Approches Déclaratives pour la Fouille de Données

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Work done in the framework of Project DAG ANR Défis 2009
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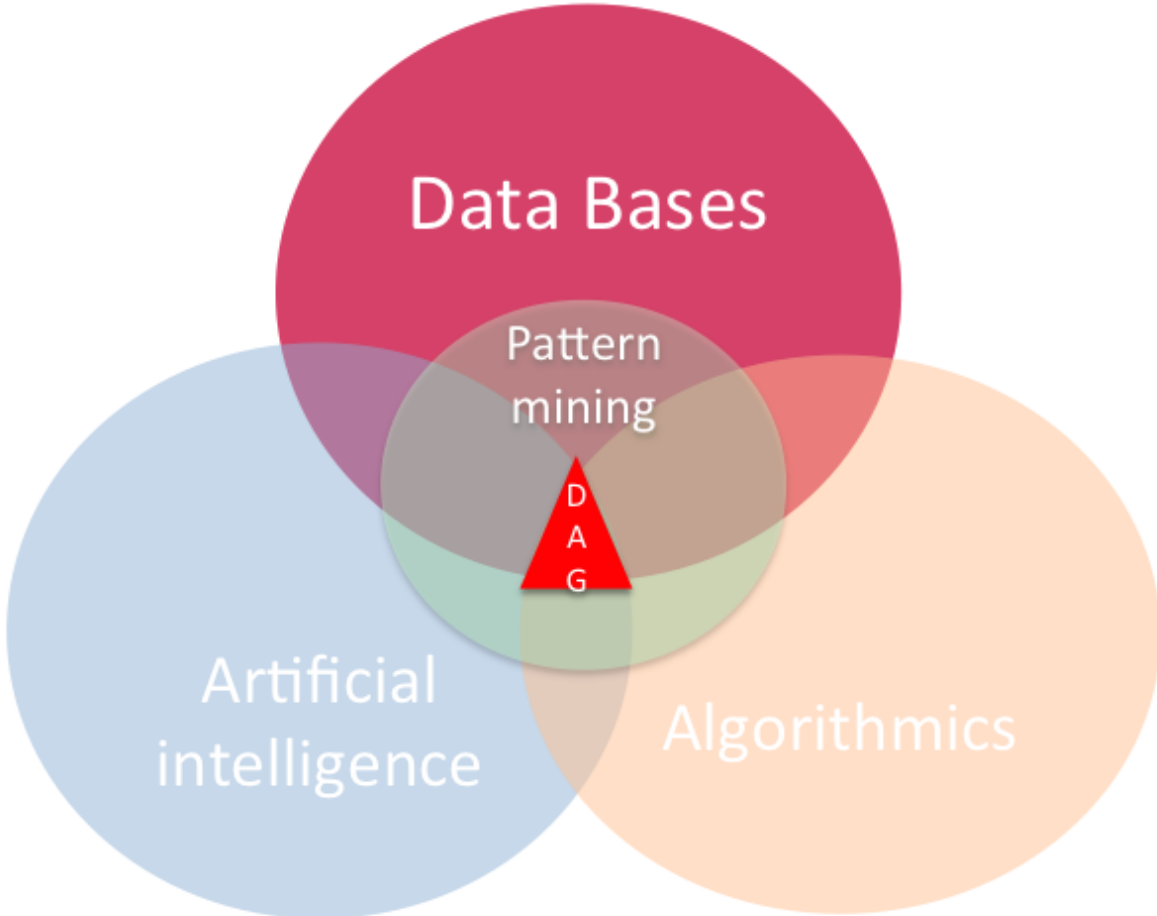
Outline

DAG: Declarative Approaches for Enumerating Interesting Patterns



`http://liris.cnrs.fr/dag/`

- CRIL, University d'Artois, Lens
- LIMOS, University of Blaise Pascal, Clermont-Ferrand
- LIRIS, University Claude Bernard, Lyon 1, Lyon

DAG: Objectives and Challenges

- Objectives
  - Cross-fertilization between artificial intelligence (CSP, SAT), combinatorial algorithmics and databases
  - Bringing original solutions to fundamental pattern mining problems
  - Definition of high level declarative languages for describing interesting pattern enumeration problems
- Two major challenges
  - Challenge 1: Definition of classes of problems for enumerating interesting patterns
  - Challenge 2: Design of declarative, efficient and generic pattern mining systems

Motivation

- Lot of contributions in pattern mining:
  - Different kind of patterns
  - Efficiency of enumeration algorithms
  - Various predicates
  - ...

- Most are dedicated approaches
  - Lack of declarativity

  $\Rightarrow$ Changing slightly the problem statement requires deep changes in implementations.

Some Results

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7. Penalty-based Preferences in Itemset Mining. S. Jabbour, S. Kaci, L. Sais, Y. Salhi [Submitted]

Outline

Sequences and patterns

- Alphabet: a set of items Σ
- Wildcard (or Joker): $\circ \notin \Sigma$
- Sequence of items S: a word $S_0 S_1 \ldots S_{n-1}$ in Σ^*
- Pattern P: a word $P_0 P_1 \ldots P_{m-1}$ in $(\Sigma \cup \{\circ\})^*$
  - $P_0 \neq \circ$ and $P_{m-1} \neq \circ$
  - Sequences are patterns
  - Examples:

$$abbac,\ ab \circ c \circ \circ d,\ \circ ab \circ c,\ ab \circ c \circ$$

Inclusion

Let $P = P_0 P_1 \dots P_{m-1}$ and $P' = P'_0 P'_1 \dots P'_{n-1}$

- $P \preceq_l P'$ if $\forall i \in \{0, \dots, m-1\}$:
  - either $P_i = P'_{l+i}$
  - or $P_i = \circ$
- $P \preceq P'$ if $\exists l$ st. $P \preceq_l P'$
- $L_S(P) = \{l \mid P \preceq_l S\}$
- Shift: $L_S(P) + d = \{p + d \mid p \in L_S(P)\}$

$a \circ b \preceq_2 aaabbbaabab$

$a \circ \circ b \preceq_0 a \circ ab \circ b$

$a \circ \circ b \preceq_2 a \circ ab \circ b$

$a \circ bb \not\preceq a \circ \circ b$

$L_{aaabbbaabab}(a \circ b) = \{1, 2, 5\}$

$L_{aaabbbaabab}(a \circ b) + 3 = \{4, 5, 8\}$

Frequent Patterns

Definition (Finding frequent patterns in a sequence)

Input: a sequence S and a minimal support threshold λ
Output: all patterns P s.t. $|L_S(P)| \geqslant \lambda$

Property (Anti-monotonicity)

If $P \preceq P'$ then, for some d:

$$L_S(P') + d \subseteq L_S(P)$$

Closed Frequent Patterns

Definition (Finding closed frequent patterns)

Input: a sequence S and a minimal support threshold λ
Output: all patterns P s.t.:

- $|L_S(P)| \geqslant \lambda$
- there is no pattern Q and no integer d s.t.:
  - $P \prec Q$
  - $L_S(Q) + d = L_S(P)$

Maximal Frequent Patterns

Definition (Maximal frequent patterns)

Input: a sequence S and a minimal support threshold λ
Output: all patterns P st.:

- $|L_S(P)| \geqslant \lambda$
- there is no pattern Q s.t.:
  - $P \prec Q$
  - $|L_S(Q)| \geqslant \lambda$

Outline

An Encoding of Frequent Pattern Mining

- Boolean variables: for each a in Σ, we associate k_a Boolean variables

$$p_{a,0}, \ldots, p_{a,k_a-1}$$

where $k_a = min(max(L_S(a)) + 1, n - \lambda + 1)$

$\Rightarrow$ $p_{a,i}$: the item a is at the location i in the candidate pattern p

