

# Fourth Pseudo-Boolean Competition PB09

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- ▶ Pseudo-Boolean formalism
- ▶ Judges
- ▶ Benchmarks and Solvers
- ▶ Evaluation Environment
- ▶ Results

# Linear Pseudo-Boolean Constraints

- ▶ A **linear** pseudo-Boolean constraint may be defined over boolean variables by

$$\sum_i a_i \cdot l_i \geq d \text{ with } a_i, d \in \mathbb{Z}, l_i \in \{x_i, \bar{x}_i\}, x_i \in \mathbb{B}$$

Example:  $3x_1 - 3x_2 + 2\bar{x}_3 + \bar{x}_4 + x_5 \geq 5$

- ▶ Extends both clauses and cardinality constraints
  - ▶ cardinalities: all  $a_i = 1$  and  $d > 1$
  - ▶ clauses: all  $a_i = 1$  and  $d = 1$
- ▶ PB constraints are more expressive than clauses (one PB constraint may replace an exponential number of clauses)
- ▶ A pseudo-Boolean instance is a conjunction of PB constraints

# Non-Linear Pseudo-Boolean Constraints

- ▶ A **non-linear** pseudo-Boolean constraint may be defined over boolean variables by

$$\sum_i a_i \left( \prod_j l_{i,j} \right) \geq d \text{ with } a_i, d \in \mathbb{Z}, l_{i,j} \in \{x_{i,j}, \bar{x}_{i,j}\}, x_{i,j} \in \mathbb{B}$$

Example:  $3x_1\bar{x}_2 - 3x_2x_4 + 2\bar{x}_3 + \bar{x}_4 + x_5x_6x_7 \geq 5$

- ▶ A product is a AND
- ▶ Compact encoding for several problems (e.g. factoring problem encoded by one constraint)
- ▶ Can be easily translated into linear pseudo-Boolean by introducing new variables and constraints s.t.

$$p \leftrightarrow x_0 \wedge x_1 \wedge \dots \wedge x_n$$

(requires 2 PB constraints or  $n+1$  clauses)

- ▶ Another difference with SAT is that most PB problems contain a linear cost function to optimize. For example,

$$\text{minimize } f = \sum_i c_i \cdot x_i \text{ with } c_i \in \mathbb{Z}, x_i \in \mathbb{B}$$

- ▶ Example of an optimization instance

$$\left\{ \begin{array}{l} \text{minimize} \quad 5x_1 + x_2 + 8x_3 + 2x_4 + 3x_5 \\ \text{subject to} \quad x_1 + \bar{x}_2 + x_3 \geq 1 \\ \quad \quad \quad \bar{x}_1 + x_2 + \bar{x}_3 + x_4 \geq 3 \\ \quad \quad \quad 2\bar{x}_1 + 4x_2 + 2x_3 + x_4 + 5x_5 \geq 5 \\ \quad \quad \quad 5x_1 + 4x_2 + 6x_3 + x_4 + 3x_5 \geq 10 \end{array} \right.$$

Optimum: 8

$$x_1 = x_2 = x_4 = 1$$

$$x_3 = x_5 = 0$$

- ▶ The cost function may contain products (no such instance in the PB09 evaluation)

- ▶ 2 judges
  - ▶ Heidi Dixon (pbChaff solver)
  - ▶ Peter Barth (opbdp solver)
- ▶ decided of the selection of instances
  - all PB07 instances except the reduced ones and all instances submitted this year
- ▶ approved the ranking scheme

Based on the objective function

**DEC** No objective function to optimize (decision problem). The solver must simply find a solution.

**OPT** An objective function is present. The solver must find a solution with the best possible value of the objective function.

## Benchmark categories (2)

Based on the size of coefficients

**SMALLINT** small integers: no constraint with a sum of coefficients greater than  $2^{20}$  (20 bits)

- ▶ Expected to be safe for solvers using 32 bits integers and simple techniques (be careful with learning)
- ▶ Strong limit to the encoding of concrete problems.

**BIGINT** big integers: at least one constraint with a sum of coefficients greater than  $2^{20}$  (20 bits)  
requires handling of big integers

## Benchmark categories (3)

Based on the linearity of constraints

**LIN** All constraints are linear

**NLC** At least one constraint is non linear (contains products of literals)

Additional special category

**PURE-SAT** All constraints are clauses.

This is a proper subset of DEC-SMALLINT-LIN

Contains pigeon-hole instances and some easy instances from the SAT09 competition (which were solved by a majority of solvers in the first phase).

