In finding models in Logic Programming

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1 abstract

In the present paper, we prove a new fixed point theorem and also a multi-valued fixed point theorem in ultrametric spaces. This latter is a generalization of fixed point theorem given by Crampe and Ribenboim. Then,by means of this results, we prove the existence of an answer set for locally startified disjunctive programs and the existence of a minimal supported model for non disjunctive locally stratified programs..
mot-cls: Logic Programs, supported models, answer set, ultrametric spaces.

2 Introduction

The expository paper is a systematic study of a generalized ultramatric space (X, d, Γ) where X is a set, Γ a partially ordered set with a smallest element and d a distance which follows certain conditions . The theory of ultrametric spaces lies in valuation and dates back to M. Krasner and A. Monna who developed this theory for ultrametric distances with real values (non archimedian analysis) but this was extended to an arbitrary partially ordered set(with a least element). In this sens, S.Priess Crampe and Paulo Ribenboim ,developed further the general theory of ultrametric spaces in several papers.
The following ideas form the basis of this work:
(1) A result analougous to Banach contraction principle in ultrametric spaces but in the multi-valued case. An application of this theorem is given in Logic Programming: the distance was defined

on Herbrand interpretations with values in a countable ordinal γ. Then we prove the existence of an answer set for a class of disjunctive programs called locally stratified . This result is in the continuation of stable semantics(Gelfond and Lifschitz 1988).
(2) The resulting theorem in the single valued case is also a generalization of the theorem given by Crampe and Ribenboim [8]. We give an application of this theorem which consists on proving the existence of supported model for non disjunctive programs. Our study is a mathematical approach of syntax of Logic Programs. It's based on level mappings with values in a countable ordinal γ. Works are already given in this direction. Seda and hitzler proposed a constructive approach of the model and which coincide with perfect semantics given by Przymusinski. This is based on Scott Topology But this method becomes really complex for disjunctive programs as it's incorporate a larger number of connectives . Our aim in this work is to present a new convincing method which does'nt going into syntactical details. We consider the set of Herbrand interpretations as an ultrametric space which is spherically complete and apply results of fixed point theory in Logic Programming both for non disjunctive and disjunctive programs.

3 Ultrametric space and a generalization of the multivalued fixed theorem

In this section, we prove a fixed point theorem for multivalued mappings in ultrametric spaces which generalizes the fixed point theorem given by Priess-Crampe and Ribenboim [9].

let X be a set and let $(\Gamma, <=)$ be a partially ordered set which has 0 as its smallest element. Let $d : X * X \longrightarrow \Gamma$ be a mapping which satisfies the following properties for all $x, y, z \in X$ and $\gamma \in \Gamma$:
(0) $d(x, y) = 0$ if and only if $x = y$
(1) $d(x, y) = d(y, x)$
(2) if $d(x, y) \leq \gamma$ and $d(y, z) \leq \gamma$, then $d(x, z) \leq \gamma$

The triple (X, d, Γ) is called an ultrametric space.

Let $0 < \gamma \in \Gamma$ and $a \in X$. The set $B_\gamma(a) = \{x \in X; d(x, a) \leq \gamma\}$ is called a ball.

We denote the following property for an ultametric space (X, d, Γ).
(1.1) If $0 \neq \alpha \leq \beta$ and $B_\alpha(a) \cap B_\beta(b) \neq \emptyset$, then $B_\alpha(a) \subseteq B_\beta(b)$.

An ultrametric space (X, d, Γ) is said to be spherically complete, if for any chain $\mathcal{C}$ of balls (that is if $B, B' \in \mathcal{C}$, then either $B \subseteq B'$ or $B' \subseteq B$) the interection $\cap\mathcal{C}$ is non-empty.