- The first symbol must be in Σ:

$$\bigvee_{a \in \Sigma} p_{a,0} \tag{1}$$

An Encoding of Frequent Pattern Mining

- The locations where the candidate pattern does not appear:

$$\bigwedge_{a\in\Sigma,0\leqslant l\leqslant n-1,0\leqslant i\leqslant k_a-1} (p_{a,i} \wedge s_{l+i} \neq a) \rightarrow b_l \tag{2}$$

- Frequency Constraint:

$$\sum_{l=0}^{n-1} b_l \leqslant n - \lambda \tag{3}$$

Example

Sequence ($\lambda = 2$):

$$a \quad a \quad b \quad b$$

Boolean formula:

$$\begin{aligned}
&p_{a,0} \vee p_{b,0} \\
&p_{a,0} \rightarrow (b_2 \wedge b_3) \\
&p_{a,1} \rightarrow (b_1 \wedge b_2 \wedge b_3) \\
&p_{a,2} \rightarrow (b_0 \wedge b_1 \wedge b_2 \wedge b_3) \\
&p_{b,0} \rightarrow (b_0 \wedge b_1) \\
&p_{b,1} \rightarrow (b_0 \wedge b_3) \\
&p_{b,2} \rightarrow (b_2 \wedge b_3) \\
&b_0 + b_1 + b_2 + b_3 \leqslant 2
\end{aligned}$$

Models: $\{p_{a,0}\} \leftrightarrow a$, $\{p_{b,0}\} \leftrightarrow b$ and $\{p_{a,0}, p_{b,2}\} \leftrightarrow a \circ b$.

Closed Patterns

- All the locations where the candidate pattern appears (iff with (2)): $b_l = \mathit{false}$ iff $P \preceq_l S$

$$\bigwedge_{l=0}^{n-1} (b_l \rightarrow \bigvee_{a \in \Sigma, 0 \leqslant i \leqslant k_a - 1} (p_{a,i} \wedge s_{l+i} \neq a)) \tag{4}$$

- Maximizing the number of symbols different from wildcard on the right side:

$$\bigwedge_{a \in \Sigma, 0 \leqslant i \leqslant k_a - 1} (\bigwedge_{l=0}^{n-1} \overline{b_l} \rightarrow s_{l+i} = a) \rightarrow p_{a,i} \tag{5}$$

Closed Patterns

- Maximizing the number of symbols different from wildcard on the left side (negative indices):

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k'_a} (\bigwedge_{l=0}^{n-1} \overline{b_l} \rightarrow s_{l-i} = a) \leftrightarrow p_{a,-i} \tag{6}$$

where $k'_a = n - min(L_S(a)) - 1$

$\Rightarrow$ Without negative indices:

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k'_a} \neg(\bigwedge_{l=0}^{n-1} \overline{b_l} \rightarrow s_{l-i} = a) \tag{7}$$

- Frequent Closed Patterns: (1), (2), (3), (4), (5) and (7)

Maximal Patterns

- Maximizing the number of symbols different from wildcard on the right side:

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k_a-1}(\sum_{l=0}^{n-1}\overline{b_l}\wedge s_{l+i}=a\geqslant\lambda)\rightarrow p_{a,i} \tag{8}$$

- Maximizing the number of symbols different from wildcard on the left side:

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k'_a}\sum_{l=0}^{n-1}\overline{b_l}\wedge s_{l-i}=a\leqslant\lambda-1 \tag{9}$$

- Frequent Maximal Patterns: (1), (2), (3), (4), (8) and (9)

Flexibility of the proposed approach

- The frequent patterns with at least *min* items:

$$\sum_{a \in \Sigma, 0 \leqslant i \leqslant k_a - 1} p_{a,i} \geqslant min \tag{10}$$

- The frequent patterns with at most *max* items:

$$\sum_{a \in \Sigma, 0 \leqslant i \leqslant k_a - 1} p_{a,i} \leqslant max \tag{11}$$

Outline

Sequences of Itemsets

- Sequences of itemsets S: $s_0 \dots s_{n-1}$ s.t. $s_i \subseteq \Sigma$
- Wildcard: $\emptyset$
- Inclusion: $P_0 P_1 \dots P_{m-1} \preceq_I P'_0 P'_1 \dots P'_{n-1}$ if,
  $\forall i \in \{0, \dots, m-1\}$, $P_i \subseteq P'_{I+i}$
- Examples:
  $S = \{a, b\}, \{a, b\}, \{c, d\}, \{c, e\}, \{f\}, \{g\}, \{d\}, \{a, b, d\}, \{f\}, \{c\}$
  $P = \{a, b\}, \{\}, \{c\}$
  $L_S(P) = \{0, 1, 7\}$
- Frequent (Closed/Maximal) patterns are defined in the same way as in the case of the sequences of items

Frequent Pattern Mining

- The Boolean variable $p_{a,i}$ means that the symbol a is in the itemset at the location i in the candidate pattern
- We only have to replace $s_{l+i} \neq a$ with $a \notin s_{l+i}$:

$$\bigvee_{a \in \Sigma} p_{a,0} \tag{12}$$

$$\bigwedge_{a \in \Sigma, 0 \leqslant l \leqslant n-1, 0 \leqslant i \leqslant k_a - 1} (p_{a,i} \wedge a \notin s_{l+i}) \rightarrow b_l \tag{13}$$

$$\sum_{l=0}^{n-1} b_l \leqslant n - \lambda \tag{14}$$

Closed Patterns

Constraints of closeness:

$$\bigwedge_{l=0}^{n-1}(b_l \rightarrow \bigvee_{a\in\Sigma,0\leqslant i\leqslant k_a-1}(p_{a,i} \wedge a \notin s_{l+i})) \tag{15}$$

$$\bigwedge_{a\in\Sigma,0\leqslant i\leqslant k_a-1}(\bigwedge_{l=0}^{n-1}\overline{b_l} \rightarrow a \in s_{l+i}) \rightarrow p_{a,i} \tag{16}$$

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k'_a}\neg(\bigwedge_{l=0}^{n-1}\overline{b_l} \rightarrow a \in s_{l-i}) \tag{17}$$

Maximal Patterns

Constraints of maximality:
we add the following two constraints to (15):

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k_a-1} (\sum_{l=0}^{n-1} \overline{b_l} \wedge a \in s_{l+i} \geqslant \lambda) \rightarrow p_{a,i} \tag{18}$$

$$\bigwedge_{a\in\Sigma,1\leqslant i\leqslant k'_a} \sum_{l=0}^{n-1} \overline{b_l} \wedge a \in s_{l-i} \leqslant \lambda - 1 \tag{19}$$

Outline

Symmetries

- A fundamental concept (structural knowledge) in Computer Science, Mathematics, Physics and many other domains.
  - Many human artifacts (e.g. classrooms in a university, aircraft seats, circuit patterns) and entities in nature (e.g. plants, molecules, DNA sequences, atoms) exhibits symmetries.
  - $\Rightarrow$ Useful for reasoning and understanding complex entities and systems.

Frequent Itemset Mining

- Essential problem in data mining, knowledge discovery and data analysis.
- Many related problems: Association rules, frequent pattern mining in sequence data, data clustering, episode mining, etc.
- Various applications

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Main challenges

- Output of huge size, difficulty to retrieve relevant information
- Computational issues

Frequent Itemset Mining: Problem definition and notations

- Let $\mathcal{I}$ be a set of *items*.
- A set $I \subseteq \mathcal{I}$ is called an **itemset**.
- A **transaction** is a couple (t_i, I) where t_i is the *transaction identifier* and I is an itemset.
- A **transaction database** is a finite set of transactions over $\mathcal{I}$ where for each two different transactions, they do not have the same transaction identifier.
- **Cover**: $\mathcal{C}(I, \mathcal{D}) = \{t_i \mid (t_i, J) \in \mathcal{D} \text{ and } I \subseteq J\}$.
- **Support**: $\mathcal{S}(I, \mathcal{D}) = |\mathcal{C}(I, \mathcal{D})|$.
- **Frequency**: $\mathcal{F}(I, \mathcal{D}) = \frac{\mathcal{S}(I,\mathcal{D})}{|\mathcal{D}|}$.