- ▶ **DEC-SMALLINT-LIN (371 instances)**
- ▶ **DEC-SMALLINT-NLC (100 instances)**
- ▶ DEC-BIGINT-LIN (14 instances)
- ▶ DEC-BIGINT-NLC
- ▶ **OPT-SMALLINT-LIN (554 instances)**
- ▶ **OPT-SMALLINT-NLC (405 instances)**
- ▶ OPT-BIGINT-LIN (452 instances) only 1 solver!
- ▶ OPT-BIGINT-NLC
- ▶ PURE-SAT (189 instances)

# Submitted solvers

5 teams, 6 solvers

**BoolVar** Olivier Bailleux

no optimization, translation to SAT, see paper SAT09

**bsolo** Vasco Manquinho and José Santos

3 versions run, one is hors-concours

**pbclasp** Benjamin Kaufmann

**SAT4J Pseudo** Daniel Le Berre and Anne Parrain

2 versions, handles big integers

**SCIPclp/spx** Timo Berthold, Stefan Heinz, Marc Pfetsch, and Michael Winkler

2 versions, see paper SAT09

**wbo** Vasco Manquinho

see paper SAT09

- ▶ feature subscription problems (Luis Quesada)  
actually weighted MaxSAT,  
20 instances, OPT-SMALLINT-LIN
- ▶ FlexRay scheduling instances (Martin Lukasiewicz)  
special two-dimensional bin packing problem  
9 instances, OPT-SMALLINT-LIN
- ▶ optical routing problems (Miroslav Velev)  
70 instances, OPT-BIGINT-LIN

- ▶ Cluster of bi-Xeon 3 GHz, 2MB cache, 2GB RAM (all solvers were run in 32 bits mode)  
*kindly provided by the CRIL, University of Artois, France*  
The same environment was used for the SAT competition
- ▶ Each solver was given a time limit of 30 minutes (1800s) and a memory limit of 1800 MB (to avoid swapping).
- ▶ 157 days of CPU time used

# Verification of results

- ▶ The environment performs the following, efficient checks:
  - ▶ for SATISFIABLE answers, solvers must output a complete instantiation and the system checks that it satisfies all constraints
  - ▶ for UNSATISFIABLE answers, the system only checks that no other solver proved satisfiability
  - ▶ for OPTIMUM FOUND answers, the system only checks that no other solver found a better solution
- ▶ UNSATISFIABLE and OPTIMUM FOUND answers cannot be completely checked efficiently and therefore should be taken with caution.
- ▶ Solvers giving a wrong answer in a category are disqualified in that category.

# Ranking of solvers and Virtual Best Solver (VBS)

Ranking based on two criteria:

1. the number of solved instances
2. ties are broken by considering the cumulated time on solved instances

The Virtual Best Solver (VBS)

- ▶ is the virtual solver obtained by combining the best results of all submitted solvers.
- ▶ could be obtained by running in parallel all submitted solvers
- ▶ represents the current state of the art (SOTA)
- ▶ is a reference for the evaluation of the other solvers



# Results for DEC-SMALLINT-LIN

371 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	363	145 S/218 U	98%	100%
3	wbo	309	137 S/172 U	83%	85%



# Results for DEC-SMALLINT-LIN

371 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	363	145 S/218 U	98%	100%
1	bsolo	325	137 S/188 U	88%	90%
2	SCIPspx	310	119 S/191 U	84%	85%
3	wbo	309	137 S/172 U	83%	85%

# Results for DEC-SMALLINT-LIN

371 instances

Rank	Solver	solved	detail	%inst	%VBS
	VBS	363	145 S/218 U	98%	100%
1	bsolo	325	137 S/188 U	88%	90%
2	SCIPspx	310	119 S/191 U	84%	85%
3	wbo	309	137 S/172 U	83%	85%
4	SCIPclp	308	119 S/189 U	83%	85%
5	pbclasp	300	126 S/174 U	81%	83%
	<i>bsolo cl (*)</i>	290	136 S/154 U	78%	80%
6	BoolVar	288	111 S/177 U	78%	79%
7	bsolo pb	284	136 S/148 U	77%	78%
8	SAT4J Resol.	278	106 S/172 U	75%	77%
9	SAT4J CP	164	72 S/92 U	44%	45%



# Results for DEC-SMALLINT-NLC

100 instances

Rank	Solver	solved	detail	%inst	%VBS
	VBS	75	55 S/20 U	75%	100%
3	pbclasp	65	50 S/15 U	65%	87%

