# Qualitative constraints representation for the time and space in SAT

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## Abstract

*In this paper we consider the consistency problem of temporal or spatial qualitive constraint networks. A new encoding making it possible to represent and solve this problem in the framework of the propositional logic is proposed. The definition of this encoding presupposes the existence of a particular order on the basic relations of the qualitative calculus such as that of the conceptual lattice of the interval algebra of Allen.*

## 1 Introduction

Reasoning about temporal or spatial information is an important task in many applications of various domains of Artificial Intelligence. Many formalisms, called qualitative calculi, have been developped these last years. A qualitative calculus [1, 3, 8] uses particular elements (subsets of a topological space, points of a line, intervals of a line, rectangles of a plan, tuples of points ...) for representing the spatial or temporal entities of the system and considers relations between these elements. Each one of these relations corresponds to a particular temporal or spatial situation and does not consider metric aspects. Constraint networks called qualitative constraint networks (QCN in short) can be used to represent constraints concerning the relative positions of a set of temporal or spatial entities. Each constraint of a QCN is defined by a set of basic relations allowed for the relative position of the concerned entities. Given a QCN, the main problem which we must consider is the consistency problem. To solve efficiently this problem, search methods using the method of closure by weak composition as method of local propagation of constraints on the one hand, and a decomposition of the constraints in relations of a class known as tractable on the other hand, have been defined [4].

Other works have also been devoted to the representation of the QCN in propositional logic encoding. The QCN consistency problem becomes a satisfiability problem. We can cite the encoding given by Nebel and Bürckert [5] allowing to represent each constraint of the interval algebra as a SAT clause. Roughly, this set of SAT clauses symbolize a linear order corresponding to the interval bounds configurations of the system. Other translations, more generic and adaptative to the frameworks using any qualitative calculus have been recently defined and studied [7]. One of these encodings consists in the SAT representation of all the possible combinations of basic relations between each triplet of variables. These possible combinations are provided by the composition table. Another approach consists in the characterization of all banned combinations. The main interest of such encodings is the use of SAT solvers to solve efficiently the QCN consistency problem.

We can notice that there do not exist in the litterature SAT encodings using definition of tractable classes. We propose, in this paper, an encoding that takes advantage of the convex relations definition. More precisely, the proposed SAT representation uses a lattice defined on basic relations having similar features with the lattice Allen's formalism. Using this new encoding, the QCN whose constraints are defined by the intervals of such a lattice are represented using a set of Horn clauses.

This article is organized as follows. After a short technical background about qualitative formalisms, we give the definition of our encoding allowing to translate a QCN in propositionnal logic in Section 3. In Section 4, we study this encoding. Finally, we conclude with some perspectives and future works.

## 2 Qualitative formalisms overview

In the sequel, we assume a given qualitative formalism for the time and the space. This formalism is defined on a finite set B of binary basic relations on the domain D. We make the hypothesis that the basic relations are jointly exhaustive and pairwise disjoint. For an illustration, consider the well-known Allen's calculus [1]: the interval algebra (AI). AI is based on 13 binary relations defined on a set of intervals (see Figure 1). Each basic relation corresponds to a configuration between the 4 bounds of two intervals.

