

# Sixth Pseudo-Boolean Competition PB11

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- ▶ Pseudo-Boolean constraints
- ▶ PBS, PBO, WBO
- ▶ Benchmarks and Solvers
- ▶ Evaluation Environment
- ▶ Results

# Linear Pseudo-Boolean Constraints

- ▶ A **linear** pseudo-Boolean (PB) constraint may be defined over Boolean variables by

$$\sum_i a_i \cdot l_i \geq d \text{ with } a_i, d \in \mathbb{Z}, l_i \in \{x_i, \bar{x}_i\}, x_i \in \mathbb{B}$$

Example:  $3x_1 - 3x_2 + 2\bar{x}_3 + \bar{x}_4 + x_5 \geq 5$

- ▶ Extends both clauses and cardinality constraints
  - ▶ cardinalities: all  $a_i = 1$  and  $d > 1$
  - ▶ clauses: all  $a_i = 1$  and  $d = 1$
- ▶ PB constraints are more expressive than clauses (one PB constraint may replace an exponential number of clauses)
- ▶ A pseudo-Boolean instance is a conjunction of PB constraints

# Non-Linear Pseudo-Boolean Constraints

- ▶ A **non-linear** pseudo-Boolean constraint may be defined over Boolean variables by

$$\sum_i a_i \left( \prod_j l_{i,j} \right) \geq d \text{ with } a_i, d \in \mathbb{Z}, l_{i,j} \in \{x_{i,j}, \bar{x}_{i,j}\}, x_{i,j} \in \mathbb{B}$$

Example:  $3x_1\bar{x}_2 - 3x_2x_4 + 2\bar{x}_3 + \bar{x}_4 + x_5x_6x_7 \geq 5$

- ▶ A product is a AND
- ▶ Compact encoding for several problems (e.g. factoring problem encoded by one constraint)
- ▶ Can be easily translated into linear pseudo-Boolean by introducing new variables and constraints such that

$$p \leftrightarrow x_0 \wedge x_1 \wedge \dots \wedge x_n$$

(requires 2 PB constraints or n+1 clauses)

- ▶ **PBS (Pseudo Boolean Satisfaction)**  
decide of the satisfiability of a conjunction of PB constraints
- ▶ **PBO (Pseudo Boolean Optimization)**  
find a model of a conjunction of PB constraints which optimizes one objective function

$$\begin{cases} \text{minimize} & f = \sum_i c_i \cdot x_i \text{ with } c_i \in \mathbb{Z}, x_i \in \mathbb{B} \\ \text{subject to} & \text{the conjunction of constraints} \end{cases}$$

## WBO (Weighted Boolean Optimization)

- ▶ generalization of maximum satisfiability for PB constraints
- ▶ hard constraints **must** be satisfied
- ▶ soft constraints may be violated, but this has a cost
- ▶ the cost of an interpretation is the sum of the costs of violated soft constraints
- ▶ as in WCSP, there is a top cost. Interpretations with a cost greater or equal to the top cost are non admissible.
- ▶ the goal is to find an admissible interpretation with the smallest cost
- ▶ to avoid any intersection with the Max-SAT competition, at least one constraint must not be a clause.

# Benchmark categories (1)

For PBS/PBO, classification based on the objective function

**DEC** No objective function to optimize (decision problem). The solver must simply find a solution.

**OPT** An objective function is present. The solver must find a solution with the best possible value of the objective function.

For WBO, classification based on the existence of hard clauses

**SOFT** No hard clause at all.

**PARTIAL** At least one hard clause.

## Benchmark categories (2)

Classification based on the size of coefficients

**SMALLINT** small integers: no constraint with a sum of coefficients greater than  $2^{20}$  (20 bits): expected to be safe for solvers using 32 bits integers and simple techniques (be careful with learning), but strong limit to the encoding of concrete problems.

**BIGINT** big integers: at least one constraint with a sum of coefficients greater than  $2^{20}$  (20 bits): requires arbitrary precision.