# Results for DEC-SMALLINT-NLC

100 instances

Rank	Solver	solved	detail	%inst	%VBS
VBS		75	55 S/20 U	75%	100%
2	SCIPspx	74	54 S/20 U	74%	99%
3	pbclasp	65	50 S/15 U	65%	87%

# Results for DEC-SMALLINT-NLC

100 instances

Rank	Solver	solved	detail	%inst	%VBS
VBS		75	55 S/20 U	75%	100%
1	SCIPclp	74	54 S/20 U	74%	99%
2	SCIPspx	74	54 S/20 U	74%	99%
3	pbclasp	65	50 S/15 U	65%	87%

# Results for DEC-SMALLINT-NLC

100 instances

Rank	Solver	solved	detail	%inst	%VBS
	VBS	75	55 S/20 U	75%	100%
1	SCIPclp	74	54 S/20 U	74%	99%
2	SCIPspj	74	54 S/20 U	74%	99%
3	pbclasp	65	50 S/15 U	65%	87%
4	SAT4J CP	65	50 S/15 U	65%	87%
5	bsolo pb	64	49 S/15 U	64%	85%
6	SAT4J Resol.	64	51 S/13 U	64%	85%
7	bsolo	61	46 S/15 U	61%	81%
8	wbo	59	44 S/15 U	59%	79%
9	BoolVar	57	42 S/15 U	57%	76%
	<i>bsolo cl (*)</i>	26	16 S/10 U	26%	35%

# Results for OPT-SMALLINT-LIN

554 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	343	330 O/13 U	62%	100%

# Results for OPT-SMALLINT-LIN

554 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	343	330 O/13 U	62%	100%
3	pbclasp	188	175 O/13 U	34%	55%

# Results for OPT-SMALLINT-LIN

554 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	343	330 O/13 U	62%	100%
2	bsolo pb	231	218 O/13 U	42%	67%
	<i>bsolo cl (*)</i>	230	217 O/13 U	42%	67%
3	pbclasp	188	175 O/13 U	34%	55%

# Results for OPT-SMALLINT-LIN

554 instances

Rank	Solver	solved	detail	%inst	%VBS
	VBS	343	330 O/13 U	62%	100%
1	SCIPspx	295	282 O/13 U	53%	86%
2	bsolo pb	231	218 O/13 U	42%	67%
	<i>bsolo cl (*)</i>	230	217 O/13 U	42%	67%
3	pbclasp	188	175 O/13 U	34%	55%

# Results for OPT-SMALLINT-LIN

554 instances

Rank	Solver	solved	detail	%inst	%VBS
	VBS	343	330 O/13 U	62%	100%
1	SCIPspx	295	282 O/13 U	53%	86%
2	bsolo pb	231	218 O/13 U	42%	67%
	<i>bsolo cl (*)</i>	230	217 O/13 U	42%	67%
3	pbclasp	188	175 O/13 U	34%	55%
4	SAT4J Resol.	169	158 O/11 U	31%	49%
5	SAT4J CP	140	131 O/9 U	25%	41%
6	wbo	136	124 O/12 U	25%	40%

# Results for OPT-SMALLINT-NLC

405 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	290	290 O	72%	100%

# Results for OPT-SMALLINT-NLC

405 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	290	290 O	72%	100%
3	pbclasp	263	263 O	65%	91%

# Results for OPT-SMALLINT-NLC

405 instances

Rank	Solver	solved	detail	%inst	%VBS
<i>VBS</i>		290	290 O	72%	100%
2	SCIPclp	286	286 O	71%	99%
3	pbclasp	263	263 O	65%	91%

# Results for OPT-SMALLINT-NLC

405 instances

Rank	Solver	solved	detail	%inst	%VBS
<i>VBS</i>		290	290 O	72%	100%
1	SCIPspx	289	289 O	71%	100%
2	SCIPclp	286	286 O	71%	99%
3	pbclasp	263	263 O	65%	91%

# Results for OPT-SMALLINT-NLC

405 instances

Rank	Solver	solved	detail	%inst	%VBS
	<i>VBS</i>	290	290 O	72%	100%
1	SCIPspx	289	289 O	71%	100%
2	SCIPclp	286	286 O	71%	99%
3	pbclasp	263	263 O	65%	91%
	<i>bsolo cl (*)</i>	232	232 O	57%	80%
4	SAT4J Resol.	226	226 O	56%	78%
5	wbo	223	223 O	55%	77%
6	SAT4J CP	118	118 O	29%	41%

- ▶ Several solvers of the PB07 evaluation were not submitted this year.
- ▶ What if they had been submitted?
- ▶ Here's a comparison **on the set of instances common to PB07 and PB09**

