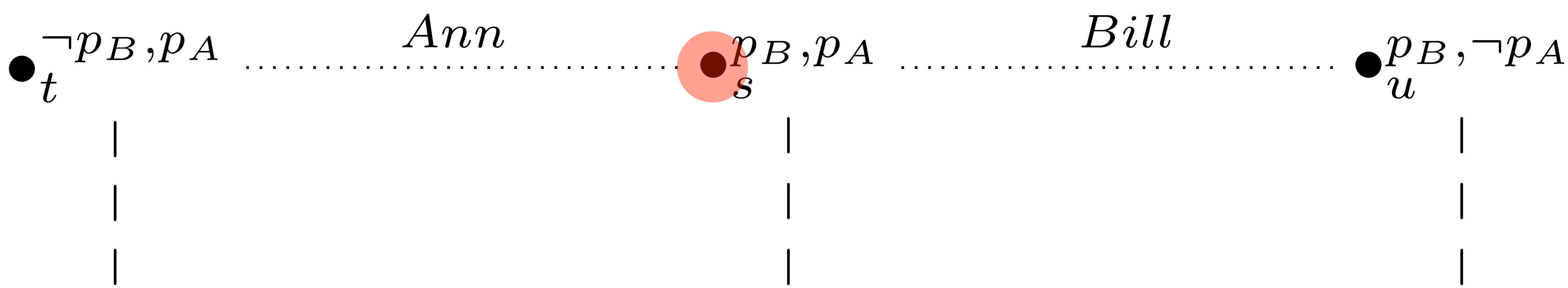
- Ann has a weakly dominant strategy de re (and, by implication, de dicto)

Weakly dominant strategies

There are goal formulae which give the following



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Weakly dominant strategies

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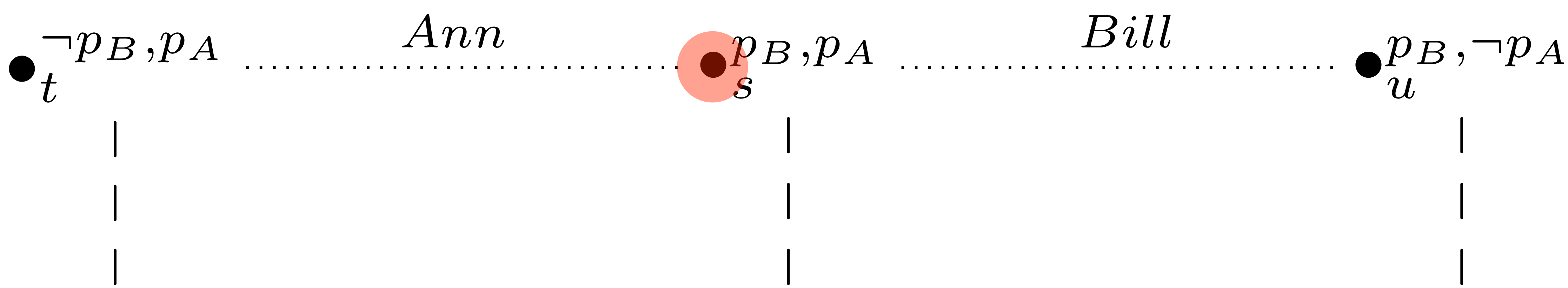

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$\top$	11	00
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- Ann has a weakly dominant strategy de dicto, but not de re

Positive Goals

The positive fragment of PAL:

$$\phi ::= p \mid \neg p \mid \phi \wedge \phi \mid \phi \vee \phi \mid K_i\phi \mid [\phi]\phi$$

Positive Goals

The positive fragment of PAL:

$$\phi ::= p \mid \neg p \mid \phi \wedge \phi \mid \phi \vee \phi \mid K_i \phi \mid [\phi]\phi$$

Theorem
If the goal of an agent is in the positive fragment, then that agent has a weakly dominant strategy *de re* in any state.

Nash Equilibrium

- De dicto/de re distinction not as clear:
  - several agents involved
  - what does it mean that they know that an outcome is a NE?
- Common assumption: common knowledge
- Thus: let us say that there is a Nash equilibrium de re in a PAG if there is a strategy profile which it is commonly known is a NE

Nash Equilibrium de re

$$\bullet_t^{\neg p_B, p_A} \;\overset{Ann}{\cdots\cdots\cdots}\; \bullet_s^{p_B, p_A} \;\overset{Bill}{\cdots\cdots\cdots}\; \bullet_u^{p_B, \neg p_A}$$

World t:

	$\top$	$\neg \mathbf{p_B}$
$\top$	01	11
$\mathbf{p_A}$	01	11

World s:

	$\top$	$\mathbf{p_B}$
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$\mathbf{p_A}$	01	11

World u:

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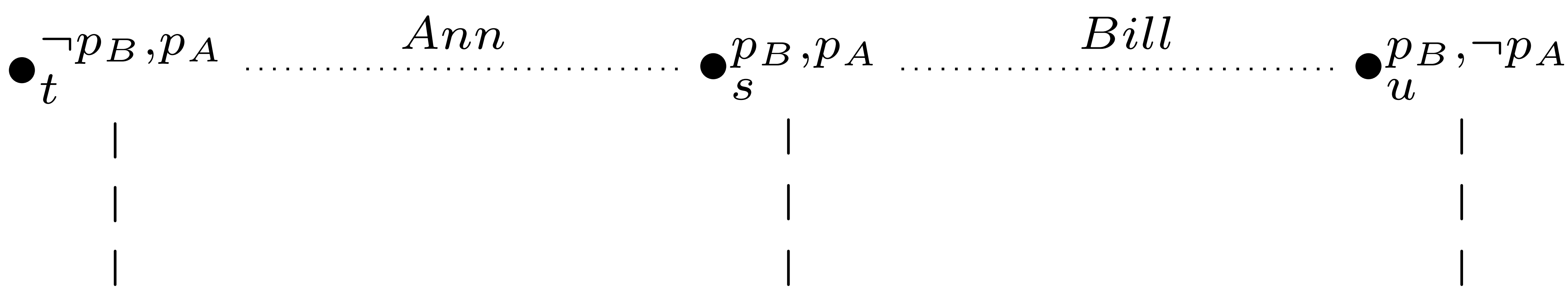
Nash Equilibrium de dicto, but not de re