Example

t_i	itemset
001	A, B, E, F
002	A, B, C, D
003	C, D, E, F
004	A, C
005	A, E
006	C, E
007	B, D
008	B, F
009	D, F

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$\mathcal{I} = \{A, B, C, D, E, F\}$

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- **Support:** $\mathcal{S}(\{A, C\}, \mathcal{D})$= $|\{002, 004\}|$=2

Example

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001	A, B, E, F
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- **Frequency:** $\mathcal{F}(\{A, C\}, \mathcal{D})$=$2/9 \equiv 0.22$

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Definition (Frequent Itemset Mining Problem)

Given a minimum support λ ($0 < \lambda \leqslant |\mathcal{D}|$), the frequent itemset mining problem consists in computing the set of itemsets

$$\mathcal{FIM}(\mathcal{D}, \lambda) = \{I \subseteq \mathcal{I} \mid \mathcal{S}upp(I, \mathcal{D}) \geqslant \lambda\}$$

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Proposition (Anti-Monotonicity)

Let I_1 and I_2 be two itemsets such that $I_1 \subseteq I_2$.

$$\textit{If } \mathcal{S}(I_2, \mathcal{D}) \geqslant \lambda \textit{ then } \mathcal{S}(I_1, \mathcal{D}) \geqslant \lambda.$$

Outline

Definition (Permutation)

A permutation σ over $\mathcal{I}$ is a bijective mapping from $\mathcal{I}$ to $\mathcal{I}$.

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Definition (Symmetry)

A permutation σ over $\mathcal{I}$ is a symmetry if $\sigma(\mathcal{D}) = \mathcal{D}$ where $\sigma(\mathcal{D}) = \{\sigma(t_i, l) = (\sigma(t_i), \sigma(l)), (t_i, l) \in \mathcal{D}\}$

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$\sigma = c_1 \ldots c_n$ where each cycle $c_i = (a_1, \ldots, a_k)$ is a list of elements of $\mathcal{I}$ such that $\sigma(a_j) = a_{j+1}$ for $j = 1, \ldots, k-1$, and $\sigma(a_k) = a_1$.

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$\sigma = c_1 \ldots c_n$ where each cycle $c_i = (a_1, \ldots, a_k)$ is a list of elements of $\mathcal{I}$ such that $\sigma(a_j) = a_{j+1}$ for $j = 1, \ldots, k-1$, and $\sigma(a_k) = a_1$.

Proposition

Let σ a symmetry of $\mathcal{D}$, λ a minimal support threshold and I an itemset. $I \in \mathcal{FIM}(\mathcal{D}, \lambda)$ iff $\sigma(I) \in \mathcal{FIM}(\mathcal{D}, \lambda)$.

Example

σ = (C,E)(D,F) is a symmetry

t_i	itemset
001	A, B, E, F
002	A, B, C, D
003	A, C, E, F
004	A, C,
005	A, E,
006	C, E,
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007	B, D,
008	B, F,
009	D, F,

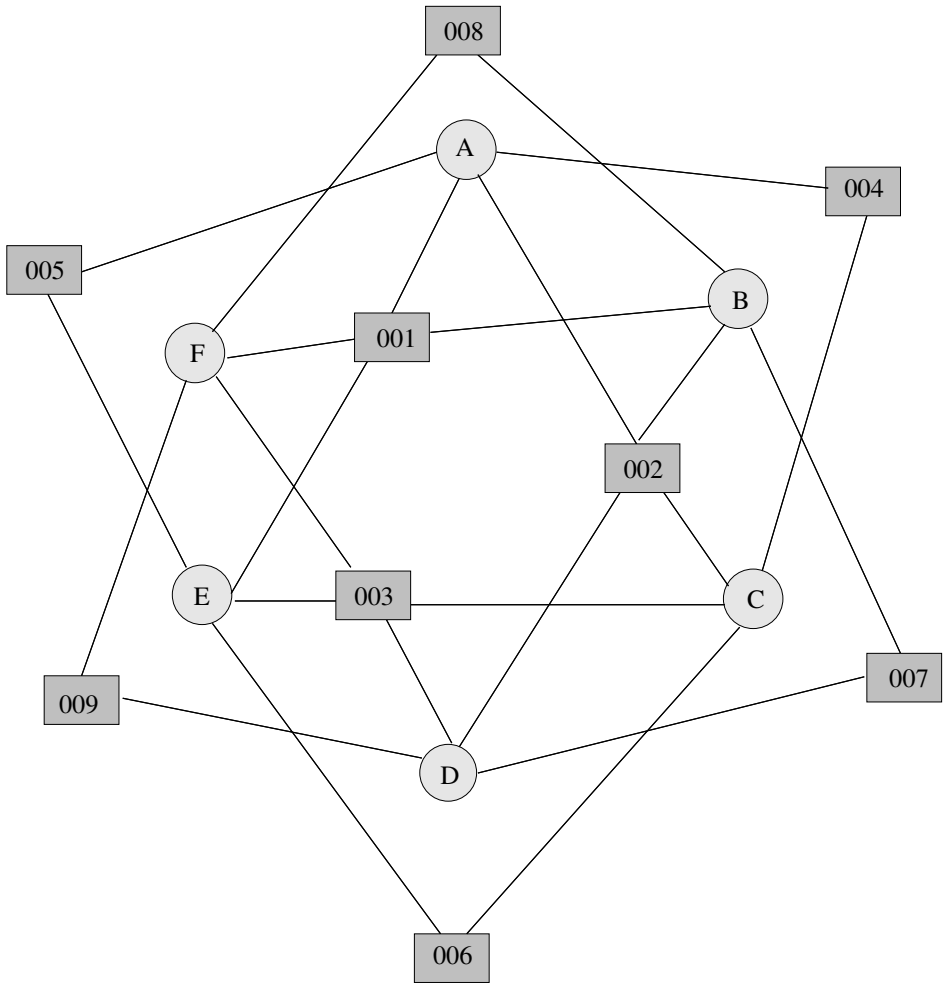
$$\sigma(t_i) = \begin{cases} 001 & \text{if } t_i\text{=002} \\ 002 & \text{if } t_i\text{=001} \\ 003 & \text{if } t_i\text{=003} \\ 004 & \text{if } t_i\text{=005} \\ 005 & \text{if } t_i\text{=004} \\ 006 & \text{if } t_i\text{=006} \\ 007 & \text{if } t_i\text{=008} \\ 008 & \text{if } t_i\text{=007} \\ 009 & \text{if } t_i\text{=009} \end{cases}$$

Outline

Symmetry Detection in Transaction Databases

- Convert the original problem $\mathcal{D}$ into a colored undirected graph $\mathcal{G}$, where vertices are labeled with colors.
- Look for the automorphism group of $\mathcal{G}$.
- Symmetries of $\mathcal{D}$ are equivalent to the automorphisms of the colored undirected graph $\mathcal{G}$ ([Jabbour et al, ECAI'12]);
- Employ a general-purpose graph symmetry tool to uncover the symmetries [Mckay'81, Aloul'03].

Symmetry Detection in Transaction Databases: Example



t_i	itemset			
001	A,	B,	E,	F,
002	A,	B,	C,	D
003	C,	D,	E,	F
004	A,	C		
005	A,	E		
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Outline

How to exploit symmetries in itemset mining?

1. By rewriting the transaction databases in a preprocessing step (items elimination). [Jabbour et al, ECAI'12]
   - $\rightarrow$ New transaction database D' + symmetry group S.
   - $\rightarrow$ Condensed representation of the output.
2. By dynamic integration in Apriori-like algorithms for search space pruning.