Let (X, d, Γ) be an ultrametric space . A mapping $F : X \longrightarrow 2^X$ is called multivalued . An element $x \in X$ with $x \in F(x)$ is called a fixed point of F. F is said to be contracting , if for every $x \in X$, for every $y \in X$ and for every $a \in F(x)$ there exists an element $b \in F(y)$ such that $d(a, b) \leq d(x, y)$.
The contraction is said to be strict, if for every $x, y \in X$ with $x \neq y$ and for every $a \in F(x)$, there exists $b \in F(y)$ such that $d(a, b) < d(x, y)$.
The mapping F is called strictly contracting on orbits , if for every $x \in X$ and for every $a \in F(x)$ with $a \neq x$ there exist an element $b \in F(a)$ with $d(a, b) < d(a, x)$. If $F : X \longrightarrow 2^X$ is a contraction and non-empty (i.e there exists $x \in X$ with $F(x) \neq \emptyset$), then $F(x) \neq \emptyset$ for all $x \in X$. Indeed, let $x \in X$ such that $F(x) \neq \emptyset$ and let $y \in X$. Choose $a \in F(x)$. Since F is contracting, there corresponds to x, y and $a \in F(x)$, an element $b \in F(y)$ with $d(a, b) \leq d(x, y)$. Hence in particular, $F(y) \neq \emptyset$. In the following, we will use this fact without mentioning it explicitly.

Before we state the theorem mentioned below, we introduce some notations. For $F : X \longrightarrow 2^X$, let $\Pi_x = \{d(x, y)/y \in F(x)\}$ and for a subset $\Delta \subseteq \Gamma$ denote by $\min \Delta$ the set of minimal elements of Δ

Theorem 1 *Let (X, d, Γ) be an ultrametric space spherically complete. Let*
$F : X \longrightarrow 2^X$ a nonempty contracting mapping. If moreover we suppose that for all $x \in X$, $\min \Pi_x$ is finite and all elements of Π_x has a lower bound in $\min \Pi_x$, then F has a fixed point or there exist a ball $B_\pi(z)$ such that $\pi \in \min \Pi_y$ for all $y \in B_\pi(z)$.

Proposition 1 *Let (X, d, Γ) an ultrametric space. Let $\emptyset \neq Y \subseteq X$ spherically complete. For all $x \in X$ we put $\Pi_x = \{d(x, y)/y \in Y\}$ then $\min \Pi_x \neq \emptyset$ and moreover each element of Π_x has a lower bound in $\min \Pi_x$*

For the proof see[9]

Corollary 1 *Let* (X, d, Γ) *an ultrametric space spherically complete with* Γ *totally ordered. Let* $F : X \longrightarrow 2^X$ *a nonempty contraction. Suppose that for all* $x \in X$, $F(X)$ *is spherically complete, then* F *has a fixed point or there exist a ball* $B_\pi(z)$ *such that* $\pi \in \Pi_y$ *for all* $y \in B_\pi(z)$

4 Application to Logic Programming

We consider normal logic programs, i.e programs which consists of finite number of clauses of the form :$A \longleftarrow A_1, ..., A_k, \neg B_1, ... \neg B_m$ where A, A_i and B_j are atoms.

We denote by $\mathcal{B}_p$ the Herbrand base of a normal logic program P, i.e. the set of all ground atoms in L, the underlying first order language of P.
Next, we let $\mathcal{I}_p$ denote the set of herbrand interpretations for P, this means $\mathcal{I}_p$ is the power set $\mathcal{P}(\mathcal{B}_p)$ of $\mathcal{B}_p$.
We also use $ground(P)$ to denote the set of all ground instances of clauses of P. One of the most important concepts in the study of these programs is that of the immediat consequence operator T : $\mathcal{I}_p \longrightarrow \mathcal{I}_p$ defined by $T_p(I) = \{A \in \mathcal{B}_p;$ there exists a clause $A \longleftarrow A_1, ..., A_k, \neg B_1, ... \neg B_m \in ground(P)$ such that
$A_1, ..., A_k \in I$ and $B_1, ..., B_m \notin I\}$