Classification based on the linearity of constraints

**LIN** All constraints are linear

**NLC** At least one constraint is non linear (contains products of literals)

# Instances submitted this year

## PBS-PBO

- ▶ instances of MIPLIB 2010 (S. Heinz and M. Winkler)
  - 4 DEC-SMALLINT-LIN instances, all selected
  - 57 OPT-SMALLINT-LIN instances, 25 selected randomly
  - 27 OPT-BIGINT-LIN instances, 25 selected randomly
- ▶ AES minimum components benchmarks (O. Kullmann and M. Gwynne)
  - 7 OPT-SMALLINT-LIN instances, all selected
- ▶ Multiple Constant Multiplication problem (N. Lopes)
  - 193 DEC-SMALLINT-LIN, 25 selected randomly
- ▶ haplotyping with pedigrees (HwP) (A. Graça, I. Lynce, J. Marques-Silva)
  - 100 instances unfortunately forgotten in the selection!*

## WBO

- ▶ no submission at all !! (second year w/o submission)

## Submitted solvers:

- ▶ PBS/PBO: 8 different solvers, 12 versions by 6 different teams
- ▶ WBO: 4 solvers by 4 teams
- ▶ only two solvers with support for BIGINT

## Selected instances:

- ▶ PBS/PBO: same as PB10 + selection of new benchmarks
- ▶ WBO: same as PB10

- ▶ **DEC-SMALLINT-LIN (481 instances)**
- ▶ **DEC-SMALLINT-NLC (100 instances)**
- ▶ DEC-BIGINT-LIN
- ▶ DEC-BIGINT-NLC
- ▶ **OPT-SMALLINT-LIN (731 instances)**
- ▶ **OPT-SMALLINT-NLC (409 instances)**
- ▶ **OPT-BIGINT-LIN (557 instances)**
- ▶ OPT-BIGINT-NLC
- ▶ **PARTIAL-SMALLINT-LIN (536 instances)**
- ▶ PARTIAL-BIGINT-LIN (263 instances)
- ▶ **SOFT-SMALLINT-LIN (201 instances)**
- ▶ SOFT-BIGINT-LIN (46 instances)

*kindly provided by CRIL, University of Artois, France*

Same environment as the SAT competition

- ▶ Cluster of bi-Xeon quad-core 2.66 GHz, 8 MB cache, 32 GB RAM
- ▶ Each solver was given a time limit of 30 minutes (1800s) and a memory limit of 15500 MB (to avoid swapping).
- ▶ 2 solvers per node (limited interactions because of the 2 CPU and the memory limit)

# Verification of results

- ▶ The environment performs the following, efficient checks:
  - ▶ for SATISFIABLE answers, solvers must output a complete instantiation and the system checks that it satisfies all constraints
  - ▶ for UNSATISFIABLE answers, the system only checks that no other solver proved satisfiability
  - ▶ for OPTIMUM FOUND answers, solvers must output a complete instantiation; the system checks if all constraints are satisfied and that no other solver found a better solution
- ▶ UNSATISFIABLE and OPTIMUM FOUND answers cannot be completely checked efficiently and therefore should be taken with caution.
- ▶ Solvers giving a wrong answer in a category are disqualified in that category.

# Ranking of solvers and Virtual Best Solver (VBS)

Ranking based on two criteria:

1. the number of solved instances
2. ties are broken by considering the cumulated time on solved instances

The Virtual Best Solver (VBS)

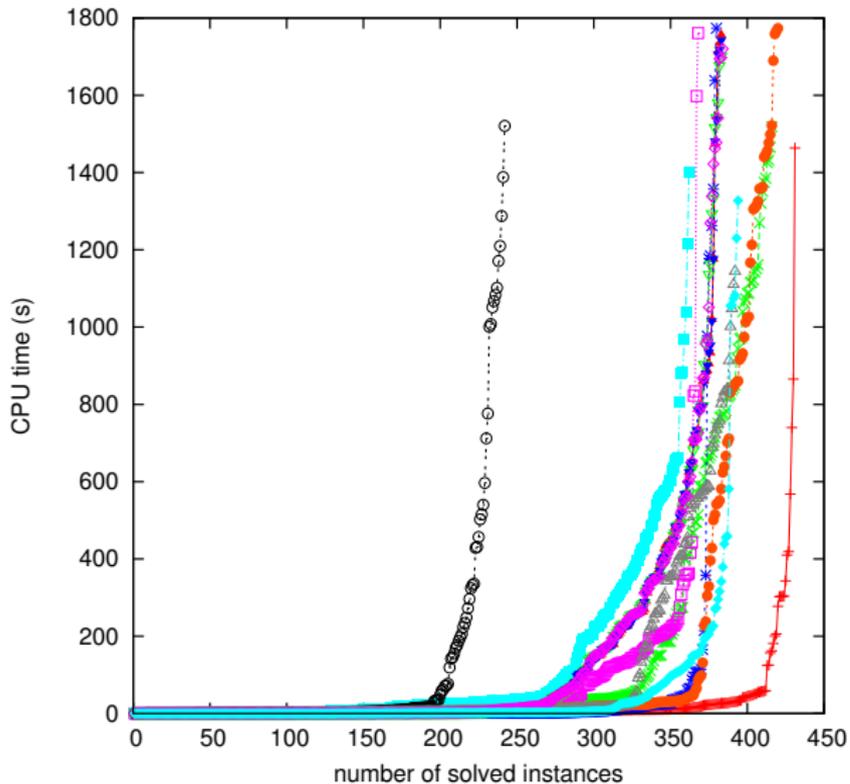
- ▶ is the virtual solver obtained by combining the best results of all submitted solvers.
- ▶ could be obtained by running in parallel all submitted solvers
- ▶ represents the current state of the art (SOTA)
- ▶ is a reference for the evaluation of the other solvers