## PB09/PB07: DEC-SMALLINT-LIN (371 inst.)

Rank	Solver	solved	detail
1	PB07: Pueblo	341	139 S/202 U
2	PB07: PBS4_v2	327	130 S/197 U
3	PB07: PBS4	327	130 S/197 U
4	bsolo	325	137 S/188 U
5	PB07: PB-clasp	312	137 S/175 U
6	SCIPspx	310	119 S/191 U
7	wbo	309	137 S/172 U
8	SCIPclp	308	119 S/189 U
9	PB07: minisat+	305	135 S/170 U
10	pbclasp	300	126 S/174 U
11	<i>bsolo cl (*)</i>	290	136 S/154 U
12	BoolVar	288	111 S/177 U
.../...			

## PB09/PB07: DEC-SMALLINT-LIN (cont'd)

Rank	Solver	solved	detail
13	bsolo pb	284	136 S/148 U
14	PB07: PB-clasp	283	112 S/171 U
15	SAT4J Resol.	278	106 S/172 U
16	PB07: SAT4JPseudoResol.	273	106 S/167 U
17	PB07: bsolo 3.0.16	271	134 S/137 U
18	PB07: bsolo 3.0.17	271	134 S/137 U
19	PB07: glpPB	200	79 S/121 U
20	SAT4J CP	164	72 S/92 U
more PB07 solvers			

## PB09/PB07: DEC-SMALLINT-NLC (100 inst.)

Rank	Solver	solved	detail
1	SCIPclp	74	54 S/20 U
2	SCIPspx	74	54 S/20 U
3	pbclasp	65	50 S/15 U
4	PB07: minisat+	65	50 S/15 U
5	PB07: PBS4	65	50 S/15 U
6	PB07: PBS4_v2	65	50 S/15 U
7	SAT4J CP	65	50 S/15 U
8	PB07: glpPB	65	45 S/20 U
9	PB07: PB-clasp	64	49 S/15 U
10	PB07: Pueblo	64	50 S/14 U
11	bsolo pb	64	49 S/15 U
12	SAT4J Resol.	64	51 S/13 U
.../...			

## PB09/PB07: DEC-SMALLINT-NLC (cont'd)

Rank	Solver	solved	detail
13	PB07: PB-clasp	62	47 S/15 U
14	bsolo	61	46 S/15 U
15	PB07: wildcat-rnp	60	60 S
16	PB07: wildcat-skc	60	60 S
17	wbo	59	44 S/15 U
18	BoolVar	57	42 S/15 U
19	PB07: oree	55	45 S/10 U
20	PB07: absconPseudo	50	40 S/10 U
21	PB07: bsolo 3.0.17	28	18 S/10 U
22	PB07: bsolo 3.0.16	27	17 S/10 U
23	<i>bsolo cl (*)</i>	26	16 S/10 U

## PB09/PB07: OPT-SMALLINT-LIN (525 inst.)

Rank	Solver	solved	detail
1	SCIPspx	293	280 O/13 U
2	PB07: bsolo 3.0.17	231	218 O/13 U
3	bsolo pb	215	202 O/13 U
4	<i>bsolo cl (*)</i>	210	197 O/13 U
5	PB07: minisat+	201	191 O/10 U
6	PB07: glpPB	186	182 O/4 U
7	PB07: Pueblo	179	166 O/13 U
8	pbclasp	164	151 O/13 U
9	PB07: PB-clasp	155	143 O/12 U
10	PB07: SAT4JPseudoResol.	153	140 O/13 U
11	PB07: PB-clasp	150	142 O/8 U
12	SAT4J Pseudo Resol.	146	135 O/11 U
.../...			

## PB09/PB07: OPT-SMALLINT-LIN (cont'd)

Rank	Solver	solved	detail
13	SAT4J Pseudo CP	140	131 O/9 U
14	PB07: sat4jPseudoCP	140	131 O/9 U
15	PB07: sat4jPseudoCPClause	139	130 O/9 U
16	PB07: PBS4_v2	137	128 O/9 U
17	PB07: PBS4	137	128 O/9 U
18	wbo	134	122 O/12 U
more PB07 solvers			

## PB09/PB07: OPT-SMALLINT-NLC (405 inst.)

Rank	Solver	solved	detail
1	SCIPspx	289	289 O
2	SCIPclp	286	286 O
3	PB07: minisat+	275	275 O
4	pbclasp	263	263 O
5	PB07: Pueblo	248	248 O
6	PB07: bsolo 3.0.17	239	239 O
7	<i>bsolo cl (*)</i>	232	232 O
8	SAT4J Pseudo Resol.	226	226 O
9	wbo	223	223 O
10	PB07: SAT4JPseudoResol.	216	216 O
11	PB07: PBS4	171	171 O
12	PB07: PBS4_v2	171	171 O
13	SAT4J Pseudo CP	118	118 O
more PB07 solvers			

# Comparing SAT and PB solvers

Since

- ▶ PB is an extension of SAT
- ▶ the SAT competition and the PB evaluation were run in the same environment
- ▶ it's easy to translate a SAT instance in the PB syntax

we may try to compare SAT and PB solvers.