Let $(\Delta', \leq)$ be a totally ordered set which is well-ordered . A mapping
$l : \mathcal{B}_p \longrightarrow \Delta'$ is called a level mapping.
Let Δ^* be the set Δ' with the converse order and assume $0 \leq \delta$ for all $\delta \in \Delta^*$. Put $\Delta = \Delta^* \cup \{0\}$
Now, we shall consider level mapping of the following special kind : let γ be an arbitrary countable ordinal and let $(\Delta', \leq)$ be the set of ordinals less than γ. The ordered set Δ^* will be identified to the set $\{2^{-\alpha}; \alpha < \gamma\}$ endowed with the order $2^{-\alpha} \leq 2^{-\beta}$ if and only if $\beta \leq \alpha$.

Let $\mathcal{L}_0 = \emptyset$ and $\mathcal{L}_\alpha = \{A \in \mathcal{B}_p, l(A) < \alpha\}$ for $\alpha < \gamma$.
$(\mathcal{P}(\mathcal{B}_p), d, \Delta)$ with the distance d, defined by $d(I, I') = \inf\{2^{-\alpha}, I \cap$

$\mathcal{L}_\alpha = I' \cap \mathcal{L}_\alpha\}$ if $I \neq I'$ and 0 else, is an ultrametric space spherically complete.
Indeed,let $(B_{2^{-\alpha_i}}(I_i))_{i\in\mathcal{K}}$ a chain of balls with $\alpha_i \in \gamma$ for all $i \in \mathcal{K}$ and $(I_i)_{i\in\mathcal{K}} \subset \mathcal{I}_p$.
Assume that $I = \bigcup_{i\in\mathcal{K}} (I_i \cap \mathcal{L}_{\alpha_i})$.
Let $A \in I_i$ with $l(A) < \alpha_i$ then $A \in I$.
Let $A \in I$ with $l(A) < \alpha_i$ then there exists $j \in \mathcal{K}$ such that $A \in I_j \cap \mathcal{L}_{\alpha_j}$. Or $B_{2^{-\alpha_i}}(I_i)$ and $B_{2^{-\alpha_j}}(I_j)$ are comparative , so $d(I_i, I_j) \leq \sup(2^{-\alpha_i}, 2^{-\alpha_j}) = 2^{-\inf(\alpha_i,\alpha_j)}$, so $A \in I_i$ and then $d(I_i, I) \leq 2^{-\alpha_i}$.

we borrow the following definition from Seda & Hitzler [13].

Definition 1 *Let P be a normal logic program ,let $l : \mathcal{B}_p \longrightarrow \gamma$ be a level mapping and let $A \longrightarrow A_1, ..., A_k, \neg B_1, ...\neg B_m$ denote a typical clause in ground(P).*
We call P:
(1) Level decreasing if the inequalities $l(A) \geq l(A_i), l(B_j)$ hold for all i and j.
(2)Strictly Level decreasing if the inequalities $l(A) > l(A_i), l(B_j)$ hold for all i and j.
(3) Semi Strictly Level decreasing if the inequalities $l(A) \geq l(A_i)$ and $l(A) > l(B_j)$ hold for all i and j.

Example 1 *Let P be the program*

$$\begin{array}{ll} p(o) & \longleftarrow \\ p(s(x)) & \longleftarrow p(x) \end{array}$$

Consider the level mapping $l : \mathcal{B}_p \longrightarrow \omega$; $l(p(s^n(0)) = n + 1$.
P is strictly decreasing with respect to l, and the set $\{p(s^n(0)), n < \omega\}$ is a fixed point for T_p.