Symmetry-Based Pruning in Apriori-like Algos

- Let $\mathcal{D}$ be a transaction database such that $\mathcal{I}(\mathcal{D}) = \{A, B, C, D\}$ and σ is a symmetry such that σ=$(A, D)(B, C)$.
- Assume that the itemsets $\{A\}$, $\{B\}$, $\{C\}$ and $\{D\}$ are frequent. We also assume that in iteration 2, we find that the itemset $\{A, B\}$ is not frequent.

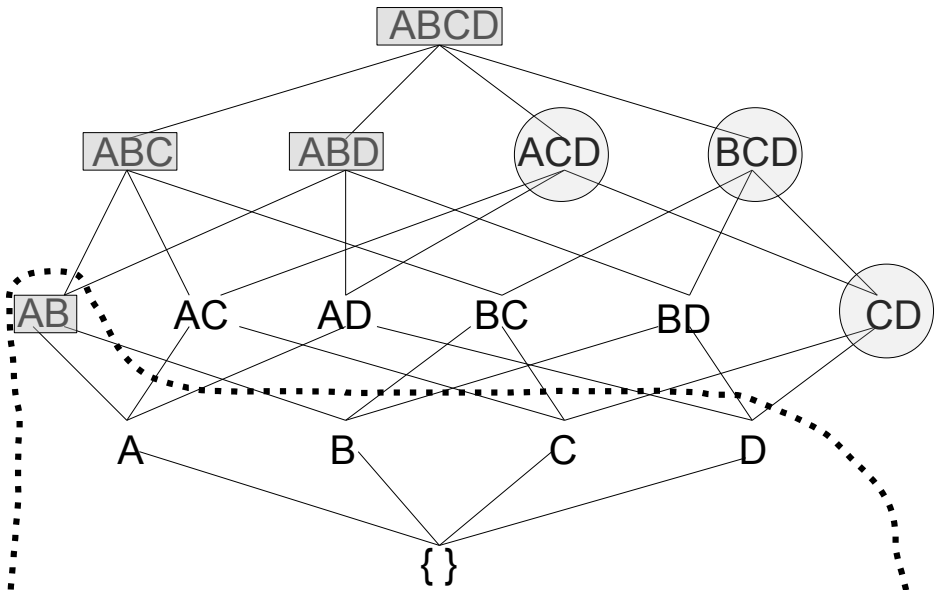


Figure : Symmetry Pruning

Symmetry Breaking

Let $\mathcal{D}$ a transaction database and $\sigma = (a, b)(c, d)$ a symmetry

$\mathcal{FIM}(\mathcal{D}, \lambda)$=$\{a, \dots\}$, $\{b, \dots\}$, $\{c, \dots\}$, $\{d, \dots\}$, $\{a, b, \dots\}$, $\{a, c, \dots\}$, $\{a, d, \dots\}$, $\{b, c, \dots\}$, $\{b, d, \dots\}$, $\{c, d, \dots\}$

$$\begin{array}{lcl} \{a, \dots\} & \rightarrow & \{b, \dots\} \\ \{a, d, \dots\} & \rightarrow & \{b, c, \dots\} \\ \{a, c, \dots\} & \rightarrow & \{b, d, \dots\} \\ \{a, b, \dots\} & \rightarrow & \{a, b, \dots\} \end{array} \qquad \begin{array}{lcl} \{b, \dots\} & \rightarrow & \{a, \dots\} \\ \{b, c, \dots\} & \rightarrow & \{a, d, \dots\} \\ \{d, \dots\} & \rightarrow & \{c, \dots\} \\ \{b, d, \dots\} & \rightarrow & \{a, c, \dots\} \end{array}$$

Symmetry Breaking

Let $\mathcal{D}$ a transaction database and $\sigma = (a, b)(c, d)$ a symmetry

$\mathcal{FIM}(\mathcal{D}, \lambda)$=$\{a, \dots\}$, $\{b, \dots\}$, $\{c, \dots\}$, $\{d, \dots\}$, $\{a, b, \dots\}$, $\{a, c, \dots\}$, $\{a, d, \dots\}$, $\{b, c, \dots\}$, $\{b, d, \dots\}$, $\{c, d, \dots\}$

$$
\begin{array}{lcl}
\{a, \dots\} & \rightarrow & \{b, \dots\} \\
\{a, d, \dots\} & \rightarrow & \{b, c, \dots\} \\
\{a, c, \dots\} & \rightarrow & \{b, d, \dots\} \\
\{a, b, \dots\} & \rightarrow & \{a, b, \dots\}
\end{array}
\qquad
\begin{array}{lcl}
\{b, \dots\} & \rightarrow & \{a, \dots\} \\
\{b, c, \dots\} & \rightarrow & \{a, d, \dots\} \\
\{d, \dots\} & \rightarrow & \{c, \dots\} \\
\{b, d, \dots\} & \rightarrow & \{a, c, \dots\}
\end{array}
$$

Symmetry Breaking

Let $\mathcal{D}$ a transaction database and $\sigma = (a, b)(c, d)$ a symmetry

$\mathcal{FIM}(\mathcal{D}, \lambda)$=$\{a, \ldots\}$, $\{b, \ldots\}$, $\{c, \ldots\}$, $\{d, \ldots\}$, $\{a, b, \ldots\}$, $\{a, c, \ldots\}$, $\{a, d, \ldots\}$, $\{b, c, \ldots\}$, $\{b, d, \ldots\}$, $\{c, d, \ldots\}$ + σ

$\{a, \ldots\}$	$\rightarrow$	$\{b, \ldots\}$	$\{b, \ldots\}$	$\rightarrow$	$\{a, \ldots\}$
$\{a, d, \ldots\}$	$\rightarrow$	$\{b, c, \ldots\}$	$\{b, c, \ldots\}$	$\rightarrow$	$\{a, d, \ldots\}$
$\{a, c, \ldots\}$	$\rightarrow$	$\{b, d, \ldots\}$	$\{d, \ldots\}$	$\rightarrow$	$\{c, \ldots\}$
$\{a, b, \ldots\}$	$\rightarrow$	$\{a, b, \ldots\}$	$\{b, d, \ldots\}$	$\rightarrow$	$\{a, c, \ldots\}$

- $\Rightarrow$ b can be removed from each $T \in \mathcal{D}$ if $\{a, b\} \not\subset T$
- $\Rightarrow$ d can be removed from each $T \in \mathcal{D}$ if $\{a, d\} \not\subset T$ and $\{c, d\} \not\subset T$

Symmetry Breaking

Proposition

Let $\mathcal{D}$ a transaction database and

$\sigma = (x_1, y_1)(x_2, y_2) \dots (x_j, y_j) \dots (x_n, y_n)$ a symmetry

$\Rightarrow y_j$ can be removed from each $T \in \mathcal{D}$ if $\{x_i, y_j\} \not\subset T, \forall i \leqslant j$

Remark

Symmetries can be broken independently

Symmetry Breaking

t_i	itemset			
001	A,	B,	E,	F,
002	A,	B,	C,	D
003	C,	D,	E,	F
004	A,	C		
005	A,	E		
006	C,	E		
007	B,	D		
008	B,	F		
009	D,	F		

σ_1 = (A C)(B, D)
σ_2 = (A B)(C, D) (E F)
σ_3 = (C,E)(D,F)

t_i	itemset			
001	A,	B,	E,	F
002	A,	B,	C,	D
003	C	D	E	F
004	A,	C,		
005	A,	E,		
006	C	E		
007	B	D		
008	B	F		
009	D	F		