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	$\top$	$\mathbf{p_B}$
$\top$	11	00
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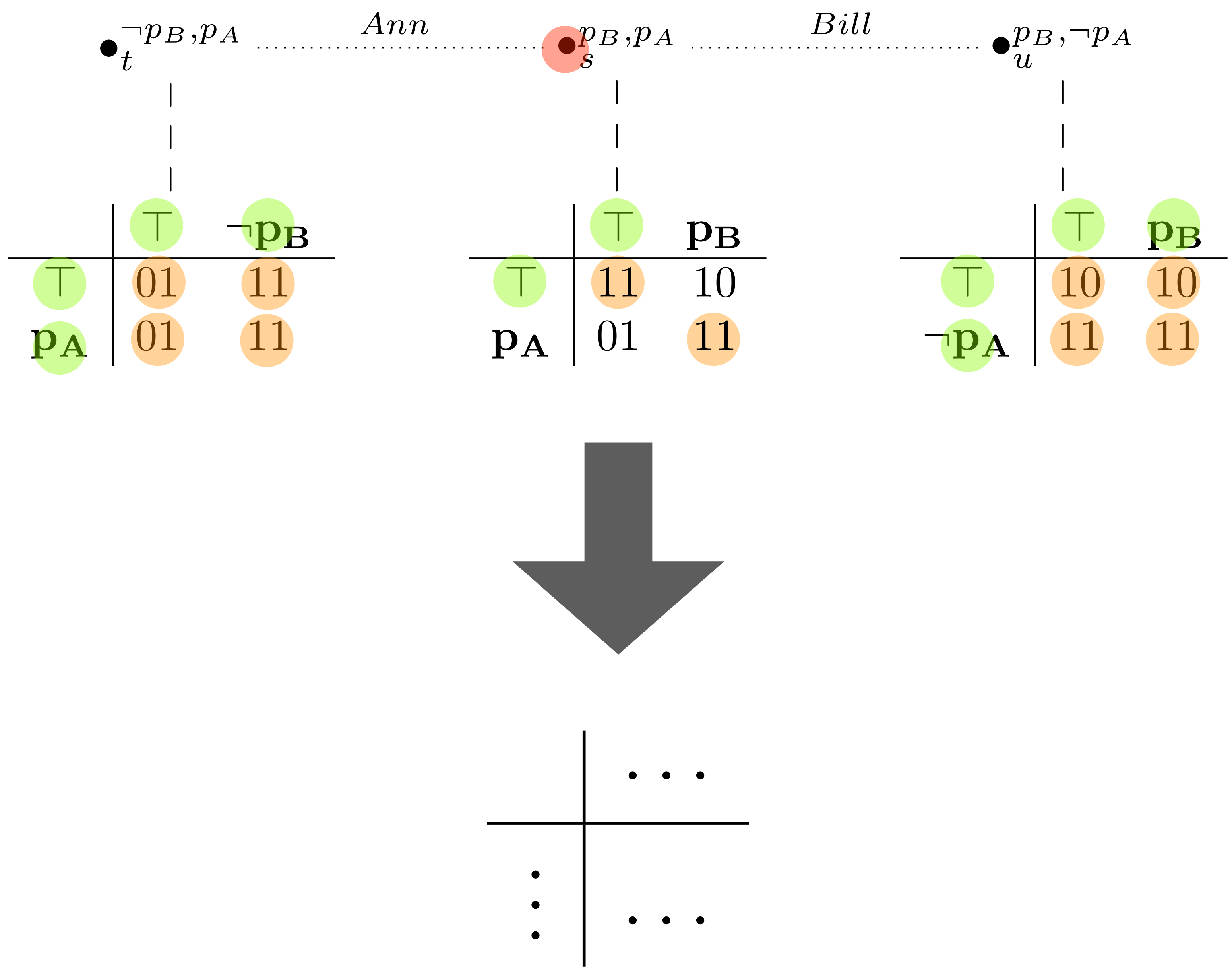
Nash equilibrium

Nash equilibrium

Theorem
If there is a Nash-equilibrium that is common knowledge, then it is non-informative

But which game are they really playing?

- Can a public announcement game be viewed as a single strategic game?



The induced game

Definition 1 *Given a PAG* $AG = \langle M, \gamma_1, \ldots, \gamma_n \rangle$ *with* $M = (S, \sim_1, \ldots, \sim_n, V)$, *the* induced game G_{AG} *is defined as follows:*

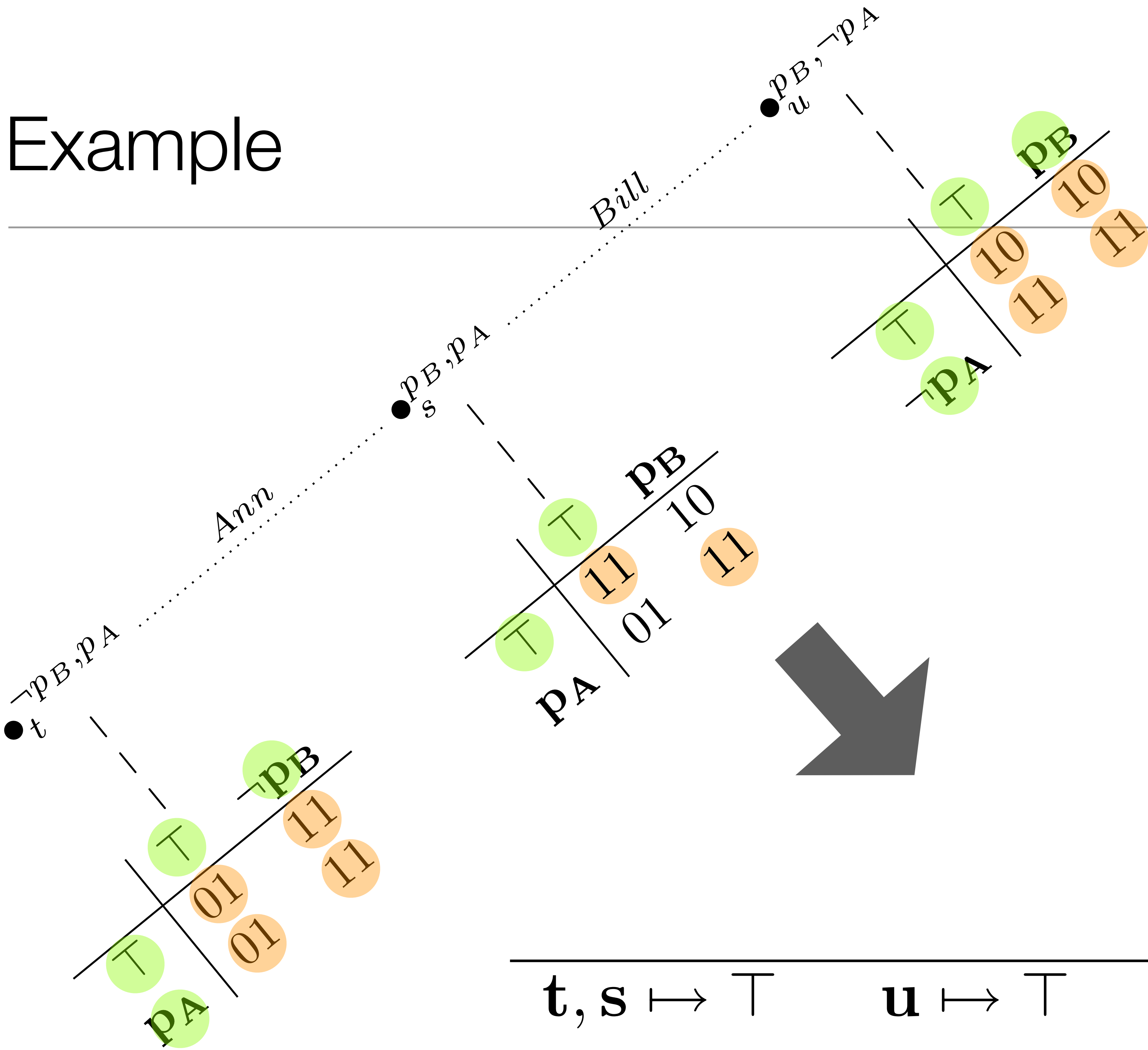
- $N = \{1, \ldots, n\}$
- A_i *is the set of functions* $a : S \to \mathcal{L}_{el}$ *with the following properties:*
  - *Truthfulness:* $M, s \models K_i a(s)$ *for any* s
  - *Uniformity:* $s \sim_i t \Rightarrow a(s) = a(t)$
- *For any state* s *in* AG, *let* $G(AG, s) = (N, \{A_i^s : i \in N\}, \{u_i^s : i \in N\})$ *be the state game associated with* s. *Let:*

$$u_i(a_1, \ldots, a_n) = \frac{\sum_{s \in S} u_i^s(a_1(s), \ldots, a_n(s))}{|S|}$$