Theorem 2 *Let P be a normal logic program which is Semi Stictly Level decreasing with respect to a level mapping $l : \mathcal{B}_p \longrightarrow \gamma$, where γ is a countable ordinal. Then P has a minimal supported model.*

Proof

Examples

Example 2 *Let P be the program:*
$p(s(x),0) \longrightarrow p(s(x),0)$ *and consider the level mapping* $l : \mathcal{B}_p \longrightarrow \omega$, $l(s^n(0), s^m(0)) = n+m+1$. *$T_p$ is contracting, we show that T_p is strictly contracting on orbits.*
First ,we remark that for all $I \in \mathcal{I}_p$, $n, m \neq 0$, $p(s^n(0), s^m(0)) \notin T_p(I)$ and also $p(0,0) \notin T_p(I)$ and if $n \neq 0$, $p(s^n(0), 0) \in T_p(I)$ if and only if $p(s^n(0), 0) \in I$. So let $d(I, T_p(I)) = i+1$ .Hence there exist $A \in \mathcal{B}_p$, $l(A) = i+1$, where I and $T_p(I)$ differ.It follows that $A = p(0,0)$ or $A = p(s^n(0), s^m(0))$, $m \neq 0$, $n+m = i$, then $A \notin T_p(I)$ and $A \notin T_p(T_p(I))$. So T_p is strictly contracting on orbits so by Theorem 2,it has a fixed point.

Example 3 *Consider the program:*

$$
\begin{array}{lll}
p(s(0)) & \longleftarrow & \neg q(0) \\
p(x) & \longleftarrow & r(x) \\
r(x) & \longleftarrow & p(x) \\
q(0) & \longleftarrow &
\end{array}
$$

Consider $l : \mathcal{B}_p \longrightarrow \omega$, $l(p(s^n(0)) = n$, $l(q(0)) = 0$, $l(r(s^n(0)) = n$
P is semi strictly decreasing with respect to l but is never strictly decreasing for any level mapping because of the cycle created by the second and third clauses.
The set $\{q(0), p(s^n(0)), r(s^n(0))\}$ is a fixed point of T_p.

5 Application in disjunctive logic programs

We consider the following programs which we called the disjunctive normal programs in which the clauses are of the form $A_1 \backslash ... \backslash A_n \longleftarrow A_{n+1}, ..., A_m, \neg B_1, ... \neg B_{l_1}$ with A_i and B_j are atoms. We put $Head(r) = \{A_1, ...A_n\}$, $Pos(r) = \{A_{n+1}, ...A_m\}$ and $Neg(r) = \{B_1, ..., B_{l_1}\}$
We denote by $\mathcal{B}_p$ the Herbrand base of a normal logic program P, i.e. the set of all ground atoms in L, the underlying first order language of P.

Definition 2 *Let Π such program, for $S \subseteq \mathbf{B_p}$, we consider the GL transformation of the program Π^S given by the set of clauses :*
If $r \in \Pi$ such that $Neg(r) \cap S \neq \emptyset$ then $r \notin \Pi^s$.

If $r \in \Pi$ such that $Neg(r) \cap S = \emptyset$ then the rule r' defined by $Head(r') = Head(r)$, $Pos(r') = Pos(r)$ and $Neg(r') = \emptyset$ bellows to Π^S.
A subset S' of $\mathbf{B_p}$ is closed of Π^S if for all the clauses $r \in \Pi$ such that $Pos(r) \subseteq S$, we have $Head(r) \cap S'$ is nonempty. We denote by $\alpha(\Pi^S)$ the set of minimal closed subsets of Π^S.
The set S is said the answer set of Π if $S \in \alpha(\Pi^S)$

We define the Gelfond Lifschitz operator $F : I_p \longrightarrow 2^{I_p}$ by $F(X) = \alpha(\Pi^X)$
By using F, the set S is an answer set of F if and only if $S \in F(S)$.
If Π is nondisjunctive then,F is single valued and S has an answer set of Π if and only if S is a fixed point of F

(2.1) The definition of an answer set follows the tradition of a stable model semantics in the sens that we remove the negative parts
(2.2) We can see easily that an answer set S of Π can be identified to a closed set of Π^S such that:
i) For all $r \in \Pi^S$ verifying $Pos(r) \subseteq S$ we have $Head(r) \cap S$ is a singleton .
ii) For all the remaining clauses of Π^S, we have $Head(r) \cap S = \emptyset$