Table : Symmetry Breaking approach

Outline

Propositional logic

- Classical propositional logic:

$$A ::= p \mid \neg A \mid A \wedge A \mid A \vee A \mid A \to A$$

- De Morgan laws:

$$A \vee B \equiv \neg(\neg A \wedge \neg B) \quad A \wedge B \equiv \neg(\neg A \vee \neg B)$$
$$A \to B \equiv \neg A \vee B \qquad \neg\neg A \equiv A$$

- Boolean interpretation:
  - $[\![\cdot]\!] : \text{Prop} \to \{0, 1\}$
  - Extension to formulae: $[\![\neg A]\!] = 1 - [\![A]\!]$, $[\![A \wedge B]\!] = min([\![A]\!], [\![B]\!])$

- Satisfiability: $\exists [\![\cdot]\!], [\![A]\!] = 1$ (NP-complete [Cook 71])

Conjunctive Normal Form (CNF) and SAT

- A conjunction of clauses:

$$\overbrace{(x_1 \vee \cdots \vee x_l)}^{clause} \wedge (y_1 \vee \cdots \vee y_m) \wedge (z_1 \vee \cdots \vee z_n) \cdots$$

- Clause: a disjunction of literals (p, $\neg p$)

- Example :

$$(p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee s) \wedge p \wedge (r \vee \neg s)$$

Conjunctive Normal Form (CNF) and SAT

- A conjunction of clauses:

$$\overbrace{(x_1 \vee \cdots \vee x_l)}^{clause} \wedge (y_1 \vee \cdots \vee y_m) \wedge (z_1 \vee \cdots \vee z_n) \cdots$$

- Clause: a disjunction of literals (p, $\neg p$)

- Example :

$$\overbrace{(p \vee \neg q \vee \neg r)}^{1} \wedge \overbrace{(p \vee \neg q \vee s)}^{1} \wedge \overbrace{p}^{1} \wedge (r \vee \neg s)$$

$$[\![p]\!] = 1$$

Conjunctive Normal Form (CNF) and SAT

- A conjunction of clauses:

$$\overbrace{(x_1 \vee \cdots \vee x_l)}^{clause} \wedge (y_1 \vee \cdots \vee y_m) \wedge (z_1 \vee \cdots \vee z_n) \cdots$$

- Clause: a disjunction of literals (p, $\neg p$)

- Example :

$$\overbrace{(p \vee \neg q \vee \neg r)}^{1} \wedge \overbrace{(p \vee \neg q \vee s)}^{1} \wedge \overbrace{p}^{1} \wedge \overbrace{(r \vee \neg s)}^{1}$$

$[\![p]\!] = 1$ et $[\![r]\!] = 1$ (Partial interpretation)

Transformation - Extension principle [G. Tseitin 1965]

- Introduce new variables to represent truth value of sub-formulae
- Example : DNF $\longrightarrow$ CNF

$$(x_1 \wedge y_1) \vee (x_2 \wedge y_2) \vee \cdots \vee (x_n \wedge y_n)$$

- Nave approach: 2^n clauses and $n \times 2^n$ literals
  $(x_1 \vee \cdots \vee x_{n-1} \vee x_n) \wedge (x_1 \vee \cdots \vee x_{n-1} \vee y_n) \wedge \cdots \wedge$
  $(y_1 \vee \cdots \vee y_{n-1} \vee y_n)$
- Tseitin approach: $2 \times n + 1$ clauses and $n + 2 \times 2 \times n$ literals
  $(z_1 \vee \cdots \vee z_n) \wedge (\neg z_1 \vee x_1) \wedge (\neg z_1 \vee y_1) \wedge \cdots \wedge (\neg z_n \vee x_n) \wedge$
  $(\neg z_n \vee y_n)$

Extended resolution proof system [G. Tseitin 1965]

- Extended resolution:

$$\frac{l \vee \alpha \in \Sigma \qquad \bar{l} \vee \beta \in \Sigma}{\alpha \vee \beta} \; [Res]$$

$$\text{Extension : } x \leftrightarrow F$$

- Shorten resolution proofs

- Open question: automatization of extended resolution proof systems ?

Modeling in SAT

- Knowledge representation using CNF formulae

		6	1		2	5		
	3	9				1	4	
				4				
9		2		3		4		1
	8						7	
1		3		6		8		9
				1				
	5	4				9	1	
		7	5		3	2		

8	4	6	1	7	2	5	9	3
1	3	9	6	5	8	1	4	2
5	2	1	3	4	9	7	6	8
9	6	2	8	3	7	4	5	1
4	8	5	9	2	1	3	7	6
1	7	3	4	6	5	8	2	9
2	9	8	7	1	4	6	3	5
3	5	4	2	8	6	9	1	7
6	1	7	5	9	3	2	8	4

- Example : $n \times n$ Sudoku
  - Associate to each cell, n propositional variables
  - Each cell contains at least one value:
    $\bigwedge_{l=1}^{n} \bigwedge_{c=1}^{n} (\bigvee_{v=1}^{n} p_{(l,c,v)}) \quad \Longrightarrow \quad n^2$ clauses of size n
- Leads usually to formulae of huge size

Modeling in SAT: an example from formal verification

Name of the CNF instance : post-cbmc-zfcp-2.8-u2.cnf (BMC)
p cnf **11 483 525** (vars) **32 697 150** (clauses)
1 -3 0
2 -3 0 $x_3 = x_1 \wedge x_2$
1 -2 3 0
. . . 1million pages later
-11482897 -11483041 -11483523 0
11482897 11483041 -11483523 0 $x_3 \leftrightarrow x_4 \leftrightarrow x_5$
11482897 -11483041 11483523 0
-11482897 11483041 11483523 0
-11483518 -11483524 0
-11483519 -11483524 0
-11483520 -11483524 0
-11483521 -11483524 0 $x_6 = (x_7 \wedge x_8 \wedge x_9 \wedge x_{10} \wedge x_{11} \wedge x_{12})$
-11483522 -11483524 0
-11483523 -11483524 0
11483518 11483519 11483520 11483521 11483522 11483523 11483524 0
-8590303 -11483524 -11483525 0
8590303 11483524 -11483525 0 $x_{13} \leftrightarrow x_{14} \leftrightarrow x_{15}$
8590303 -11483524 11483525 0
-8590303 11483524 11483525 0
-11483525 0

Frequent itemsets mining

- Transactions database

tid	itemset
001	*Joyce*, *Beckett*, *Proust*
002	*Faulkner*, *Hemingway*, *Melville*
003	*Joyce*, *Proust*
004	*Hemingway*, *Melville*
005	*Flaubert*, *Zola*
006	*Hemingway*, *Golding*

- Support: $\mathcal{S}(\{\textit{Hemingway}, \textit{Melville}\}, \mathcal{D}) = |\{002, 004\}| = 2$
- Enumerating frequent itemsets:
  $\mathcal{FIM}(\mathcal{D}, \lambda) = \{A \subseteq \mathcal{I} | \mathcal{S}(A, \mathcal{D}) \geqslant \lambda\}$
- Example: $\mathcal{FIM}(\mathcal{D}, 2) =$
  $\{\{\textit{Hemingway}\}, \{\textit{Melville}\}, \{\textit{Hemingway}, \textit{Melville}\}, \{\textit{Joyce}\},$
  $\{\textit{Proust}\}, \{\textit{Joyce}, \textit{Proust}\}\}$

Frequent itemsets mining

Condensed representations of frequent itemsets

- Maximal frequent itemsets:

$$Max(\mathcal{D}, \lambda) = \{A \in \mathcal{FIM}(\mathcal{D}, \lambda) | \forall B \supset A, B \notin \mathcal{FIM}(\mathcal{D}, \lambda)\}$$

- Closed frequent itemsets:

$$Cl(\mathcal{D}, \lambda) = \{A \in \mathcal{FIM}(\mathcal{D}, \lambda) | \forall B \supset A, \mathcal{S}(B, \mathcal{D}) \neq \mathcal{S}(A, \mathcal{D})\}$$