# Results for DEC-SMALLINT-LIN

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 481</i>					
<i>Virtual Best Solver (VBS)</i>		448	191 S, 257 U	93%	100%
1	borg	431	183 S, 248 U	90%	96%
2	Sat4j Res//CP	420	183 S, 237 U	87%	94%
3	bsolo	416	179 S, 237 U	86%	93%
4	wbo	394	180 S, 214 U	82%	88%
5	Sat4j Res.	392	184 S, 208 U	81%	88%
6	SCIP spx E_2	384	149 S, 235 U	80%	86%
7	SCIP spx 2	383	148 S, 235 U	80%	85%
8	clasp	380	168 S, 212 U	79%	85%
9	MinisatID	368	169 S, 199 U	77%	82%
10	MinisatID <i>gmp</i>	362	165 S, 197 U	75%	81%
11	Sat4j CP	242	114 S, 128 U	50%	54%

# DEC-SMALLINT-LIN

Time to solve an instance  
(SAT/UNSAT answers, category DEC-SMALLINT-LIN)

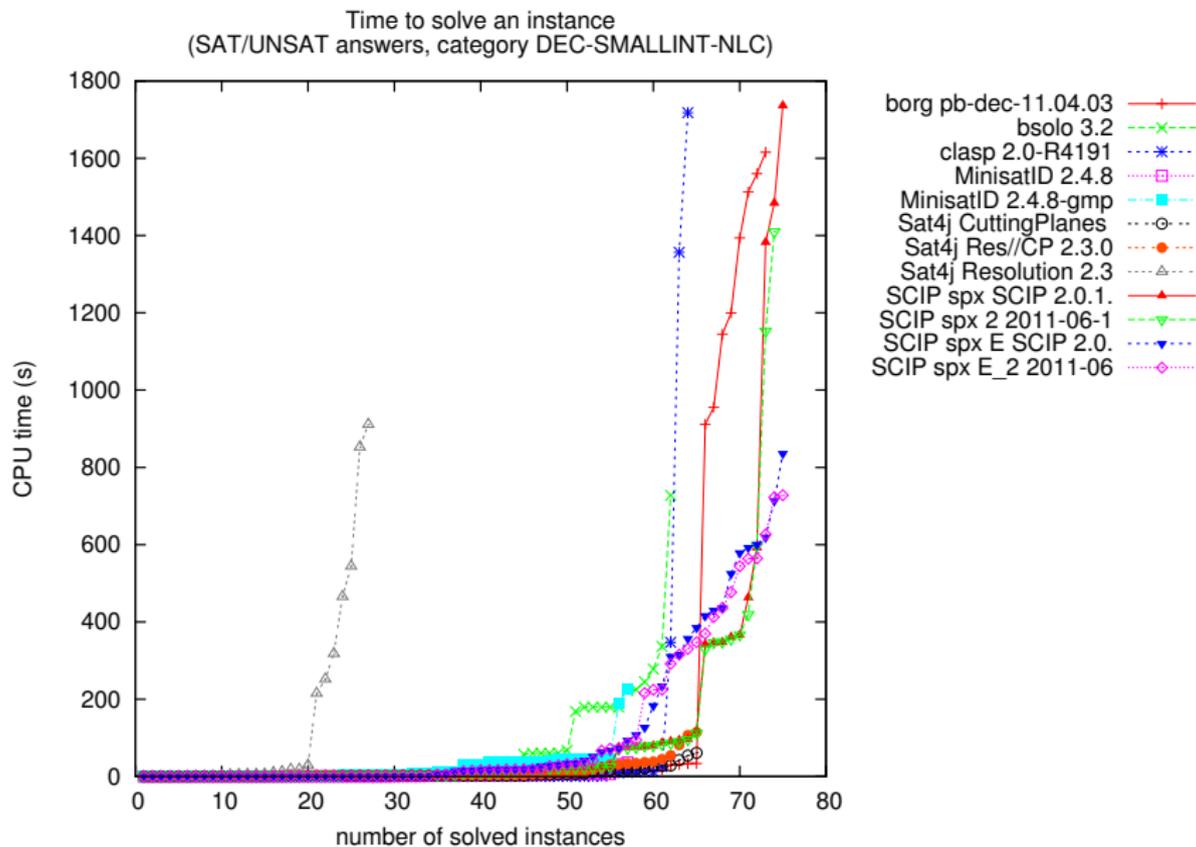


- borg pb-dec-11.04.03
- bsolo 3.2
- clasp 2.0-R4191
- MinisatID 2.5.2 (fix)
- MinisatID 2.5.2-gmp
- Sat4j CuttingPlanes
- Sat4j Res//CP 2.3.0
- Sat4j Resolution 2.3
- SCIP spx SCIP 2.0.1.
- SCIP spx 2 2011-06-1
- SCIP spx E SCIP 2.0.
- SCIP spx E\_2 2011-06
- wbo 1.6

# Results for DEC-SMALLINT-NLC

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 100</i>					
<i>Virtual Best Solver (VBS)</i>		76	55 S, 21 U	76%	100%
1	SCIP spx E_2	75	55 S, 20 U	75%	99%
2	SCIP spx 2	74	54 S, 20 U	74%	97%
3	borg	73	53 S, 20 U	73%	96%
4	Sat4j CP	65	50 S, 15 U	65%	86%
5	Sat4j Res//CP	65	50 S, 15 U	65%	86%
6	clasp	64	49 S, 15 U	64%	84%
7	bsolo	62	47 S, 15 U	62%	82%
8	Sat4j Res.	27	12 S, 15 U	27%	36%
9	MinisatID	0		0%	0%
10	MinisatID <i>gmp</i>	0		0%	0%