The induced game

- A strategy is a complete plan of action for any state (even those that the agent knows are not the actual one)
  - One agent might not know which states another agent considers possible, and must therefore consider what the other agent will do in a range of circumstances
- Payoffs are computed by taking the average over all states in the model
  - Corresponds to expected payoffs computed by a common knower - someone whose knowledge is exactly what is common knowledge in the game
  - If we alternatively, e.g., computed an agent's payoff by taking the average over the set of states she considers possible, the game wouldn't be common knowledge
- It follows that the induced game is a model property rather than a pointed model property

Example



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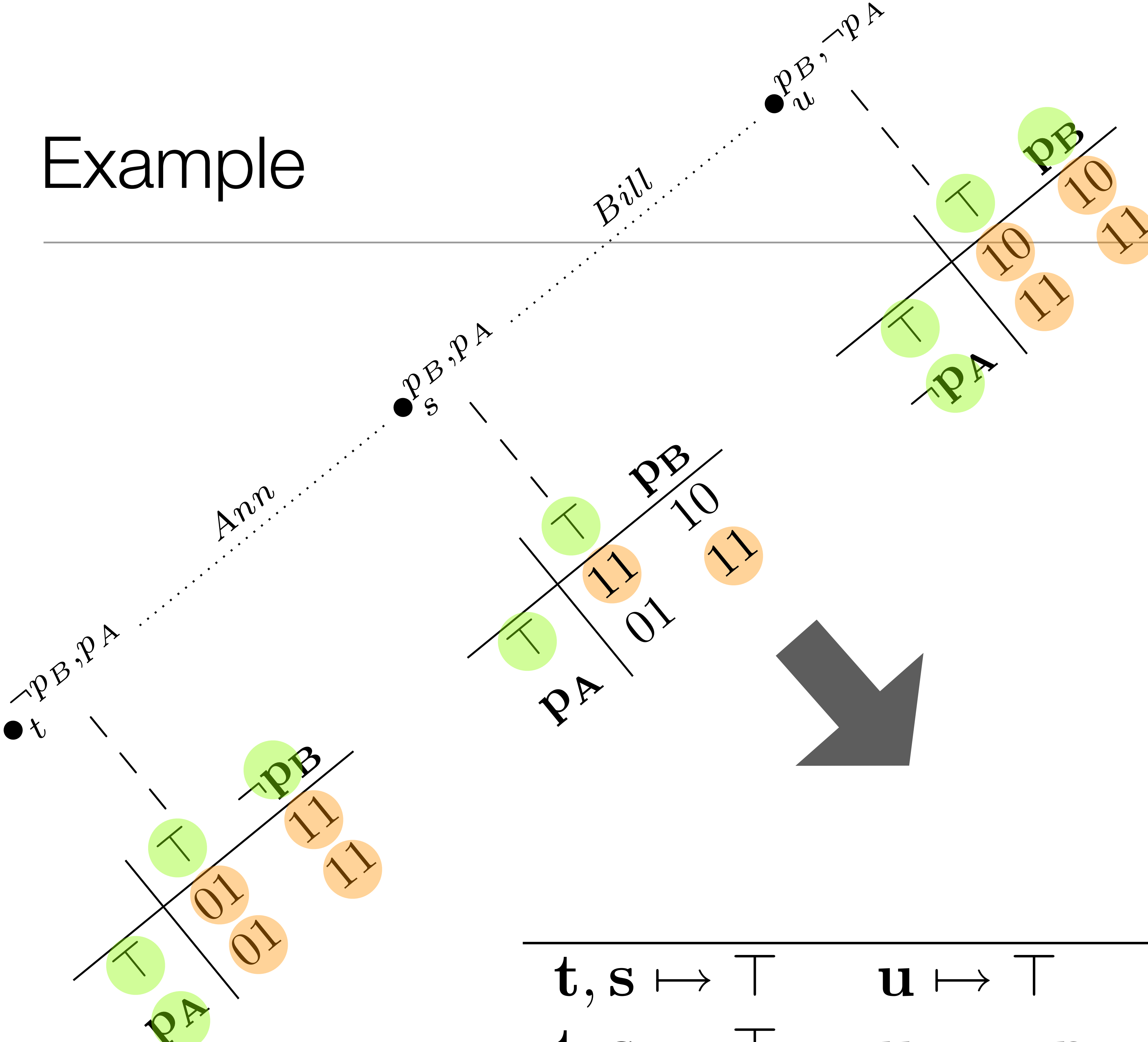
$\mathbf{t, s \mapsto p_A \quad u \mapsto \top}$	12	22	22	32
$\mathbf{t, s \mapsto p_A \quad u \mapsto \neg p_a}$	13	23	23	33

Nash Announcement Equilibrium

- Definition: a Nash Announcement Equilibrium of a Public Announcement Game is a Nash equilibrium of the induced game

Example



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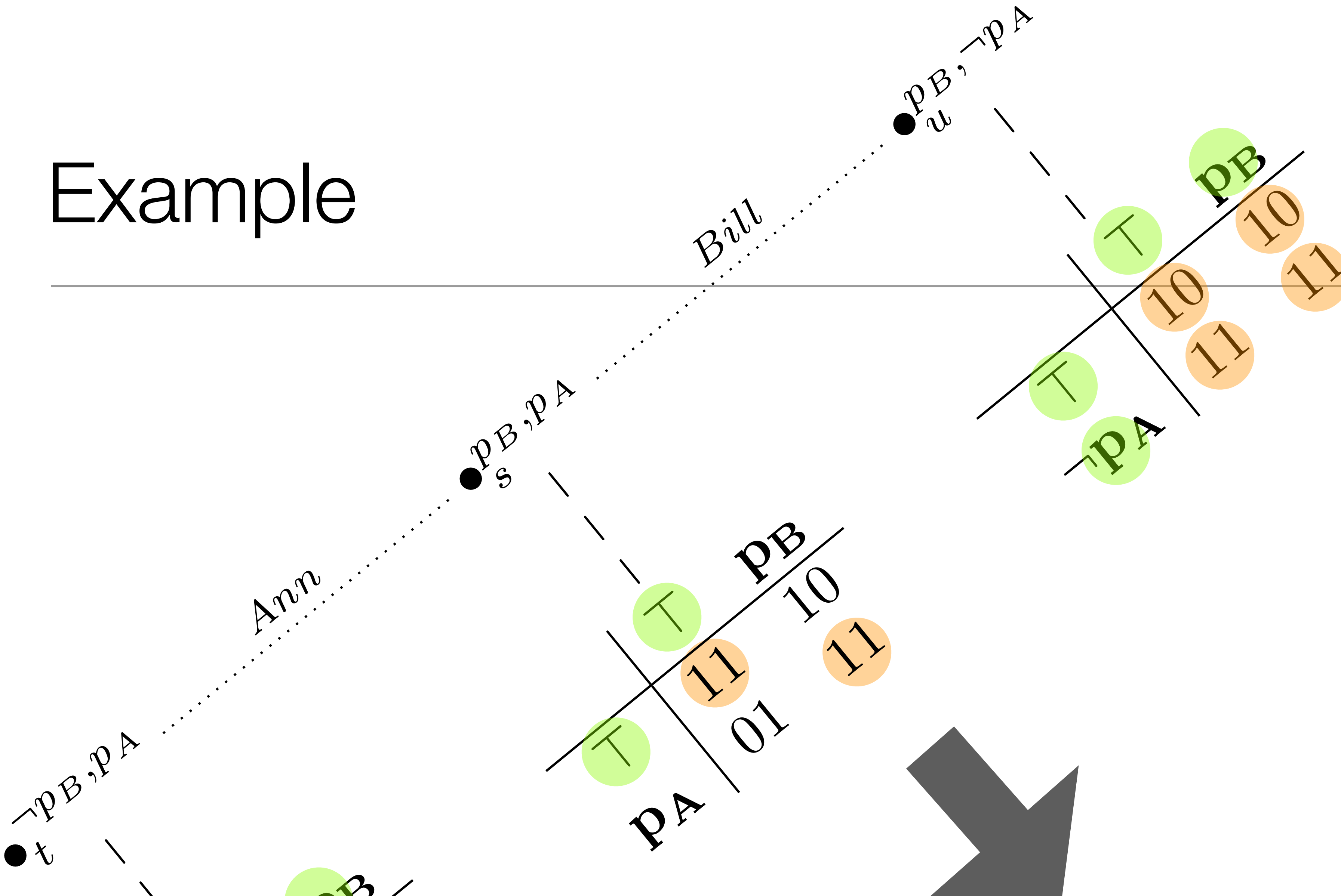