Definition 3 *Let Π is a disjunctive normal logic program and $l : \mathbf{B_p} \longrightarrow \gamma$, a level mapping, let $A_1 \backslash ... \backslash A_n \longleftarrow A_{n+1}, ...A_m, \neg B_1, ..., \neg B_{l_1}$ a typical clause in $ground(\Pi)$. We say that Π is*
1) decreasing with respect to l if $l(A) \geq l(A_i), l(B_j)$ for all $A \in Head(r)$, $i = n+1..m$ and $j = 1..l_1$
2) strictly decreasing with respect to l if $l(A) > l(A_i)$ and $l(A) > l(B_j)$ for all $A \in Head(r)$ and i, j
3) semi-strictly decreasing with respect to l if $l(A) \geq l(A_i)$ and $l(A) > l(B_j)$ for all $A \in Head(r)$ and i, j.

Theorem 3 *Let Π a disjunctive normal logic program strictly decreasing with respect to a level mapping , then Π has an answer set*

Proof
Let us proove that F is strictly contracting . Let X and Y in $\mathbf{B_p}$ such that $X \neq Y$. Then there there exist $U \in F(Z)$ as $F(Z) \neq \emptyset$. We suppose that $d(X, Y) = 2^{-\alpha}$. Let $Z \in F(X)$. Let r a clause of Π^Y such that $Pos(r) \subseteq Y$. If there exist $A \in Head(r)$ such

that $l(A) \leq \alpha$. Then $Pos(r) \subseteq \mathcal{L}_\alpha$, but $d_l(X,Y) = 2^{-\alpha}$. Then $Pos(r) \subseteq X$ and r is a clause in Π^X. We take $Head(r) \cap V = Head(r) \cap Z = singleton$ as $Z \in F(X)$. Else, we take $Head(r) \cap V = Head(r) \cap U = singleton$ /subseteq$\mathcal{L}_{\alpha+1}$. Then $V \in F(Y)$ and $d_l(V,Z) = 2^{-(\alpha+1)} < 2^{-\alpha} = d_l(X,Y)$. Then F is strictly contracting , so from the theorem 2, the program Π has an answer set.

Theorem 4 *Let Π a disjunctive normal logic program semi strictly decreasing with respect to a level mapping , then Π has an answer set*

Proof
1)$F$is contracting. We bring the proof above and replace α by $\alpha+1$.

2) Suppose that F hasn't any fixed point . Then there exist a ball $B = B_{2^{-\alpha}}(X)$ such that $2^{-\alpha} = \min\limits_{Y' \in F(Y)} d(Y,Y')$

3) Let $Y = \bigcap\limits_{X'} X'$. Since for each $X' \in B$, we have $\mathcal{L}_\alpha \cap X' = \mathcal{L}_\alpha \cap X$, then $Y \in B$

4) We prove $Y \cap \mathcal{H}_\alpha = \emptyset$ where $\mathcal{H}_\alpha = \{A \in \mathcal{B}_p; l(A) = \alpha\}$. Indeed if we assume the contrary, put $X' = X \cap \mathcal{L}_\alpha$. It is easy to verify that $d_l(X,X') \leq 2^{-\alpha}$. Therefore $X' \in B$ and $X' = X$