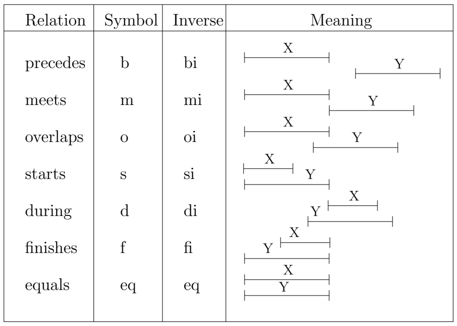


**Figure 1. The basic relations of** $\mathsf{AI}$

We denote by $\mathsf{A}$, the set $2^{\mathsf{B}}$ of all the sets built on $\mathsf{B}$. For $r \in \mathsf{A}$, two entities $x, y \in \mathsf{D}$ satisfy $r$, denote by $x\ r\ y$, iff there exists a basic relation $a \in r$ such as $(x, y) \in a$. Thereby, each element $r$ in $\mathsf{A}$ may be represented as the union of all the basic relations that compose it. We will use the term of "relation" to name the union of basic relations. The set $\mathsf{A}$ is equipped with the intersection ($\cap$), the union ($\cup$) and the weak composition ($\circ$) operations. It is also fitted up with the unary converse operation ($^{-1}$). The temporal or spatial information on the configuration between a set of entities may be represented using a constraints network called qualitative constraints network ($\mathsf{QCN}$). Formally, a $\mathsf{QCN}$ $\mathcal{N}$ is described as a pair $(V, C)$ where $V$ is a finite set of variables $v_0, \ldots, v_{n-1}$ (with $n$ a positive integer) and $C$ is a map that, for each pair $(v_i, v_j)$ of variables in $V$, assigns a subset $C(v_i, v_j)$ of basic relations, such as $C(v_i, v_j) \subseteq \mathsf{B}$ ($C(v_i, v_j) \in \mathsf{A}$). Regarding the $\mathsf{QCN}$, we will use the following definitions:

**Definition 1.** *Let $\mathcal{N} = (V, C)$ be a* $\mathsf{QCN}$*, with $V = \{v_0, \ldots, v_{n-1}\}$. A* partial solution *of $\mathcal{N}$ on $V' \subseteq V$ is a map $\sigma$ from $V'$ to $\mathsf{D}$ such as $\sigma(v_i)\ C(v_i, v_j)\ \sigma(v_j)$, for all $v_i, v_j \in V'$. A* solution *of $\mathcal{N}$ is a partial solution on $V$. $\mathcal{N}$ is* consistent *if and only if there exists a solution. $\mathcal{N}$ is* $\circ$-closed *(weak composition closed) if and only if for all $v_k, v_i, v_j \in V$, $C(v_i, v_j) \subseteq C(v_i, v_k) \circ C(v_k, v_j)$ and $C(v_i, v_j) \neq \emptyset$ (note that we could do a restriction on the triplets $v_k, v_i, v_j \in V$ with $i < j$). A* sub-$\mathsf{QCN}$ $\mathcal{N}'$ *of $\mathcal{N}$ is a* $\mathsf{QCN}$ *$(V, C')$ where $C'(v_i, v_j) \subseteq C(v_i, v_j)$ for all $v_i, v_j \in V$. The notation $\mathcal{N}' \subseteq \mathcal{N}$ will denote that $\mathcal{N}'$ is a sub-*$\mathsf{QCN}$ *of $\mathcal{N}$. A* scenario *of $\mathcal{N}$ is a* sub-$\mathsf{QCN}$ *$(V, C')$ of $\mathcal{N}$ such as $C'(v_i, v_j) = \{a\}$ with $a \in \mathsf{B}$. A* consistent scenario *of $\mathcal{N}$ is a scenario of $\mathcal{N}$ which admits a solution. $\mathcal{N}' = (V, C')$ and $\mathcal{N}$ are* equivalent *if and only if the two* $\mathsf{QCN}$ *have the same solutions.*

## 3 An encoding from $\mathsf{QCN}$ to $\mathsf{SAT}$

In this section, we describe and we study an encoding to represent a $\mathsf{QCN}$ in a set of propositionnal clauses. To define this encoding, we assume defined a partial order $\preceq$ on $\mathsf{B}$ satisfying some properties. First of all, $(B, \preceq)$ must be a lattice. Hence for all $a, b, c \in \mathsf{B}$ we have: (1) $a \preceq a$, (2) if $a \preceq c$ and $c \preceq b$ then $a \preceq b$, (3) if $a \preceq b$ and $b \preceq a$ then $a = b$, (4) there exists two elements in $\mathsf{B}$ matching with $\mathsf{Inf}\{a, b\}$ and $\mathsf{Sup}\{a, b\}$. Let two basic relations $a, b \in \mathsf{B}$, the interval $[a, b]$ will represent the relation in $\mathsf{A}$ defined by $\{c \in \mathsf{B} : a \preceq c \text{ and } c \preceq b\}$. $\mathcal{C}_{\preceq}$ will denote the subset of $\mathsf{A}$ corresponding to the intervals of $(\mathsf{B}, \preceq)$. Formally, $\mathcal{C}_{\preceq} = \{[a, b] : a, b \in \mathsf{B}\}$. Later in this paper, we call by simple $\mathsf{QCN}$ ($\mathsf{SQCN}$), all $\mathsf{QCN} = (V, C)$ for which each constraint $C_{ij}$ is defined by a relation $[C^-_{ij}, C^+_{ij}]$ in $\mathcal{C}_{\preceq}$.
Regarding $(\mathsf{B}, \preceq)$, we suppose that the following properties for the operations of inverse and weak composition hold:
- for all $a, b \in \mathsf{B}$, if $a \preceq b$ then $b^{-1} \preceq a^{-1}$ ;
- for all $a, b, c, d \in \mathsf{B}$, $[a, b] \circ [c, d] = [\mathsf{Inf}(a \circ c), \mathsf{Sup}(b \circ d)]$.