But of course

- ▶ PB solvers generally are disadvantaged (must handle coefficients)
- ▶ PB solvers may not be optimized for clauses

Context of the comparison

- ▶ subset of instances from the CRAFTED and INDUSTRIAL categories which were solved by a majority of solvers in the first phase of the SAT competition
- ▶ 20 minutes timeout for all solvers
- ▶ all SAT solvers from the Competition or Demonstration division

## PB vs SAT solvers (1/4)

Rank	Solver	Version	Solved	Time
1	picosat	913	123	8033
2	ManySAT 1.1	aimd 1/2009-03-20	122	7833
3	precosat	236	122	10257
4	glucose	1.0	121	6677
5	ManySAT 1.1	aimd 2/2009-03-20	121	9567
6	MXC	2009-03-10	120	7011
7	minisat_cumr	p-2009-03-18	120	9534
8	SATzilla2009_I	2009-03-22	120	10621
9	MiniSat	2.1 (Sat-race'08 Edition)	119	3883
10	Rsat	2009-03-22	119	7073
11	LySAT	i/2009-03-20	119	8711
12	MiniSAT 09z	2009-03-22	118	7843
13	ManySAT 1.1	aimd 0/2009-03-20	118	8010
14	kw	2009-03-20	118	8445
15	SApperloT	base	118	8738

## PB vs SAT solvers (2/4)

Rank	Solver	Version	Solved	Time
16	HydraSAT-Flat	2009-03-22-Flat	118	11389
17	clasp	1.2.0-SAT09-32	117	5809
18	CircUs	2009-03-23	117	8522
19	LySAT	c/2009-03-20	117	9182
20	HydraSAT	2009-03-22-Base	117	10614
21	VARSAT-industrial	2009-03-22	116	11412
22	IUT_BMB_SAT	1.0	115	10042
23	HydraSAT-Multi	2009-03-22-Multi	114	10582
24	pbclasp	2009-04-24	114	12758
25	SApperloT	hrp	112	9847
26	SATzilla2009_C	2009-03-22	108	8544
27	satake	2009-03-22	105	11364
28	VARSAT-crafted	2009-03-22	102	10986
29	SAT4J CORE	2.1 RC1	101	14186
30	SAT4J Pseudo Resol.	2.1.1	101	14920

## PB vs SAT solvers (3/4)

Rank	Solver	Version	Solved	Time
31	CSat	2009-03-22	99	10307
32	SATzilla2009_R	2009-03-22	96	5676
33	wbo	1.0	93	13808
34	SAT4J Pseudo CP	2.1.1	90	15233
35	bsolo	3.1 cl	86	16275
36	VARSAT-random	2009-03-22	76	12816
37	bsolo	3.1 pb	74	12989
38	bsolo	3.1	74	14338
39	march_hi	hi	52	6503
40	march_nn	nn	52	7250
41	adaptg2wsat2009	2009-03-23	50	4370
42	iPAWS	2009-03-22	48	1852
43	gnovelty+	2009-03-22	45	3540
44	gNovelty+-T	2009-03-22	44	4100
45	Hybrid2	2009-03-22	44	4605

## PB vs SAT solvers (4/4)

Rank	Solver	Version	Solved	Time
46	TNM	2009-03-22	44	4621
47	adaptg2wsat2009++	2009-03-23	43	2136
48	hybridGM3	3	41	4388
49	hybridGM	1	39	1465
50	NCVWr	2009-03-22	37	6720
51	gnovelty+2	2009-03-22	35	2405
52	SCIPspx	with SoPLEX	35	5554
54	hybridGM7	7	34	1199
53	tts	tts-5-0	34	2906
55	slstc	1.0	33	3202
56	SCIPclp	with CLP	32	3795
57	ttsth-5-0	2009-03-19	32	4048

- ▶ All details are on the web site  
<http://www.cril.univ-artois.fr/PB09>
- ▶ Thanks to all participants!