- Example :

$$Max(\mathcal{D}, 2) = \{\{Joyce, Proust\}, \{Hemingway, Melville\}\}$$

$$Cl(\mathcal{D}, 2) = \{\{Hemingway\}, \{Joyce, Proust\}, \{Hemingway, Melville$$

CNF formula as transactions database

- Goal : reduce the number of literals using the frequent sets of literals: similar to Tseitin approach (introduce new Boolean variables)
- Items: literals
- Transactions: clauses > 2

Reduce the number of literals

- Introduce new Boolean variables:

$$(x_1 \vee \cdots \vee x_n \vee \alpha_1) \wedge \cdots \wedge (x_1 \vee \cdots \vee x_n \vee \alpha_k)$$

equivalent w.r.t. *SAT*

$$\Longrightarrow$$

$$(y \vee \alpha_1) \wedge \cdots \wedge (y \vee \alpha_k) \wedge (x_1 \vee \cdots \vee x_n \vee \neg y)$$

- $n \geqslant 2$ et $k > \frac{n+1}{n-1}$

- $n \times k$ literals substituted by $k + n + 1$ literals
- Quorum : $k \begin{cases} \geqslant 4 & \text{si } n = 2 \\ \geqslant 3 & \text{si } n = 3 \\ \geqslant 2 & \text{otherwise} \end{cases}$
- Not interesting to associate new variables to subsets of $\{x_1, \ldots, x_n\}$: use of condensed representation

Closed Vs. Maximal

- Maximal $\subseteq$ Closed : more informations with closed

$$(x_1 \vee \ldots \vee x_k \vee \ldots \vee x_n \vee \alpha_1) \wedge \cdots \wedge (x_1 \vee \ldots \vee x_k \vee \ldots \vee x_n \vee \alpha_m) \wedge$$
$$(x_1 \vee \ldots \vee x_k \vee \beta_1) \wedge \cdots \wedge (x_1 \vee \ldots \vee x_k \vee \beta_{m'})$$

with $k \geqslant 2$ and $m, m' \geqslant 4$
we suppose that the set of itemsets are frequent
$\mathcal{P}(\{x_1, \ldots, x_n\})$
$\Rightarrow$ Max $= \{\{x_1, \ldots, x_n\}\}$ and closed
$= \{\{x_1, \ldots, x_k\}, \{x_1, \ldots, x_n\}\}$

- Use of $\{x_1, \ldots, x_n\}$:

$$(y \vee \alpha_1) \wedge \cdots \wedge (y \vee \alpha_m) \wedge$$
$$(x_1 \vee \ldots \vee x_k \vee \beta_1) \wedge \cdots \wedge (x_1 \vee \ldots \vee x_k \vee \beta_{m'}) \wedge$$
$$(x_1 \vee \ldots \vee x_n \vee \neg y)$$

- Use of $\{x_1, \ldots, x_k\}$:

$$(y \vee \alpha_1) \wedge \cdots \wedge (y \vee \alpha_m) \wedge$$
$$(z \vee \beta_1) \wedge \cdots \wedge (z \vee \beta_{m'}) \wedge$$
$$(z \vee x_{k+1} \vee \ldots \vee x_n \vee \neg y) \wedge (x_1 \vee \ldots \vee x_k \vee \neg z) \ldots$$

Subsets

- X and Y ($Y \subset X$) are both interesting if

$$\mathcal{S}(Y) - \mathcal{S}(X) > \frac{|Y|+1}{|Y|-1} - 1$$

- The best:
  - X if
    $|X| \times \mathcal{S}(X) - (\mathcal{S}(X) + |X| + 1) \geqslant |Y| \times \mathcal{S}(Y) - (\mathcal{S}(Y) + |Y| + 1)$
  - Y otherwise

- Associates a weight to frequent itemsets:

$$|X| \times \mathcal{S}(X) - (\mathcal{S}(X) + |X| + 1)$$

Overlaps

- X overlaps with Y ($X \sim Y$): $X \cap Y \neq \emptyset$
- overlaps class (Overlap class) : an equivalence class (transitive closure of $\sim$)

$$Y \in [X] \;\; ssi \;\; Y = Y_1 \sim Y_2 \sim \ldots \sim Y_k = X$$

- Optimal solution $\longrightarrow$ optimal solution in an overlaps class

Overlap

- Problem :
  - $\{x_1, x_2, x_3\}$ et $\{x_2, x_3, x_4\}$ two frequents itemsets s.t. $\mathcal{S}(\{x_1, x_2, x_3\}) = 3$, $\mathcal{S}(\{x_2, x_3, x_4\}) = 3$ and $\mathcal{S}(\{x_1, x_2, x_3, x_4\}) = 2$
  - Use of $\{x_1, x_2, x_3\} \longrightarrow \mathcal{S}(\{x_2, x_3, x_4\}) = 2$

- X and Y ($Y \sim X$) are both interesting if
  - $\mathcal{S}(X) - \mathcal{S}(X \cup Y) > \frac{|X|+1}{|X|-1} - 1$,
  - $\mathcal{S}(Y) - \mathcal{S}(X \cup Y) > \frac{|Y|+1}{|Y|-1} - 1$, or
  - $|X \setminus Y| \geqslant k$ (resp. $|Y \setminus X| \geqslant k$) where
    - $k = 2$ if $\mathcal{S}(X) \geqslant 4$ (resp. $\mathcal{S}(Y) \geqslant 4$)
    - $k = 3$ if $\mathcal{S}(X) = 3$ (resp. $\mathcal{S}(Y) = 3$)
    - $k = 4$ otherwise

    $\Rightarrow$ Use of $X \setminus Y$ (resp. $Y \setminus X$)

Problems summary

- Choose of the quorum ?
  - 2 $\longrightarrow$ lot of useless itemsets
  - 3 and 4 $\longrightarrow$ loss of interesting itemsets
- Overlap (subsets) :
  - Simplification: overlaps classes
  - Need to compute $\mathcal{S}(X \cup Y)$
  - Optimal solution in one class?
- A gready algorithm: loss of interesting itemsets

Gready Algorithm

Require: A formula ϕ, an overlap class of closed frequent itemsets C

1: **while** $C \neq \emptyset$ **do**
2: $I \leftarrow C.MostInterstingElement()$;
3: $\phi.replace(I, x)$;
4: $\phi.Add(I, x)$:
5: $C.remove(I)$;
6: $C.replaceSubset(I, x)$;
7: $C.removeUninterestingElements()$;
8: $C.updateSupports()$;
9: **end while**
10: **return** ϕ

Experiments: Industrial SAT instances

Instance	orig.	comp.	% red
1dlx_c_iq57_a	190 Mb	164 Mb	13.68 %
6pipe_6_ooo.*-as.sat03-413	11 Mb	7.7 Mb	30.00 %
9dlx_vliw_at_b_iq6.*-*04-347	76 Mb	65 Mb	14.47 %
abb313GPIA-9-c.*.sat04-317	21 Mb	6.9 Mb	67.14 %
E05F18	3.7 Mb	2.2 Mb	40.54 %
eq.atree.braun.11.unsat	120 Kb	72 Kb	40.00 %
eq.atree.braun.12.unsat	144 Kb	88 Kb	38.88 %
k2mul.miter.*-as.sat03-355	1.5 Mb	1.3 Mb	13.33 %
korf-15	1.2 Mb	752 Kb	37.33 %
rbcl_xits_08_UNSAT	1.1 Mb	856 Kb	22.18 %
SAT_dat.k45	3.5 Mb	2.6 Mb	25.71 %
traffic_b_unsat	18 Mb	12 Mb	33.33 %
x1mul.miter.*-as.sat03-359	1.1 Mb	928 Kb	15.63 %
9dlx_vliw_at_b_iq3	19 Mb	15 Mb	21.05 %
9dlx_vliw_at_b_iq4	31 Mb	26 Mb	16.12 %
AProVE07-09	2.8 Mb	2.7 Mb	3.57 %
eq.atree.braun.10.unsat	96 Kb	56 Kb	41.66 %
goldb-heqc-frg1mul	348 Kb	328 Kb	5.74 %
minand128	7.7 Mb	2.6 Mb	66.23 %
ndhf_xits_09_UNSAT	2.6 Mb	2.1 Mb	19.23 %
velev-pipe-o-uns-1.1-6	5.5 Mb	4.4 Mb	20.00 %