For many qualitative calculi [6, 5, 3, 2, 8], such lattice exists, and it is sometimes called qualitative or conceptual lattice, whereas the relations of $\mathcal{C}_{\preceq}$ are called convex relations. The conceptual lattice for the interval algebra is represented in Figure 2 and a $\mathsf{SQCN}$ defined on this lattice may also be seen in the same figure. Note that the constraint $C_{ij}$ is not represented when $C_{ji}$ is already present and when the relation corresponds to the total relation (*i.e.* $\mathsf{B}$) or if indexes $i$ and $j$ are equals. Now, it is possible to define our encoding

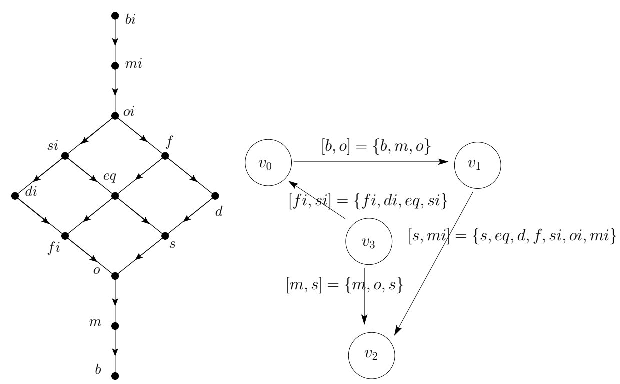


**Figure 2. The conceptual lattice** $(\mathsf{B}, \preceq)$ **for** $\mathsf{AI}$ **and a consistent** $\mathsf{SQCN}$

$\mathsf{Sat}$ representing a $\mathsf{QCN}$ with a set of clauses. These clauses represent properties linking the lattice $(\mathsf{B}, \preceq)$ and the basic relations which must be satisfied.

**Definition 2.** *Let $\mathcal{N} = (V, C)$ be a* $\mathsf{SQCN}$ *and $n = |V|$. Using the set of propositions $\{C_{ij} \preceq a$ with $a \in \mathsf{B}$ and $i, j \in \{0, \ldots, n-1\}\} \cup \{a \preceq C_{ij}$ with $a \in \mathsf{B}$ and $i, j \in \{0, \ldots, n-1\}\}$, we define $\mathsf{Sat}(\mathcal{N})$ by the following set of clauses:*
- *for each constraint $C_{ij} = [a_{ij}, b_{ij}]$ with $i, j \in \{0, \ldots, n-1\}$, two unit clauses bounding the possible basic*

*relations are introduced:*

$$a_{ij} \preceq C_{ij} \text{ and } C_{ij} \preceq b_{ij} \quad (I)$$

• *We also introduce some clauses describing a property on the infima and suprema of $\preceq$ for all $a, b \in \mathsf{B}$:*

$$\neg(a \preceq C_{ij}) \vee \neg(b \preceq C_{ij}) \vee \mathsf{Sup}\{a, b\} \preceq C_{ij} \quad (II\ a)$$

$$\neg(C_{ij} \preceq a) \vee \neg(C_{ij} \preceq b) \vee C_{ij} \preceq \mathsf{Inf}\{a, b\} \quad (II\ b)$$

• *Some clauses describing the property of transitivity of $\preceq$ are introduced for all $a, b \in \mathsf{B}$ such as $a \not\preceq b$:*

$$\neg(a \preceq C_{ij}) \vee \neg(C_{ij} \preceq b) \quad (III)$$

• *Two clauses for the converse operation are introduced for all $a \in \mathsf{B}$:*

$$\neg(a \preceq C_{ij}) \vee C_{ji} \preceq a^{-1}, \neg(C_{ij} \preceq a) \vee a^{-1} \preceq C_{ji} \quad (IV)$$