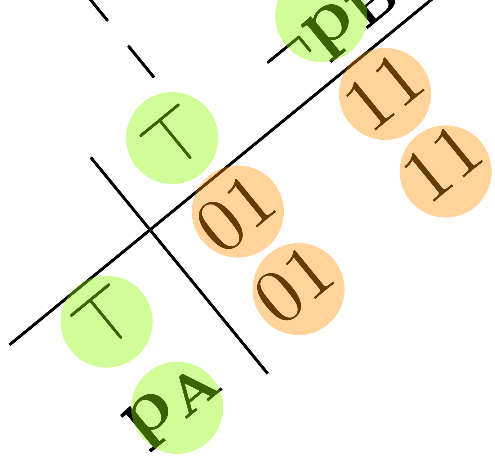

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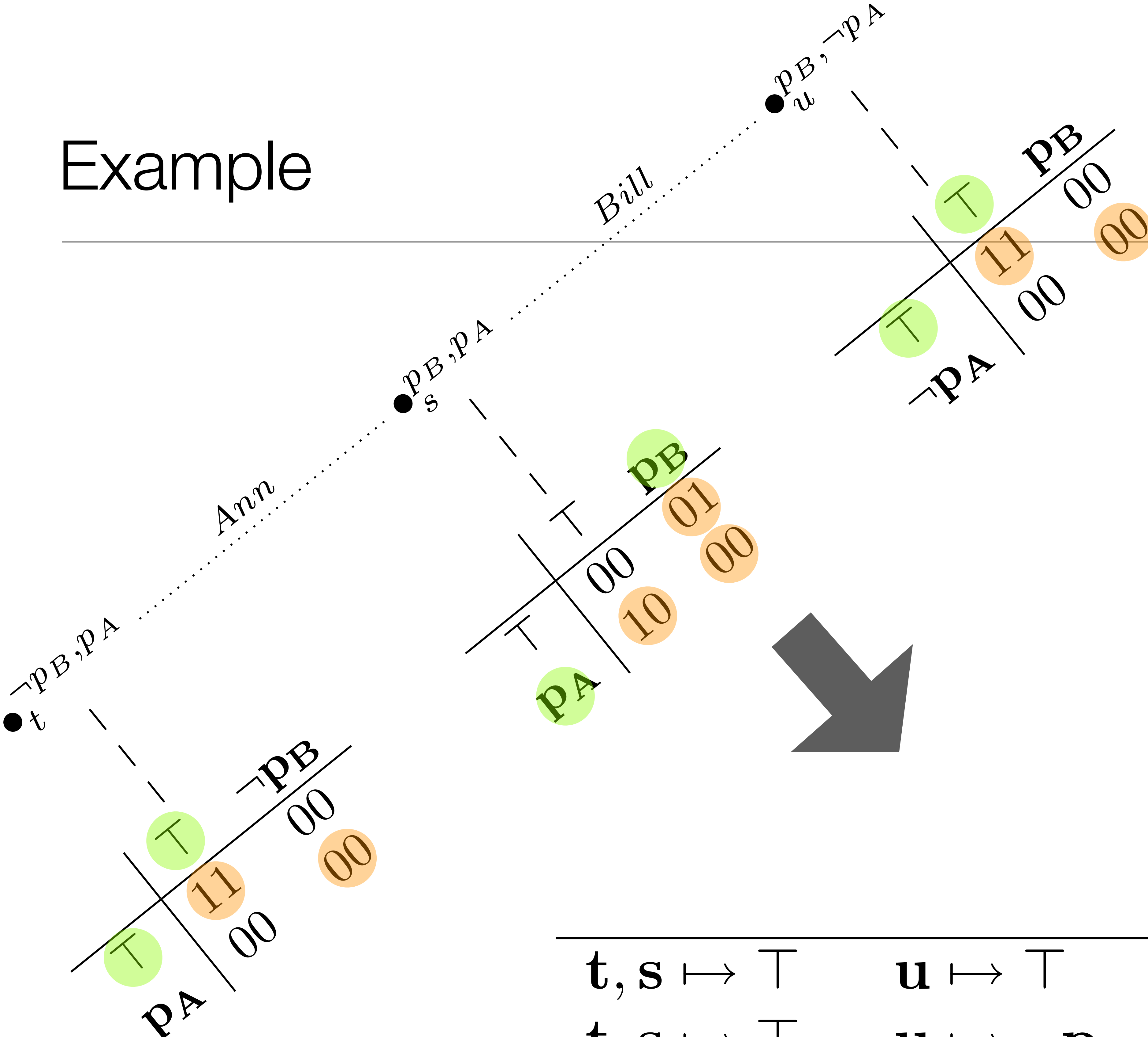