Table : Results of Mining4SAT : a general approach

Application: A compact representation of 2-CNF

instance	#cls	#bin	(%) bin
velev-pipe-o-uns-1.1-6	304026	268354	88,26 %
9dlx_vliw_at_b_iq2	542253	500227	92,24 %
1dlx_c_iq57_a	8562505	7567948	88,38 %
7pipe_k	751116	722278	96,16 %
SAT_dat.k100.debugged	670701	523153	78,00 %
BM_FV_2004_rule_batch	445444	339588	76,23 %
sokoban-sequential-p145-*.040-*	1413816	1364160	96,48 %
openstacks-*-p30_1.085-*	1621926	1601145	98,71 %
aaai10-planning-ipc5-*-12-step16	1029036	991140	96,31 %
k2fix_gr_rcs_w8.shuffled	271393	270136	99,53 %
homer17.shuffled	1742	1716	98,50 %
gripper13u.shuffled-as.sat03-395	38965	35984	92,34 %
grid-strips-grid-y-3.045-*	2750755	2695230	97,98 %

Table : Ratio of binary clauses in some SAT instances

Application: A compact representation of 2-CNF

Example

Let us consider the following 2-CNF Φ:

$$\begin{aligned}\Phi = \quad & (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_1 \vee x_4) \wedge (x_1 \vee x_5) \quad \wedge \\ & (x_1 \vee x_6) \wedge (x_1 \vee x_7) \wedge (x_2 \vee x_3) \wedge (x_2 \vee x_4) \quad \wedge \\ & (x_2 \vee x_5) \wedge (x_2 \vee x_6) \wedge (x_2 \vee x_7) \wedge (x_3 \vee x_4) \quad \wedge \\ & (x_3 \vee x_6) \wedge (x_3 \vee x_7) \wedge (x_3 \vee x_5) \wedge (x_4 \vee x_5) \quad \wedge \\ & (x_4 \vee x_6) \wedge (x_4 \vee x_7) \wedge (x_5 \vee x_6) \wedge (x_5 \vee x_7) \quad \wedge \\ & (x_6 \vee x_7)\end{aligned}$$

Definition (B-implication)

A *B-implication* is a Boolean formula of the following from : $x \vee \beta(x)$ where $\beta(x)$ is a conjunction of literals.

Application: A compact representation of 2-CNF

Using the complete order relation $x_1 \prec \ldots \prec x_7$ over $\mathcal{L}_\Phi$ rewrite Φ as set of B-implications $B^1_{[\vee(\wedge)]}(\Phi)$:

$\{[x_1 \vee (x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7)],$
$[x_2 \vee (x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7)],$
$[x_3 \vee (x_4 \wedge x_5 \wedge x_6 \wedge x_7)],$
$[x_5 \vee (x_6 \wedge x_7)],$
$[x_6 \vee (x_7)]\}$

tid	itemset					
tid_{x_1}	x_2	x_3	x_4	x_5	x_6	x_7
tid_{x_2}		x_3	x_4	x_5	x_6	x_7
tid_{x_3}			x_4	x_5	x_6	x_7
tid_{x_4}				x_5	x_6	x_7
tid_{x_5}					x_6	x_7
tid_{x_6}						x_7

Application: A compact representation of sets of 2-CNF

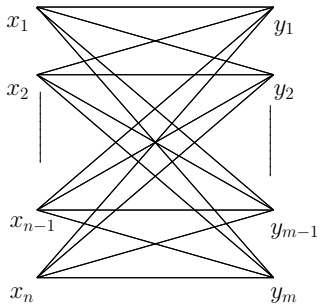
FIM process on the conjunctive part of $B^1_{\vee[\wedge]}(\Phi)$

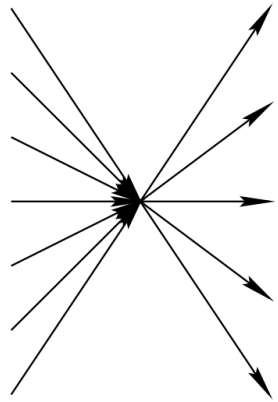
Using $\{x_5, x_6, x_7\}$ a 4-frequent itemset, we can rewrite $B^1_{[\vee(\wedge)]}(\Phi)$ as:

$$
\begin{aligned}
B^2_{\vee[\wedge]}(\Phi) = \quad & \{[x_1 \vee (x_2 \wedge x_3 \wedge y)] \,, \\
& [x_2 \vee (x_3 \wedge x_4 \wedge y)] \,, \\
& [x_3 \vee (x_4 \wedge y)] \,, \\
& [x_5 \vee (x_6 \wedge x_7)] \,, \\
& [x_6 \vee (x_7)] \,, \\
& [\neg y \vee (x_5 \wedge x_6 \wedge x_7)]\}
\end{aligned}
$$

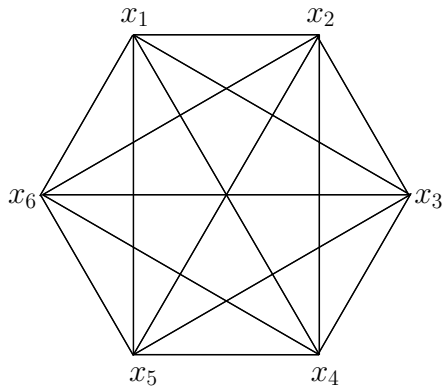
$$
\begin{aligned}
\mathrm{CNF}(B^2_{[\vee(\wedge)]}(\Phi)) = \\
& (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_1 \vee y) && \wedge \\
& (x_2 \vee x_3) \wedge (x_2 \vee x_4) \wedge (x_2 \vee y) && \wedge \\
& (x_3 \vee x_4) \wedge (x_3 \vee y) && \wedge \\
& (x_5 \vee x_6) \wedge (x_5 \vee x_7) && \wedge \\
& (x_6 \vee x_7) && \wedge \\
& (\neg y \vee x_5) \wedge (\neg y \vee x_6) \wedge (\neg y \vee x_7)
\end{aligned}
$$

Two particular cases: bi-cliques and cliques





$n \times m$ binary clauses $\Rightarrow$ $n + m$ binary clauses and 1 new variable



$\mathcal{O}(n^2)$ binary clauses $\Rightarrow$ $\mathcal{O}(n)$ binary clauses and $\mathcal{O}(n)$ new variables

More details on bi-cliques

Let $\Phi = [(x_1 \vee y_1) \wedge (x_1 \vee y_2) \wedge \cdots \wedge (x_1 \vee y_m)] \ldots [(x_n \vee y_1) \wedge (x_n \vee y_2) \wedge \cdots \wedge (x_n \vee y_m)]$

- Using a complete order relation defined by: $f(x_i) = i, f(y_j) = n + j$.
- $B_{[\vee(\wedge)]}(\Phi)$ corresponds exactly to $\{(x_i \vee [y_1 \wedge y_2 \wedge \cdots \wedge y_m]) | 1 \leqslant i \leqslant n\}$
- Using a single closed frequent itemset $\{y_1, y_2, \ldots, y_m\}$

$\Phi' = [\bigwedge_{1 \leqslant i \leqslant n}(x_i \vee z)] \wedge [\bigwedge_{1 \leqslant j \leqslant m}(\neg z \vee y_j)]$.