• *Finally, we introduced, for each triplet of constraints $C_{ik}, C_{kj}, C_{ij}$, with $i, j, k \in \{0, \ldots, n-1\}$ and $i < j$, two clauses resulting from the weak composition property:*

$$\neg(a \preceq C_{ik}) \vee \neg(b \preceq C_{kj}) \vee \mathsf{Inf}(a \circ b) \preceq C_{ij} \quad (Va)$$

$$\neg(C_{ik} \preceq a) \vee \neg(C_{kj} \preceq b) \vee C_{ij} \preceq \mathsf{Sup}(a \circ b) \quad (Vb)$$

Note that $\mathsf{Sat}(\mathcal{N})$ contains only Horn clauses. Consequently, it is possible to answer to the satisfiability of $\mathcal{N}$ in polynomial time. Moreover, we remark that the encodings of two SQCN build on an unique qualitative calculus and on a same set of variables differ only for the set of clauses $(I)$.

Consider the SQCN represented in Figure 2. Its encoding into SAT contains, for example, the clauses $eq \preceq C_{00}$, $C_{00} \preceq eq$, $b \preceq C_{01}$, $C_{01} \preceq o$, $oi \preceq C_{10}$, $C_{10} \preceq bi$, $b \preceq C_{02}$, $C_{02} \preceq bi$, $b \preceq C_{20}$, $C_{20} \preceq bi$ for the clauses $(I)$. The set of clauses $(II)$ will contain, for example, $\neg(fi \preceq C_{01}) \vee \neg(d \preceq C_{01}) \vee f \preceq C_{01}$ since $f = \mathsf{Sup}\{fi, d\}$. In the set $(III)$ and $(IV)$, we will obtain some clauses whereof $\neg(di \preceq C_{01}) \vee \neg(C_{01} \preceq eq)$ and $\neg(f \preceq C_{10}) \vee C_{01} \preceq fi$ respectively. Since $m \circ f = \{o, s, d\} = [o, d]$, the clauses $\neg(m \preceq C_{01}) \vee \neg(f \preceq C_{12}) \vee o \preceq C_{02}$ and $\neg(C_{01} \preceq m) \vee \neg(C_{12} \preceq f) \vee C_{02} \preceq d$ belong to the clauses $(V)$.

Previously, we have proposed an encoding from SQCN into propositional logic. This encoding may easily be generalised to the general QCN case. Indeed, since a constraint defined by a relation $C_{ij} \in \mathsf{A}$, we can always split $C_{ij}$ in a set of $k$ relations $\{C^1_{ij}, \ldots, C^{k_{ij}}_{ij}\}$, with $k_{ij}$ an integer smaller than $|\mathsf{B}|$, such as $C_{ij} = \bigcup_{l \in 1,\ldots,k_{ij}} C^l_{ij}$ and $C^l_{ij}$ is a relation in $\mathcal{C}_{\preceq}$ for all $l \in \{1, \ldots, k_{ij}\}$. Consequently, $C^l_{ij} = [a^1_{ij}, b^1_{ij}] \cup \ldots \cup [a^{k_{ij}}_{ij}, b^{k_{ij}}_{ij}]$ with $a^1_{ij}, \ldots, a^{k_{ij}}_{ij}, b^1_{ij}, \ldots, b^{k_{ij}}_{ij} \in \mathsf{B}$. The description of such constraints will be performed using a substitution of the two unit clauses introduced in $(I)$ by a set of clauses corresponding to the formula $(a^1_{ij} \preceq C_{ij} \wedge C_{ij} \preceq b^1_{ij}) \vee \ldots \vee (a^{k_{ij}}_{ij} \preceq C_{ij} \wedge C_{ij} \preceq b^{k_{ij}}_{ij})$. For example, assume that for the QCN in Figure 2, the constraint $C_{02}$ is not defined by the total relation but by the relation $\{m, o, s, eq, f, d\}$. Since we have $C_{02} = \{m, o, s, eq, f, d\} = \{m, o\} \cup \{s, eq, f, d\} = [m, o] \cup [s, d]$, we can split this constraint like $(m \preceq C_{02} \wedge C_{02} \preceq o) \vee (s \preceq C_{02} \wedge C_{02} \preceq d)$ and so we obtain the clauses $m \preceq C_{02} \vee s \preceq C_{02}$, $m \preceq C_{02} \vee C_{02} \preceq d$ and $C_{02} \preceq o \vee s \preceq C_{02}$, $C_{02} \preceq o \vee C_{02} \preceq d$.