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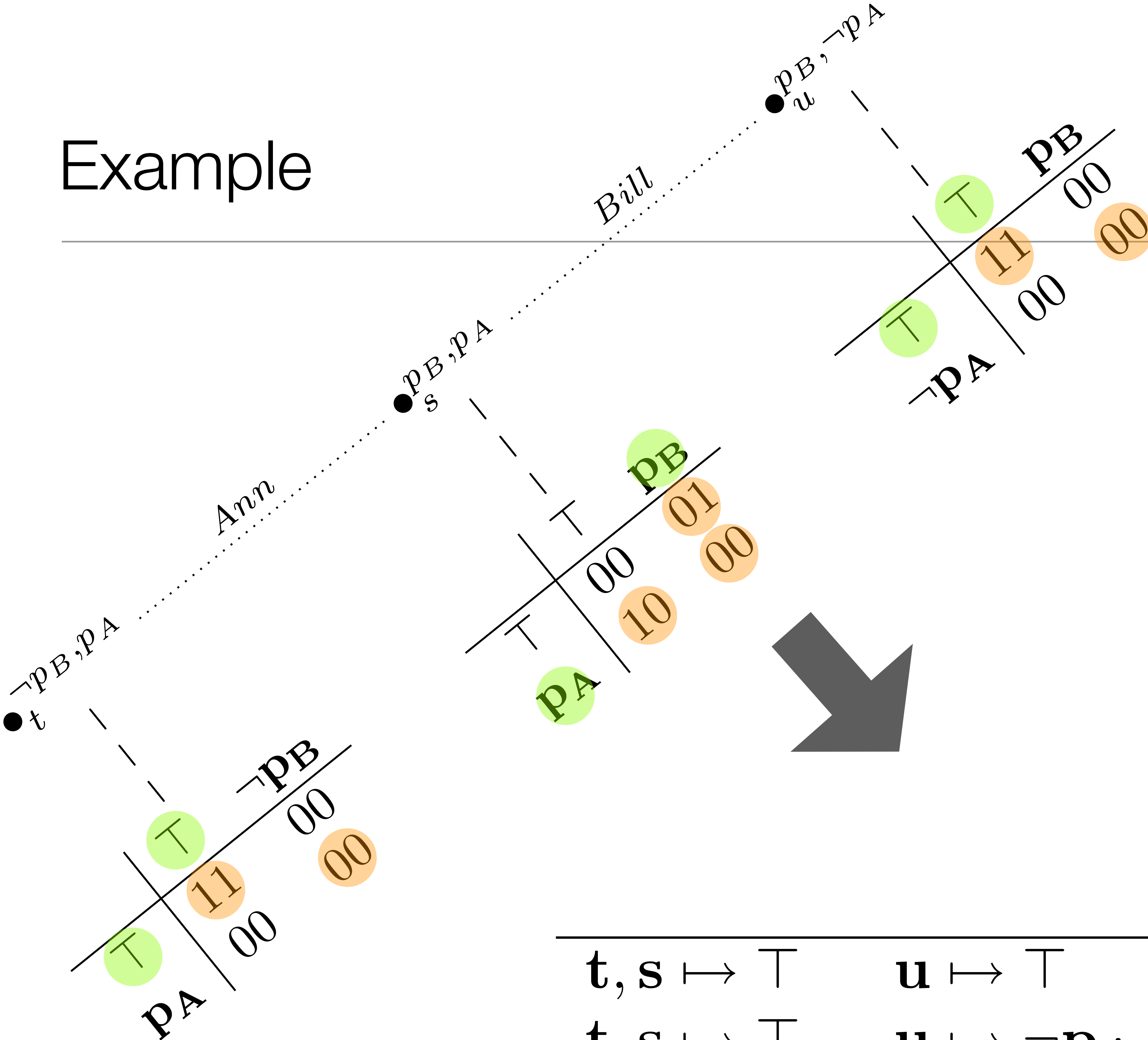
$\mathbf{t,s} \mapsto \mathbf{p_A} \quad \mathbf{u} \mapsto \neg \mathbf{p_a}$	10	10	00	00

Example



	$\mathbf{u,s} \mapsto \top$ $\mathbf{t} \mapsto \top$	$\mathbf{u,s} \mapsto \top$ $\mathbf{t} \mapsto \neg\mathbf{p_B}$	$\mathbf{u,s} \mapsto \mathbf{p_B}$ $\mathbf{t} \mapsto \top$	$\mathbf{u,s} \mapsto \mathbf{p_B}$ $\mathbf{t} \mapsto \neg\mathbf{p_B}$
$\mathbf{t,s} \mapsto \top$ $\quad \mathbf{u} \mapsto \top$	22	11	12	01
$\mathbf{t,s} \mapsto \top$ $\quad \mathbf{u} \mapsto \neg\mathbf{p_A}$	11	00	12	01
$\mathbf{t,s} \mapsto \mathbf{p_A}$ $\quad \mathbf{u} \mapsto \top$	21	21	00	00
$\mathbf{t,s} \mapsto \mathbf{p_A}$ $\quad \mathbf{u} \mapsto \neg\mathbf{p_a}$	10	10	00	00

Bayesian Games

- Nash Announcement Equilibria = Bayes-Nash equilibria of a certain class of Bayesian Games (Harsanyi)
    - Induced Public Announcement Games *are* Bayesian Games

Some properties

Theorem

If an agent has a weakly dominant strategy de re in every state of a EGS, then she has a weakly dominant strategy in the induced game.

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Theorem
If an agent has a weakly dominant strategy de re in every state of a EGS, then she has a weakly dominant strategy in the induced game.

Theorem
If an agent has a positive goal, then she has a weakly dominant strategy in the induced game.

Part II: Questions and Answers

Introduction

- *Do you have the queen of spades?*
- What is the right question?
    - Depends on: the information revealed by possible answers, your goal, the questions you think others will ask, others' goals, ...
- Besides individual decisions, scenarios that require genuine interactive rationality are very frequent not only in parlour games but also in everyday life

Motivation

- Modelling the dynamics of strategic questioning and answering
- Providing new links between game theory and dynamic logics of information
- Exploiting the dynamic/strategic structure that lies implicitly inside standard epistemic models
- Relevant earlier work:
    - Inquisitive semantics (Groenendijk, 2008)
    - Questioning dynamics by issue management (van Benthem and Minica, 2009)
    - Knowledge Games (van Ditmarsch, 2002, 2004)

Starting point

Standard pointed epistemic model:

$$(M, s)$$

$$M = (S, \sim_1, \ldots, \sim_n, V) \qquad \sim_i \text{ equivalence rel. over S}$$

What are questions, answers and games in this setting?

Questions

We model a question as a formula of standard multi-agent epistemic logic. For example:

$$K_a p?$$

is the question "does *a* know that *p*?"

Questions: pragmatic preconditions

$$K_a p?$$

It can possibly be assumed that before the question is answered:

$$\neg K_a p \wedge \neg K_a \neg p$$

$$\neg K_a \neg (K_b \vee K_b \neg p)$$

Answers

- We assume that:
  - questions are answered truthfully
  - the person questioned is obliged to answer
  - the answer is publicly announced

Answers

a asks *b*:

$$\phi?$$

Answers

a asks *b*:

$$\phi?$$

3 possible answers; the announcements:

$K_b\phi!$ ("yes!")

$K_b\neg\phi!$ ("no!")

$\neg(K_b\phi \vee K_b\neg\phi)!$ ("I don't know!")