5) We have $Y \cap \mathcal{H}_\alpha = \emptyset$, then there exist $Y_1 \in F(Y) \cap B$ such that $d(Y,Y_1) = 2^{-\alpha}$, then $\emptyset = Y \cap \mathcal{H}_\alpha$ and $Y_1 \cap \mathcal{H}_\alpha \neq \emptyset$ as $d(Y,Y') = 2^{-\alpha}$. We want to built a sequence (Y_n) in B such that $Y_n \in F(Y_{n-1})$ and $\emptyset = Y \cap \mathcal{H}_\alpha \overset{\subset}{\neq} Y_1 \cap \mathcal{H}_\alpha \subseteq Y_2 \cap \mathcal{H}_\alpha \subseteq ...$
$Y_1 \in B$ then there exist $U_2 \in F(Y_1)$ such that $d(Y_1,U_2) = 2^{-\alpha}$
Let r a clause in Π^{Y_1} such that $Pos(r) \subseteq Y_1$. If there exists $A \in Head(r)$ such that $A \in Y_1 \cap \mathcal{H}_\alpha$, then from (2.2) r is necessary a clause in Π^Y such that $Pos(r) \subseteq Y$ and $Head(r) \cap Y_1 = \{A\}$. We take $Head(r) \cap Y_2 = \{A\}$. Else we take $Head(r) \cap Y_2 = Head(r) \cap U_2$. Thus $Y_2 \in F(Y_1)$ and $Y_2 \in B$.
Let us show that Thus $Y_1 \cap \mathcal{H}_\alpha \subseteq Y_2 \cap \mathcal{H}_\alpha$ and $Y_2 \in B$. Let $A \in Y_1 \cap \mathcal{H}_\alpha$, then A appear in the head of clause r in Π^Y such that $Pos(Y) \subseteq Y \cap \mathcal{L}_{\alpha+1} = Y \cap \mathcal{L}_\alpha$. Then r is a clause in Π^{Y_1} such that $Pos(r) \subseteq Y_1 \cap \mathcal{L}_{\alpha+1}$ as $d(Y,Y_1) \leq 2^{-\alpha}$ and $Y \cap \mathcal{H}_\alpha = \emptyset$. So $Head(r) \cap Y_1 = \{A\} = Head(r) \cap Y_2$ from (2.2) and the definition of Y_2. We built in the same way the sequence (Y_n) .

6) We put $Y_\omega = \cup Y_n$ (ω stands for the first limit ordinal), then $Y_\omega \in B$. Let r a clause in Π^{Y_ω} such that $Pos(r) \subseteq Y_\omega$. r is of the form $A_1 \backslash ... \backslash A_n \longleftarrow A_{n+1}, ...A_m$. If there exist k and $A_k \in Head(r)$ such that $A \in Y_k \cap \mathcal{H}_\alpha$, then we take $Head(r) \cap Y_{\omega+1} = \{A\} = Head(r) \cap Y_k$. If there exist another m and $A_m \in Head(r)$ such that $A_m \in Y_m \cap \mathcal{H}_\alpha$, (we suppose that $k \geq m$), then from (2.2), we have $\{A_m\} = Head(r) \cap Y_m \subseteq Head(r) \cap Y_k = \{A_k\}$ as $l(A_m) = l(A_k) = \alpha$. Thus, $Head(r) \cap Y_{\omega+1}$ is well defined.
Else Let $\{k_{n+1}, ...k_r\}$ a subset of $\{n+1, ...m\}$ such that $l(A_{k_i}) = \alpha$ for every $i \in \{n+1, ...m\}$, then there exist $m_{k_{n+1}}, ...m_{k_r}$ such that $A_{k_i} \in Y_{m_{k_i}}$ for every $i \in \{n+1, ...r\}$. Put $m_k = \max_{n+1 \leq i \leq r} m_{k_i}$. Then $A_{k_{n+1}}, ...A_{k_r} \in Y_{m_k}$ and since for $i \notin \{k_{n+1}, ...K_r\}$, $l(A_j) < \alpha$ we have $A_j \in Y_{m_k}$. let m the least integer such that $Pos(r) \subseteq Y_m$. r is a clause in Π^{Y_m} as $Y_m \in B$, then we take $Head(r) \cap Y_{\omega+1} = Head(r) \cap Y_m \not\subseteq \mathcal{H}_\alpha$. So $Y_{\omega+1} \in F(Y_\omega)$ from (2.2) and $Y_{\omega+1} \in B$.
We show that $Y_{\omega+1} \cap \mathcal{H} = Y_\omega \cap \mathcal{H}_\alpha$.
Let $A \in Y_\omega \cap \mathcal{H}_\alpha$; then there exist n such that $A \in Y_n \cap \mathcal{H}_\alpha$. So A appear in a head of a clause r in $\Pi^{Y_{n-1}}$ such that $Pos(r) \subseteq Y_{n-1}$ and $Head(r) \cap Y_n = \{A\}$ from (2.2). So $Pos(r) \in Y_\omega$ and r is a clause in Π^{Y_ω}. Then from the definition of $Y_{\omega+1}$, we have $Head(r) \cap Y_{\omega+1} = Head(r) \cap Y_n = \{A\}$. So $A \in Y_{\omega+1} \cap \mathcal{H}_\alpha$.
Let $A \in Y_{\omega+1} \cap \mathcal{H}_\alpha$. Then A appear in the head of a clause r in Π^{Y_ω} such that $Pos(r) \in Y_\omega$ and $Head(r) \cap Y_{\omega+1} = \{A\}$. So from the definition of $Y_{\omega+1}$ there exist n such that $\{A\} = Head(r) \cap Y_{\omega+1} = Head(r) \cap Y_n$. So $A \in Y_n \cap \mathcal{H}_\alpha$. Then $A \in Y_\omega \cap \mathcal{H}_\alpha$.
So $d(Y_{\omega+1}, Y_\omega) \leq 2^{-(\alpha+1)}$ Contradiction with $2^{-\alpha} = \min_{Z \in F(Y_\omega)} d(Y_\omega, Z)$