Remark that, using the previous definition, all the clauses are not Horn clauses. We can also remark that, in the case of the Allen's calculus, the number of relations of $\mathcal{C}_{\preceq}$ used to the spliting is in average of 3.55 [4]. In the general case, the spliting of any relation of A into relations of $\mathcal{C}_{\preceq}$ is not unique. In the sequel, we will discard this non-deterministic case fixing an unique parsing for each relation $r \in \mathsf{A}$.

## 4 Encoding Sat completeness

In this section, we prove that the satisfiability of the obtained set of clauses from the encoding Sat applied to a QCN allows to determine the consistency or not of this QCN. Before that, let's prove a property about the lattice $(\mathsf{B}, \preceq)$ that will be helpful in the sequel:

**Proposition 1.** *Let $a, b, c, d \in \mathsf{B}$. If $a \preceq c$ and $b \preceq d$ then $\mathsf{Inf}(a \circ b) \preceq \mathsf{Inf}(c \circ d)$ and $\mathsf{Sup}(a \circ b) \preceq \mathsf{Sup}(c \circ d)$.*

*Proof.* Consider the relations $[a, \mathsf{Sup}(\mathsf{B})]$, $[b, \mathsf{Sup}(\mathsf{B})]$, $[c, \mathsf{Sup}(\mathsf{B})]$ et $[d, \mathsf{Sup}(\mathsf{B})]$. Since $a \preceq c$ and $b \preceq d$ we have $[c, \mathsf{Sup}(\mathsf{B})] \subseteq [a, \mathsf{Sup}(\mathsf{B})]$ and $[d, \mathsf{Sup}(\mathsf{B})] \subseteq [b, \mathsf{Sup}(\mathsf{B})]$. Consequently, $[c, \mathsf{Sup}(\mathsf{B})] \circ [d, \mathsf{Sup}(\mathsf{B})] \subseteq [a, \mathsf{Sup}(\mathsf{B})] \circ [b, \mathsf{Sup}(\mathsf{B})]$. As $[c, \mathsf{Sup}(\mathsf{B})] \circ [d, \mathsf{Sup}(\mathsf{B})] = [\mathsf{Inf}(c \circ d), \mathsf{Sup}(\mathsf{Sup}(\mathsf{B}) \circ \mathsf{Sup}(\mathsf{B}))]$ and $[a, \mathsf{Sup}(\mathsf{B})] \circ [b, \mathsf{Sup}(\mathsf{B})] = [\mathsf{Inf}(a \circ b), \mathsf{Sup}(\mathsf{Sup}(\mathsf{B}) \circ \mathsf{Sup}(\mathsf{B}))]$, it follows that $\mathsf{Inf}(a \circ b) \preceq \mathsf{Inf}(c \circ d)$. Using a similar reasoning, we can demonstrate that $\mathsf{Sup}(a \circ b) \preceq \mathsf{Sup}(c \circ d)$. ⊣

Now, let's prove that an encoded weak composition closed SQCN is satisfiable:

**Proposition 2.** *Let $\mathcal{N} = (V, C)$ be a SQCN defined on $(\mathsf{B}, \preceq)$. If $\mathcal{N}$ admits a $\circ$-closed scenario then $\mathsf{Sat}(\mathcal{N})$ is satisfiable.*