Answers

- In dynamic epistemic logic, a public announcement is interpreted as a model restriction

- Answers can be seen as rough sets:

"yes!"		$\underline{\sim_b}([[\phi]])$
"no!"	*the actual state is in*	$\overline{\overline{\sim_b}([[\phi]])}$
"don't know!"		$\overline{\sim_b}([[\phi]]) \setminus \underline{\sim_b}([[\phi]])$

$$[[\phi]] = \{s \in S : M, s \models \phi\}$$

Answers

Let:

$$\overline{K}_i\phi = \begin{cases} K_i\phi & M, s \models K_i\phi \\ K_i\neg\phi & M, s \models K_i\neg\phi \\ \neg(K_i\phi \vee K_i\neg\phi) & \text{otherwise} \end{cases}$$

Games

- Assumptions
  - preferences are modelled as (typically epistemic) goal formulae, in the style of Boolean games
    - e.g., Ann's goal is to get to know the secret without Bill knowing it
  - each agent asks a single question, at the same time
  - 2 players

Games

Given a pointed epistemic structure M, s and goals γ_a and γ_b, we define the following *pointed question-answer game*:

- $N = \{a, b\}$
- Strategies: $A_i = \{\phi? : \phi \in \mathcal{L}\}$
- Payoffs:

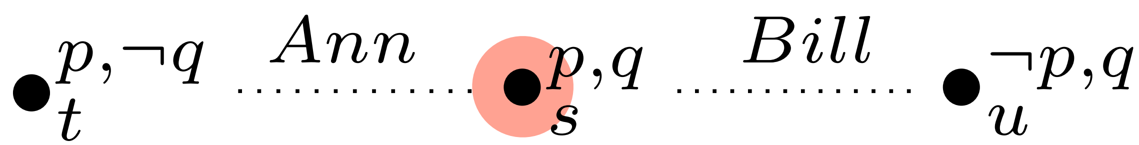
$$u_i(\langle \phi?, \psi? \rangle) = \begin{cases} 1 & M, s \models \langle \overline{K}_b \phi \wedge \overline{K}_a \psi \rangle \gamma_i \\ 0 & \text{otherwise} \end{cases}$$

Taking pragmatic preconditions into account

- This definition is easily modified for pragmatic preconditions of questions:
    - Restricting the strategy space
    - Updating not only with the answers to the questions, but also with the preconditions
- Will disregard pragmatic preconditions in the following

$A_i = \{\phi? : \phi \in \mathcal{L}\}$

When are two questions the same?



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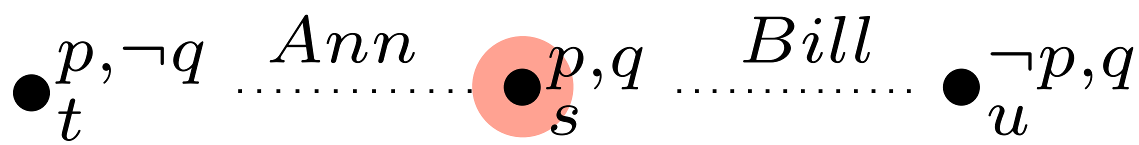

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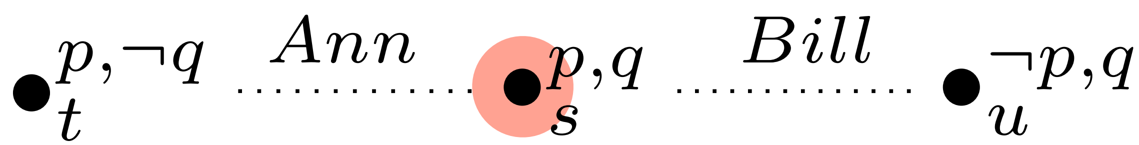

$q?$ and $q \wedge q?$ are the same question (for Ann)

$q?$ and $p?$ are the same question (for Ann)

Equivalence of questions

We say that $\phi?$ and $\psi?$ are equivalent when:

$$\{\ [[K_i\phi]],\ [[K_i\neg\phi]],\ [[\neg(K_i\phi \vee K_i\neg\phi)]]\ \}$$
$$=$$
$$\{\ [[K_i\psi]],\ [[K_i\neg\psi]],\ [[\neg(K_i\psi \vee K_i\neg\psi)]]\ \}$$

Note that it is common knowledge when two questions are equivalent

$$[[\phi]] = \{s \in S : M, s \models \phi\}$$

Dichotomous games

- We call a game dichotomous if agents can only ask questions (equivalent to) of the form ”do you know that ...”?
- Formally: every strategy for a is equivalent to a strategy of the form $K_b\phi$, and similarly for b
- Special case: restrict allowed questions to be only of this form
- This rules out the third answer alternative

$$\overline{K}_i\phi = \begin{cases} K_i\phi & M, s \models K_i\phi \\ K_i\neg\phi & M, s \models K_i\neg\phi \\ \neg(K_i\phi \vee K_i\neg\phi) & \text{otherwise} \end{cases}$$

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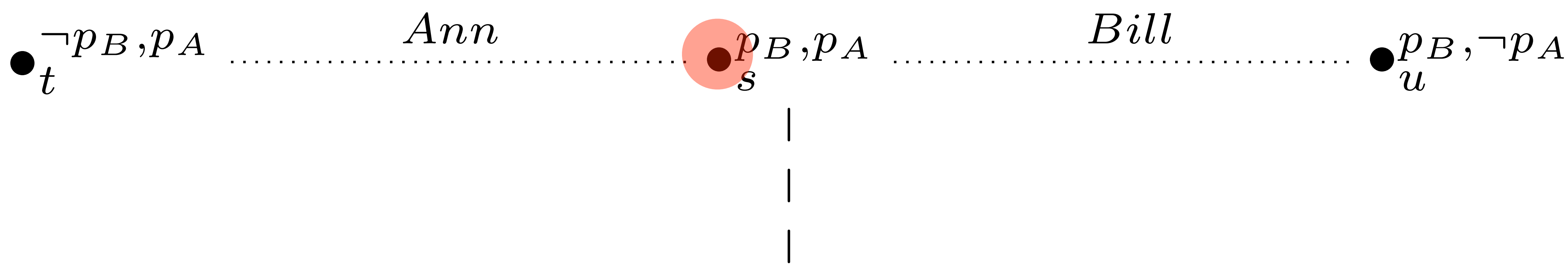
Strategies

In finite dichotomous games in bisimulation contracted structures, i has

$$2^{m_j m_i - m_i}$$

different non-equivalent questions to ask j, where m_i, m_j are the number of equivalence classes for i and j respectively

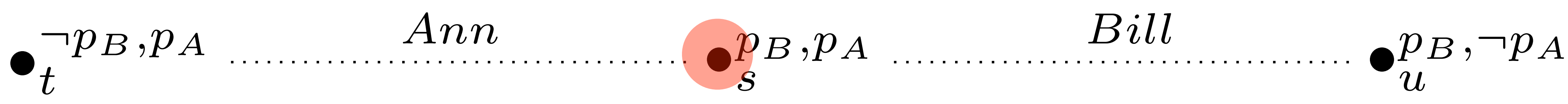
Example



	$\top?$	$\mathbf{p_A}?$
$\top?$	11	01
$\mathbf{p_B}?$	10	11

$$\gamma_{Ann} = (K_B p_A \vee K_B \neg p_A) \rightarrow (K_A p_B \vee K_A \neg p_B)$$
$$\gamma_{Bill} = (K_A p_B \vee K_A \neg p_B) \rightarrow (K_B p_A \vee K_B \neg p_A)$$