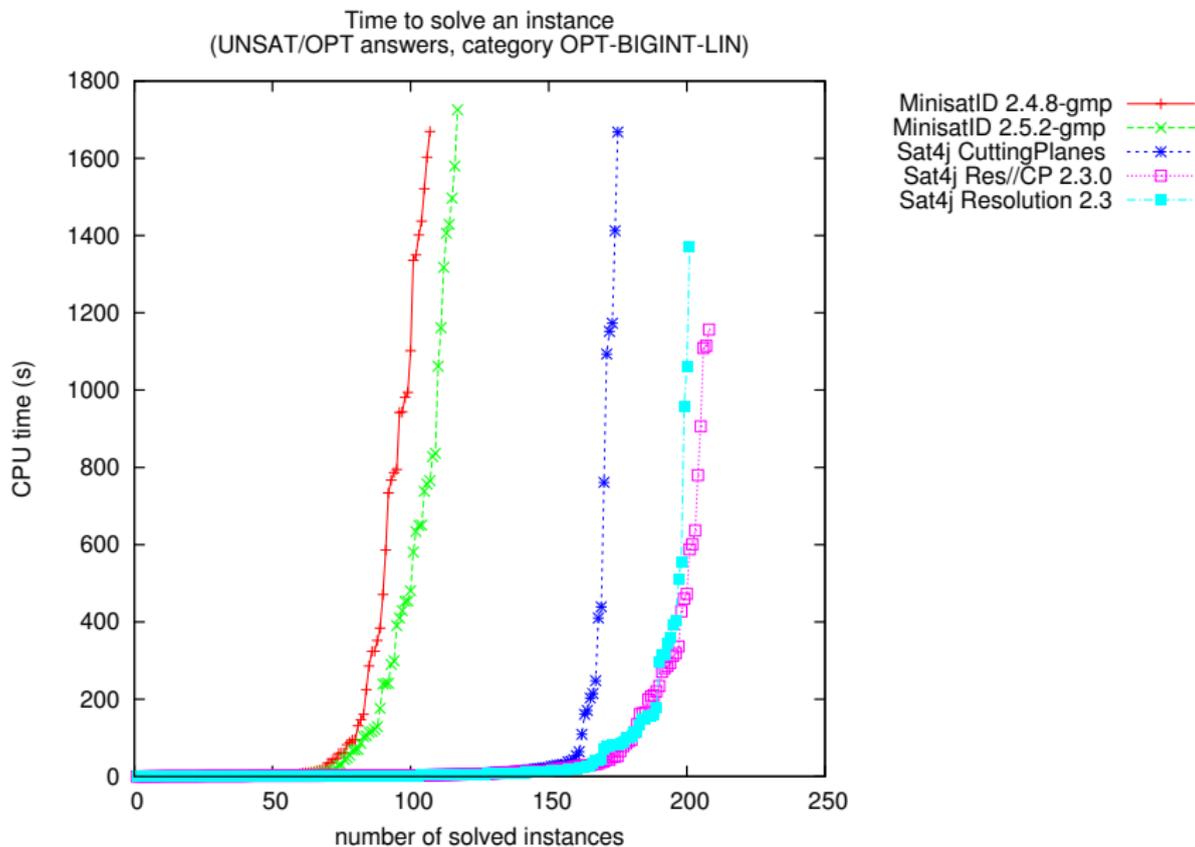
# DEC-SMALLINT-NLC



# Results for OPT-BIGINT-LIN

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 557</i>					
<i>Virtual Best Solver (VBS)</i>		213	154 OPT, 59 U	38%	100%
1	Sat4j Res//CP	208	149 OPT, 59 U	37%	98%
2	Sat4j Res.	201	144 OPT, 57 U	36%	94%
3	Sat4j CP	175	116 OPT, 59 U	31%	82%
4	MinisatID <i>gmp</i>	117	60 OPT, 57 U	21%	55%