*Proof.* Let $\mathcal{S}$ be a $\circ$-closed scenario of $\mathcal{N}$. Let's note $s_{ij}$ the corresponding basic relation to the constraint between the variables $v_i$ and $v_j$ in $\mathcal{S}$ for all $i, j \in \{0, \ldots, n-1\}$ with $n = |V|$. We will define an interpretation $I$ of $\mathsf{Sat}(\mathcal{N})$ as, for each constraint $C_{ij}$ and each relation $a \in \mathsf{B}$: if $a \preceq s_{ij}$ then $I(a \preceq C_{ij}) = true$, $I(a \preceq C_{ij}) = false$ otherwise ; if $s_{ij} \preceq a$ then $I(C_{ij} \preceq a) = true$, $I(C_{ij} \preceq a) = false$ otherwise. As $\mathcal{S}$ is a scenario of $\mathcal{N}$, we have $a_{ij} \preceq s_{ij} \preceq b_{ij}$, with $C_{ij} = [a_{ij}, b_{ij}]$. If follows that $I$ satisfies the clauses $(I)$. $s_{ij} \in \mathsf{B}$ and $(\mathsf{B}, \preceq)$ is a lattice, it results that the clauses $(II)$ and $(III)$ are satisfied. $s_{ij} = s_{ji}^{-1}$ and we know that if $a \preceq b$ then $b^{-1} \preceq a^{-1}$. Consequently, the clauses $(IV)$ are satisfied using $I$. Since $\mathcal{S}$ is $\circ$-closed, we have $s_{ij} \in [s_{ik}, s_{ik}] \circ [s_{kj}, s_{kj}]$ for all $i, j, k \in \{0, \ldots, n-1\}$. Thus, $\mathsf{Inf}(s_{ik} \circ s_{kj}) \preceq s_{ij} \preceq \mathsf{Sup}(s_{ik} \circ s_{kj})$. For all $a, b \in \mathsf{B}$, if $a \preceq s_{ik}$ and $b \preceq s_{kj}$ then $\mathsf{Inf}(a \circ b) \preceq \mathsf{Inf}(s_{ik} \circ s_{kj})$ (Prop. 1). It follows that $\mathsf{Inf}(a \circ b) \preceq s_{ij}$. Also, for all $a, b \in \mathsf{B}$, if $s_{ik} \preceq a$ and $s_{kj} \preceq b$ then $\mathsf{Sup}(s_{ik} \circ s_{kj}) \preceq \mathsf{Sup}(a \circ b)$ (Prop. 1). Finally, we have $s_{ij} \preceq \mathsf{Sup}(a \circ b)$ and the clauses $(V)$ are satisfied. Therefore, we can conclude that $I$ is a model of $\mathsf{Sat}(\mathcal{N})$. $\dashv$

From an interpretation satisfying $\mathsf{Sat}(\mathcal{N})$, we can define a $\circ$-closed sub-QCN of $\mathcal{N}$ defined on $\mathcal{C}_{\preceq}$. That SQCN is formally defined as follows:

**Definition 3.** *Let $\mathcal{N} = (V, C)$ be a* SQCN *and $I$ a model satisfying* $\mathsf{Sat}(\mathcal{N})$. $\mathsf{sqcn}(\mathsf{Sat}(\mathcal{N}))$ *is the* SQCN $(V, C')$ *defined by $C'_{ij} = [l_{ij}, u_{ij}]$, with $l_{ij} = \mathsf{Sup}\{a \in \mathsf{B} : I(a \preceq C_{ij}) = true\}$ and $u_{ij} = \mathsf{Inf}\{a \in \mathsf{B} : I(C_{ij} \preceq a) = true\}$, for all $i, j \in \{0, \ldots, |V|-1\}$.*

This SQCN ownes as well the following features:

**Proposition 3.** *Let $\mathcal{N} = (V, C)$ be a* SQCN *and $I$ a model satisfying* $\mathsf{Sat}(\mathcal{N})$. *We have:* $(a)$ $\mathsf{sqcn}(\mathsf{Sat}(\mathcal{N})) \subseteq N$, $(b)$ $\mathsf{sqcn}(\mathsf{Sat}(\mathcal{N}))$ *is a $\circ$-closed* QCN.