Example



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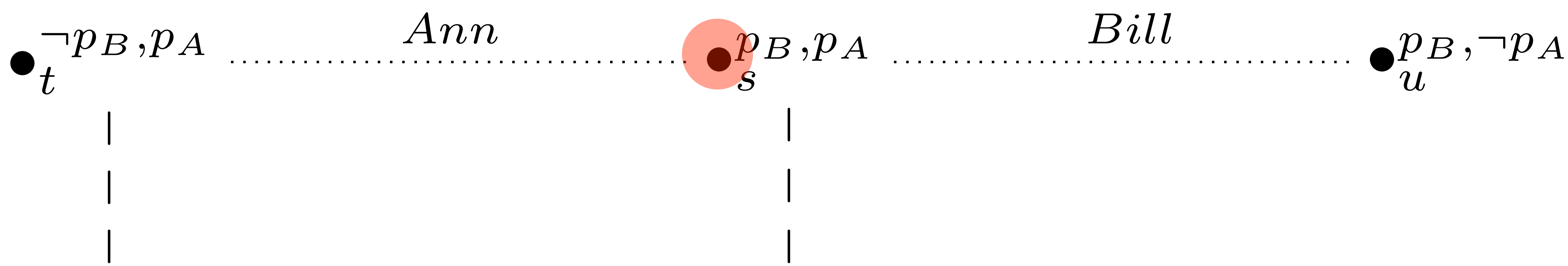
Example



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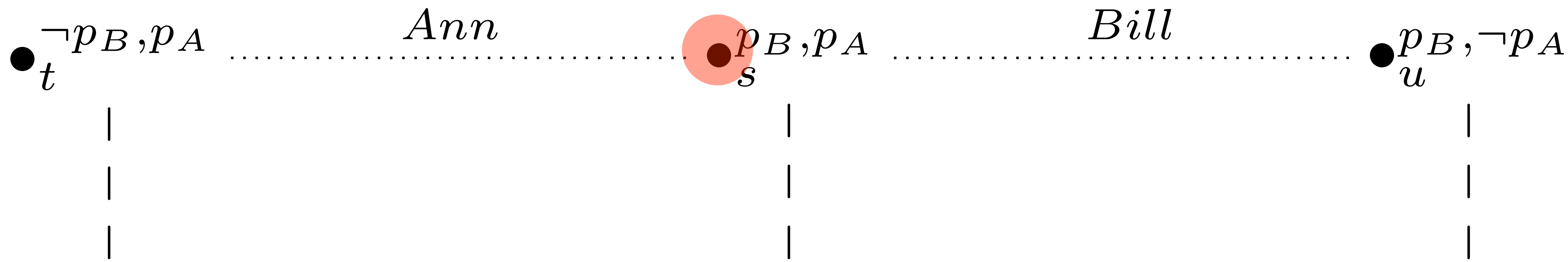

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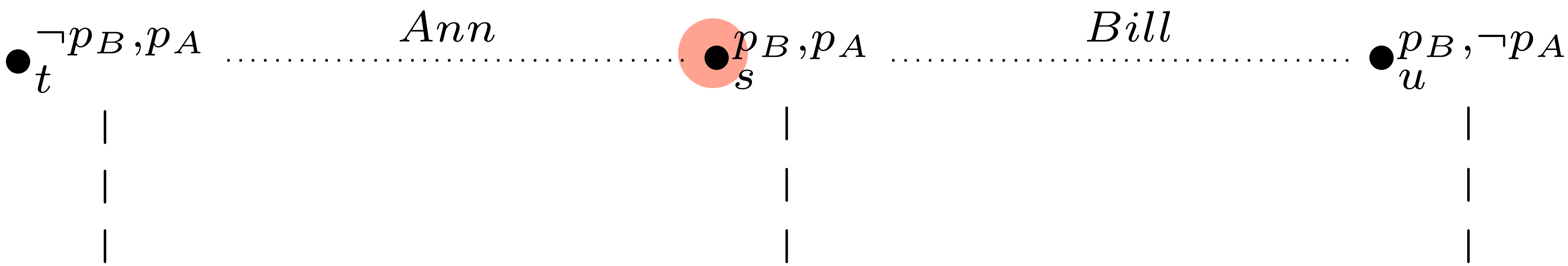
Solution concepts

- Again, intimate connection between information, strategies and payoff
- What are reasonable solution concepts?
- Let us consider some possibilities

Weakly dominant strategies

- It might be that there is a dominant question, but that the agent does not know it
- In the case that the agent knows that there is a dominant question, it might be that:
  - The agent has a weakly dominant question de dicto: there is a weakly dominant question in every state she considers possible
  - The agent has a weakly dominant question de re: there is a question which is weakly dominant in every state she considers possible

Weakly dominant strategies



	$\top?$	$\mathbf{p_A}?$
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$\top?$	10	11
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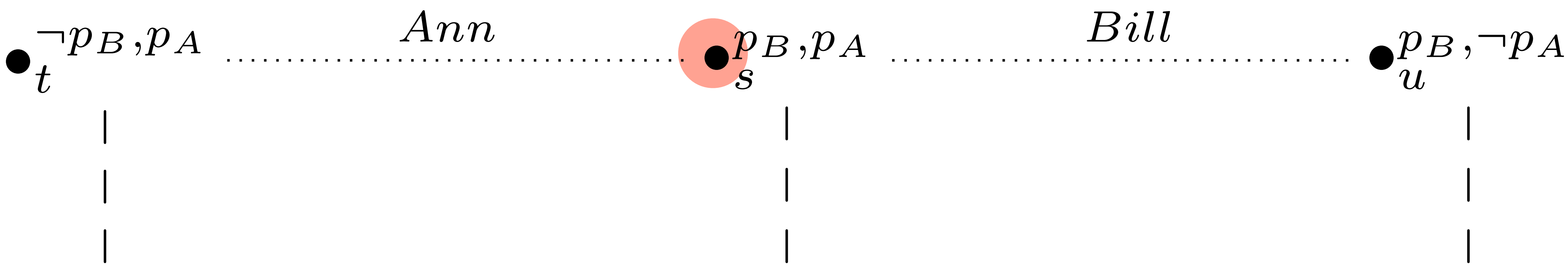
Weakly dominant strategies



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$\mathbf{p_B}$?	10	11

- Ann has a weakly dominant strategy de re (and, by implication, de dicto)

The most informative question

Proposition:

There is always a most informative question that can be asked, making the opponent reveal all she knows

If the questioner's goal is in the positive fragment, asking the most informative question is always a dominant strategy

The positive fragment: $\phi ::= p \mid \neg p \mid \phi \wedge \phi \mid \phi \vee \phi \mid K_i \phi \mid [\phi]\phi$

The most informative question

Proposition:

There is always a most informative question that can be asked, making the opponent reveal all she knows

If the questioner's goal is in the positive fragment, asking the most informative question is always a dominant strategy

Thus, if all goals are positive, there is a NE in every state

However, she may only know *de dicto* that she has a dominant strategy

The positive fragment: $\phi ::= p \mid \neg p \mid \phi \wedge \phi \mid \phi \vee \phi \mid K_i \phi \mid [\phi]\phi$

Common knowledge Nash equilibrium

$$\bullet_t^{\neg p_B, p_A} \quad \overset{Ann}{\cdots\cdots\cdots\cdots} \quad \bullet_s^{p_B, p_A} \quad \overset{Bill}{\cdots\cdots\cdots\cdots} \quad \bullet_u^{p_B, \neg p_A}$$