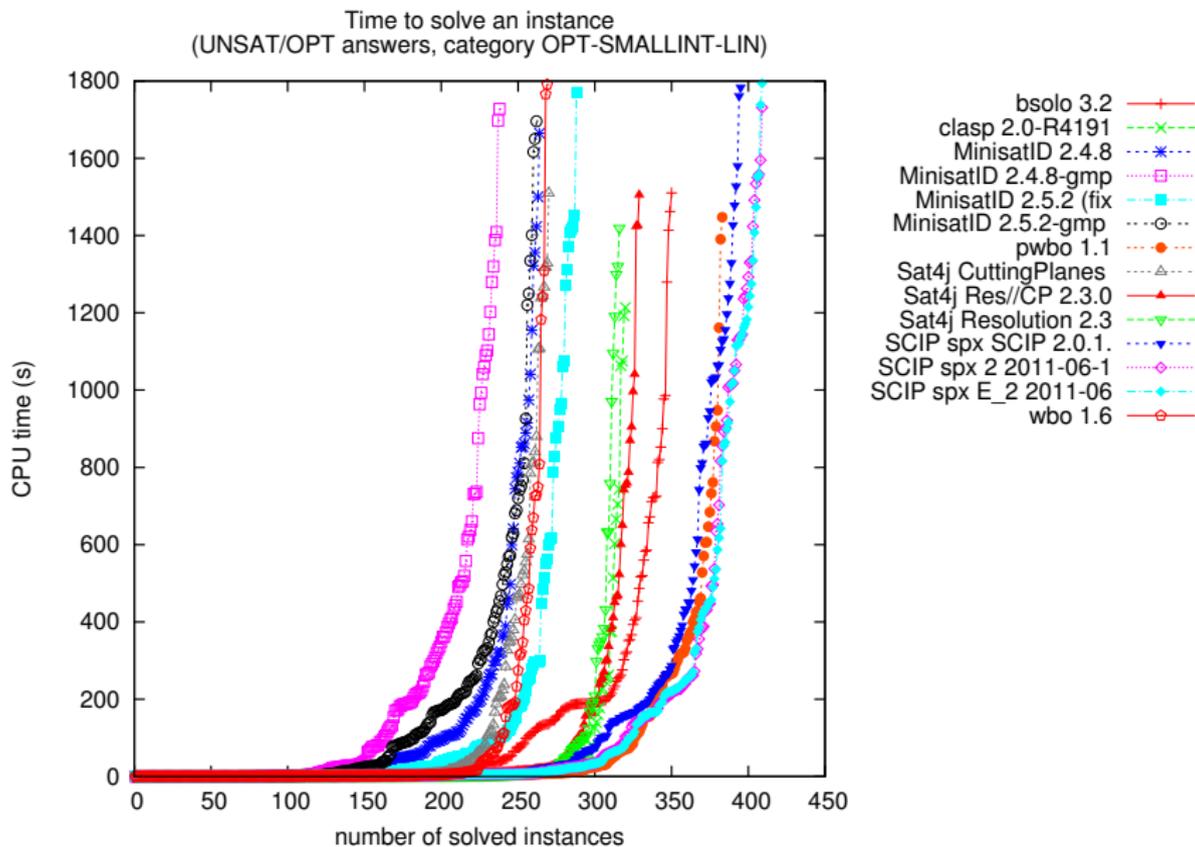
# OPT-BIGINT-LIN



# Results for OPT-SMALLINT-LIN

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 731</i>					
<i>Virtual Best Solver (VBS)</i>		494	459 OPT, 35 U	68%	100%
1	SCIP spx E.2	409	374 OPT, 35 U	56%	83%
2	SCIP spx 2	409	374 OPT, 35 U	56%	83%
3	pwbo	383	350 OPT, 33 U	52%	78%
4	bsolo	350	316 OPT, 34 U	48%	71%
5	Sat4j Res//CP	329	295 OPT, 34 U	45%	67%
6	clasp	320	286 OPT, 34 U	44%	65%
7	Sat4j Res.	316	282 OPT, 34 U	43%	64%
8	MinisatID	288	256 OPT, 32 U	39%	58%
9	Sat4j CP	270	240 OPT, 30 U	37%	55%
10	wbo	269	236 OPT, 33 U	37%	54%
11	MinisatID <i>gmp</i>	262	232 OPT, 30 U	36%	53%