*Proof.* $(a)$ Let us show that, for all $i, j \in \{0, \ldots, n-1\}$ with $n = |V|$, $a_{ij} \preceq l_{ij} \preceq u_{ij} \preceq b_{ij}$ with $C_{ij} = [a_{ij}, b_{ij}]$, $l_{ij} = \mathsf{Sup}\{a \in \mathsf{B} : I(a \preceq C_{ij}) = true\}$ and $u_{ij} = \mathsf{Inf}\{a \in \mathsf{B} : I(C_{ij} \preceq a) = true\}$. Using the clauses $(I)$, we know that $I$ satisfies $a_{ij} \preceq C_{ij}$ and $C_{ij} \preceq b_{ij}$. By definition of $l_{ij}$ and $u_{ij}$, we have $a_{ij} \preceq l_{ij}$ and $u_{ij} \preceq b_{ij}$. Let's show that $l_{ij} \preceq u_{ij}$. $I$ satisfies the clauses $(II)$, we can deduct that $I$ satisfies $l_{ij} \preceq C_{ij}$ and $C_{ij} \preceq u_{ij}$. If $l_{ij} \not\preceq u_{ij}$ $I$ would not satisfy the clauses $(III)$, it results that $l_{ij} \preceq u_{ij}$. $(b)$ Let us show that, for all $i, j, k \in \{0, \ldots, n-1\}$ with $i < j$, we have $[l_{ij}, u_{ij}] \subseteq [l_{ik}, u_{ik}] \circ [l_{kj}, u_{kj}]$. We know that $I$ satisfies the literals $l_{ik} \preceq C_{ik}$, $l_{kj} \preceq C_{kj}$, $C_{ik} \preceq u_{ik}$ and $C_{kj} \preceq u_{kj}$. Since $I$ satisfies the clauses $(V)$, we affirm that $\mathsf{Inf}(l_{ik} \circ l_{kj}) \preceq C_{ij}$ and $C_{ij} \preceq \mathsf{Sup}(u_{ik} \circ u_{kj})$ are two satisfied variables. By definition of $l_{ij}$ and $u_{ij}$, we deduce that $\mathsf{Inf}(l_{ik} \circ l_{kj}) \preceq l_{ij}$ and $u_{ij} \preceq \mathsf{Sup}(u_{ik} \circ u_{kj})$. Since $[l_{ik}, u_{ik}] \circ [l_{kj}, u_{kj}] = [\mathsf{Inf}(l_{ik} \circ l_{kj}), \mathsf{Sup}(u_{ik} \circ u_{kj})]$, we have $[l_{ij}, u_{ij}] \subseteq [l_{ik}, u_{ik}] \circ [l_{kj}, u_{kj}]$. $\mathsf{sqcn}(\mathsf{Sat}(\mathcal{N}))$ is therefore a $\circ$-closed QCN. $\dashv$

From the propostions 2 and 3 we have:

**Proposition 4.** *Let $\mathcal{N} = (V, C)$ be a* SQCN. *If the weak composition closure method is complete for the* SQCN *then $\mathcal{N}$ is consistent iff* $\mathsf{Sat}(\mathcal{N})$ *is satisfiable.*

Remember that the encoding $\mathsf{Sat}$ of a general QCN $\mathcal{N}$ is based on a splitting in relations of $\mathcal{C}_{\preceq}$ for each of its constraints: $C_{ij} = [a^1_{ij}, b^1_{ij}] \cup \ldots \cup [a^{k_{ij}}_{ij}, b^{k_{ij}}_{ij}]$ with $a^1_{ij}, \ldots, a^{k_{ij}}_{ij}, b^1_{ij}, \ldots, b^{k_{ij}}_{ij} \in \mathsf{B}$ and $k_{ij}$ an integer. From this and the previous result, it is possible to extend the previous proposition to the general case of the QCNs:

**Theorem 1.** *Let $\mathcal{N} = (V, C)$ be a* QCN. *If the weak composition closure method is complete for the* SQCN *then $\mathcal{N}$ is consistent iff* $\mathsf{Sat}(\mathcal{N})$ *is satisfiable.*

## 5 Conclusion and future works

In this paper, we defined an encoding allowing to transform qualitative constraints networks into propositional logic using a structure corresponding to a conceptual lattice on the basic relations. In the specific case of the SQCNs we obtain a set of Horn clauses. We proved that the satisfiability problem of the resulting set of clauses allows to solve the consistency problem of QCNs. In order to prove the effectivness of our approach many experimentations are in process. This paper presents a new encoding which takes into account the convex relations, one of our future works is to define new encodings using other tractable relations as the preconvex relations.

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