	$\top$?	$\mathbf{p_A}$?
$\top$?	01	01
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$\top$?	10	11
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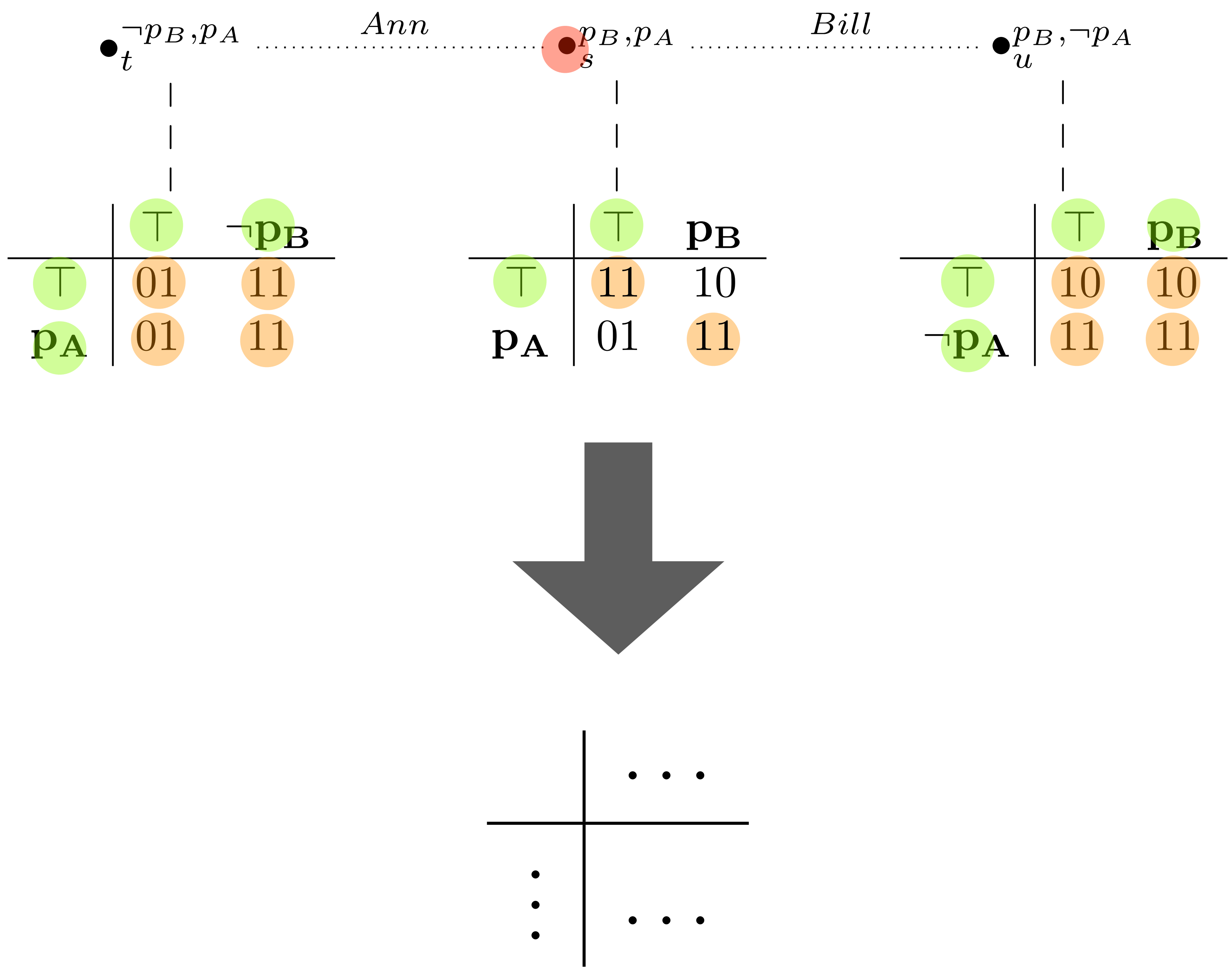
Which game are they really playing?

- Can a question-answer game be viewed as a single strategic game?



The induced game

Definition 1 *Given* M, s *with* $M = (S, \sim_1, \dots, \sim_n, V)$ *and* γ_a *and* γ_b, *the* induced game *is defined as follows:*

- $N = \{a, b\}$
- A_i *is the set of uniform functions* $a : S \to \mathcal{L}$
  - *Uniform:* $s \sim_i t \Rightarrow a(s) = a(t)$
- $$u_i(a_1, a_2) = \frac{\sum_{s \in S} u_i^s(a_a(s), a_b(s))}{|S|}$$

where u_i^s *is the payoff function in the "local" game in* s

Strategies in the induced game

Proposition *If the structure is finite, dichotomous and bisimulation contracted structures, a has*

$$2^{m_a m_b - m_a}$$

non-equivalent strategies in the induced game, where m_a *and* m_b *is the number of a- and b-equivalence classes, respectively*

Bayesian Games

- Equilibria in the induced game = Bayes-Nash equilibria of Bayesian Games (Harsanyi) under some natural assumptions
  - Induced Q-A games *are* Bayesian Games

A practical tool

- We have implemented a tool:
  - **input**: pointed epistemic model + goal formulae
  - **output**: induced game
- Based on van Eijk's DEMO model checker for DEL

Illustrations

```
*QAGM> displayS5 m78

[0,1,2,3]
[(0,[]),(1,[p]),(2,[q]),(3,[p,q])]
(a,[[0,2],[1,3]])
(b,[[0,1],[2,3]])
[0,1,2,3]

*QAGM> display 4 (qagame m78 (K a (dimp p q),K b (dimp q p)))

(0,0)(0,1)(0,1)(0,2)
(1,0)(1,1)(1,1)(1,2)
(1,0)(1,1)(1,1)(1,2)
(2,0)(2,1)(2,1)(2,2)

*QAGM> (profiles m78)!!15

[[([0,2],v[n,n1]),([1,3],v[n,n1])],[([0,1],v[n,n2]),([2,3],v[n,n2])]]
```

Illustrations

```
*QAGM> display 4 (qagame m78 (Disj [Conj [Neg (K a q),Neg (K b p)],Conj [K a (dimp p q),K b (dimp p q)]],Neg(Disj [Conj [Neg (K a q),Neg (K b p)],Conj [K a (dimp p q),K b (dimp p q)]])))

(4,0)(3,1)(3,1)(2,2)
(3,1)(4,0)(2,2)(3,1)
(3,1)(2,2)(2,2)(1,3)
(2,2)(3,1)(1,3)(2,2)
```

Question-and-answer games: further research

- model theory and axioms for appropriate logics describing our games; including issues like bisimulation invariance and fixed-point definability;
- extensive games with longer sequences of moves;
- a richer account of questions as possible moves of inquiry;
- connections with existing logics of inquiry and learning;
- non-uniform probability distributions;
- structured goals for agents, ordered goal-sets, etc.

Time to wrap up

Announcement Games: current and future work

- Sequential announcements, extensive form games
- Coalitional games
- More sophisticated goal models
- More sophisticated DELs
- Relation to argumentation theory?
- **Lying games**

Coming soon...


a lost twin
a dark secret
a deadly game
FROM THE CREATOR OF PRETTY LITTLE LIARS
THE LYING GAME

For more details:

- T. Ågotnes and H. van Ditmarsch, *What will they say? - Public Announcement Games*, Synthese 179(1), 2011.
- T. Ågotnes, J. van Benthem, H. van Ditmarsch and S. Minica, *Question-answer games*, to appear in Journal of Applied Non-Classical Logic
- T. Ågotnes, P. Balbiani, H. van Ditmarsch and P. Seban, *Group Announcement Logic*, Journal of Applied Logic **8**(1), 2010

Any ϕ?