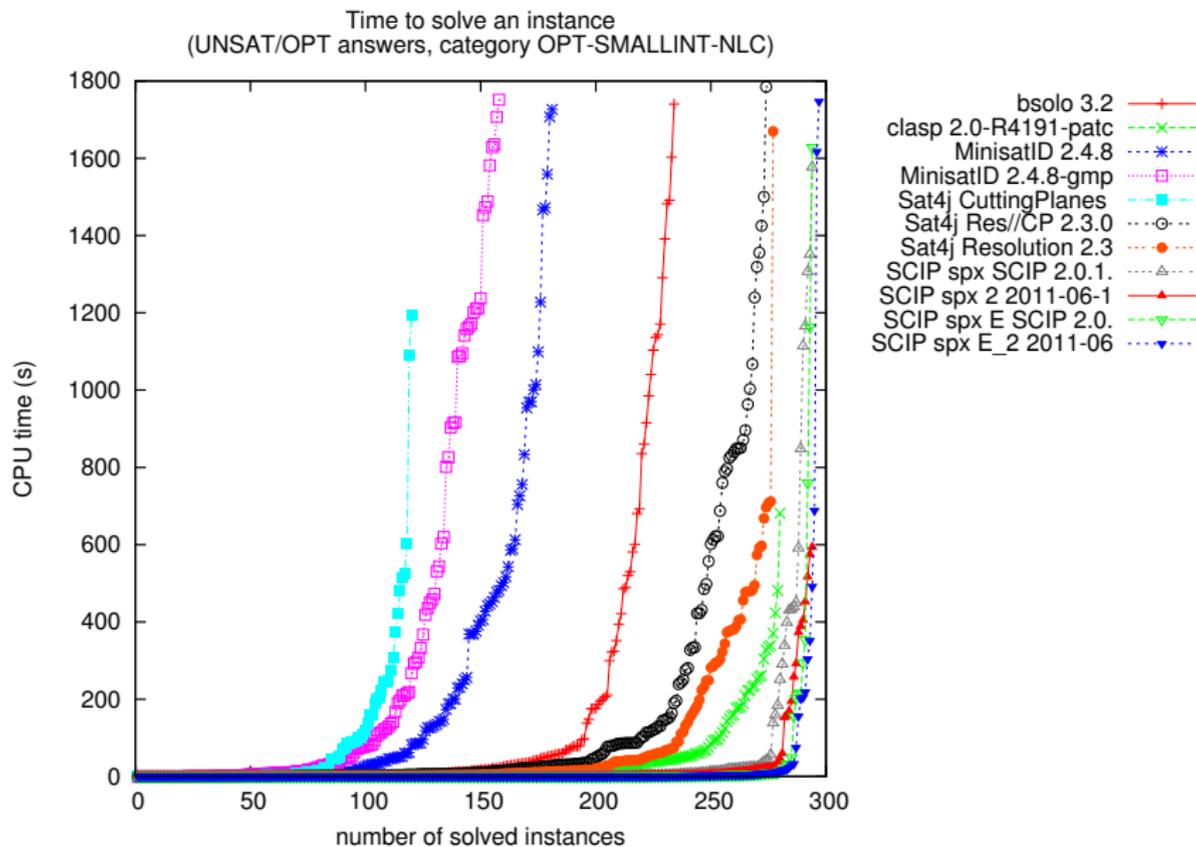
# OPT-SMALLINT-LIN



# Results for OPT-SMALLINT-NLC

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 409</i>					
<i>Virtual Best Solver (VBS)</i>		298	298 OPT	73%	100%
1	SCIP spx E_2	297	297 OPT	73%	100%
2	SCIP spx 2	294	294 OPT	72%	99%
3	clasp	280	280 OPT	68%	94%
4	Sat4j Res.	277	277 OPT	68%	93%
5	Sat4j Res//CP	274	274 OPT	67%	92%
6	bsolo	234	234 OPT	57%	79%
7	Sat4j CP	120	120 OPT	29%	40%
8	MinisatID	0		0%	0%
9	MinisatID <i>gmp</i>	0		0%	0%

# OPT-SMALLINT-NLC



# Results for PARTIAL-SMALLINT-LIN

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 536</i>					
<i>Virtual Best Solver (VBS)</i>		530	529 MOPT, 1 U	99%	100%
1	clasp	455	454 MOPT, 1 U	85%	86%
2	Sat4j Res.	448	447 MOPT, 1 U	84%	85%
3	SCIP spx	383	382 MOPT, 1 U	71%	72%
4	wbo	373	372 MOPT, 1 U	70%	70%

# Results for SOFT-SMALLINT-LIN

Rank	Solver	#solved	Detail	%inst.	%VBS
<i>Total number of instances: 201</i>					
<i>Virtual Best Solver (VBS)</i>		201	201 MOPT	100%	100%
1	clasp	163	163 MOPT	81%	81%
2	Sat4j Res.	162	162 MOPT	81%	81%
3	wbo	160	160 MOPT	80%	80%
4	SCIP spx	120	120 MOPT	60%	60%

## Some lessons

- ▶ Linear programming techniques seem particularly relevant for optimization, less for the decision problem.
- ▶ A portfolio approach is valuable
- ▶ CDCL solvers working with PB constraints from end to end doesn't seem competitive.
- ▶ WBO not considered by the community

*<Add you own conclusion here>*

# What's a competition worth?

The goal of a competition is to:

- ▶ evaluate solvers in the same conditions
- ▶ help collecting publicly available benchmarks
- ▶ help identifying new solvers on the market
- ▶ help the community identify good ideas and strange results: the goal is to raise questions and get new ideas!

Competitions should not be misunderstood:

- ▶ The results are not an absolute truth: they depend on the benchmark selection, experimental condition,...
- ▶ A competition is not limited to a ranking: rankings are just an over-simplified view, but still relevant to motivate authors
- ▶ There are a lot of data collected and published to benefit the whole community
- ▶ Competitions must be driven by the community: benchmark submission/selection advices, suggestions for improvements...

- ▶ All details are on the web site  
**<http://www.cril.univ-artois.fr/PB11>**
- ▶ Thanks to all participants!

Since a few points were not perfect in this edition, a **second round will be organized in September**:

- ▶ **warm encouragements to submit new solvers and new benchmarks**
- ▶ a new selection will be made
- ▶ the best solvers of the previous competitions will be run for comparison purpose
- ▶ possible redefinition of the BIGINT category (switch to 64 bits)