Experiments: Industrial SAT instances

Instance	orig.	comp.	% red
velev-pipe-o-uns-1.1-6	5.5 Mb	3.2 Mb	41.81 %
9dlx_vliw_at_b_iq2	11 Mb	6 Mb	44.45 %
1dlx_c_iq57_a	190 Mb	124 Mb	34.73 %
7pipe_k	14 Mb	5.4 Mb	61.42 %
SAT_dat.k100.debugged	16 Mb	13 Mb	18.75 %
IBM_FV_2004_rule_batch _2_31_1_SAT_dat.k80.debugged	9.7 Mb	7.5 Mb	22.68 %
sokoban-sequential-p145-*.040-*	24 Mb	14 Mb	41.66 %
openstacks-*-p30_1.085-*	30 Mb	26 Mb	13.33 %
aaai10-planning-ipc5-*-12-step16	17 Mb	12 Mb	29.41 %
k2fix_gr_rcs_w8.shuffled	3.4 Mb	1.7 Mb	50.00 %
homer17.shuffled	20 Kb	16 Kb	20.00 %
gripper13u.shuffled-as.sat03-395	524 Kb	364 Kb	30.35 %
grid-strips-grid-y-3.045-*	52 Mb	42 Mb	19.23 %

Table : Results of Mining4Binary: a 2-CNF approach

Application: A compact Graph Representation

For free, we can apply our approach for graphs.

- 2-CNF $\leftrightarrow$ graphs
- Adjacency lists $\leftrightarrow$ A set of B-implications
- $2 \rightarrow [4, 6, 8, 12] \leftrightarrow 2 \vee [4 \wedge 6 \wedge 8 \wedge 12]$

Compression as an Optimisation Problem

The compression problem can be formulated as an optimisation problem

Problem : $\mathcal{C}omp(\mathcal{F}, \mathcal{P})$

- **Input**: $\mathcal{F}$ a CNF formula, and $\mathcal{P}$ a set of patterns
- **Output**: a compressed formula $\mathcal{F}$ of minimal size using $\mathcal{P}$

Compression as an Optimisation Problem

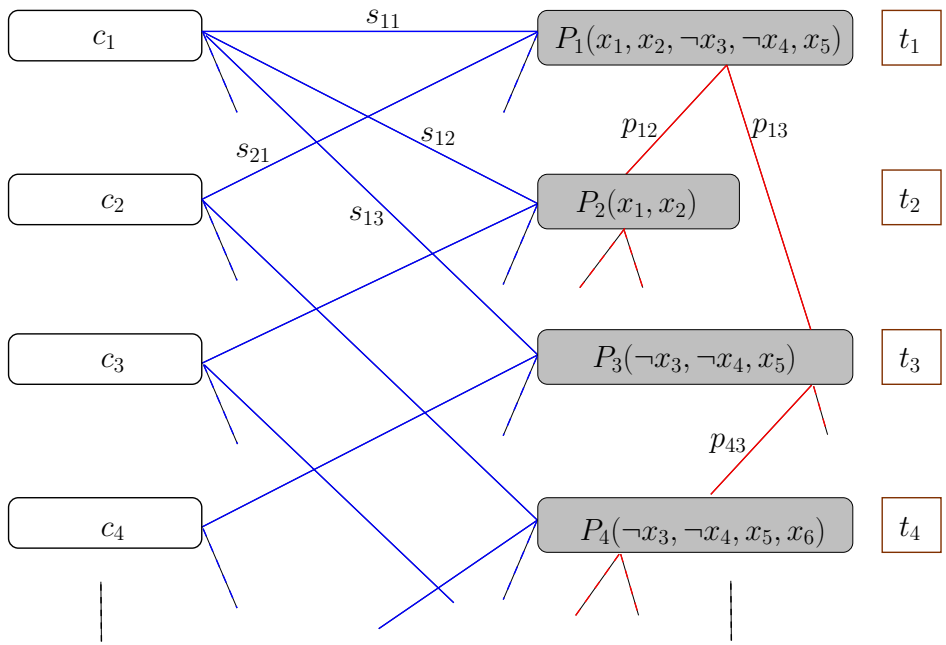
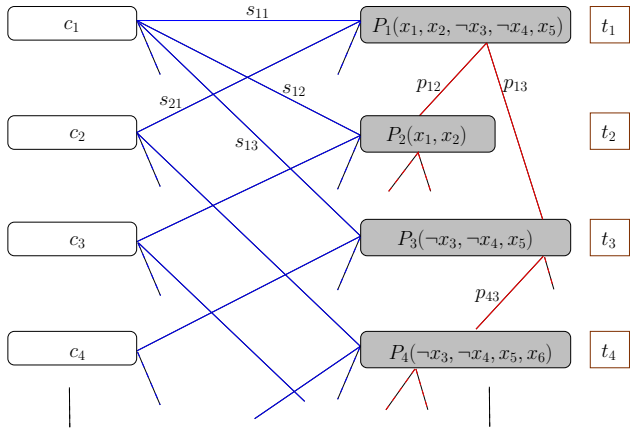


Figure : Compression using location problem

- If we use pattern P_j, we set t_j to 1, otherwise t_j is 0
- If we replace the literals in c_i by P_i, then we set s_{ij} to 1, and 0 otherwise.

Compression as an Optimisation Problem

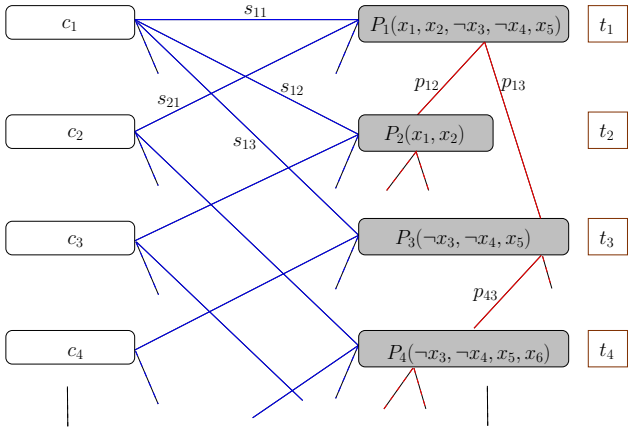


A **first formulation** as 0/1 linear program

$Max \sum(|P_j| - 1)s_{ij} - \sum(|P_j| + 1)t_j$ Maximize the reduction

1. $s_{ij} \leqslant t_j$ (C_i is replaced by P_j only when P_j is used)
2. $\sum_j s_{ij} \leqslant 1$ (C_i is replaced by only one pattern)
3. $s_{ij} \in \{0, 1\}$, $t_j \in \{0, 1\}$

Compression as an Optimisation Problem



A **second formulation** as 0/1 linear program

$Max \sum(|P_j| - 1)s_{ij} - \sum(|P_j| + 1)t_j$ Maximize the reduction

1. $s_{ij} \leqslant t_j$ (C_i is replaced by P_j only when P_j is used)
2. $s_{ij} + s_{ik} \leqslant 1$ if $P_j \cap P_k \neq \emptyset$ (C_i can be replaced by a set of disjoint patterns)
3. $s_{ij} \in \{0, 1\}$, $t_j \in \{0, 1\}$

Outline

Conclusions

- Theoretical foundations for discovering and using symmetries in itemset mining problems.
- Using symmetries to prune a search space.
- Integration of symmetry-based pruning in Apriori-like algorithms.

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- Theoretical foundations for discovering and using symmetries in itemset mining problems.
- Using symmetries to prune a search space.
- Integration of symmetry-based pruning in Apriori-like algorithms.

Futur works

- Extend the symmetry-based framework to other data mining algorithms and problems : sequence, tree or graph mining, etc.
- Investigate other forms of symmetries such as approximate symmetries.