References

[1] M.Fitting, *Metric methods: Three Examples and a Theorem, J.Logic Programming* 21 (3) (1994), 113-127.

[2] E.Jawhari, D.Misane and M.Pouzet, , *Retracts, Graphs and Ordered Sets from Metric Point of View*, Contemporary Mathematics, volume 57, 1986.

[3] M.Khamsi, V.Kreinovitch, D.Misane, *A new method of proving the existence of answer sets for disjunctive logic programs*, Proceedings ILPS' 93 Workshop on logic programming with incomplete information. Vancouver, British-columbia, Canada 1993.

[4] M.A.Khamsi and Driss Misane, *Fixed Point Theorems In Logic Programming* Annals of Mathematics and Artificial Intelligence 21 (1997) 231-243.

[5] J.W. Lloyd, *Foundations of Logic Programming.Springer-Verlag*, 2nd Edition, Berlin, 1988.

[6] S.Priesse-crampe and P. Ribenboim, *Fixed points , combs and generalized power series*, Abh. Math. Sem. Univ. Hambourg **63**(1993),227-244.

[7] S. Priesse-crampe and P. Ribenboim, *Generalized Ultrametric Spaces I*,Abh. Math. Sem. Univ. Hamburg **66** (1997), 19-31.

[8] S. Priesse-crampe and P. Ribenboim, *Generalized Ultrametric Spaces II*, Abh. Math. sem. Univ. Hamburg **67** (1996), 19-31.

[9] S. Priess Crampe, P.Ribenboim, *Ultrametric Spaces and Logic Programming* , Preprint submitted to Elsevier Preprint (1998).

[10] H. Przymusinska and T. Przymusinski, *Semantics Issues in Deductive Databases and Logic Programs*, Elsevier Science Publishers B.V.(1990), 321-366.

[11] P. Ribenboim, , *The new theory of ultrametric spaces*, Periodica Math. Hung. **32 (1996), 103-111.**

[12] A.K.Seda**,** *Quasi-metrics and the semantics of Logic Programs. Fundamenta Informaticae*, **29 (1997), 97-117.**

[13] Seda, A.K. and Hitzler, P.**,** *Topology and Iterates in Computational Logic*, **Preprint, November 1997, 24 pages.**